Glauber Gluons and Multiple Parton Interactions.

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Parton Shower and Resummation
Workshop 2014, Münster, 10th June 2014

Based on arXiv:1405.2080

Outline

- 'Standard factorisation formulae' with soft, hard and collinear functions. Glauber gluons, and the need for their cancellation to obtain a standard factorisation formula.
- Analysis of Glauber gluons for the observable E_T, with E_T << Q, and demonstration of lack of cancellation at the level of two Glauber gluon exchanges between spectators (plus review of successful cancellation for p_T).
- Connection of these noncancelled Glauber exchange diagrams to MPI and (at higher orders) Regge physics.
- Other MPI sensitive variables for which standard factorisation fails.
- Problems with applying standard factorisation formula to MPI sensitive observables even when observable is ~ Q.

Factorisation Formulae

Factorisation formulae are essential to make predictions at LHC.

Separate out short distance interaction of interest from long-distance QCD-dominated interactions. Low-momentum part of long-distance piece will not be calculable perturbatively, but is (hopefully) universal.

Examples of factorisation formulae:

Collinear factorisation for pp → V + X inclusive total cross section, V colourless

PDFs (long distance physics, universal)

$$\sigma = \int dx_A dx_B \hat{\sigma}_{ij\to X}(\hat{s} = x_A x_B s) f_i(x_A) f_j(x_B) + \mathcal{O}\left(\frac{\Lambda^2_{QCD}}{Q^4}\right)$$

Parton-level cross section/coefficient function (short distance physics)

Corrections suppressed by Λ^2/Q^2

Factorisation Formulae

TMD factorisation for pp → V + X cross section differential in p_T, p_T << Q, V colourless

Hadronic tensor

Approximated momenta (transverse momenta → 0)

Rapidity regulator

$$W^{\mu\nu} = \frac{8\pi^2 s}{Q^2} \sum_f C_f^{\mu\nu}\left(\hat{k}_A, \hat{k}_B\right) \int d^2\mathbf{b}_T e^{i\mathbf{p}_T\cdot\mathbf{b}_T} \tilde{f}_f(x_A, \mathbf{b}_T; \zeta_A)\, \tilde{f}_{\bar{f}}(x_B, \mathbf{b}_T; \zeta_B) + \mathcal{O}\left(\frac{p_T^2}{Q^2}\right)$$

Transverse momentum dependent PDFs (long distance physics)

Coefficient function (short distance physics)

Corrections suppressed by p_T^2/Q^2 (can be augmented to Λ^2/Q^2 by adding matching to fixed order)

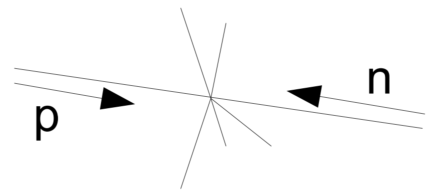
Both formula rigorously proved to leading power by Collins, Soper and Sterman

Bodwin Phys. Rev. 31 (1985) 2616
Collins, Soper, Sterman Nucl. Phys. B261 (1985) 104, Nucl. Phys. B308 (1988) 833
Collins, pQCD book

Momentum regions

Consider observable *O* in process pp → V + X with associated scale *Q*.

First step in attempt to prove factorisation for the observable *O* is to identify momentum regions for involved particles that result in leading contributions. Relevant regions in QCD are:



n/- component

p/+ component

transverse components

1) Hard region – momentum with large virtuality (order *Q*)

$$k \sim Q\,(1, 1, 1)$$

2) Collinear region – momentum close to some beam/jet direction

$$k \sim Q\,(1, \lambda^2, \lambda)$$ (for example)

$$\lambda \ll 1$$

Momentum regions

3) (Central) soft region – all momentum components small and of same order

$$k \sim Q\left(\lambda^n, \lambda^n, \lambda^n\right)$$

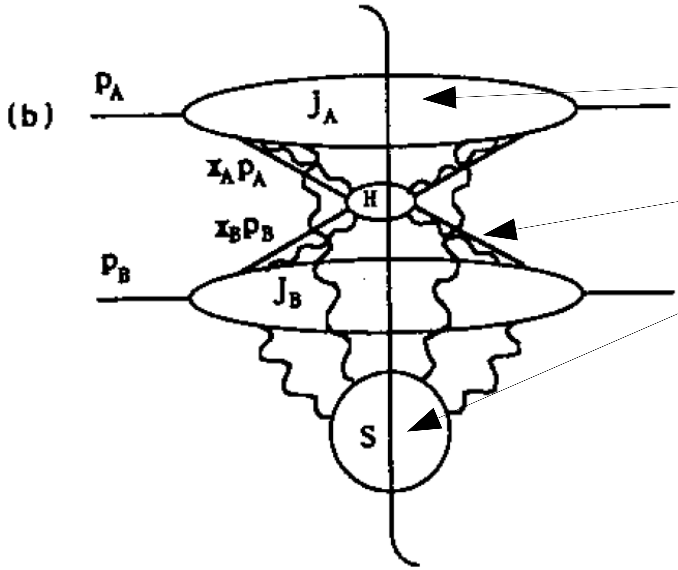
4) Glauber region – all momentum components small, but transverse components much larger than longitudinal ones

$$|k^+ k^-| \ll \mathbf{k}_T^2 \ll Q^2$$

Canonical example: $k \sim Q\left(\lambda^2, \lambda^2, \lambda\right)$

Glauber Gluons and Factorisation

Deriving a factorisation formula that includes Glauber gluons is problematic.

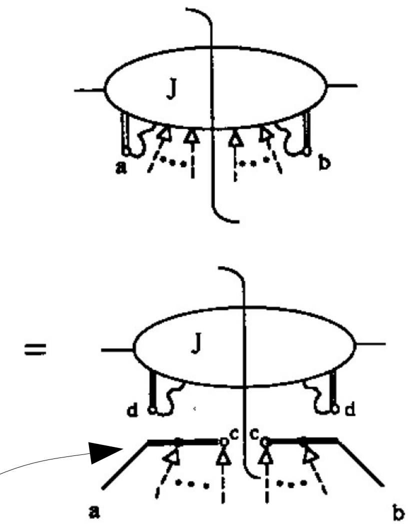


Starting picture (colourless V)

Collinear to proton A

Single parton + extra scalar gluon attachments into hard

Soft + Glauber particles



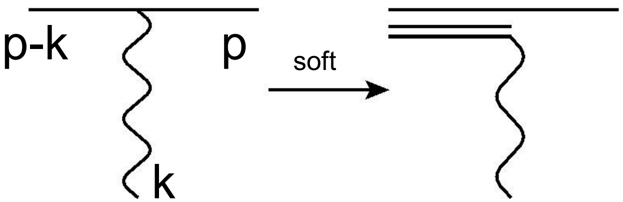
If blob S only contained central soft, then we could strip soft attachments to collinear J blobs using Ward identities.

