Towards the Higgs Cross-Section at N3LO

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PSR14

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THE ERA OF HIGGS-MEASUREMENTS

- Experimental era of Higgs physics is just beginning
- Pinning down precisely the properties of the Higgs poses an exciting challenge for years to come
- Demand for precision measurement and prediction is evident

$$\sigma = \sum \int dx_1 dx_2 f(x_1) f(x_2) \boxed{\hat{\sigma}(x_1 x_2)}$$

So, what is new in QCD theory?

SOFT-VIRTUAL **N3LO** CROSS-

$$
\begin{aligned}
\hat{\eta}^{(3)}(z) = \delta(1-z) \Bigg\{ & C_A^3 \left(-\frac{2003}{48}\zeta_6 + \frac{413}{6}\zeta_3^2 - \frac{7579}{144}\zeta_5 + \frac{979}{24}\zeta_2\zeta_3 - \frac{15257}{864}\zeta_4 - \frac{819}{16}\zeta_3 + \frac{16151}{1296}\zeta_2 + \frac{215131}{5184}\right) \\
& + N_F \left[C_A^2 \left(\frac{869}{72}\zeta_5 - \frac{125}{12}\zeta_3\zeta_2 + \frac{2629}{432}\zeta_4 + \frac{1231}{216}\zeta_3 - \frac{70}{81}\zeta_2 - \frac{98059}{5184}\right) \right. \\
& \left. + N_F^2 \left[C_A \left(-\frac{19}{36}\zeta_4 + \frac{43}{108}\zeta_3 - \frac{133}{324}\zeta_2 + \frac{2515}{1728}\right) + C_F \left(-\frac{1}{36}\zeta_4 - \frac{7}{6}\zeta_3 - \frac{23}{72}\zeta_2 + \frac{4481}{2592}\right) \right] \right] \Bigg\} \\
+ \left[\frac{1}{1-z}\right]_+ \Bigg\{ & C_A^3 \left(186\zeta_5 - \frac{725}{6}\zeta_3\zeta_2 + \frac{253}{24}\zeta_4 + \frac{8941}{108}\zeta_3 + \frac{8563}{324}\zeta_2 - \frac{297029}{23328}\right) + N_F^2 C_A \left(\frac{5}{27}\zeta_3 + \frac{10}{27}\zeta_2 - \frac{58}{729}\right) \\
& + N_F \left[C_A^2 \left(-\frac{17}{12}\zeta_4 - \frac{475}{36}\zeta_3 - \frac{2173}{324}\zeta_2 + \frac{31313}{11664}\right) + C_A C_F \left(-\frac{1}{2}\zeta_4 - \frac{19}{18}\zeta_3 - \frac{1}{2}\zeta_2 + \frac{1711}{864}\right) \right] \Bigg\} \\
+ \left[\frac{\log(1-z)}{1-z}\right]_+ \Bigg\{ & C_A^3 \left(-77\zeta_4 - \frac{352}{3}\zeta_3 - \frac{152}{3}\zeta_2 + \frac{30569}{648}\right) + N_F^2 C_A \left(-\frac{4}{9}\zeta_2 + \frac{25}{81}\right) \\
& + N_F \left[C_A^2 \left(\frac{46}{3}\zeta_3 + \frac{94}{9}\zeta_2 - \frac{4211}{324}\right) + C_A C_F \left(6\zeta_3 - \frac{63}{8}\right) \right] \Bigg\} \\
+ \left[\frac{\log^2(1-z)}{1-z}\right]_+ \Bigg\{ & C_A^3 \left(181\zeta_3 + \frac{187}{3}\zeta_2 - \frac{1051}{27}\right) + N_F \left[C_A^2 \left(-\frac{34}{3}\zeta_2 + \frac{457}{54}\right) + \frac{1}{2} C_A C_F \right] - \frac{10}{27} N_F^2 C_A \Bigg\} \\
+ \left[\frac{\log^3(1-z)}{1-z}\right]_+ \Bigg\{ & C_A^3 \left(-56\zeta_2 + \frac{925}{27}\right) - \frac{164}{27} N_F C_A^2 + \frac{4}{27} N_F^2 C_A \Bigg\} \\
+ \left[\frac{\log^4(1-z)}{1-z}\right]_+ & \left(\frac{20}{9} N_F C_A^2 - \frac{110}{9} C_A^3\right) + \left[\frac{\log^5(1-z)}{1-z}\right]_+ 8 C_A^3 .
\end{aligned}
$$

Soft-Virtual σ at NNNLO

Uncharted territory in QCD - perturbation theory

What is in the formula?

