# Higgs production in gluon fusion beyond NNLO

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Based on:

#### Richard Ball, M.B., Stefano Forte, Simone Marzani, Giovanni Ridolfi

arXiv:1303.3590

arXiv:1404.3204 (see also arXiv:1306.6633)

and M.B., Simone Marzani

arXiv:1405.3654

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## Perturbative (in)stability to NNLO



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Higgs cross section: gluon fusion



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Higgs cross section: gluon fusion



#### Higgs production: inclusive cross section

$$\sigma(\tau) = \tau \,\sigma_0 \sum_{ij} \int_{\tau}^1 \frac{dz}{z} \,\mathscr{L}_{ij}\left(\frac{\tau}{z}\right) C_{ij}(z, \alpha_s), \qquad \tau = \frac{m_{\rm H}^2}{s}, \qquad z = \frac{m_{\rm H}^2}{\hat{s}}$$

 $C_{ij}(z,\alpha_s) = \delta_{ig}\delta_{jg}\delta(1-z) + \alpha_s C_{ij}^{(1)}(z) + \alpha_s^2 C_{ij}^{(2)}(z) + \alpha_s^3 C_{ij}^{(3)}(z) + \dots$ 

## Higgs production: inclusive cross section

$$\sigma( au) = au \, \sigma_0 \sum_{ij} \int_{ au}^1 rac{dz}{z} \, \mathscr{L}_{ij}\left(rac{ au}{z}
ight) rac{C_{ij}(z,lpha_s)}{G_{ij}(z,lpha_s)}, \qquad au = rac{m_{
m H}^2}{\hat{s}}, \qquad z = rac{m_{
m H}^2}{\hat{s}}$$

 $C_{ij}(z,\alpha_s) = \delta_{iq}\delta_{jq}\delta(1-z) + \alpha_s C_{ii}^{(1)}(z) + \alpha_s^2 C_{ij}^{(2)}(z) + \alpha_s^3 C_{ij}^{(3)}(z) + \dots$ 



• NLO  $C_{ii}^{(1)}(z)$ :

- large  $m_t \ (\gg m_{\rm H})$  approximation
- full  $m_t$  ( $m_b$ ,  $m_c$ , ...) dependence

[Dawson 1991; Djouadi, Spira, Zerwas 1991] [Spira, Djouadi, Graudenz, Zerwas 1995]

• NNLO  $C_{ii}^{(2)}(z)$ :

- large  $m_t$  approximation [Harlander, Kilgore 2002; Anastasiou, Melnikov 2002]
- expansion in  $m_{\rm H}/m_t$  and in 1-z [Harlander, (Mantler, Marzani,) Ozeren 2009(10)]
  - [Pak, Rogal, Steinhauser 2010]
  - [Marzani, Ball, Del Duca, Forte, Vicini 2008]

#### • NNLO + NNLL soft resummation

• finite  $m_t$  small-z behavior

• expansion in  $m_{\rm H}/m_t$ 

[de Florian, Grazzini 2012]

## The $N^3$ revolution

- N<sup>3</sup>LO  $C_{ij}^{(3)}(z)$ :
  - large  $m_t$  limit:
    - soft approximation (only log terms)
    - three loops
       [Baikov, Chetyrkin, Smirnov, Smirnov, Steinhauser 2009]
    - Gehrmann, Glover, Huber, Ikizlerli, Studerus 2010
       one emission at two loops
       [Gehrmann, Jaquier, Glover, Koukoutsakis 2012]
      - [Duhr, Gehrmann 2013] [Li, Zhu 2013]
    - one emission at one loop [Anastasiou, Duhr, Dulat, Herzog, Mistlberger 2013] [Kilgore 2013]
    - three emissions (soft expansion) [Anastasiou, Duhr, Dulat, Mistlberger 2013]
    - scale dependent terms [Anastasiou, Bühler, Duhr, Herzog 2012] [Höschele, Hoff, Pak, Steinhauser, Ueda 2012] [Bühler, Lazopoulos 2013]
    - Wilson coefficient at N<sup>3</sup>LO [Chetyrkin, Kniehl, Steinhauser 1998]
    - two emissions at one loop [Li, von Manteuffel, Schabinger, Zhu 2014]
    - complete soft approximation at N<sup>3</sup>LO [Anastasiou, Duhr, Dulat, Furlan, Gehrmann, Herzog, Mistlberger 2014]
  - with finite  $m_t$  dependence:
    - soft + high-energy approximation [Ball, MB, Forte, Marzani, Ridolfi 2013]
- NNLO + N $^{3}$ LL improved soft resummation