In SCET this is achieved by a field redefinition.
Bauer, Pirjol, Stewart Phys. Rev. D 65 (2002) 054022

Eikonal line in direction of J

Glauber Gluons and Factorisation

Simple example:



Propagator denominator:

$$(p-k)^2 = -2p\cdot k + k^2 \xrightarrow{\text{soft}} -2p\cdot k$$

Eikonal piece

This manipulation is NOT POSSIBLE for Glauber gluons – two terms in denominator are of same order in Glauber region

How do we get around this problem?

One approach: try and show that that contribution from the Glauber region cancels (used by CSS for total cross section and p_T of V)

Possibility of factorisation formulae including Glaubers? (Glaubers and central soft treated differently). Not yet developed. But work ongoing by Stewart, Rothstein

Glauber Gluons and Factorisation

Quite common – derive factorisation/resummation formula in absence of Glaubers – SCET, resummation approach

$$\frac{d\sigma}{dO} = \underbrace{H}_{\text{Hard}} \times \Big[\underbrace{B_a \otimes B_b \otimes J_1 \otimes \ldots \otimes J_n}_{\text{Collinear}} \otimes \underbrace{S}_{\text{Soft}}\Big](O)$$

Here we will take this sort of formula as the 'standard' factorisation, and say that factorisation breaks/fails for an observable if such a formula does not completely capture the leading contribution.

Other definitions of factorisation breaking are possible: breakdown of factorisation of QCD scattering amplitudes, failure of universality for collinear functions,...

Hadronic Transverse Energy

In p + p → V + X: $E_T = \sum_{i \in X} \sqrt{\mathbf{p}_{Ti}^2 + m_i^2}$

Standard factorisation/resummation formula for E_T obtained by Papaefstathiou, Smillie, Webber:

See talk by Andreas

$$\left[\frac{d\sigma_H}{dQ^2\, dE_T}\right]_{\text{res.}} = \frac{1}{2\pi} \sum_{a,b} \int_0^1 dx_1 \int_0^1 dx_2 \int_{-\infty}^{+\infty} d\tau\; \mathrm{e}^{-i\tau E_T}\; f_{a/h_1}(x_1,\mu)\; f_{b/h_2}(x_2,\mu) \cdot W_{ab}^H(x_1 x_2 s; Q, \tau, \mu)$$

$$W_{ab}^H(s; Q, \tau, \mu) = \int_0^1 dz_1 \int_0^1 dz_2\; C_{ga}(\alpha_S(\mu), z_1; \tau, \mu)\; C_{gb}(\alpha_S(\mu), z_2; \tau, \mu)\; \delta(Q^2 - z_1 z_2 s) \cdot \sigma_{gg}^H(Q, \alpha_S(Q))\; S_g(Q, \tau)\;.$$

Papaefstathiou, Smillie, Webber JHEP 1004 (2010) 084
Grazzini, Papaefstathiou, Smillie, Webber arXiv:1403.3394

Contains effects of collinear, soft, hard radiation, could be rearranged into HxJxS form:

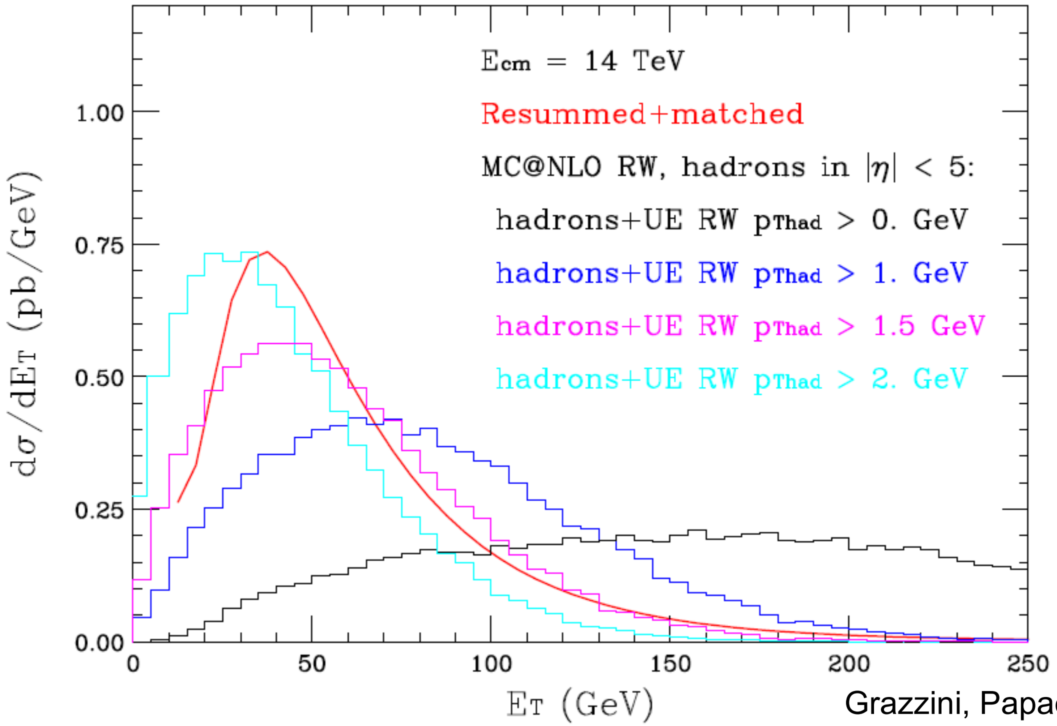
$$\frac{\mathrm{d}\sigma}{\mathrm{d}E_T} = \sigma_0\, H_{gg}(m_H, \mu) \int \mathrm{d}Y \int \mathrm{d}k_a\, \mathrm{d}k_b \times B_g(m_H, k_a, x_a, \mu, \nu)\, B_g(m_H, k_b, x_b, \mu, \nu) \times S_T^{gg}\Big(E_T - k_a - k_b, \mu, \nu\Big)\,.$$

Tackmann, Walsh, Zuberi
Phys. Rev. D 86 (2012) 053011

Does this give the leading contribution?

