Fixed order + parton shower(s)

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Parton showers and resummation 2014, Münster, June 11, 2014



Outline

Introduction

- ♦ The ME+PS problem.
- ♦ The MLM prescription.

ME+PS merging

- ◊ Traditional (CKKW-L) approach for combining ME and PS.
- ◊ Unitarised merging.
- Next-to-leading order multi-jet merging.
- ◊ NNLO matching.

Summary

The ME+PS problem

Problem: We want to describe soft/collinear and hard jets in one sample, ... but "soft" and "hard" are not well-defined terms.
... also, embed this in a larger event generation framework!
MCEGs are built for describing soft/collinear jets and soft/hadronic event features.
Fixed-order ME generators are good for well-separated jets.

- Just adding several showered MEG samples gives massive over-counting! Need to remove overlap!
- \rightarrow MEPS merging:

Use ME above some cut, PS below cut. Minimise cut dependence.

\rightarrow MEPS matching:

Interpolate between two multipicities by "adapting" NLO subtraction.

The MLM method

The MLM¹ prescription is:

- ♦ Calculate the tree-level MEs, using running α_s and PDFs. Cut away every state with $t(S_{+n}) > t_{MS}$.
- ◊ Count partons before shower.
- ♦ Start the PS on the ME configuration. After shower, count jets. Veto the event if n_{jets} does not match $n_{partons}^2$.
- Combine by adding all accepted events.

$$\langle \mathcal{O} \rangle = \mathsf{B}_0 \mathcal{O}(S_{+0\rho}) \times \operatorname{VETO}(S_{+n\rho}) + \int \mathsf{B}_1 w_f^0 w_{\alpha_{\mathrm{S}}}^0 \Theta(t(S_{+1}) - t_{\mathrm{MS}}) \mathcal{O}(S_{+1\rho}) \times \operatorname{VETO}(S_{+m\rho})$$

¹ Mangano, http://www-cpd.fnal.gov/personal/mrenna/tuning/nov2002/mlm.pdf. Talk presented at the Fermilab ME/MC Tuning Workshop, Oct 4, 2002.

² Glossing over many details here!

MLM results from PYTHIA 8



Figure: y and p_{\perp} of the Z-boson in by CMS (Phys.Rev. D85 (2012) 032002). MLM matching available since 8.170 (Richard Corke, Steve Mrenna). Shower-kT-MLM available since 8.185 (Simon de Visscher, SP). Comment: CKKW-L available since 8.157 (Leif Lönnblad, SP).

ME+PS merging

The $VETO(S_{+np})$ factors in MLM do not directly correspond to a QCD object.

 \implies Reasonable base line, but accuracy iffy.

To improve, remember that the PS generates exclusive cross sections

$$\mathbf{PS}\left[\sigma_{+0}^{\mathsf{ME}}\right] = \underbrace{\sigma_{+0}^{\mathsf{PS}}}_{\mathsf{exclusive due to Sudakov factor}} + \underbrace{\sigma_{+1}^{\mathsf{PS}}}_{\mathsf{exclusive due to Sudakov factors}} + \underbrace{\sigma_{+\geq 2}^{\mathsf{PS}}}_{\mathsf{inclusive}}$$

by multiplying PS Sudakov factors.

- \Rightarrow Convert the inclusive states of the ME calculation into *exclusive* states by multiplying PS no-emission probabilities.
- \Rightarrow CKKW-L

CKKW-L¹ merging

... so the merging prescription is:

- ◊ Calculate the tree-level MEs.
- $\diamond~$ Reweight with Sudakovs, $\alpha_{s}\text{-}$ and PDF-ratios.
- Start the PS on the reweighted ME configuration. Throw away the event if the PS produced another state in the ME region.
- ◊ Combine by adding all accepted events.

$$\begin{aligned} \langle \mathcal{O} \rangle &= \mathsf{B}_{0} \quad \times \; \mathsf{\Pi}_{\mathcal{S}_{+0}}(\rho_{0}, t_{\mathsf{MS}}) \quad \mathcal{O}(\mathcal{S}_{+0j}) \\ &+ \; \int \mathsf{B}_{1} \Theta \left(t \left(\mathcal{S}_{+1} \right) - t_{\mathsf{MS}} \right) \, w_{f}^{0} \, w_{\alpha_{s}}^{0} \, \mathsf{\Pi}_{\mathcal{S}_{+0}}(\rho_{0}, \rho_{1}) \mathcal{O}(\mathcal{S}_{+1j}) \end{aligned}$$

¹ JHEP 0111 (2001) 063 (Catani, Krauss, Kuhn, Webber), JHEP 0205 (2002) 046 (Lönnblad), JHEP 0507 (2005) 054 (Lavesson, Lönnblad), JHEP 0911 (2009) 038 (Hamilton, Tully, Richardson), JHEP 0905 (2009) 053 (Höche, Krauss, Schumann, Siegert), JHEP 1203 (2012) 019 (Lönnblad, SP), ...all differ in the details.

CKKW-L results from PYTHIA 8



Figure: k_{\perp} -separation between the first and second jet for W+jets, when clustering to exactly two jets. The bands are obtained by varying the PS starting scale in $\mu_Q \in [\frac{1}{2}M_W, 2M_W]$.

CKKW-L available since 8.157 (Leif Lönnblad, SP).

CKKW-L and BSM



Figure: Exclusion limits for squarks+jets. PS bands are obtained by varying between "wimpy" and "power shower", merged bands by varying the merging scale from 50 - 200 GeV (taken from Phys.Rev.D87(2013)3,035006 (Dreiner, Krämer, Tattersall)).