[MB, Marzani 2014]

[Moch. Vogt. 2005]

## Ingredients of our N<sup>3</sup>LO prediction

gg channel only (~ 97\% of the full NNLO):

$$\begin{array}{ll} C_{gg}(z,\alpha_s) \;\simeq\; C_{\rm soft}(z,\alpha_s) \;+\; C_{\rm high-energy}(z,\alpha_s) \\ & z \to 1 \qquad \qquad z \to 0 \end{array}$$

 $C_{\rm soft}(z,\alpha_s)$  from threshold resummation  $C_{\rm high-energy}(z,\alpha_s) \text{ from BFKL resummation}$ 

## Ingredients of our N<sup>3</sup>LO prediction

gg channel only (  $\sim 97\%$  of the full NNLO):

$$C_{gg}(z, \alpha_s) \simeq C_{\text{soft}}(z, \alpha_s) + C_{\text{high-energy}}(z, \alpha_s)$$
$$z \to 1 \qquad z \to 0$$
$$N \to \infty \qquad N \to 1$$

 $C_{
m soft}(z, lpha_s)$  from threshold resummation  $C_{
m high-energy}(z, lpha_s)$  from BFKL resummation

$$\begin{array}{ll} \mbox{Mellin space:} & C_{gg}(N,\alpha_s) = \int_0^1 dz \; z^{N-1} \, C_{gg}(z,\alpha_s) \\ (\mbox{ordinary funct}) & (\mbox{distribution}) \end{array}$$

Known analytic structure:

 $\bullet\,$  logarithmic growth at large N

• poles in 
$$N = 1, 0, -1, -2, ...$$

[Regge theory, BFKL]

#### Soft part: naive construction

In the soft  $N \to \infty$  limit soft-gluon (threshold) resummation gives [Sterman 1987] [Catani, Trentadue 1989] [Forte, Ridolfi 2003]

$$\begin{split} C_{gg}(N,\alpha_s) &\stackrel{N\to\infty}{=} g_0\left(\alpha_s,\frac{m_{\rm H}}{m_t}\right) \times \exp \mathcal{S}(\alpha_s,N) \\ \mathcal{S}(\alpha_s,N) &= \exp\left[\frac{1}{\alpha_s}g_1(\alpha_s \ln N) + g_2(\alpha_s \ln N) + \alpha_s g_3(\alpha_s \ln N) + \alpha_s^2 g_4(\alpha_s \ln N) + \dots\right] \end{split}$$

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$$C_{gg}(N,\alpha_s) \stackrel{N\to\infty}{=} g_0\left(\alpha_s, \frac{m_{\rm H}}{m_t}\right) \times \exp \mathcal{S}(\alpha_s, N)$$
$$\mathcal{S}(\alpha_s, N) = \exp\left[\frac{1}{\alpha_s}g_1(\alpha_s \ln N) + g_2(\alpha_s \ln N) + \alpha_s g_3(\alpha_s \ln N) + \alpha_s^2 g_4(\alpha_s \ln N) + \dots\right]$$

Expansion in powers of  $\alpha_s$  leads to (we call it *N*-soft)

$$C_{N-\text{soft}}(N,\alpha_s) = \sum_{n=0}^{\infty} \alpha_s^n \sum_{k=0}^{2n-1} c_{n,k} \ln^k N$$

 $\ln N$  has a cut in  $N \leq 0$ , not compatible with the analytic structure of  $C_{gg}(N, \alpha_s)$  which has only poles!

The N-soft approximation is <u>not</u> expected to be good at finite N!