Hadronic Transverse Energy

Monte Carlo study with Herwig++:

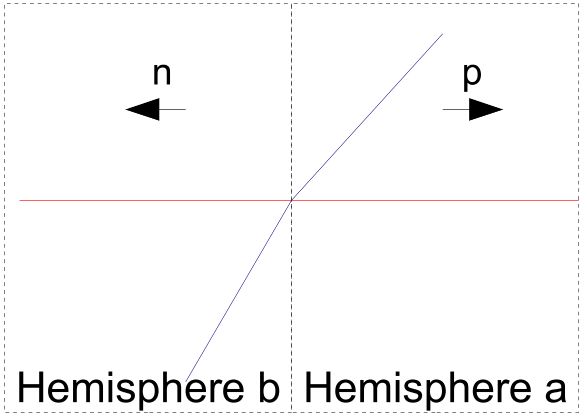


Grazzini, Papaefstathiou, Smillie, Webber
arXiv:1403.3394

Apparently not! Once we turn on UE, event shape completely changes

Can we see this from a factorisation point of view? Must be related to Glauber gluons.

Beam thrust



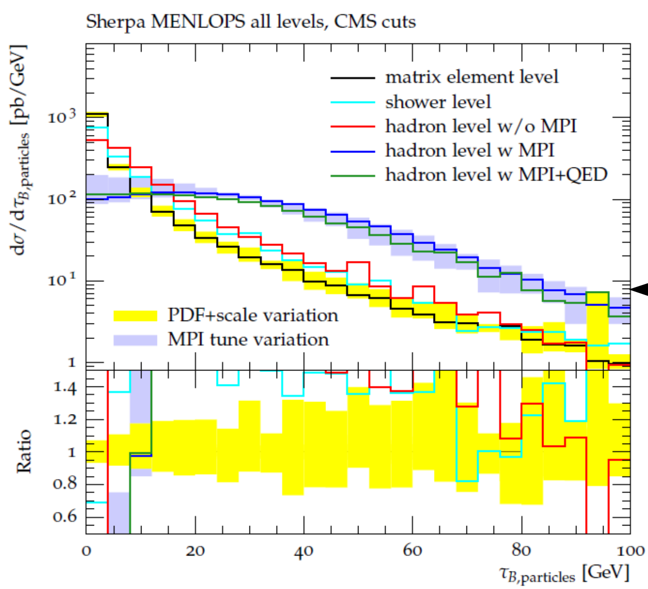
In p + p → $l^+ l^-$ + X $\quad B_a = \sqrt{2} \sum_{i \in X,a} p_i \cdot p \quad B_b = \sqrt{2} \sum_{i \in X,b} p_i \cdot n$

Standard factorisation formula for beam thrust derived in SCET by Tackmann, Waalewijn, Stewart

$$\frac{1}{\sigma_0} \frac{d\sigma}{dq^2 dY dB_a^+ dB_b^+} = \sum_{ij} H_{ij}(q^2, \mu) \int dk_a^+ \, dk_b^+ \qquad (19)$$
$$\times \, q^2 B_i[\omega_a(B_a^+ - k_a^+), x_a, \mu] B_j[\omega_b(B_b^+ - k_b^+), x_b, \mu]$$
$$\times \, S_{\text{ihemi}}(k_a^+, k_b^+, \mu) \left[1 + \mathcal{O}\left(\frac{\Lambda_{\text{QCD}}}{Q}, \frac{\omega_{a,b} B_{a,b}^+}{Q^2}\right)\right].$$

Stewart, Tackmann, Waalewijn
Phys. Rev. D 81 (2010) 094035

Derivation also included an argument to rule out contribution from Glauber modes



When we investigate variable using MC – same qualitative picture as E_T

J. Alcaraz Maestre et. al.
arXiv:1203.6803

Should revisit the factorisation properties of these variables, looking particularly at Glauber contributions.

Model Setup

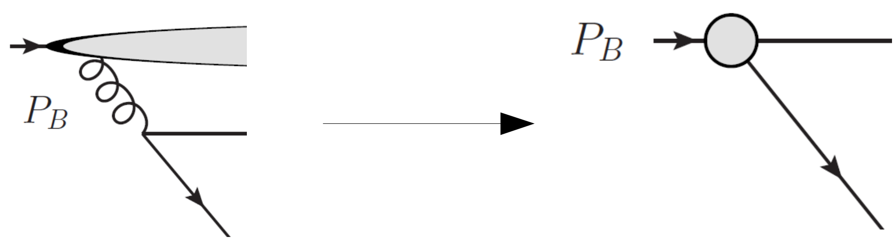
We will see if the effects of Glauber gluons can be cancelled for E_T, as was demonstrated to occur for p_T by CSS, when E_T << Q

Model setup: Each 'proton' is composed from a quark-antiquark pair

Central 'hard process' is $q\bar{q} \to V$ with V colourless and associated scale Q

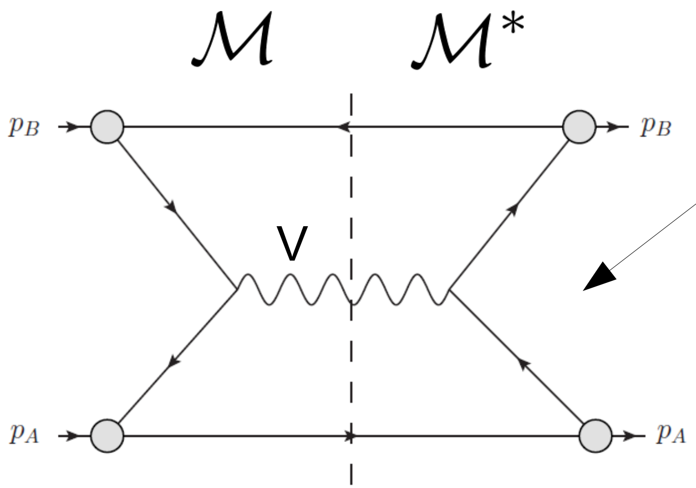
Assume momentum of proton A mainly along p/+ direction and that of B mainly along n/- direction, but with small masses. All partons taken to be massless.

