Matching NLO with parton shower in Monte Carlo scheme

Andrzej Siódmok

MANCHESTER 1824
The University of Manchester

in collaboration with:
S. Jadach, W. Płaczek, S. Sapeta and M. Skrzypek

Parton Showers, Event Generators and Resummation

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Outline

- Motivation/notation.
- Our approach to NLO+PS matching
- Results, comparison to:
  - fixed order
  - other matched calculations (MCatNLO and POWHEG)
- Final remarks and outlook

- MC@NLO and POWHEG are by now well established and mature techniques.
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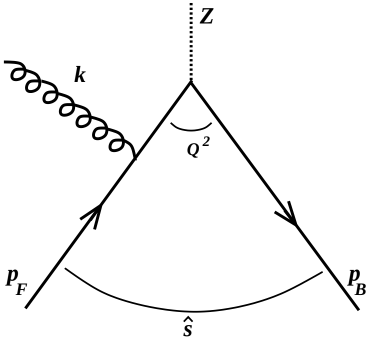
- **Why would you like another method of NLO+PS matching?**
  - The method is extremely simple.
  - No negative weight events.
  - In angular ordered PS - no need for a truncated shower.
  - Simple at NLO $\Rightarrow$ you may hope that pushing the method to NNLO+NLO PS should be possible.



$\hat{s} = (p_F + p_B)^2$

$z = \frac{Q^2}{\hat{s}}$

$$\alpha = \frac{2k \cdot p_B}{\sqrt{\hat{s}}} = \frac{2k^+}{\sqrt{\hat{s}}}$$
$$\beta = \frac{2k \cdot p_F}{\sqrt{\hat{s}}} = \frac{2k^-}{\sqrt{\hat{s}}}$$

$$z = 1 - \alpha - \beta$$
$$k_T^2 = \hat{s}\alpha\beta$$
$$y = \frac{1}{2}\ln\frac{\alpha}{\beta}$$

Basic idea of the MC scheme

DY cross section at NLO in collinear $\overline{\text{MS}}$ factorization for the $q\bar{q}$ channel:

$$\sigma^1_{\text{DY}} - \sigma^B_{\text{DY}} \quad = \quad \sigma^B_{\text{DY}}\, D_1^{\overline{\text{MS}}}(x_1, \mu^2) \otimes \frac{\alpha_s}{2\pi} C_q^{\overline{\text{MS}}}(z) \otimes D_2^{\overline{\text{MS}}}(x_2, \mu^2)\,,$$

where

$$C_q^{\overline{\text{MS}}}(z) = C_F \left[4\,(1+z^2) \left(\frac{\ln(1-z)}{1-z} \right)_+ - 2\frac{1+z^2}{1-z} \ln z + \delta(1-z) \left(\frac{2}{3}\pi^2 - 8 \right) \right].$$

All solutions for NLO + PS matching which use $\overline{\text{MS}}$ PDFs, need to implement terms of the type $4\,(1+z^2) \left(\frac{\ln(1-z)}{1-z} \right)_+$ that are technical artefacts of $\overline{\text{MS}}$ scheme.

The implementation is not easy since those terms correspond to the collinear limit but Monte Carlo lives in 4 dimensions and not in the phase space restricted by $\delta(k_T^2)$.
The idea behind the MC scheme is to absorb those terms to PDF.
[Staszek's talk at previous PSR conference]

KRK method [Jadach, Kusina, Płaczek, Skrzypek & Sławińska '13]

1. Take a parton shower that covers the (α, β) phase space completely (no gaps, no overlaps) and produces emissions according to approx. real matrix element K.
2. Upgrade the real emissions to exact ME R by reweighting the PS events by $W_R = R/K$.
3. We define the coefficion function $C_2^R(z) = \int (R - K)$. To avoid unphysical artifacts of $\overline{\text{MS}}$.
4. Transform PDF for $\overline{\text{MS}}$ scheme to this new physical MC factorization scheme.
5. As a result the virtual+soft correction, Δ_{S+V}, is just a constant now. Multiply the whole result by $1 + \Delta_{S+V}$ to achieve complete NLO accuracy.

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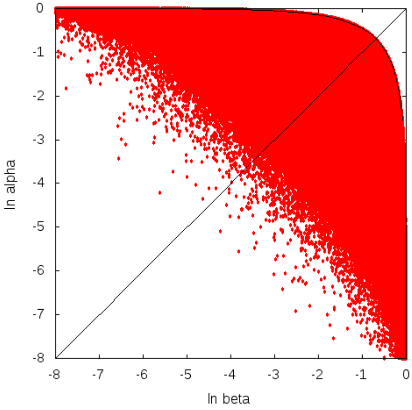
This has been shown to reproduce exactly the NLO result of fixed order collinear factorization, for the case of simplistic PS by means of analytical integration.
[S. Jadach at al. Phys.Rev. D87], or see Staszek's talk at PSR2012 DESY

Could we implement the method in a popular, general purpose MC?