 \implies Merging pins down jet momenta, which reduces uncertainties from MC modelling.

. . however



Figure: Ratio of the inclusive cross section after merging, compared to the LO inclusive cross section.

Note the red bars!

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Bug vs. Feature in $\mathsf{ME}+\mathsf{PS}$

The ME includes terms that are not compensated by the PS approximate virtual corrections (i.e. Sudakov factors).

These are the improvements that we need to describe multiple hard jets!

If we simply add samples, the "improvements" will degrade the inclusive cross section: σ_{inc} will contain $\ln(t_{MS})$ terms.

INCLUSIVE CROSS SECTIONS DO NOT KNOW ABOUT HIGHER MULTIPLICITIES. INCLUSIVE IS INCLUSIVE!

Traditional approach: Don't use a too small merging scale.

 \rightarrow Uncancelled terms numerically not important.

New approach¹:

Use a (PS) unitarity inspired approach exactly cancel the dependence of the inclusive cross section on t_{MS} .

 1 JHEP1302(2013)094 (Leif Lönnblad, SP), JHEP1308(2013)114 (Simon Plätzer) $^{11/32}$

Unitarised merging

We can use parton shower unitarity to rewrite $\mathrm{C}\mathrm{K}\mathrm{K}\mathrm{W}\text{-}\mathrm{L}$ as

$$\begin{aligned} \langle \mathcal{O} \rangle &= \mathsf{B}_{0} \Pi_{S_{+0}}(\rho_{0}, \rho_{\mathrm{MS}}) \mathcal{O}(S_{+0j}) \mathcal{O}(S_{+0j}) \\ &+ \int \mathsf{B}_{1} \Theta(t(S_{+1}) - t_{\mathrm{MS}}) \, w_{f}^{0} \, w_{\alpha_{\mathrm{s}}}^{0} \Pi_{S_{+0}}(\rho_{0}, \rho_{1}) \mathcal{O}(S_{+1j}) \end{aligned}$$

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and replace

$$\begin{aligned} \langle \mathcal{O} \rangle &= \mathsf{B}_{0} - \int d\rho \; w_{f}^{0} w_{\alpha_{s}}^{0} \mathsf{B}_{1} \Pi_{\mathcal{S}_{+0}}(\rho_{0}, \rho) \Theta(t(S_{+1}) - t_{\mathsf{MS}}) \, \mathcal{O}(S_{+0j}) \\ &+ \int \mathsf{B}_{1} \Theta(t(S_{+1}) - t_{\mathsf{MS}}) \, w_{f}^{0} w_{\alpha_{s}}^{0} \Pi_{\mathcal{S}_{+0}}(\rho_{0}, \rho_{1}) \mathcal{O}(S_{+1j}) \end{aligned}$$

 \implies UMEPS!

Comments on UMEPS

This sketch can directly be extended to the case when we have

 $\widehat{B}_2 = LO$ cross section, weighted with w_f , w_{α_s} and Π 's $\int \widehat{B}_{n \to m} =$ integrated LO cross section, weighted with w_f , w_{α_s} and Π 's.

For example two-jet merging:

$$\begin{split} \langle \mathcal{O} \rangle &= \int d\phi_0 \bigg\{ \mathcal{O}(S_{+0j}) \left[\mathsf{B}_0 - \int \widehat{\mathsf{B}}_{1 \to 0} - \int \widehat{\mathsf{B}}_{2 \to 0} \right] \\ &+ \int \mathcal{O}(S_{+1j}) \left[\widehat{\mathsf{B}}_1 - \int \widehat{\mathsf{B}}_{2 \to 1} \right] \\ &+ \int \int \mathcal{O}(S_{+2j}) \, \widehat{\mathsf{B}}_2 \bigg\} \end{split}$$

Integrated configurations are available anyway since we need them to perform the Sudakov weighting... free lunch for once.

UMEPS result



Figure: Ratio of the inclusive cross section after merging, compared to the tree-level inclusive cross section.

 \Rightarrow UMEPS preserves the inclusive cross section. However, the statistical error can be larger than in CKKW-L (due to positive and negative weights).

UMEPS results



Figure: p_{\perp} of the W-boson in the Sudakov region (for 2-jet merging, $E_{CM} = 7$ TeV). Lower inset shows the comparison to default PYTHIA 8.

- \Rightarrow CKKW-L overshoots for (very) low merging scales due to uncancelled terms.
- \Rightarrow UMEPS describes the Sudakov peak nicely.

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Any leading-order method ${\bf X}$ only ever contains approximate virtual corrections.

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To do NLO multi-jet merging for your preferred LO scheme X, do:

- \diamond Subtract approximate X $\mathcal{O}(\alpha_{\rm s})$ -terms, add multiple NLO calculations.
- Make sure fixed-order calculations do not overlap by cutting, vetoing events and/or vetoing emissions.
- Adjust higher orders to suit other needs.
- \Rightarrow X@NLO

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 \ldots where NLO = cross section w/o phase space of "hard" real corrections, and Born-level divergences regulated with a suitable jet cut.

 \Rightarrow Can be taken from NLO-matched calculation.