We assume little about the coupling of the quark-antiquark pair to the 'proton' – could either represent some soft nonperturbative coupling (appropriate when E_T ~ Λ) or the perturbative quark-antiquark-gluon coupling (appropriate when E_T is perturbative)



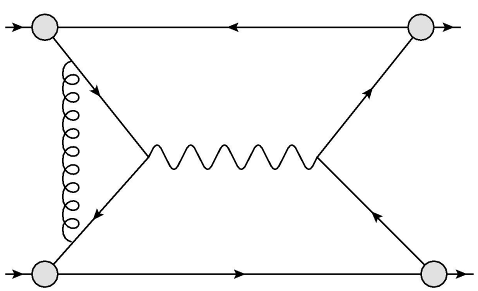
Calculation follows closely discussion in Collins pQCD book

One-Glauber Exchange

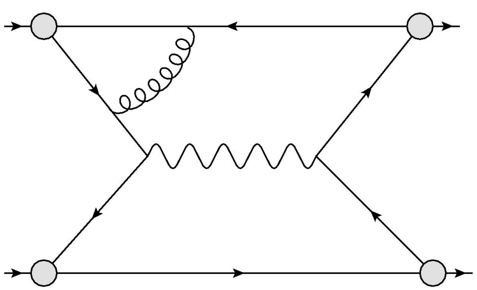


Lowest order 'parton model' process for p + p $\rightarrow$ V + X

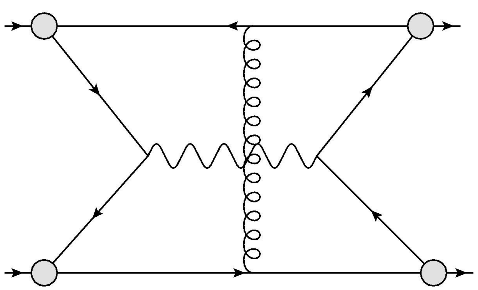
Consider one gluon corrections to this picture. Various possibilities:



Active-active interaction

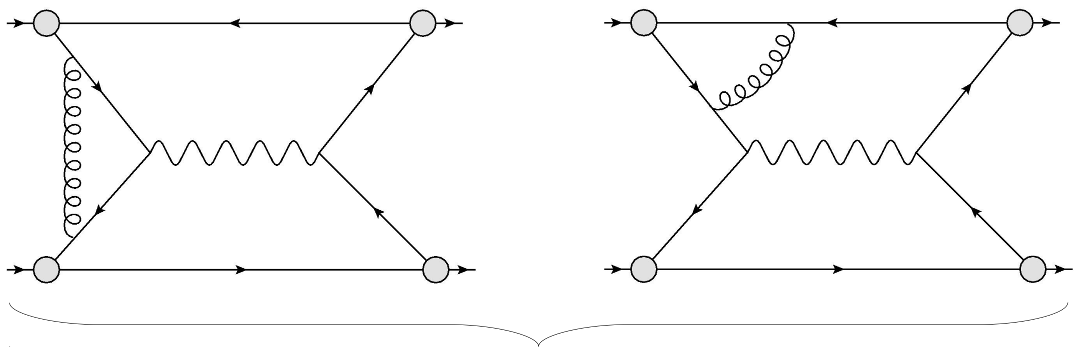


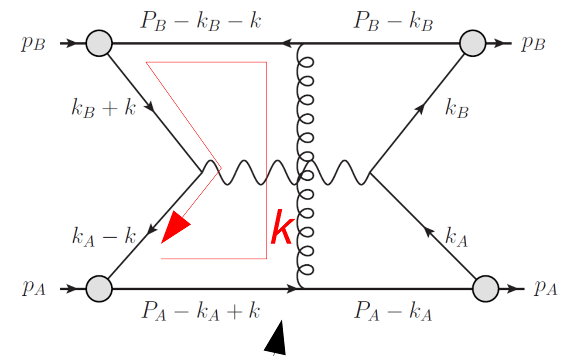
Active-spectator interaction



Spectator-spectator interaction

One-Glauber Exchange





For these graphs, gluon momentum is not trapped in the Glauber region however → can deform loop integration contour and handle these using usual soft/collinear/hard regions.

Collins, Phys.Rev. D57 (1998) 3051–3056
Collins, Metz, Phys.Rev.Lett. 93 (2004) 252001

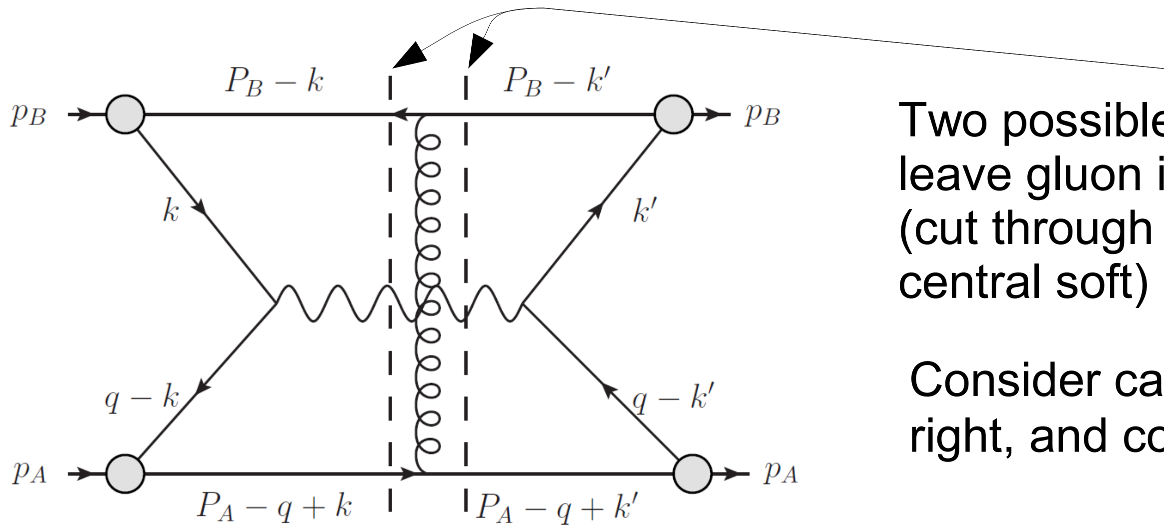
$$\int \frac{\mathrm{d}k^+ \mathrm{d}k^-}{(2\pi)^2} \frac{\text{numerator}}{2k^+k^- - \mathbf{k}_T^2 + i0}$$
$$\times \frac{1}{[-2k^+(P_B^- - k_B^-) + ... + i0][2k^+k_B^- + ... + i0]}$$
$$\times \frac{1}{[-2k^-k_A^+ + ... + i0][2k^-(P_A^+ - k_A^+) + ... + i0]}$$

In this graph gluon is trapped in Glauber region

Traps k^+ small

Traps k^- small

One-Glauber Exchange



Two possible cuts of graph that leave gluon in Glauber region (cut through gluon forces it into central soft)

Consider case where cut is to the right, and consider k^+ integral.