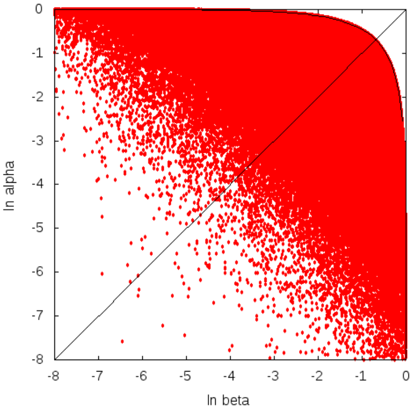
1. Take a PS that covers the (α, β) phase space

Herwig++ (Dipole Shower)



The evolution variable:
$q^2 = k_T^2 = \alpha\,\beta\,s.$

Sherpa 2.0



The evolution variable:
$q^2 = (\alpha + \beta)\,\beta\,s.$

2. Upgrade the real emissions to exact ME by reweighting.

The hardest real emission is upgraded to ME by reweighting:

$$W_R = R/K$$

Real part:

$$\begin{aligned} W_R^{q\bar{q}}(\alpha,\beta) &= 1 - \frac{2\alpha\beta}{1+(1-\alpha-\beta)^2} \\ W_R^{qg}(\alpha,\beta) &= 1 + \frac{\alpha(2-\alpha-2\beta)}{1+2\,(1-\alpha-\beta)(\alpha+\beta)} \end{aligned}$$

<u>Note:</u>
Very simple weight dependent only on the kinematics α, β.

The coefficient function $C_2^R(z) = \int (R - K)$.

- The full MC coefficient for the $q\bar{q}$ channel:

$$C_2^{\text{R+VS}}(z) = C_2^{\text{R}}(z) + C_2^{\text{VS}}(z) = \frac{\alpha_s}{2\pi} C_F \left[-2(1-z) + \delta(1-z) \left(\frac{4}{3}\pi^2 - \frac{5}{2} \right) \right] .$$

- Quark and anti-quark PDFs are redefined by:
  - subtracting $-\frac{\alpha_s}{2\pi} C_F \, (1-z)$,
  - absorbing $\frac{\alpha_s}{2\pi} \, C_F \left[\frac{1+z^2}{1-z} \ln \frac{(1-z)^2}{z} \right]_+$, coming from $\overline{\text{MS}}$ coeff. function

4. Redefine PDFs: MC PDF

Recipe: Make convolution of the LO PDF in $\overline{\mathrm{MS}}$ (q and $\bar{q}$) with the difference of collinear counterterms in $\overline{\mathrm{MS}}$ and MC schemes:

$$q_{\mathrm{MC}}(x, Q^2) \quad = \quad q_{\overline{\mathrm{MS}}}(x, Q^2) + \int_x^1 \frac{dz}{z}\, q_{\overline{\mathrm{MS}}}\left(\frac{x}{z}, Q^2\right) \Delta C_{2q}(z)$$

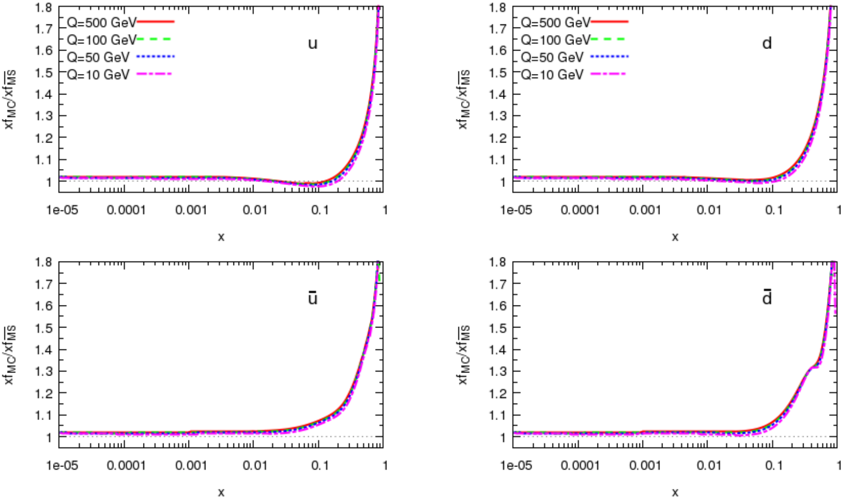
$$\Delta C_{2q}(z) \quad = \quad \frac{\alpha_s}{2\pi} C_F \left[\frac{1+z^2}{1-z} \ln \frac{(1-z)^2}{z} + 1 - z\right]_+$$

Notes:

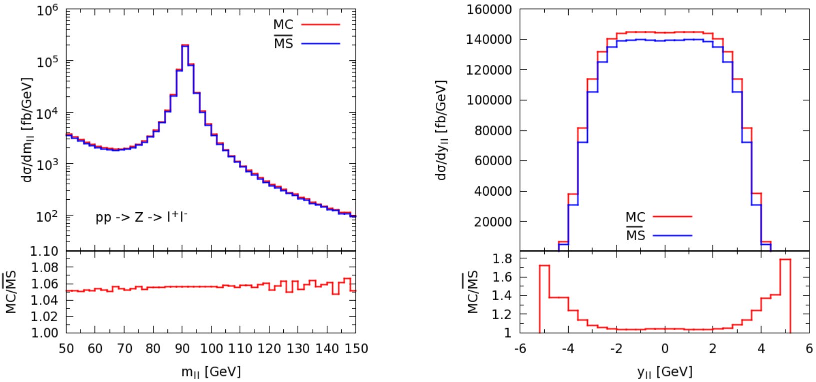
- The MC scheme has been validated by reproducing the scheme-independent relations between DY and DIS.
  [S. Jadach at al. Phys.Rev. D87]
- We constructed the LHAPDF grid (easy to use by all PS MC) for the MC PDF.
  (As a source we used MSTW2008lo, other $\overline{\mathrm{MS}}$ PDF possible).
- How big is the difference?