NLO matching

For NLO matching, we start out with a seed cross section and Sudakov

$$\overline{B}_{n} = B_{n} + V_{n} + I_{n} + \int d\Phi_{rad} \left(B'_{n+1} - D_{n+1} \right)$$
$$\Delta^{B}(t_{0}, t_{min}) = \exp\left(-\int^{t_{0}} d\Phi_{rad} \frac{B'_{n+1}}{B_{n}}\right)$$

and perform a PS step on \overline{B}_n^1

$$\overline{\mathsf{B}}_{n}\Delta^{B}(t_{0}, t_{min})\mathcal{O}_{0}(\Phi_{n}) + \int_{0}^{t_{0}} d\Phi_{\mathsf{rad}}\overline{\mathsf{B}}_{n}\frac{\mathsf{B}_{n+1}'}{\mathsf{B}_{n}}\Delta^{B}(t_{0}, t)\mathcal{O}_{1}(\Phi_{n+1}) \\ + \left(\mathsf{B}_{n+1} - \mathsf{B}_{n+1}'\right)\mathcal{O}_{1}(\Phi_{n+1})$$

Expanded to $\mathcal{O}(\alpha_s^{n+1})$, this gives back the NLO cross section. The common NLO matching schemes are

POWHEG:
$$B'_{n+1} = B_{n+1} \cdot \frac{h^2}{h^2 + p_{\perp}^2}$$
, $t_0 = s$
MC@NLO: $B'_{n+1} = D_{n+1} \cdot \Theta(\mu_Q - t(S_{+1}))$, $\mu_Q = kQ^2$

 1 Glossing over subtleties with the PS interface here. \$17/32\$

W+b+jet@NLO with aMC@NLO + PYTHIA8



 \implies Differences between the showers become smaller after NLO matching.

(Work in collaboration with Stefano Frixione, Andrew Papanasasiou, Paolo Torrielli) 18/32

UNLOPS = UMEPS @NLO

UMEPS is a leading-order method, i.e. it contains only approximate virtual corrections.

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UNLOPS = UMEPS **@NLO**

UMEPS is a leading-order method, i.e. it contains only approximate virtual corrections.

We want to use the full NLO results whenever possible.

Basic idea: Do NLO multi-jet merging for UMEPS:

- \diamond Subtract approximate UMEPS $\mathcal{O}(\alpha_{\rm s})$ -terms, add back full NLO.
- $\diamond~$ To preserve the inclusive (NLO) cross section, add approximate NNLO.
- \Rightarrow UNLOPS¹.

For UNLOPS merging, we need exclusive NLO inputs:

$$\widetilde{\mathsf{B}}_{n} = \mathsf{B}_{n} + \mathsf{V}_{n} + \mathsf{I}_{n+1|n} + \int d\Phi_{\mathsf{rad}} \left(\mathsf{B}_{n+1|n} \Theta \left(\rho_{\mathrm{MS}} - t \left(S_{+n+1}, \rho \right) \right) - \mathsf{D}_{n+1|n} \right)$$

We can get these e.g. from POWHEG-BOX or MC@NLO output.

Start with UMEPS:

$$\begin{split} \langle \mathcal{O} \rangle &= \int d\phi_0 \bigg\{ \mathcal{O}(S_{+0j}) \bigg(\begin{array}{ccc} \mathsf{B}_0 + & & - & \int \widehat{\mathsf{B}}_{1 \to 0} & & - & \int \widehat{\mathsf{B}}_{2 \to 0} \bigg) \\ & & + & \int \mathcal{O}(S_{+1j}) \bigg(& & \widehat{\mathsf{B}}_1 & & - & \int \widehat{\mathsf{B}}_{2 \to 1} \end{array} \bigg) & + & \int \int \mathcal{O}(S_{+2j}) \widehat{\mathsf{B}}_2 \bigg\} \end{split}$$

Remove all unwanted $\mathcal{O}(\alpha_{\mathrm{s}}^n)$ - and $\mathcal{O}(\alpha_{\mathrm{s}}^{n+1})$ -terms:

$$\begin{split} \langle \mathcal{O} \rangle &= \int d\phi_0 \bigg\{ \mathcal{O}(S_{+0j}) \bigg(\qquad \qquad - \left[\int \widehat{\mathsf{B}}_{1\to0} \right]_{-1,2} \qquad - \int \widehat{\mathsf{B}}_{2\to0} \bigg) \\ &+ \int \mathcal{O}(S_{+1j}) \left(\qquad \left[\widehat{\mathsf{B}}_1 \right]_{-1,2} - \left[\int \widehat{\mathsf{B}}_{2\to1} \right]_{-2} \right) + \int \int \mathcal{O}(S_{+2j}) \widehat{\mathsf{B}}_2 \bigg\} \end{split}$$

Add full NLO results:

$$\langle \mathcal{O} \rangle = \int d\phi_0 \left\{ \mathcal{O}(S_{+0j}) \begin{pmatrix} \widetilde{B}_0 & -\left[\int \widehat{B}_{1\to 0}\right]_{-1,2} & -\int \widehat{B}_{2\to 0} \end{pmatrix} \right. \\ \left. + \int \mathcal{O}(S_{+1j}) \left(\left. \widetilde{B}_1 + \left[\widehat{B}_1 \right]_{-1,2} - \left[\int \widehat{B}_{2\to 1} \right]_{-2} \right) \right. + \int \int \mathcal{O}(S_{+2j}) \widehat{B}_2 \right\}$$