- Inclusive Gluon - Fusion Higgs production in large top-mass limit at N3LO

$$\hat{\sigma}(z) = \hat{\sigma}^{LO}(z) + \alpha_S \hat{\sigma}^{NLO}(z) + \alpha_S^2 \hat{\sigma}^{NNLO}(z) + \boxed{\alpha_S^3 \hat{\sigma}^{N3LO}(z)} + \mathcal{O}(\alpha_S^4)$$

- Threshold Expansion

$$z = \frac{m_H^2}{\hat{s}} \sim 1 \quad \Rightarrow \quad \hat{\sigma}(z) = \sigma^{SV} + \sigma^{(0)} + (1-z)\sigma^{(1)} + \ldots$$

- Soft - Virtual term contains all 3-loop contributions + soft gluon radiation

Let's ask 2 questions

1. What is the method

Analytic Advancements

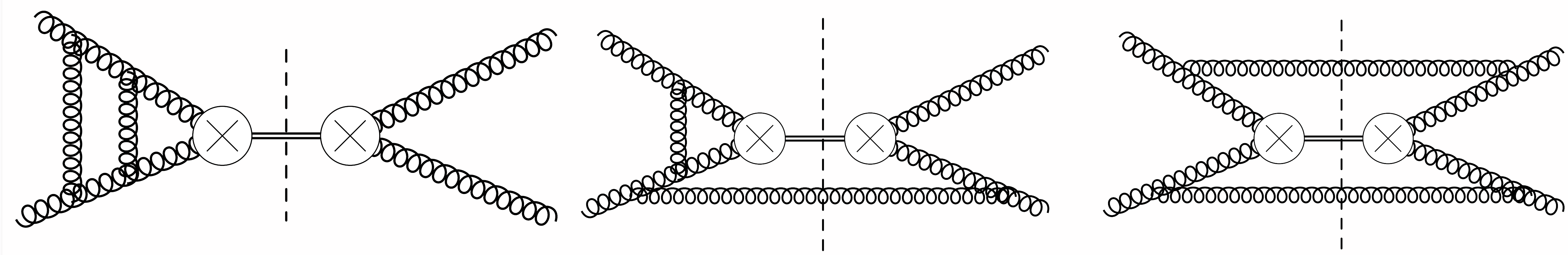
2. What is the benefit

Phenomenological Implications

FEYNMAN DIAGRAMS

- Combining real and virtual contributions calculated with Feynman diagrams is the only way for analytic calculation at N3LO

@ NNLO ~1000 Interference diagrams



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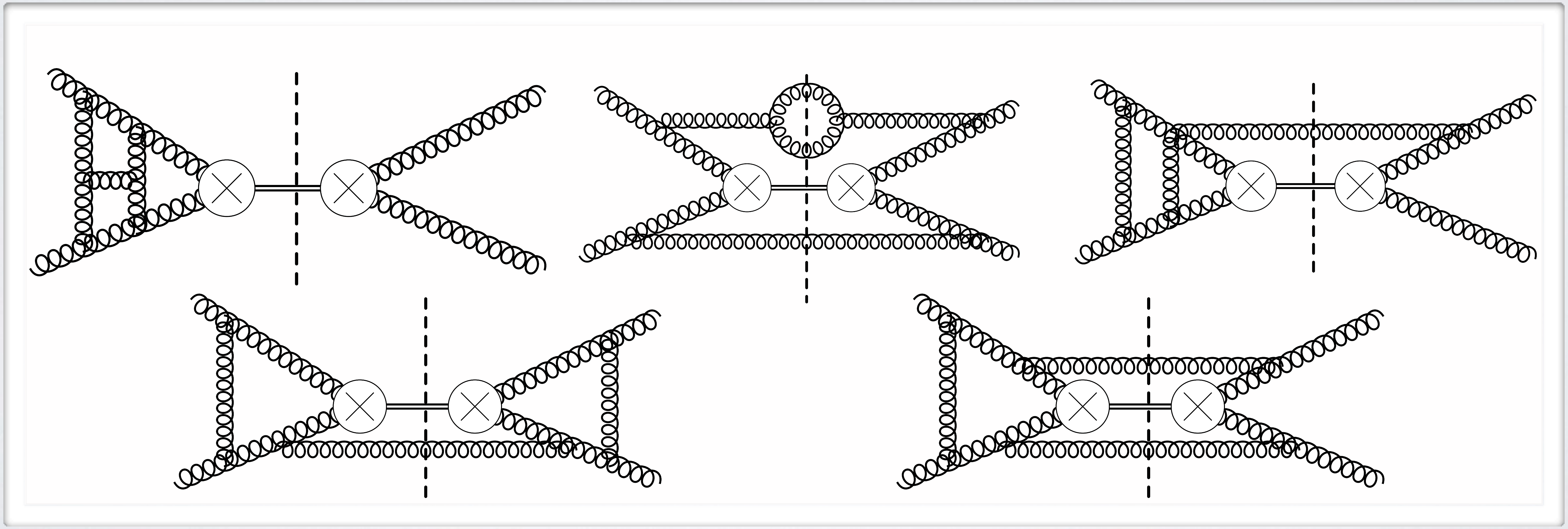
@ N3LO: ~**100 000** Interference Diagrams