4. Redefine PDFs: MC PDFs

- Ratios with respect to standard $\overline{\mathrm{MS}}$ PDFs for light quarks.



Introductory exercise:



- 5% effect at central rapidities
- pronounced difference at large y coming from the $x \sim 1$ region

$$x_{1,2} = \frac{m_Z}{\sqrt{s}} e^{\pm y_Z}$$

MCFM $\overline{\text{MS}}$ vs MCFM modified MC scheme at NLO

Fixed order cross-check
(using modified MCFM: using MC PDF and MC C_2)

$$
\begin{aligned}
\sigma_{\text{tot}}^{\overline{\text{MS}}} &= f_q \otimes (1 + \alpha_s\, C_q^{\overline{\text{MS}}}) \otimes f_{\bar{q}} \\
\sigma_{\text{tot}}^{\text{MC}} &= (f_q + \alpha_s \Delta f_q) \otimes (1 + \alpha_s\, C_q^{\text{MC}}) \otimes (f_{\bar{q}} + \alpha_s \Delta f_{\bar{q}}) \\
&= f_q \otimes f_{\bar{q}} + \alpha_s \left(\Delta f_q \otimes f_{\bar{q}} + \Delta f_{\bar{q}} \otimes f_q + C_q^{\text{MC}} \otimes f_q \otimes f_{\bar{q}} \right) + \mathcal{O}(\alpha_s^2) + \mathcal{O}(\alpha_s^3)
\end{aligned}
$$

At $\mathcal{O}(\alpha_s)$:

$$
C_q^{\overline{\text{MS}}} \otimes f_q \otimes f_{\bar{q}} = \Delta f_q \otimes f_{\bar{q}} + \Delta f_{\bar{q}} \otimes f_q + C_q^{\text{MC}} \otimes f_q \otimes f_{\bar{q}}
$$

Drell-Yan, $q\bar{q}$ channel, $\alpha_s = \alpha_s(m_Z)$, MCFM, MSTW2008LO

$$
(336.36 \pm 0.09)\,\text{pb} = \underbrace{25.79\,\text{pb} + 25.79\,\text{pb} + 284.77\,\text{pb}}_{(336.35 \pm 0.09)\,\text{pb}}
$$

- Final result is scheme independent up to $\mathcal{O}(\alpha_s)$.
- Terms $\mathcal{O}(\alpha_s^2) \simeq 16\,\text{pb}$, for this example; $\mathcal{O}(\alpha_s^3) \simeq 0.2\,\text{pb}$.

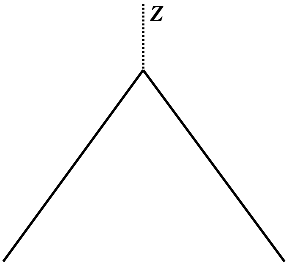
5. Virtual+soft correction, Δ_{S+V}

Virtual + soft:

$$
\begin{aligned}
W^{q\bar{q}}_{V+S} &= \frac{\alpha_s}{2\pi} C_F \left[\frac{4}{3}\pi^2 - \frac{5}{2} \right] \\
W^{qg}_{V+S} &= 0
\end{aligned}
$$

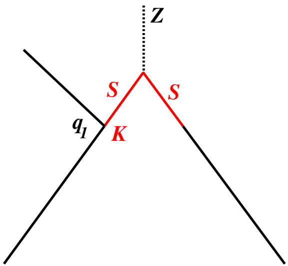
<u>Notes:</u>

- Simple, kinematics independent!



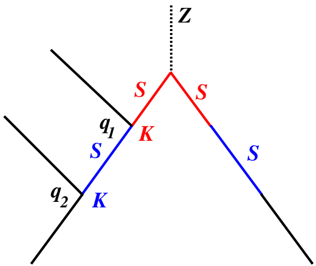
$$\sigma^{\text{LO}} \quad = \quad \sigma_B \otimes D_{\oplus}(Q^2, x_{\oplus}) \otimes D_{\ominus}(Q^2, x_{\ominus})$$

Upgrading to NLO: the hardest emission



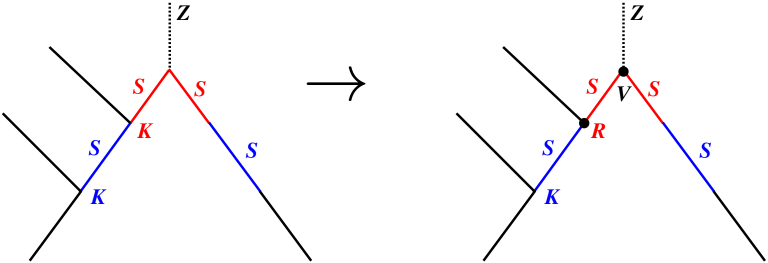
$$
\begin{aligned}
\sigma_{1+}^{\mathrm{PS}} &= \sigma_B \otimes D_\oplus(Q^2, x_\oplus) \otimes D_\ominus(Q^2, x_\ominus) \\
&\otimes \left\{ S_\oplus(Q^2, q_1^2) K_\oplus(q_1^2, z_1) S_\ominus(Q^2, q_1^2) + S_\ominus(Q^2, q_1^2) K_\ominus(q_1^2, z_1) S_\oplus(Q^2, q_1^2) \right\}
\end{aligned}
$$