Unitarise:

$$\begin{split} \langle \mathcal{O} \rangle &= \int d\phi_0 \bigg\{ \mathcal{O}(S_{+0j}) \bigg(\qquad \widetilde{B}_0 - \int_s \widetilde{B}_{1 \to 0} + \int_s B_{1 \to 0} - \left[\int \widehat{B}_{1 \to 0} \right]_{-1,2} - \int_s B_{2 \to 0}^{\uparrow} - \int \widehat{B}_{2 \to 0} \bigg) \\ &+ \int \mathcal{O}(S_{+1j}) \left(\left[\widetilde{B}_1 + \left[\widehat{B}_1 \right]_{-1,2} - \left[\int \widehat{B}_{2 \to 1} \right]_{-2} \right) + \int \int \mathcal{O}(S_{+2j}) \widehat{B}_2 \bigg\} \end{split}$$

UNLOPS merging of zero and one parton at NLO:

$$\begin{split} \langle \mathcal{O} \rangle = & \int d\phi_0 \bigg\{ \mathcal{O}(S_{+0j}) \bigg(\qquad \widetilde{B}_0 - \int_s \widetilde{B}_{1 \to 0} + \int_s B_{1 \to 0} - \left[\int \widehat{B}_{1 \to 0} \right]_{-1,2} - \int_s B_{2 \to 0}^{\uparrow} - \int \widehat{B}_{2 \to 0} \bigg) \\ & + \int \mathcal{O}(S_{+1j}) \left(\left[\widetilde{B}_1 + \left[\widehat{B}_1 \right]_{-1,2} - \left[\int \widehat{B}_{2 \to 1} \right]_{-2} \right) + \int \int \mathcal{O}(S_{+2j}) \widehat{B}_2 \bigg\} \end{split}$$

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Iterate for the case of M different NLO calculations, and N tree-level calculations:

$$\begin{split} \langle \mathcal{O} \rangle &= \sum_{m=0}^{M-1} \int d\phi_0 \int \cdots \int \mathcal{O}(S_{+mj}) \left\{ \left[\widetilde{B}_m + \left[\widehat{B}_m \right]_{-m,m+1} + \int_s B_{m+1 \to m} \right. \\ &\left. - \sum_{i=m+1}^M \int_s \widetilde{B}_{i \to m} - \sum_{i=m+1}^M \left[\int \widehat{B}_{i \to m} \right]_{-i,i+1} - \sum_{i=m+1}^M \int_s B_{i+1 \to m}^{\uparrow} - \sum_{i=M+1}^N \int \widehat{B}_{i \to m} \right\} \\ &\left. + \int d\phi_0 \int \cdots \int \mathcal{O}(S_{+Mj}) \left\{ \left[\widetilde{B}_M + \left[\widehat{B}_M \right]_{-M,M+1} - \left[\int \widehat{B}_{M+1 \to M} \right]_{-M} - \sum_{i=M+1}^N \int \widehat{B}_{i+1 \to M} \right] \right\} \\ &\left. + \sum_{n=M+1}^N \int d\phi_0 \int \cdots \int \mathcal{O}(S_{+nj}) \left\{ \left[\widehat{B}_n - \sum_{i=n+1}^N \int \widehat{B}_{i \to n} \right] \right\} \right] \end{split}$$

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UNLOPS results (W+jets)



Inclusive sample containing (W + no resolved)@NLO, (W + one resolved)@NLO and (W + two resolved)@LO.

NLO merged results (H+jets)



Figure: Higgs p_{\perp} and dijet invariant mass for gg \rightarrow H after merging (H+0,1,2)@NLO and (H+3)@LO in UNLOPS, compared to different schemes.

Uncertainties remain large for exclusive observables that are used to design cuts.

FxFx: Jet matching @ NLO

- Start from MC@NLO calculations. Reweight with CKKW-type α_s -running, Sudakov factors (or suppression functions)
- Remove double-counted $\mathcal{O}(\alpha_s^{+1})$ -terms
- Match "ME jets" to "PS jets" (not "ME partons" to "PS jets")



Figure: Transverse momentum of the Z-boson (PRD85(2012)032002) in aMC@NLO + PYTHIA8.

So what's next? NNLO+PS?

Note that in UNLOPS, the lowest-multiplicity input cross section is *not* reweighted. To go to NNLO, we have to identify terms of...

$$\begin{aligned} \langle \mathcal{O} \rangle &= \int d\phi_0 \bigg\{ \mathcal{O}(S_{+0j}) \left(\widetilde{\mathsf{B}}_0 - \int_s \widetilde{\mathsf{B}}_{1 \to 0} + \int_s \mathsf{B}_{1 \to 0} - \left[\int \widehat{\mathsf{B}}_{1 \to 0} \right]_{-1,2} \\ & \mathcal{O}(\alpha_s) &- \int_s \mathsf{B}_{2 \to 0}^{\uparrow} - \int \widehat{\mathsf{B}}_{2 \to 0} \bigg) \\ & \mathcal{O}(\alpha_s^2) &+ \int \mathcal{O}(S_{+1j}) \left(\widetilde{\mathsf{B}}_1 + \left[\widehat{\mathsf{B}}_1 \right]_{-1,2} - \left[\int \widehat{\mathsf{B}}_{2 \to 1} \right]_{-2} \right) \\ &+ \int \int \mathcal{O}(S_{+2j}) \, \widehat{\mathsf{B}}_2 \end{aligned}$$

Terms entering due to PS weights are $\mathcal{O}(\alpha_s^2)_{\text{PS}} \times \mathcal{O}(\alpha_s^1)_{\text{ME}}$ and $\mathcal{O}(\alpha_s^0)_{\text{PS}} \times \mathcal{O}(\alpha_s^2)_{\text{ME}}$

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... now remove these terms ... and instead use the exclusive NNLO result¹ $\widetilde{\widetilde{B}}_0 \approx \overline{\overline{B}}_0 - \overline{B}_1$ (more formal definition, see back-up)

Comments on UNLOPS@NNLO

For UNLOPS@NNLO, the construction principle was that the exclusive 0-parton cross section should be given by NNLO + UMEPS higher orders, and nothing else.