#### Soft part: one step backward

#### The previous resummed expression comes from

$$C_{gg}(N,\alpha_s) \stackrel{N\to\infty}{=} \bar{g}_0\left(\alpha_s, \frac{m_{\rm H}}{m_t}\right) \times \exp\bar{\mathcal{S}}(\alpha_s, N)$$
$$\bar{\mathcal{S}}(\alpha_s, N) = \int_0^1 dz \, \frac{z^{N-1} - 1}{1 - z} \left\{ \int_{\mu_{\rm F}^2}^{M^2(1-z)^2} \frac{d\mu^2}{\mu^2} 2A(\alpha_s(\mu^2)) + D\left(\alpha_s\left(M^2(1-z)^2\right)\right) \right\}$$

#### Soft part: one step backward

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$$= \sum_{n=0}^\infty \alpha_s^n \sum_{k=0}^n b_{n,k} \mathcal{D}_k(N)$$

$$\mathcal{D}_k(N) = \int_0^1 dz \, z^{N-1} \mathcal{D}_k(z) \qquad \qquad \mathcal{D}_k(z) = \left(\frac{\log^k(1-z)}{1-z}\right)_+$$

 $\mathcal{D}_k(N)$  is a sum of polygamma functions  $\psi_j(N)$ , which have poles in  $N=0,-1,-2,\ldots$ 

 $\mathcal{D}_k(N)$  respects the analytic structure of  $C_{gg}(N, \alpha_s)!$ 

#### Good! Can we do better?

## Soft part: improvements

Single gluon emission:  $P_{gg}^{(0)}(z) \int_{\mu_{\rm F}^2}^{M^2 \frac{(1-z)^2}{z}} \frac{dk_{\rm T}^2}{k_{\rm T}^2} = \frac{A_g^{(0)}(z)}{1-z} \ln \frac{M^2(1-z)^2}{\mu_{\rm F}^2 z}$ Regular terms in z = 1 usually dropped:  $A_g^{(0)}(z) \to A_g^{(0)}(1), \ \frac{(1-z)^2}{z} \to (1-z)^2$ We keep them!

## Soft part: improvements

Single gluon emission:  $P_{qq}^{(0)}(z)$ 

$$\int_{\mu_{\rm F}^2}^{M^2 \frac{(1-z)^2}{z}} \frac{dk_{\rm T}^2}{k_{\rm T}^2} = \frac{A_g^{(0)}(z)}{1-z} \ln \frac{M^2 (1-z)^2}{\mu_{\rm F}^2 z}$$

Regular terms in z = 1 usually dropped:  $A_g^{(0)}(z) \to A_g^{(0)}(1)$ ,  $\frac{(1-z)^2}{z} \to (1-z)^2$ 

We keep them! Effectively:

• we replace 
$$\mathcal{D}_k(z) = \left(\frac{\log^k(1-z)}{1-z}\right)_+ \rightarrow \hat{\mathcal{D}}_k(z) = \left(\frac{\log^k \frac{1-z}{\sqrt{z}}}{1-z}\right)_+$$

• we include collinear contributions

$$A_g^{(0)}(z) = A_g^{(0)}(1) \frac{1 - 2z + 3z^2 - 2z^3 + z^4}{z} = A_g^{(0)}(1) \left[1 - (1 - z) + 2(1 - z)^2 + \dots\right]$$

expanded to first (soft<sub>1</sub>) or second (soft<sub>2</sub>) order to avoid double counting with high-energy part.