Upgrading to NLO: the hardest emission



$$
\begin{aligned}
\sigma_{2+}^{\mathrm{PS}} \quad = \quad & \sigma_B \otimes D_{\oplus}(Q^2, x_{\oplus}) \otimes D_{\ominus}(Q^2, x_{\ominus}) \\
& \otimes \Big\{ S_{\oplus}(Q^2, q_1^2) K_{\oplus}(q_1^2, z_1) S_{\ominus}(Q^2, q_1^2) \\
& \quad \otimes \Big\{ S_{\oplus}(q_2^2, q_1^2) K_{\oplus}(q_2^2, z_2) S_{\ominus}(q_2^2, q_1^2) + S_{\oplus}(q_2^2, q_1^2) K_{\ominus}(q_2^2, z_2) S_{\ominus}(q_2^2, q_1^2) \Big\} \\
& \quad + S_{\ominus}(Q^2, q_1^2) \otimes K_{\ominus}(q_1^2, z_1) \otimes S_{\oplus}(Q^2, q_1^2) \\
& \quad \otimes \Big\{ S_{\oplus}(q_2^2, q_1^2) K_{\oplus}(q_2^2, z_2) S_{\ominus}(q_2^2, q_1^2) + S_{\oplus}(q_2^2, q_1^2) K_{\ominus}(q_2^2, z_2) S_{\ominus}(q_2^2, q_1^2) \Big\} \Big\}
\end{aligned}
$$

Upgrading to NLO: the hardest emission



$$
\begin{aligned}
\sigma_{2+}^{\mathrm{NLO+PS}} \quad = \quad & \sigma_B \,(1+V) \otimes D_\oplus(Q^2, x_\oplus) \otimes D_\ominus(Q^2, x_\ominus) \\
& \otimes \Big\{ S_\oplus(Q^2, q_1^2) K_\oplus(q_1^2, z_1) S_\ominus(Q^2, q_1^2)\, R_\oplus(q_1^2, z_1)/K_\oplus(q_1^2, z_1) \\
& \quad \otimes \Big\{ S_\oplus(q_2^2, q_1^2) K_\oplus(q_2^2, z_2) S_\ominus(q_2^2, q_1^2) + S_\oplus(q_2^2, q_1^2) K_\ominus(q_2^2, z_2) S_\ominus(q_2^2, q_1^2) \Big\} \\
& \quad + S_\ominus(Q^2, q_1^2) \otimes K_\ominus(q_1^2, z_1) \otimes S_\oplus(Q^2, q_1^2)\, R_\ominus(q_1^2, z_1)/K_\ominus(q_1^2, z_1) \\
& \quad \otimes \Big\{ S_\oplus(q_2^2, q_1^2) K_\oplus(q_2^2, z_2) S_\ominus(q_2^2, q_1^2) + S_\oplus(q_2^2, q_1^2) K_\ominus(q_2^2, z_2) S_\ominus(q_2^2, q_1^2) \Big\} \Big\}
\end{aligned}
$$

Upgrading to NLO: the hardest emission

Steps:

1. Run LO PS[1] (Herwig/Sherpa) using MC PDF (via LHAPDF interface)
2. Get and an event record (for example in the HepMC format).

```
________________________________________________________________________________
GenEvent: #8 ID=0 SignalProcessGenVertex Barcode: 0
 Momenutm units:     GEV     Position units:      MM
 Cross Section: 697.653 +/- 206.627
 Entries this event: 1 vertices, 5 particles.
 Beam Particles are not defined.
 RndmState(0)=
 Wgts(9)=(0,3023.17) (1,0.17886) (2,3023.17) (3,9) (4,0) (5,1.14371) (6,0) (7,1) (8,1)
 EventScale -1 [energy]         alphaQCD=0.139387       alphaQED=-1
                                   GenParticle Legend
        Barcode   PDG ID      ( Px,       Py,       Pz,     E ) Stat  DecayVtx
________________________________________________________________________________
GenVertex:       -1 ID:    0 (X,cT):0
 I: 2     10001        1 +0.00e+00,+0.00e+00,+6.26e+02,+6.26e+02   2        -1
          10002       21 +0.00e+00,+0.00e+00,-1.84e+01,+1.84e+01   2        -1
 O: 3     10003        1 -1.82e+00,+5.68e-01,-1.50e+01,+1.51e+01   1
          10004       11 +2.58e+01,+9.16e+00,+5.71e+02,+5.71e+02   1
          10005      -11 -2.40e+01,-9.73e+00,+5.17e+01,+5.78e+01   1
```

3. Book a histograms (for example using Rivet) with MC weight calculated from the event record (and information on α_s).

It is almost as fast as LO+PS calculation!

[1]Cover full Phase Space.