- Pro: This makes the assessment of the log-accuracy very easy.
- Con: Different from most schemes used in analytic fixed-order + resummation matching. UNLOPS contains the terms $\left[\int \widehat{B}_{1\to 0}\right]_{-1,2}$ which basically contain integration over PS kernels. Numerical issues possible.
- \implies Improve, to make small t_{MS} possible.
- \implies UN²LOPS *matching* to NNLO calculations.

From UNLOPS to UN^2LOPS^1

- ♦ Take UNLOPS merging of zero- and one-jet NLO calculations.
- $\diamond~$ Rewrite the one-jet calculation as MC@NLO, multiplying the 0 \rightarrow 1 no-emission probability $\Pi_0.$
- $\diamond~$ Fortunately, this means that the ${\cal O}(\alpha_s^{+1})$ -subtractions in the one-jet part should be multiplied by Π_0 as well.
- ◊ Unitarise.
- \implies Merging scale can, even in practice, be chosen very small.
- ⇒ Use $q_T \approx 1$ GeV as merging scale, and use fast, approximate NNLO. ⇒ UN²LOPS

 1 arXiv:1405.3607 (Stefan Höche, Ye Li, SP). I can't pronouce this. Just UNLOPS is fine :).

UN²LOPS results: Z-production in SHERPA



Figure: Rapidity and invariant mass spectra of the electron pair.

- \diamond NNLO implementation agrees with FEWZ.
- $\diamond \mu_{R/F}$ -variation reduced at NNLO.

UN²LOPS results: Z-production in SHERPA



Figure: Transverse momentum and rapidity spectrum of the electron.

- ♦ UN²LOPS agrees with NNLO.
- \diamond Shower starting scale uncertainty not larger than $\mu_{R/F}$ -variation.

UN²LOPS results: W-production in SHERPA



Figure: Transverse momentum and rapidity spectrum of the positron-neutrino pair.

 \diamond Shower starting scale uncertainty not larger than $\mu_{R/F}$ -variation, except at small p_{\perp} , where resummation *is* important.



Figure: Rapidity spectrum of the electron. Left panel: NLO w/ NLO PDFs, NNLO w/ NNLO PDFs. Right panel: Everything with NNLO PDFs.

 \diamond Also for other inclusive \mathcal{O} 's: MC@NLO with NNLO PDFs looks like NNLO :)

Summary

- $\diamond~$ To describe LHC data, we need to infuse parton showers with matrix elements.
- ◊ Tree-level merging is standard nowadays
 - ... but can still teach us a lot
 - ... PYTHIA8 contains CKKW-L and UMEPS.
 - ... but that's not NLO ...
- ◊ Multiple NLO merging schemes have appeared, and are being used.
 ... PYTHIA8 contains NL³, UNLOPS and FxFx.
- ◊ Ideas for NNLO matching are around¹. MiNLO-NNLOPS publicly available². UN²LOPS in coming soon, with implementation in PYTHIA8 coming shortly thereafter.

Thanks for your time.

² JHEP1310(2013)222 (Hamilton, Nason, Re, Zanderighi)

 1 A combined framework can be found arXiv:1311.0286 (Alioli, Bauer, Berggren, Tackmann, Walsh, Zuberi) $^{32/32}$

Back-up

One solution: MLM jet matching

The MLM^1 prescription is:

- ♦ Calculate the tree-level MEs, cut away every state with $t(S_{+n}) > t_{MS}$.
- $\diamond~$ Reweight with $\alpha_{\rm s}\text{-}$ and PDF-ratios.
- ◊ Count partons before shower.
- Start the PS on the ME configuration. After shower, count jets above cut, and veto the event if #j ≠ #p, or if a hard parton has a ME-parton/PS-jet separation larger than t_{MS}. For highest multiplicity, veto if #j < #p, or if p^{softest ps}_⊥ > p^{softest me}_⊥
- Combine by adding all accepted events.

$$\begin{split} \langle \mathcal{O} \rangle &= \mathsf{B}_0 \ \mathcal{O}(S_{+0p}) \quad \times \ \mathrm{VETO} \ (S_{+np}) \\ &+ \ \int \mathsf{B}_1 w_p^0 w_{\alpha_{\mathrm{s}}}^0 \Theta \left(t \left(S_{+1} \right) - t_{\mathrm{MS}} \right) \mathcal{O}(S_{+1p}) \times \ \mathrm{VETO} \ (S_{+mp}) \end{split}$$

¹ Mangano, http://www-cpd.fnal.gov/personal/mrenna/tuning/nov2002/mlm.pdf. Talk presented at the Fermilab ME/MC Tuning Workshop, Oct 4, 2002.