In top half of graph can ignore $q^- - k^-$ compared to large components (q^-, $P_B^- - q^-$)

In bottom half of graph can ignore k^+ compared to large components (q^+, $P_A^+ - q^+$)

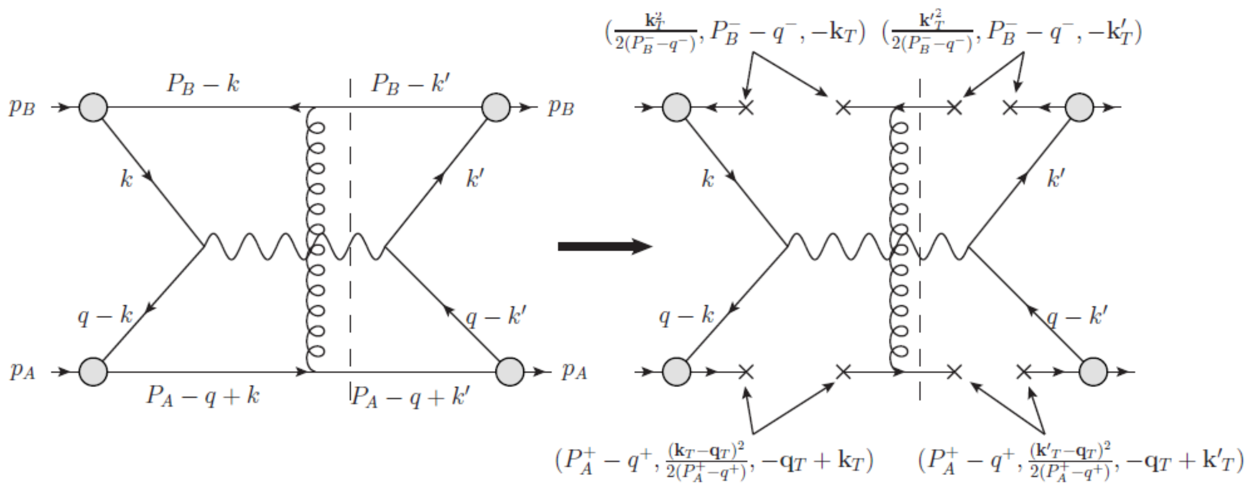
In gluon propagator can ignore lightcone components of k compared to transverse components

$$\int \frac{\mathrm{d}k^+}{2\pi} \frac{i}{(2q^-k^+ - \mathbf{k}_T^2 + i0)} \frac{i}{(2(-k^+ + P_B^+)(P_B^- - q^-) - \mathbf{k}_T^2 + i0)}$$
$$= \frac{i}{2(P_B^- - q^-)} \frac{i}{2q^-k^+_{\text{on-shell}} - \mathbf{k}_T^2 + i0}$$
$$= \int \frac{\mathrm{d}k^+}{2\pi} \frac{i}{(2q^-k^+ - \mathbf{k}_T^2 + i0)} 2\pi\delta(2(-k^+ + P_B^+)(P_B^- - q^-) - \mathbf{k}_T^2 + i0)$$

Net effect – set P_B-k line on shell!

One-Glauber Exchange

Repeat with k^-, k'^+, k'^- integrations:



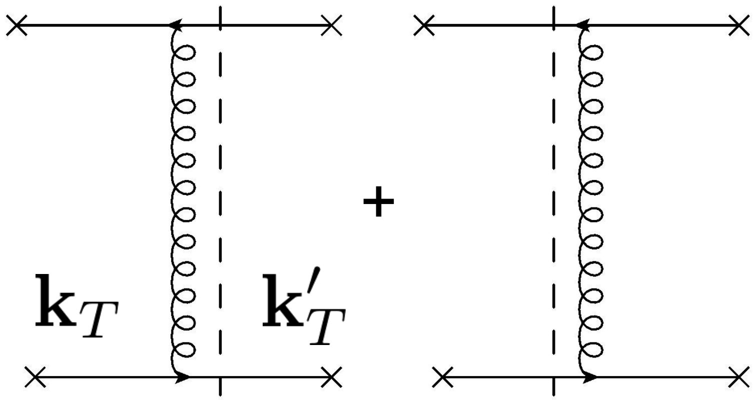
Factor out on-shell Glauber exchange graph!

Can do a similar procedure when cut is on left.

Measured Transverse Momentum

Consider case where we measure p_T of V. For given momenta in the two decomposed graphs, the value of the measurement is the same.

Therefore, we can factor out the parton model graph and measurement and add together the two Glauber subgraphs:



$\mathcal{M} = -iL$

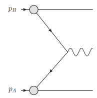
$$\left.\begin{array}{ll} \text{Graph level:} & L(\mathbf{k}_T \to \mathbf{k}'_T) + L^*(\mathbf{k}'_T \to \mathbf{k}_T) \\ \text{Amplitude level:} & i\mathcal{M}(\mathbf{k}_T \to \mathbf{k}'_T) - i\mathcal{M}^*(\mathbf{k}'_T \to \mathbf{k}_T) \end{array}\right\} = 0$$

CUTKOSKY RULE

Measured Hadronic Transverse Energy

For E_T, can't do the same thing – it equals $|\mathbf{k}_T| + |\mathbf{q}_T - \mathbf{k}_T|$ for cut to left and $|\mathbf{k}'_T| + |\mathbf{q}_T - \mathbf{k}'_T|$ for cut to right. Can still arrange cancellation as follows:

Parton model graph



$$\int \mathrm{d}^{d-2}\mathbf{k}_T\, \mathrm{d}^{d-2}\mathbf{k'}_T\, f_P(\mathbf{k}_T) f_P^*(\mathbf{k'}_T) L(\mathbf{k}_T \to \mathbf{k'}_T)\delta(E_T = |\mathbf{k'}_T| + |\mathbf{k'}_T - \mathbf{q}_T|) +$$

$$\int \mathrm{d}^{d-2}\mathbf{k}_T\, \mathrm{d}^{d-2}\mathbf{k'}_T\, f_P(\mathbf{k}_T) f_P^*(\mathbf{k'}_T) L^*(\mathbf{k'}_T \to \mathbf{k}_T)\delta(E_T = |\mathbf{k}_T| + |\mathbf{k}_T - \mathbf{q}_T|)$$