Matched results: total cross section

Schematically:

$$\sigma_{\text{tot}}^{\text{MCFM},\overline{\text{MS}}} = f_q^{\overline{\text{MS}}} \otimes (1 + \alpha_s\, C_2^{\overline{\text{MS}}}) \otimes f_{\bar{q}}^{\overline{\text{MS}}}\,,$$

$$\sigma_{\text{tot}}^{\text{MCFM,MC}} = (f_q^{\overline{\text{MS}}} + \alpha_s \Delta f_q) \otimes (1 + \alpha_s\, C_2^{\text{MC}}) \otimes (f_{\bar{q}}^{\overline{\text{MS}}} + \alpha_s \Delta f_{\bar{q}})\,,$$

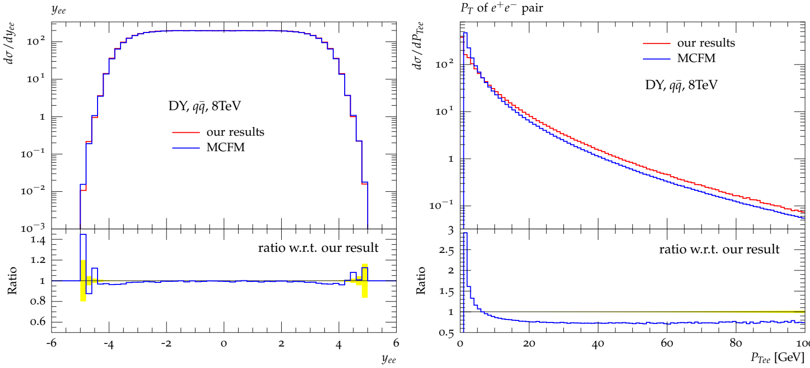
$$\sigma_{\text{tot}}^{\text{NLO+PS,MC}} = (f_q^{\overline{\text{MS}}} + \alpha_s \Delta f_q) \otimes (1 + \alpha_s \int K \tfrac{R}{K}) \otimes (1 + \alpha_s \Delta_{\text{V+S}}) \otimes (f_{\bar{q}}^{\overline{\text{MS}}} + \alpha_s \Delta f_{\bar{q}})$$

Total cross section for DY, $q\bar{q}$ channel, 8 TeV

	σ_{tot} [pb]
MCFM ($\overline{\text{MS}}$ PDFs)	1344.1 ± 0.1
MCFM (MC PDFs)	1361.6 ± 0.3
PS+full NLO (MC PDFs)	1355.9 ± 0.8

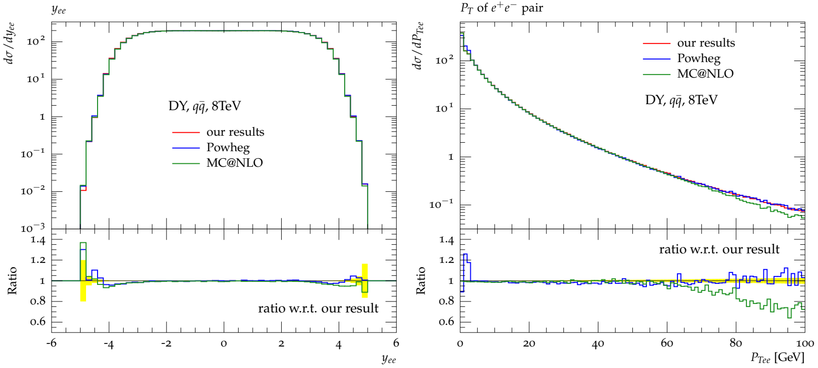
- The difference between fully corrected PS+NLO is at the level of 0.8% w.r.t. MCFM in $\overline{\text{MS}}$ scheme and 0.4% w.r.t. to MCFM in MC scheme.

Matched results: distributions (vs fixed order)



- Our results for y_Z distribution agrees with MCFM at NLO.
- As expected, p_T distribution suppressed at low p_T due to Sudakov.
- Virtual correction spread over a range of p_T.

Matched results: distributions (vs matched results)



- y_Z and p_T distributions very close to POWHEG
  (difference at low p_T due to slightly different evolution variable)
- y_Z very close to MC@NLO, same for low and intermediate p_T
  (differences for the tail of p_T distributions due to higher orders as expected)

Adding qg channel is almost finished:

- $W_R^{qg}(\alpha,\beta) = 1 + \frac{\alpha(2-\alpha-2\beta)}{1+2\,(1-\alpha-\beta)(\alpha+\beta)}$
- $\Delta C_{2g}(z) = \frac{\alpha_s}{2\pi}\,T_R\left\{\left[z^2+(1-z)^2\right]\ln\frac{(1-z)^2}{z} + 2z(1-z)\right\}$
- $W_{V+S}^{qg} = 0$
- Preliminary results:

Total cross section for DY, 8 TeV	$\sigma_{\rm tot}$ [pb]
MCFM ($\overline{\rm MS}$ PDFs)	1146.8 ± 0.1
PS+full NLO (MC PDFs)	1127.6 ± 0.5

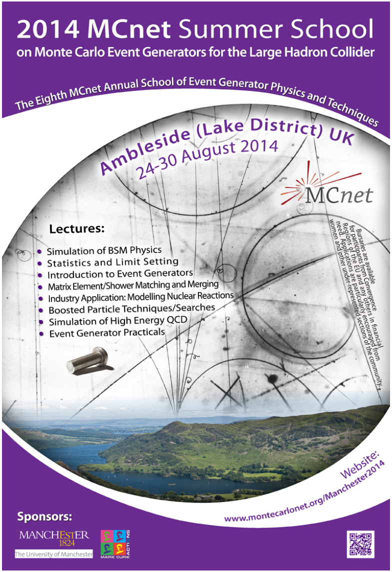
- In progress: final validation and comparison with the data.