Predicts correctly LL next-to-eikonal terms  $\alpha_s^k \ln^{2k-1}(1-z)$  to all orders. [Krämer, Laenen, Spira 1997]

 $C_{gg}(N,\alpha_s) = 1 + \alpha_s C^{(1)}(N) + \alpha_s^2 C^{(2)}(N) + \alpha_s^3 C^{(3)}(N) + \dots$ 



Higgs production in gluon fusion beyond NNLO

 $C_{gg}(N,\alpha_s) = 1 + \alpha_s C^{(1)}(N) + \alpha_s^2 C^{(2)}(N) + \alpha_s^3 C^{(3)}(N) + \dots$ 



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Higgs production in gluon fusion beyond NNLO

$$C_{gg}(N,\alpha_s) = 1 + \alpha_s C^{(1)}(N) + \alpha_s^2 C^{(2)}(N) + \alpha_s^3 C^{(3)}(N) + \dots$$



## High-energy part

Leading Log poles in 
$$C_{gg}(N, \alpha_s)$$
:  $\frac{\alpha_s^k}{(N-1)^k}$   $\left(\alpha_s^k \frac{\log^{k-1} z}{z}\right)$ 

Described by BFKL resummation:

[Marzani, Ball, Del Duca, Forte, Vicini 2008]

$$C_{\mathsf{high-energy}}(N,\alpha_s) = \sum_{k_1,k_2 \ge 0} c_{k_1,k_2}(\frac{m_{\mathrm{H}}}{m_t}) \big[\gamma_+^{k_1}\big] \big[\gamma_+^{k_2}\big]$$

 $\gamma_+(N)$ : DGLAP anomalous dimension (largest eigenvalue)

- accurate at LL (with running coupling effects)
- momentum conservation  $C_{\text{high-energy}}(N=2,\alpha_s)=0$ , but grows at  $N \to \infty$
- we use an expansion of  $\gamma_+(N)$  to NLL (removes the growth)
- we enforce mom. cons. by adding subdominant terms (poles at  $N \leq 0$ )
- we vary these subdominant terms as a measure of the uncertainty

 $C_{gg}(N,\alpha_s) = 1 + \alpha_s \, C^{(1)}(N) + \alpha_s^2 \, C^{(2)}(N) + \alpha_s^3 \, C^{(3)}(N) + \dots$ 



Higgs production in gluon fusion beyond NNLO

 $C_{gg}(N,\alpha_s) = 1 + \alpha_s \, C^{(1)}(N) + \alpha_s^2 \, C^{(2)}(N) + \alpha_s^3 \, C^{(3)}(N) + \dots$ 



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Higgs production in gluon fusion beyond NNLO

$$C_{gg}(N,\alpha_s) = 1 + \alpha_s C^{(1)}(N) + \alpha_s^2 C^{(2)}(N) + \alpha_s^3 C^{(3)}(N) + \dots$$



 $C_{gg}(N,\alpha_s) = 1 + \alpha_s C^{(1)}(N) + \alpha_s^2 C^{(2)}(N) + \alpha_s^3 C^{(3)}(N) + \dots$ 



#### Parton level to hadron level: K-factors

$$\frac{\sigma_{gg}}{\sigma_{\rm LO}} = 1 + \alpha_s \, K^{(1)} + \alpha_s^2 \, K^{(2)} + \alpha_s^3 \, K^{(3)} + \dots$$

Higgs K-factor at NLO (NNLO PDFs)



Higgs production in gluon fusion beyond NNLO

#### Parton level to hadron level: K-factors

$$\frac{\sigma_{gg}}{\sigma_{\rm LO}} = 1 + \alpha_s \, K^{(1)} + \alpha_s^2 \, K^{(2)} + \alpha_s^3 \, K^{(3)} + \dots$$





Higgs production in gluon fusion beyond NNLO

#### Parton level to hadron level: K-factors

$$\frac{\sigma_{gg}}{\sigma_{\rm LO}} = 1 + \alpha_s \, K^{(1)} + \alpha_s^2 \, K^{(2)} + \alpha_s^3 \, K^{(3)} + \dots$$

Higgs K-factor at NNNLO (NNLO PDFs)



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Higgs production in gluon fusion beyond NNLO









N-soft: [Moch, Vogt 2005]

## N<sup>3</sup>LL improved soft resummation

Same technique, to all orders. We consider here only  $soft_2$ .