Relabel $\mathbf{k}_T \leftrightarrow \mathbf{k}'_T$ in second term

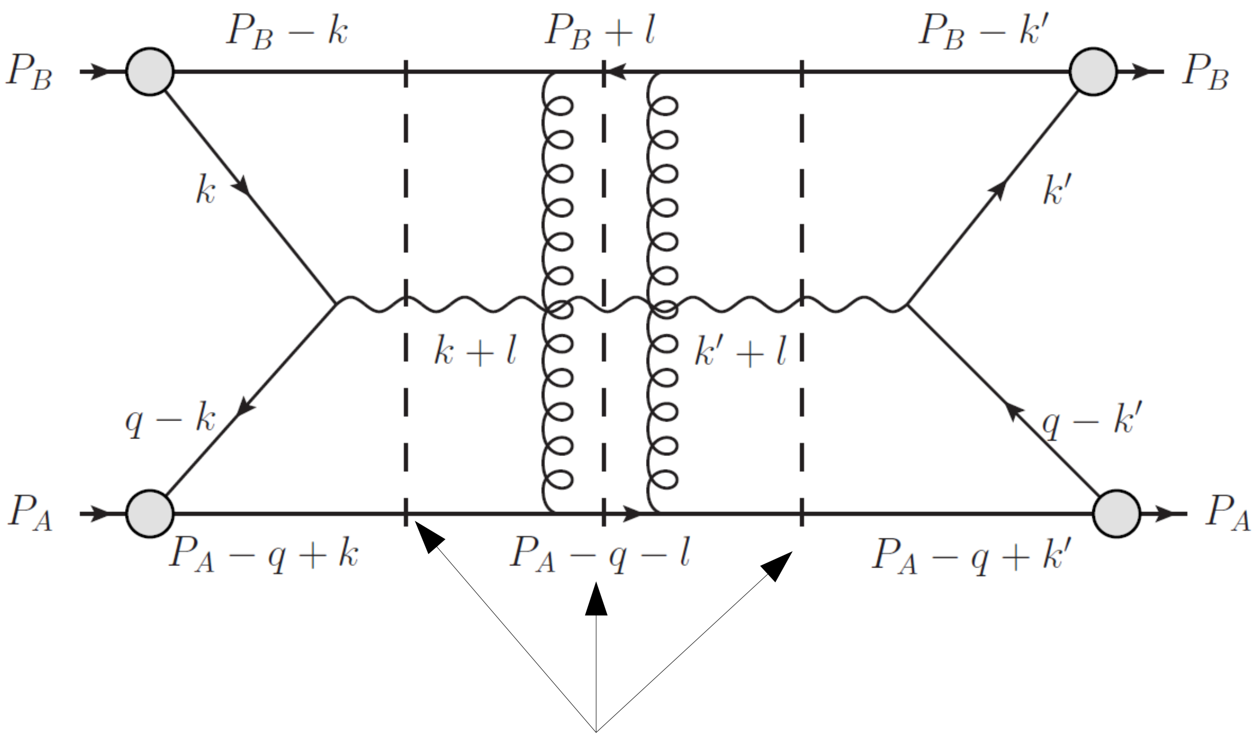
$$\int \mathrm{d}^{d-2}\mathbf{k}_T\, \mathrm{d}^{d-2}\mathbf{k'}_T\, \delta(E_T = |\mathbf{k'}_T| + |\mathbf{k'}_T - \mathbf{q}_T|) \times$$
$$\left[f_P(\mathbf{k}_T) f_P^*(\mathbf{k'}_T) L(\mathbf{k}_T \to \mathbf{k'}_T) + f_P(\mathbf{k'}_T) f_P^*(\mathbf{k}_T) L^*(\mathbf{k}_T \to \mathbf{k'}_T)\right]$$

$L(\mathbf{k}_T \to \mathbf{k}'_T) \propto 1/(\mathbf{k}_T - \mathbf{k}'_T)^2 = L(\mathbf{k}'_T \to \mathbf{k}_T)$ & $f_P = -f_P^*$

$$\int \mathrm{d}^{d-2}\mathbf{k}_T\, \mathrm{d}^{d-2}\mathbf{k'}_T\, \delta(E_T = |\mathbf{k'}_T| + |\mathbf{k'}_T - \mathbf{q}_T|) \times$$
$$f_P(\mathbf{k}_T) f_P^*(\mathbf{k'}_T) \left[L(\mathbf{k}_T \to \mathbf{k'}_T) + L^*(\mathbf{k'}_T \to \mathbf{k}_T)\right] \longrightarrow 0$$

Two-Glauber Exchange

Now add in one more (Glauber) gluon between spectators:

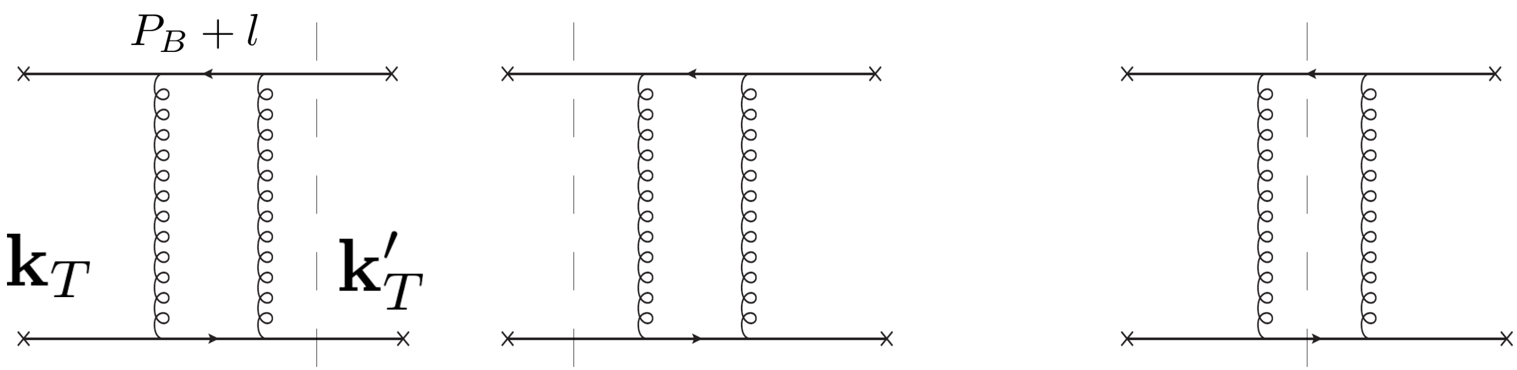


Now 3 cuts that leave both gluons in Glauber region

Factor out Glauber subgraphs as before.