Automation is vital!

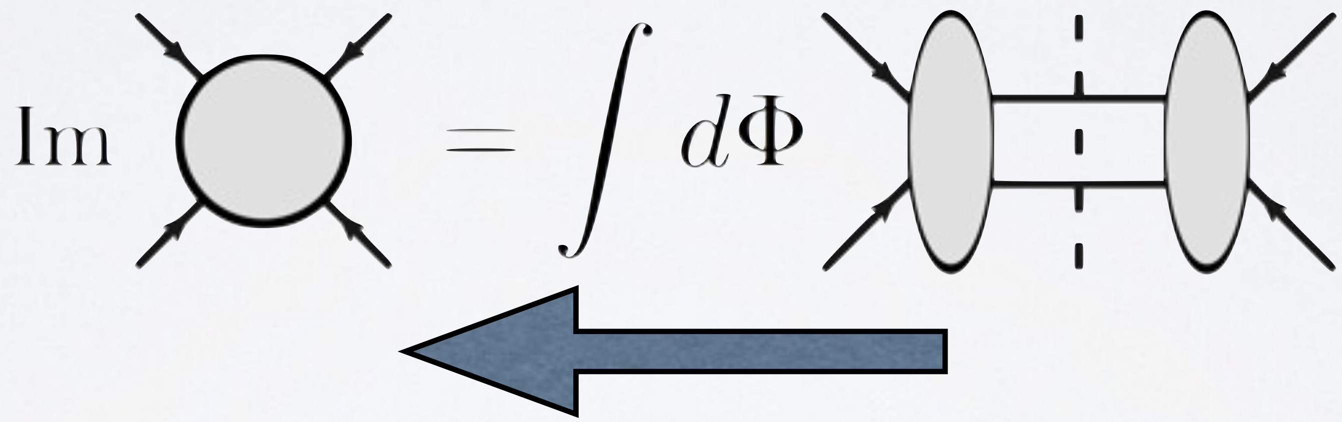
REDUCING COMPLEXITY

- Diagram calculation produces an enormous number of **combined** phase-space & loop integrals

Diagrams

Loop/Phase-Space Integrals

- Treat loop and phase-space integrals with equal methods: **Reverse Unitarity**



- Cutkosky's rule to relate on-shell constraints to cut -propagators

$$\delta^+(p^2) \to \left[\frac{1}{p^2}\right]_c \sim \frac{1}{p^2 + i\epsilon} - \frac{1}{p^2 - i\epsilon}$$