CKKW(-L)¹ merging

The CKKW-L prescription is:

- ◊ Calculate the tree-level MEs.
- $\diamond~$ Reweight with Sudakovs, $\alpha_{s}\text{-}$ and PDF-ratios.
- Start the PS on the reweighted ME configuration.
 Veto the event if the PS emission if it gives a state in the ME region.
- Combine by adding all accepted events.

$$\begin{split} \langle \mathcal{O} \rangle &= \mathsf{B}_{0} \quad \times \ \mathsf{\Pi}_{\mathcal{S}_{+0}}(\rho_{0}, t_{\mathsf{MS}}) \quad \mathcal{O}(\mathcal{S}_{+0j}) \\ &+ \ \int \mathsf{B}_{1} \Theta\left(t\left(\mathcal{S}_{+1}\right) - t_{\mathsf{MS}}\right) w_{f}^{0} w_{\alpha_{\mathrm{s}}}^{0} \mathsf{\Pi}_{\mathcal{S}_{+0}}(\rho_{0}, \rho_{1}) \mathcal{O}(\mathcal{S}_{+1j}) \end{split}$$

¹ JHEP 0111 (2001) 063 (Catani, Krauss, Kuhn, Webber), JHEP 0205 (2002) 046 (Lönnblad) ... 32/32

$CKKW(-L)^1$ merging

The CKKW-L prescription is:

- ◊ Calculate the tree-level MEs.
- \diamond Reweight with Sudakovs, α_{s} and PDF-ratios.
- ♦ Start the PS on the reweighted ME configuration. Veto the event if the PS emission if it gives a state in the ME region.
- Combine by adding all accepted events.

$$\begin{aligned} \langle \mathcal{O} \rangle &= \mathsf{B}_{0} \left(1 - \int d\rho w_{f}^{0} w_{\alpha_{\rm s}}^{0} P_{0}(\rho) \Pi_{S_{+0}}(\rho_{0},\rho) \Theta \left(t \left(S_{+1} \right) - t_{\rm MS} \right) \right) \mathcal{O}(S_{+0j}) \\ &+ \int \mathsf{B}_{1} \Theta \left(t \left(S_{+1} \right) - t_{\rm MS} \right) w_{f}^{0} w_{\alpha_{\rm s}}^{0} \Pi_{S_{+0}}(\rho_{0},\rho_{1}) \mathcal{O}(S_{+1j}) \end{aligned}$$

¹ JHEP 0111 (2001) 063 (Catani, Krauss, Kuhn, Webber), JHEP 0205 (2002) 046 (Lönnblad) ... 32/32

CKKW-L for BSM



Figure: Jet p_{\perp} s for squarks+jets. PS bands are obtained by varying between "wimpy" and "power shower", merged bands by varying the merging scale from 50 – 200 GeV (taken from Phys.Rev. D87 (2013) 3, 035006 (Dreiner, Krämer, Tattersall)).

 \implies Merging pins down the transverse momenta.

UMEPS merging

The UMEPS prescription is:

- ◊ Calculate the tree-level MEs.
- $\diamond~$ Reweight with Sudakovs, $\alpha_s\textsc{-}$ and PDF-ratios.
- ◊ Process 1j-ME again: Reweight, then project onto 0-parton state.
- Start the PS on the reweighted ME configuration.
 Veto the PS emission if it gives a state in the ME region.
- Combine by adding reweighted events, and subtracting reweighted-projected events.

$$\begin{aligned} \langle \mathcal{O} \rangle &= \mathsf{B}_{0} \left(1 - \int d\rho w_{f}^{0} w_{\alpha_{s}}^{0} \frac{\mathsf{B}_{1}}{\mathsf{B}_{0}} \Pi_{S_{+0}}(\rho_{0}, \rho) \Theta(t(S_{+1}) - t_{\mathsf{MS}}) \right) \mathcal{O}(S_{+0j}) \\ &+ \int \mathsf{B}_{1} \Theta(t(S_{+1}) - t_{\mathsf{MS}}) w_{f}^{0} w_{\alpha_{s}}^{0} \Pi_{S_{+0}}(\rho_{0}, \rho_{1}) \mathcal{O}(S_{+1j}) \end{aligned}$$

¹ JHEP1302(2013)094 (Leif Lönnblad, SP), JHEP1308(2013)114 (Simon Plätzer)

Comments on UMEPS

This sketch can directly be extended to the case when we have e.g. two-jet merging:

$$\begin{split} \widehat{B}_{0} &= B_{0} \quad , \ \widehat{B}_{1} &= B_{1} w_{f}^{0} w_{\alpha_{s}}^{0} \Pi_{S_{+0}}(\rho_{0}, \rho_{1}) \quad , \ \widehat{B}_{2} &= B_{2} w_{f}^{0} w_{\alpha_{s}}^{0} \Pi_{S_{+0}}(\rho_{0}, \rho_{1}) w_{f}^{1} w_{\alpha_{s}}^{1} \Pi_{S_{+1}}(\rho_{1}, \rho_{2}) \\ \int \widehat{B}_{1 \to 0} &= \int d\Phi \widehat{B}_{1} \quad , \ \int \widehat{B}_{2 \to 1} &= \int d\Phi \widehat{B}_{2} \quad , \ \int \widehat{B}_{2 \to 0} &= \int d\Phi \int d\Phi \widehat{B}_{2} \Theta(\rho_{\rm MS} - \rho_{1}) \end{split}$$

We can get the integrated version of the real-emission matrix elements by projecting onto an underlying Born configuration. Such configurations are available anyway since we need them to perform the Sudakov weighting.