Measured Transverse Momentum

p_T measured: Again, for given momenta in (decomposed) graphs, measurement is the same – factor measurement and parton model graphs, and combine Glauber subgraphs:



$$L(\mathbf{k}_T \to \mathbf{k'}_T; l) + L^*(\mathbf{k'}_T \to \mathbf{k}_T; l) \ + \int \Phi_2 L(\mathbf{k}_T \to \mathbf{l}_T) L^*(\mathbf{k'}_T \to \mathbf{l}_T)$$

$$i\mathcal{M}(\mathbf{k}_T \to \mathbf{k'}_T; l) - i\mathcal{M}^*(\mathbf{k'}_T \to \mathbf{k}_T; l) \ + \int \Phi_2 \mathcal{M}(\mathbf{k}_T \to \mathbf{l}_T) \mathcal{M}^*(\mathbf{k'}_T \to \mathbf{l}_T)$$

-Imaginary Part

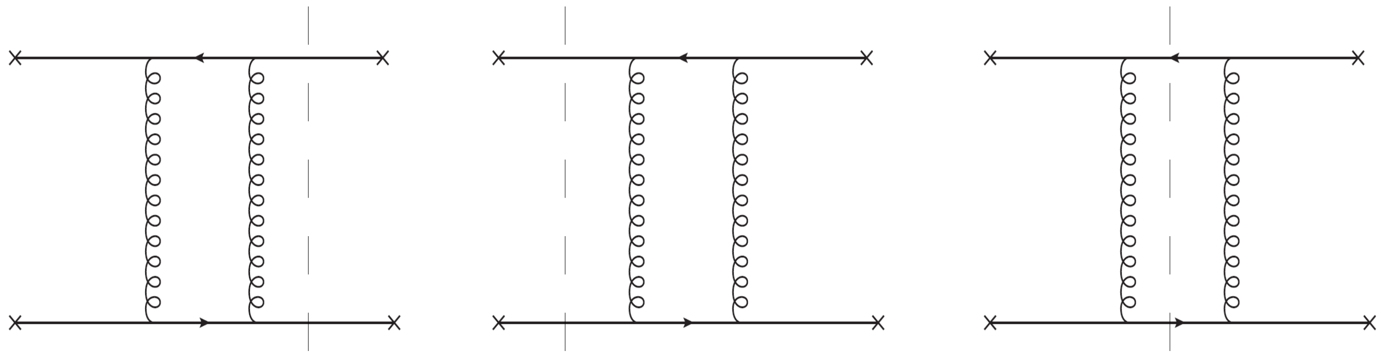
Sum over (1) cuts

=0 using Cutkosky rules

Measured Hadronic Transverse Energy

For E_T cancellation of Glauber subgraphs fails, because E_T for central/real cut depends on loop momentum, whilst same is not true for external/absorptive cuts:

$$E_T: \quad |\mathbf{k}'_T| + |\mathbf{q}_T - \mathbf{k}'_T| \qquad |\mathbf{k}_T| + |\mathbf{q}_T - \mathbf{k}_T| \qquad |\mathbf{l}_T| + |\mathbf{q}_T + \mathbf{l}_T|$$

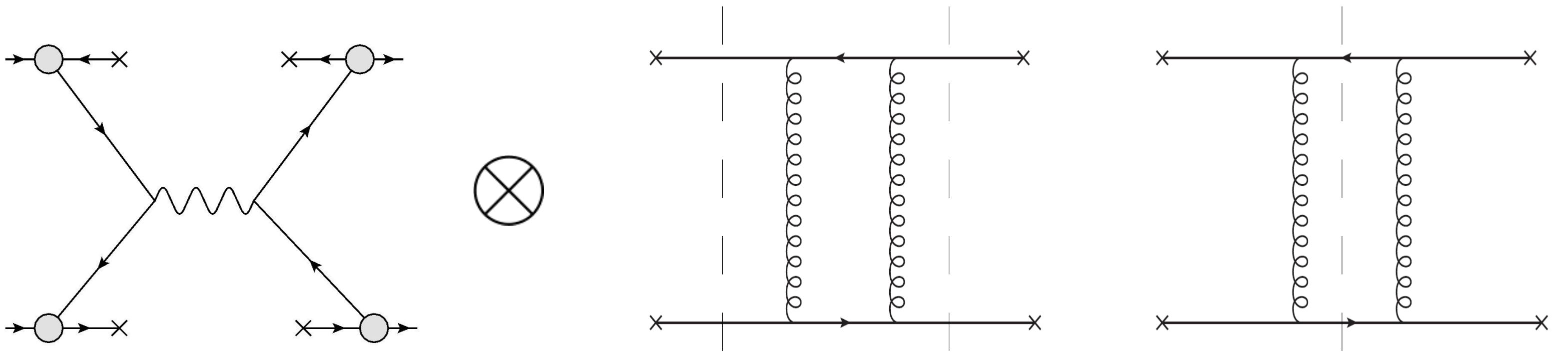


Maybe some change of loop and external variables is possible to arrange cancellation?

I argue no such change of variables is possible. Cutkosky cancellation is one that occurs point by point in spatial momentum – should match parameterisation of loop momentum between graphs. Sterman, hep-ph/9606312

Glauber Gluons and MPI

The factorisation breaking for E_T is associated with:



But this can be interpreted as:

Primary hard interaction

Secondary low scale absorptive process ('MPI did not occur')

Secondary low scale scattering ('MPI occurred')

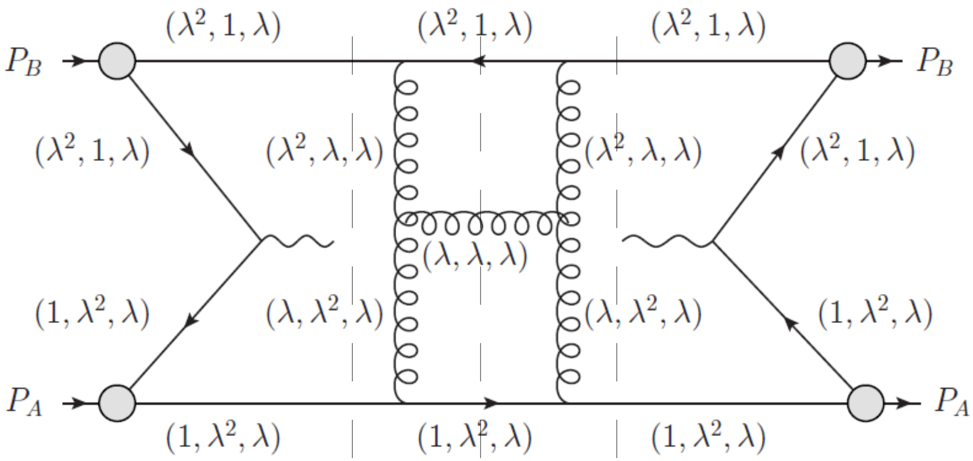
Factorisation breaking effects are due to MPI, as was also found in MC studies. Close connection between Glauber gluons and MPI!

Absorptive process included in MCs via unitarity constraints.

NOTE: Contribution from Glauber region cancels for p_T because p_T of V is insensitive to whether extra interactions occurred or not, NOT because MPI is cancelled.