[MB, Marzani 2014]

## N<sup>3</sup>LL improved soft resummation (soft2)



## N<sup>3</sup>LL improved soft resummation (soft<sub>2</sub>)

Higgs cross section: gluon fusion



N-soft: NNLL resummation [Catani, de Florian, Grazzini, Nason 2003]

## N<sup>3</sup>LL improved soft resummation (soft2)

[MB, Marzani 2014]

Higgs cross section: gluon fusion



Reasonably mild factorization scale dependence

## Conclusions

• We are predicting the inclusive Higgs cross section using

$$C_{gg}(z, \alpha_s) \simeq C_{\text{soft}}(z, \alpha_s) + C_{\text{high-energy}}(z, \alpha_s)$$

with:

- exact  $m_t$  dependence (important at large  $\sqrt{s}$ )
- improved soft approximation (kinematical logs and AP splittings)
- high-energy behavior at LL (plus some NLL elements)
- At N<sup>3</sup>LO we find  $(\mu_{\rm F} = m_{\rm H})$ :
  - an increase of  $\sim 17\%~(\sim 11\%)$  for  $\mu_{
    m R}=m_{
    m H}~(m_{
    m H}/2)$  wrt the NNLO
  - stabilization of renormalization scale dependence (at low scales)
- At N<sup>3</sup>LL (only soft) we find:
  - an increase of  $\sim 22\%~(\sim 5.5\%)$  for  $\mu_{\rm R} = \mu_{\rm F} = m_{\rm H}~(m_{\rm H}/2)$  wrt the NNLO
  - mild scale dependence
- Public codes ggHiggs, ResHiggs: http://www.ge.infn.it/~bonvini/higgs/
- Future: high-energy LL resummation; other partonic channels (qg)

# Backup slides

## Before and after the $\delta(1-z)$ term at order $\alpha_s^3$

$order\ n$	$\delta(1-z)$ coeff	$ar{g}_{0,n}$	$+$ $r_n$	=	$g_{0,n}$
1	4.9374	4.9374	3.7779	)	8.7153
2	8.94	10.92	29.18		40.10
3	44.45	2.0	114.7		116.7

Higgs hadron-level cross section



## Threshold resummation: logarithmic counting

$\infty$ 2	2n	$\infty$	n+1
$C(N, M^2) = 1 + \sum_{n=1}^{\infty} \alpha_s^n \sum_{k=1}^{\infty} \alpha_k^n \sum_{k=1}^{\infty} $	$\sum_{k=0} c_{nk} \ln^k N \qquad \ln C(N, M^2) =$	$= \sum_{n=1}^{\infty} \alpha_s^n$	$\sum_{k=0}^{k} \hat{b}_{nk} \ln^{k} N$

	$A(\alpha_s)$	$D(\alpha_s)$	$ar{g}_0(lpha_s)$	accuracy: $c_{nk}$	$\hat{b}_{nk}$
LL	1-loop	—	tree-level	k = 2n	k = n + 1
NLL	2-loop	1-loop	tree-level	$2n-1 \leq k \leq 2n$	$n \leq k \leq n+1$
NLL'	2-loop	1-loop	1-loop	$2n-2 \leq k \leq 2n$	$n \leq k \leq n+1$
NNLL	3-loop	2-loop	1-loop	$2n-3 \leq k \leq 2n$	$n-1 \leq k \leq n+1$
NNLL'	3-loop	2-loop	2-loop	$2n-4 \le k \le 2n$	$n-1 \leq k \leq n+1$
NNNLL	4-loop	3-loop	2-loop	$2n-5 \leq k \leq 2n$	$n-2 \leq k \leq n+1$
NNNLL'	4-loop	3-loop	3-loop	$2n-6 \le k \le 2n$	$n-2 \le k \le n+1$

Un-primed counting: appropriate for  $\ln C(N, M^2)$ , assumes  $\alpha_s \ln N \sim 1$ Primed counting: more appropriate for  $C(N, M^2)$ , assumes just  $\alpha_s \ln^2 N \sim 1$ 

NNLL': [Catani, de Florian, Grazzini, Nason 2003] [de Florian, Grazzini 2012] NNNLL: [Ahrens, Becher, Neubert, Yang 2008] (within SCET) NNNLL': [MB, Marzani 2014]