Conclusions

- I have discussed a method of NLO+PS matching:
  - Real emissions are corrected by simple reweighting.
  - Collinear terms are dealt with by putting them to PDFs. This amounts to change of factorization scheme from $\overline{\text{MS}}$ to MC.
  - Virtual correction is just a constant and does not depend on Born kinematics.
- The method has been implemented on top of Catani-Seymour shower.
- It has been validated against fixed order NLO for Drell-Yan process in $q\bar{q}$ channel.
- First comparisons to MC@NLO and POWHEG.

Near future: qg channel (hence full DY), correction of n emissions, public code (next Herwig++ release).

MCnet Schools



Next MCnet school: Summer 2014, UK

Monte Carlo
training studentships

3-6 month fully funded studentships for current PhD students at one of the MCnet nodes. An excellent opportunity to really understand and improve the Monte Carlos you use!

Application rounds every 3 months.

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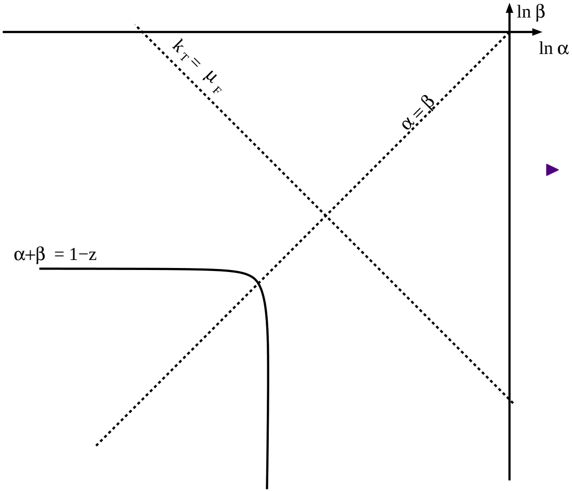
for details go to:
www.montecarlonet.org

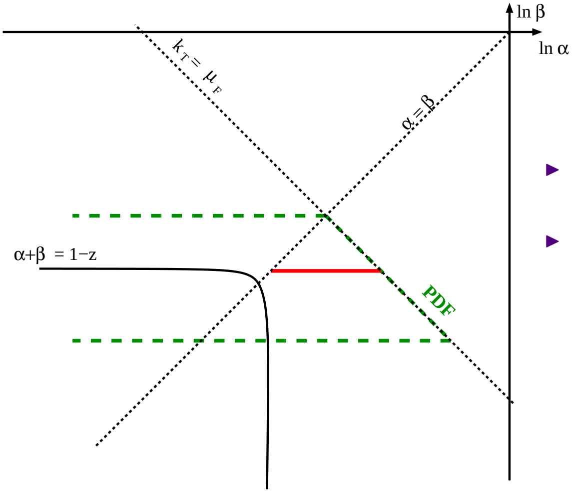
MC*net* projects
- Pythia
- Herwig
- Sherpa
- MadGraph
- Ariadne
- CEDAR

Thank you for the attention!

Origin of $4\frac{\ln(1-z)}{1-z}$ in $\overline{\text{MS}}$

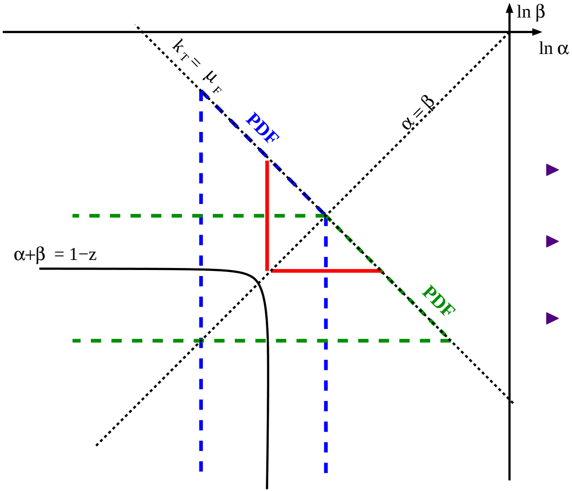


- Integration extends up to a fixed $k_T = \mu_F$.



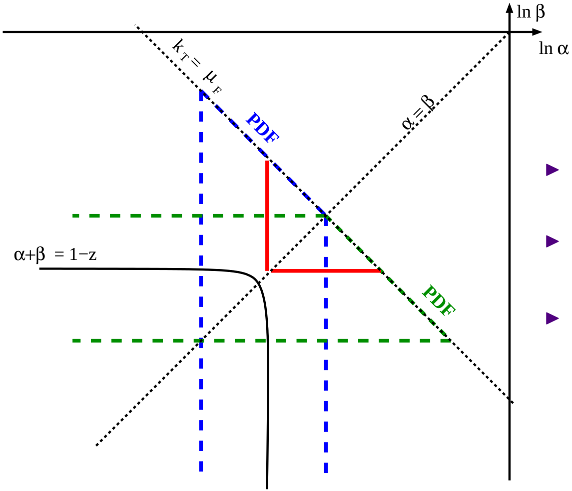
- Integration extends up to a fixed $k_T = \mu_F$.
- For one PDF we get $2\frac{\ln(1-z)}{1-z}$