UMEPS definitions

$$\begin{split} w_{n} &= \frac{x_{n}^{+} f_{n}^{+}(x_{n}^{+},\rho_{n})}{x_{n}^{+} f_{n}^{-}(x_{n}^{-},\mu_{n})} \\ &\times \prod_{i=1}^{n} \left[\frac{\alpha_{s}(\rho_{i})}{\alpha_{s}(\mu_{R})} \frac{x_{i-1}^{+} f_{i-1}^{+}(x_{i-1}^{+},\rho_{i-1})}{x_{i-1}^{+} f_{i-1}^{+}(x_{i-1}^{+},\rho_{i})} \frac{x_{i-1}^{-} f_{i-1}^{-}(x_{i-1}^{-},\rho_{i-1})}{x_{i-1}^{-} f_{i-1}(x_{i-1}^{-},\rho_{i})} \Pi_{S_{+i-1}}(x_{i-1},\rho_{i-1},\rho_{i}) \right] \\ \widehat{B}_{n} &= B_{n} w_{n} \\ \widehat{B}_{n \to m} &= \left[\prod_{a=m+1}^{n-1} \int d\rho_{a} dz_{a} d\varphi_{a} \Theta(\rho_{MS} - \rho_{a}) \right] \int d\rho_{n} dz_{n} d\varphi_{n} B_{n} w_{n} \\ &\langle \mathcal{O} \rangle &= \sum_{n=0}^{N} \int d\phi_{0} \int \dots \int \mathcal{O}(S_{+nj}) \; \left\{ \widehat{B}_{n} \; - \; \sum_{i=n+1}^{N} \int \widehat{B}_{i \to n} \right\} \; . \end{split}$$

In CKKW-L, w_n contains an additional factor $\prod_{S_{+n}}(x_n, \rho_n, \rho_{\rm MS})$. UMEPS induces this through $\int \widehat{B}_{n+1 \to n}$ instead.

How do we get \widetilde{B}_n ?

Like everyone else, we need exclusive NLO cross sections as input:

$$\widetilde{\mathsf{B}}_{n} = \mathsf{B}_{n} + \mathsf{V}_{n} + \mathsf{I}_{n+1|n} + \int d\Phi_{\mathsf{rad}} \left(\mathsf{B}_{n+1|n}\Theta\left(\rho_{\mathrm{MS}} - t\left(S_{+n+1}, \rho\right)\right) - \mathsf{D}_{n+1|n}\right)$$

First, observe that

$$\begin{split} \widetilde{\mathsf{B}}_{n} &= \mathsf{B}_{n} + \mathsf{V}_{n} + \mathsf{I}_{n+1|n} + \int d\Phi_{\mathsf{rad}} \left(\mathsf{B}_{n+1|n} - \mathsf{D}_{n+1|n} \right) - \int d\Phi_{\mathsf{rad}} \mathsf{B}_{n+1|n} \Theta\left(t\left(\mathsf{S}_{+n+1}, \rho \right) - \rho_{\mathrm{MS}} \right) \\ &= \overline{\mathsf{B}}_{n} - \int_{\rho_{\mathrm{MS}}} d\Phi_{\mathsf{rad}} \mathsf{B}_{n+1|n} \end{split}$$

and remember that the POWHEG-BOX produces

$$\overline{B}_{n}\Delta(p_{\perp min}) + \overline{B}_{n}\frac{B_{n+1}}{B_{n}}\Delta(p_{\perp}) = \overline{B}_{n}\left[1 - \int_{p_{\perp}min}\frac{B_{n+1}}{B_{n}}\Delta(p_{\perp})\right] + \overline{B}_{n}\frac{B_{n+1}}{B_{n}}\Delta(p_{\perp})$$

Thus, if we project onto an underlying Born for radiative events, we get \overline{B}_n . By having a subtraction sample $\int_{\rho_{MS}} d\Phi_{rad} B_{n+1|n}$, we get \widetilde{B}_n . Comparison of UNLOPS to the other NLO merging schemes

Other schemes don't go through the trouble of unitarisation. Why?

FxFx1: Restricts the range of merging scales. Violation numerically small.Probably fewest counter events.

MEPS@NLO²: Improved, colour-correct Sudakov of MC@NLO for the first emission. Improved Sudakov means smaller violation. Restricted merging scale range. Improved resummation in process-independent way.

MiNLO³: applies analytical (N)NLL Sudakov factors, which cancel the necessary terms when merging two multiplicities. Can be (and was!) moulded into an NNLO matching⁴.

¹JHEP1212(2012)061 (Frixione, Frederix), ²JHEP1304(2013)027 (Höche, Krauss, Schönherr, Siegert),
 ³JHEP1305(2013)082 (Hamilton, Nason, Oleari, Zanderighi), ⁴JHEP1310(2013)222 (Hamilton, Nason, Re, Zanderighi)

NLO merged results (H+jets)



Figure: Ratio of the inclusive cross section for $gg \rightarrow H$ after merging (H+0)@NLO, (H+1)@NLO and (H+2)@LO, compared to the NLO inclusive cross section.

 \Rightarrow NL^3 (=CKKW-L@NLO) has problems for processes with large, loop-driven NLO corrections.

What is $\widetilde{\widetilde{B}}_0$?

For UNLOPS@NNLO, $\widetilde{\widetilde{B}}_0$ is given by

$$\widetilde{\widetilde{\mathsf{B}}}_{0}\mathcal{O}_{+0} = \left[\overline{\overline{\mathsf{B}}}_{0} - \overline{\mathsf{B}}_{1}\Theta\left(
ho\left(\mathcal{S}_{+1},
ho
ight) -
ho_{\mathrm{MS}}
ight)
ight]\mathcal{O}_{+0}$$

where we need to project

onto 0-parton states. (Regularisation scheme irrelevant, only an example!) $E_{2|0}$ cancels spurious div's from integrating $D_{2|1}$ over the full 2-parton PS; $F_{2|0}$ cancels double-unresolved div's in ${\textstyle \iint} B_2$ that would otherwise cancel against ${\textstyle \int} V_1$;

 $G_{2|0}$ cancels double-unresolved div's in $\iint B_2$ that would cancel against W_0 ; $H_{2|0}$ cancels div's that are genuinely shared between $\iint B_2$, $\int V_1$ and W_0 (if present), or other spurious div's (if existing).