Glauber Gluons and Regge Physics

Also achieve a leading contribution by inserting central soft rung between Glauber verticals:



3 cuts again don't cancel for E_T

This graph suppressed by additional power of α_s compared to zero-rung graph, but enhanced by rapidity (BFKL) logarithm. Can insert arbitrary number of rungs (forming Pomeron type object) and still be at leading log order in BFKL sense.

→ should need good control of BFKL effects in MPI to describe E_T well.

SHRiMPS and DIPSY include Pomeron/BFKL effects in their MPI models

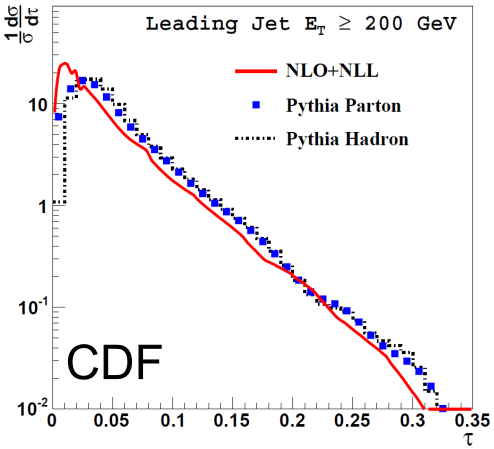
Other observables

Same effect should occur for any other observable which is sensitive to whether an additional interaction occurred or not (MPI sensitive observables)

For example:

Beam thrust

Transverse thrust $\max_{\mathbf{n}_{iT}} \sum_i |\mathbf{q}_{iT} \cdot \mathbf{n}_{iT}| / \sum_i |\mathbf{q}_{iT}|$



Phys.Rev. D83 (2011) 112007

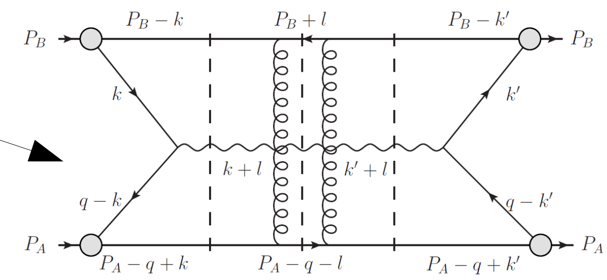
Usually global observables. Jet observables much less MPI sensitive as particles from MPI only collected up over jet radius → MPI suppressed by radius *R*.

M. Dasgupta, L. Magnea, and G. P. Salam, JHEP 0802 (2008) 055

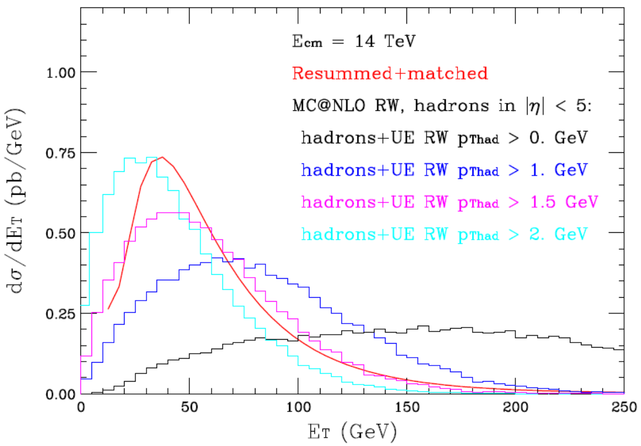
MPI sensitive variable of order of hard scale

What about when MPI sensitive observable O_s is of order of the hard scale Q? Then for the cumulant of O_s, are we inclusive enough that standard factorisation formula can be used?

Miscancellation of cuts in this graph now only smears observable by some power suppressed amount



The trouble is that we can have Glauber miscancellations on multiple spectator legs adding up to produce a big smearing of O_s, even when $O_s \sim Q$

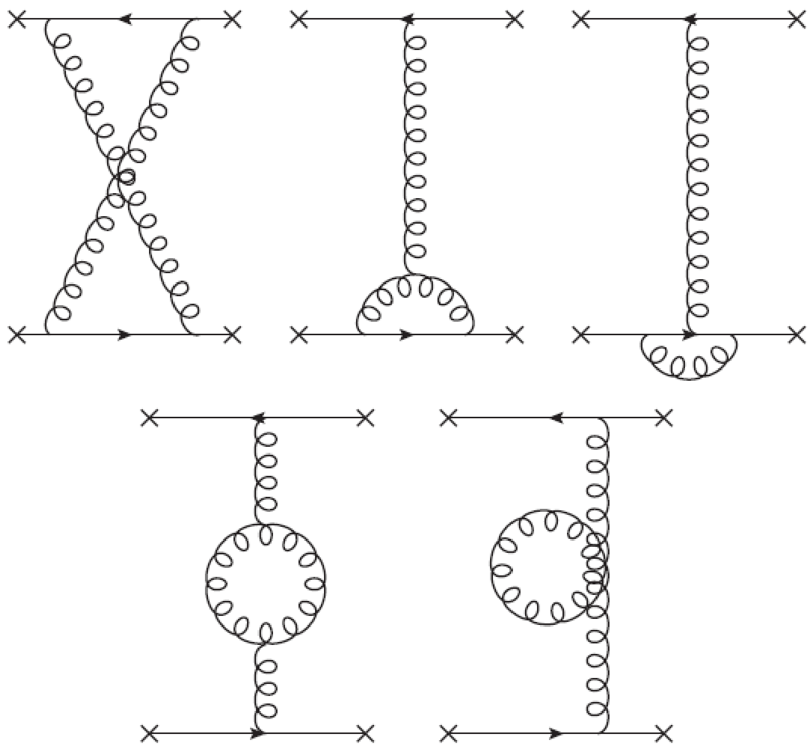


Conclusions

- CSS-style cancellation of Glauber region fails for E_T (<< Q), at the level of two Glauber gluons exchanged between spectators.
- Can connect these diagrams to events with additional soft scatterings – connection between Glauber gluons and MPI.
- Also lack of cancellation for more general Pomeron-type exchanges – connection between Glauber gluons and Regge physics.
- Standard factorisation with only collinear, soft and hard functions also fails for a wider class of MPI sensitive observables – e.g. beam thrust, transverse thrust.
- MPI sensitive observables can receive a cumulative contribution from MPI – problems with applying standard factorisation to these observables even when they are of order Q.

BACKUP SLIDES

Additional Spectator-Spectator Interactions



No real cuts (i.e. non-external ones) – can cancel contribution when gluons are in Glauber region using same argument that was applied for single gluon exchange.