Origin of $4\frac{\ln(1-z)}{1-z}$ in $\overline{\text{MS}}$



- Integration extends up to a fixed $k_T = \mu_F$.
- For one PDF we get $2\frac{\ln(1-z)}{1-z}$
- Combining two PDFs leads to overcounting by $4\frac{\ln(1-z)}{1-z}$

Origin of $4\frac{\ln(1-z)}{1-z}$ in $\overline{\text{MS}}$



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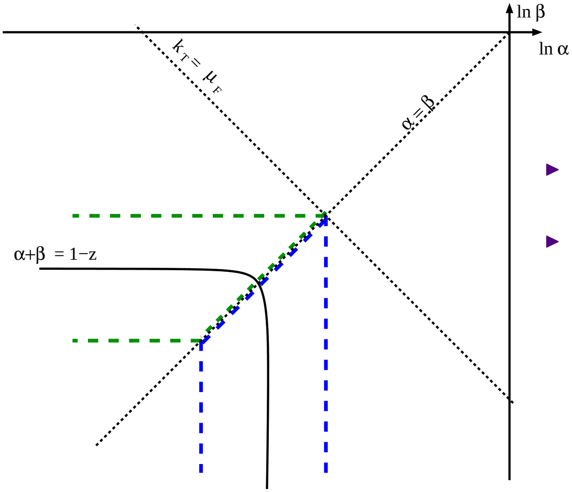
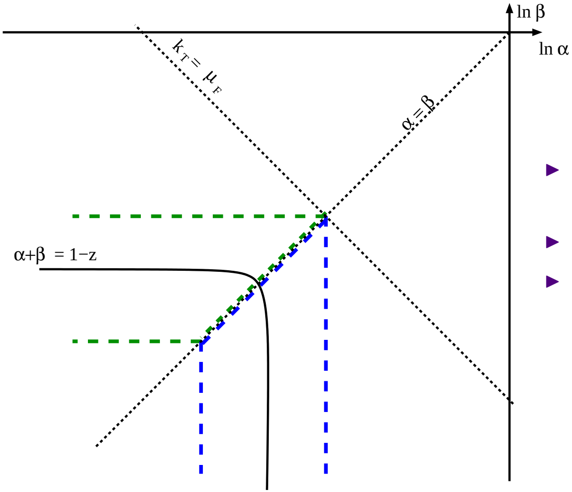
- For one PDF we get $2\frac{\ln(1-z)}{1-z}$
- Combining two PDFs leads to overcounting by $4\frac{\ln(1-z)}{1-z}$

Could we reorganize phase space integration to remove the oversubtraction?

Alternative factorization scheme

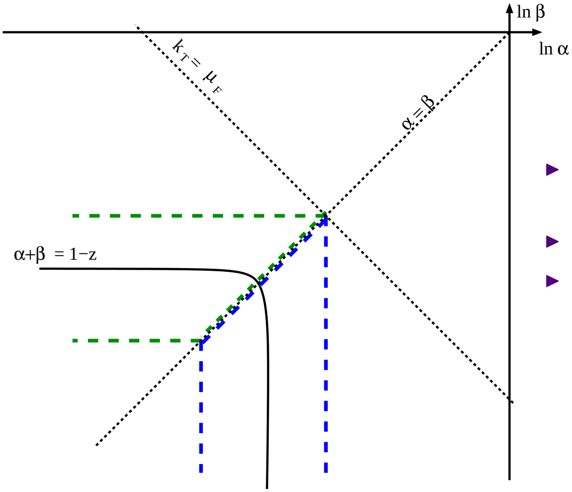


- Integration in angle rather than k_T.
- No overcounting.



- Integration in angle rather than k_T.
- No overcounting.
- This is equivalent to saying that the $4\frac{\ln(1-z)}{1-z}$ term gets absorbed into PDFs.

Alternative factorization scheme



- Integration in angle rather than k_T.
- No overcounting.
- This is equivalent to saying that the $4\frac{\ln(1-z)}{1-z}$ term gets absorbed into PDFs.

Could the change of factorization scheme help us to simplify NLO+PS matching?

More on Δ_{V+S} virtual+soft correction

$$\Delta_{V+S} = D_{DY}^{\overline{MS}}(z) - 2C_{ct}^{psMC}(z)$$

where we use $\overline{MS}$ results, eq. (89) in Altarelli+Ellis+Martinelli (1979):

$$D_{DY}^{\overline{MS}}(z), = \delta(1-z) \; + \delta(1-z)\frac{C_F\alpha_s}{\pi}\left(\frac{1}{3}\pi^2 - 4\right) +$$
$$+\, 2\frac{C_F\alpha_s}{\pi}\left(\frac{\hat{s}}{\mu^2}\right)^{\varepsilon}\left(\frac{\bar{P}(z)}{1-z}\right)_+\left(\frac{1}{\varepsilon} + \gamma_E - \ln 4\pi + [2\ln(1-z) - \ln z]\right)$$

and collinear counterterm of psMC (one gluon in psMC in $d = 4 + 2\varepsilon$):

$$C_{ct}^{psMC}(z) = \frac{C_F\alpha_s}{\pi}\int_{\beta<\alpha}\frac{d\alpha d\beta}{\alpha\beta}\int d\Omega_{1+2\varepsilon}\left(\frac{s\alpha\beta}{\mu_F^2}\right)^{\varepsilon}\bar{P}(1-\alpha,\varepsilon)\delta_{1-z=\alpha} =$$
$$= \frac{C_F\alpha_s}{\pi}\left(\frac{\bar{P}'(z,\varepsilon)}{1-z}\right)_+\left(\frac{1}{\varepsilon} + \gamma_E - \ln 4\pi + \ln\frac{s}{\mu_F^2}\right),$$
$$\bar{P}'(z,\varepsilon) = \bar{P}(z) + \frac{1}{2}\varepsilon(1-z)^2 + \varepsilon\ln(1-z).$$

NLO Monte Carlo weight
This is Yennie-Frautschi-Suura (YFS) style!