What is $\widetilde{\widetilde{B}}_0$?

$$\begin{split} \text{With } \Theta_n (>) &= \Theta \left(t \left(S_{+n}, \rho \right) - \rho_{\text{MS}} \right) \text{ and } \Theta_n \left(< \right) = \Theta \left(\rho_{\text{MS}} - t \left(S_{+n}, \rho \right) \right), \\ \widetilde{\widetilde{B}}_0 &= \left[B_0 + V_0 + \int D_{1|0} \right] \mathcal{O}_{+0} \\ &+ \int \left[B_1 \Theta_1 \left(< \right) \mathcal{O}_{+1} - D_{1|0} \mathcal{O}_{+0} \right] + \left[W_0 + \iint G_{2|0} + \iint (1-h) H_{2|0} \right] \mathcal{O}_{+0} \\ &+ \int \left[V_1 \Theta_1 \left(< \right) \mathcal{O}_{+1} + \int D_{2|1} \Theta_1 \left(< \right) \mathcal{O}_{+1} - \int E_{2|0} \mathcal{O}_{+0} + \int F_{2|0} \mathcal{O}_{+0} + \int h H_{2|0} \mathcal{O}_{+0} \right] \\ &+ \iint \left[B_2 \Theta_1 \left(< \right) \Theta_2 \left(> \right) \mathcal{O}_{+2} + B_2 \Theta_1 \left(< \right) \Theta_2 \left(< \right) \mathcal{O}_{+2} - D_{2|1} \Theta_1 \left(< \right) \mathcal{O}_{+1} \right. \\ &+ E_{2|0} \mathcal{O}_{+0} - F_{2|0} \mathcal{O}_{+0} - G_{2|0} \mathcal{O}_{+0} - H_{2|0} \mathcal{O}_{+0} \right] \end{split}$$

Note the term $B_2\Theta_1\left(<\right)\Theta_2\left(>\right)\mathcal{O}_{+2}$, which contains a state S_{+2} with two resolved partons, but no state with only one resolved parton. Alternatively, we could have defined $\widetilde{\widetilde{B}}_0$ without this term. Then, it would be, in an NNLO-matched calculation, be included e.g. through $\int \widehat{B}_{2\to 0}$. This is how we defined $\widetilde{\widetilde{B}}_0$ in the main text. Above, we also defined the sum $\overline{\overline{B}}_0 - \overline{B}_1$ to not contain such terms.

UN²LOPS matching formula

$$\begin{split} \mathcal{O}^{(\mathrm{UN}^{2}\mathrm{LOPS})} &= \int \!\! d\Phi_{0} \,\bar{\bar{\mathsf{B}}}_{0}^{q_{7},\mathrm{cut}}(\Phi_{0}) \, O(\Phi_{0}) \\ &+ \int_{q_{T,\mathrm{cut}}} \!\! d\Phi_{1} \left[1 - \Pi_{0}(t_{1},\mu_{Q}^{2}) \left(w_{1}(\Phi_{1}) + w_{1}^{(1)}(\Phi_{1}) + \Pi_{0}^{(1)}(t_{1},\mu_{Q}^{2}) \right) \right] \mathsf{B}_{1}(\Phi_{1}) \, O(\Phi_{0}) \\ &+ \int_{q_{T,\mathrm{cut}}} \!\! d\Phi_{1} \, \Pi_{0}(t_{1},\mu_{Q}^{2}) \left(w_{1}(\Phi_{1}) + w_{1}^{(1)}(\Phi_{1}) + \Pi_{0}^{(1)}(t_{1},\mu_{Q}^{2}) \right) \mathsf{B}_{1}(\Phi_{1}) \, \bar{\mathcal{F}}_{1}(t_{1},O) \\ &+ \int_{q_{T,\mathrm{cut}}} \!\! d\Phi_{1} \left[1 - \Pi_{0}(t_{1},\mu_{Q}^{2}) \right] \tilde{\mathsf{B}}_{1}^{\mathrm{R}}(\Phi_{1}) \, O(\Phi_{0}) + \int_{q_{T,\mathrm{cut}}} \!\! d\Phi_{1} \Pi_{0}(t_{1},\mu_{Q}^{2}) \, \tilde{\mathsf{B}}_{1}^{\mathrm{R}}(\Phi_{1}) \, \bar{\mathcal{F}}_{1}(t_{1},O) \\ &+ \int_{q_{T,\mathrm{cut}}} \!\! d\Phi_{2} \left[1 - \Pi_{0}(t_{1},\mu_{Q}^{2}) \right] \mathsf{H}_{1}^{\mathrm{R}}(\Phi_{2}) \, O(\Phi_{0}) + \int_{q_{T,\mathrm{cut}}} \!\! d\Phi_{2} \, \Pi_{0}(t_{1},\mu_{Q}^{2}) \, \mathsf{H}_{1}^{\mathrm{R}}(\Phi_{2}) \, \mathcal{F}_{2}(t_{2},O) \\ &+ \int_{q_{T,\mathrm{cut}}} \!\! d\Phi_{2} \, \, \mathsf{H}_{1}^{\mathsf{E}}(\Phi_{2}) \, \mathcal{F}_{2}(t_{2},O) \end{split}$$