Once LO MC is re-designed, introduction of the complete NLO to hard process part is done with help of simple positive MC weight:

$$W_{MC}^{NLO} = 1 + \Delta_{S+V} + \sum_{j\in F} \frac{\tilde{\beta}_1(\hat{s}, \hat{p}_F, \hat{p}_B; a_j, z_{Fj})}{\bar{P}(z_{Fj})\; d\sigma_B(\hat{s}, \hat{\theta})/d\Omega} + \sum_{j\in B} \frac{\tilde{\beta}_1(\hat{s}, \hat{p}_F, \hat{p}_B; a_j, z_{Bj})}{\bar{P}(z_{Bj})\; d\sigma_B(\hat{s}, \hat{\theta})/d\Omega},$$

where the IR/Col.-finite real emission part is

$$\tilde{\beta}_1(\hat{p}_F, \hat{p}_B; q_1, q_2, k) = \left[\frac{(1-\alpha)^2}{2} \frac{d\sigma_B}{d\Omega_q}(\hat{s}, \theta_{F1}) + \frac{(1-\beta)^2}{2} \frac{d\sigma_B}{d\Omega_q}(\hat{s}, \theta_{B2})\right]$$
$$- \theta_{\alpha>\beta} \frac{1+(1-\alpha-\beta)^2}{2} \frac{d\sigma_B}{d\Omega_q}(\hat{s}, \hat{\theta}) - \theta_{\alpha<\beta} \frac{1+(1-\alpha-\beta)^2}{2} \frac{d\sigma_B}{d\Omega_q}(\hat{s}, \hat{\theta}),$$

and the kinematics independent virtual+soft correction is

$$\Delta_{V+S} = \frac{C_F\alpha_s}{\pi}\left(\frac{1}{3}\pi^2 - 4\right) + \frac{C_F\alpha_s}{\pi}\frac{1}{2}$$

Next slide more on Δ_{V+S}.

Notation: CS parton shower

The “Sudakov” form factor

$$S(Q^2, \Lambda^2, x) = \int\limits_{\Lambda^2}^{Q^2} \frac{dq^2}{q^2} \int\limits_{z_{\min}(q^2)}^{z_{\max}(q^2)} dz \;\; K(q^2, z, x)\,,$$

where

$$K(q^2, z, x) = \frac{C_F \alpha_s}{2\pi} \frac{1+z^2}{1-z} \; \frac{D(q^2, x/z)/z}{D(q^2, x)}\,.$$

- z, q^2 - internal variables of the shower
- $D(q^2, x)$ - parton distribution functions

The kernel K is just a CS dipole written in terms of shower’s internal variables multiplied by the ratio of PDFs due to backward evolution.

Convolution:

$$(f \otimes g)(x) \equiv \int_0^1 dx_1 \int_0^1 dx_2 \; \delta(x - x_1 x_2)\, f(x_1) f(x_2)\,. \tag{2}$$

Eliminating x_2 and delta function we obtain[2]

$$(f \otimes g)(x) \equiv \int_x^1 \frac{dx_1}{x_1}\, f(x_1) f(x/x_1)\,. \tag{3}$$

$$C(z) = \tilde{C}(z) + \{\Delta C(z)\}_+\,. \tag{4}$$

$$\begin{aligned}
[C \otimes D_1 \otimes D_2](x) &= [\tilde{C} \otimes D_1 \otimes D_2](x) \\
&+ \frac{C_F \alpha_s}{\pi} \Big[\Big(\Big\{ \frac{1}{2} \Delta C(z) \Big\}_+ \otimes D_1 \Big) \otimes D_2 \Big](x) + \frac{C_F \alpha_s}{\pi} \Big[D_1 \otimes \Big(\Big\{ \frac{1}{2} \Delta C(z) \Big\}_+ \otimes D_2 \Big) \Big](x)
\end{aligned} \tag{5}$$

Denoting

$$\begin{aligned}
\Delta D(x) &= \frac{C_F \alpha_s}{\pi} \Big[\Big\{ \frac{1}{2} \Delta C(z) \Big\}_+ \otimes D \Big](x), \\
\tilde{D}(x) &= D(x) + \Delta D(x),
\end{aligned} \tag{6}$$

the above formula can be expressed at the NLO precision level (i.e. dropping NNLO terms) as follows:

$$\begin{aligned}
[C \otimes D_1 \otimes D_2](x) &= [\tilde{C} \otimes D_1 \otimes D_2](x) + [\Delta D_1 \otimes D_2](x) + [D_1 \otimes \Delta D_2](x) \\
&= [\tilde{C} \otimes \tilde{D}_1 \otimes \tilde{D}_2](x) + \mathcal{O}(\alpha_s^2)\,.
\end{aligned} \tag{7}$$

[2]Note the importance of $x/x_1 < 1$ condition when eliminating delta.