REDUCING COMPLEXITY

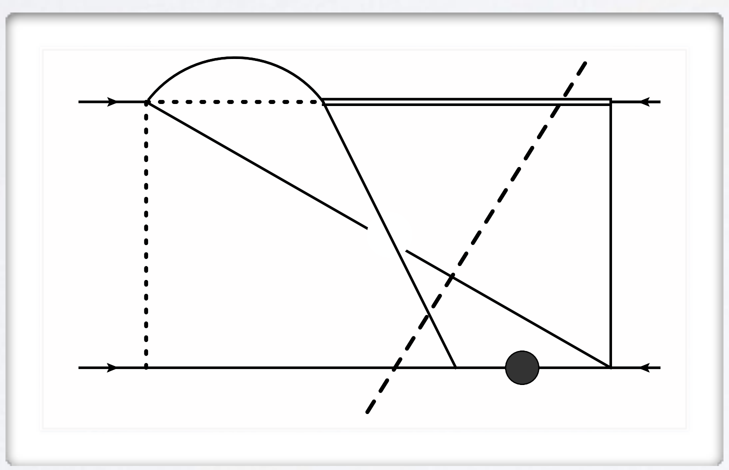
Diagrams

Loop/Phase-Space Integrals

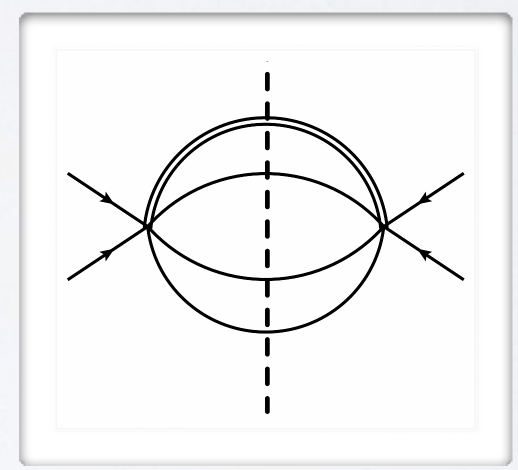
- Techniques for loop integrals applicable for phase-space integrals

$$\frac{\partial}{\partial p^\mu}\left[\frac{1}{p^2}\right]_c = -2p_\mu\left[\frac{1}{p^2}\right]_c^2$$

- Integration-By-Part identities to relate to a limited set of 'Master Integrals' proof powerful at N3LO



$$= C(\epsilon, z)$$



NNLO - Real Virtual:

Reduced 40 000 Integrals

REDUCING COMPLEXITY

Diagrams

Loop/Phase-Space Integrals

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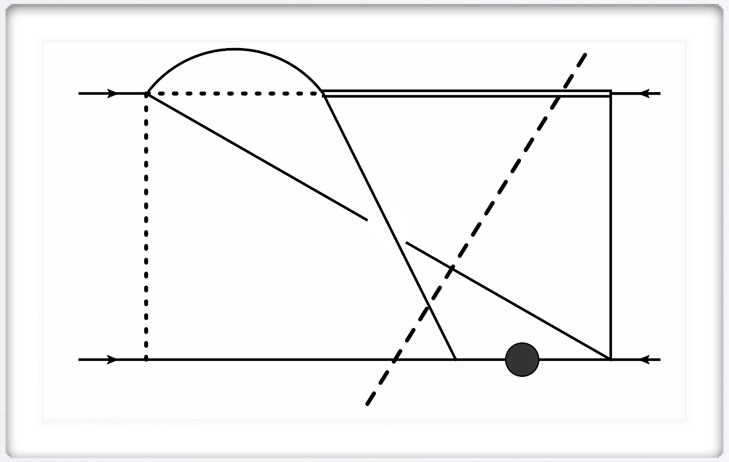

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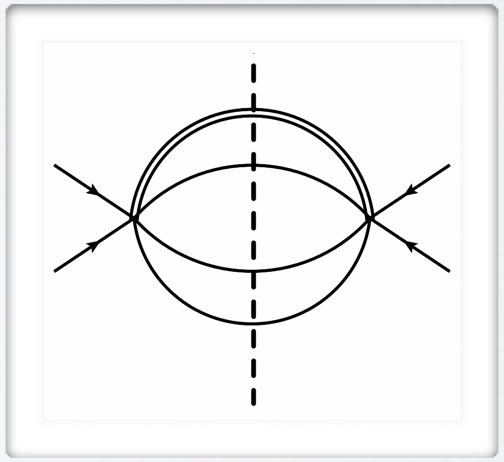


NNLO - Real Virtual:

Reduced 40 000 Integrals

N3LO - (Real Virtual)^2:

Reduced **25 000 000** Integrals

REDUCING COMPLEXITY

- Master integrals are complicated

- Again huge jump in complexity

NNLO - double Real:

18 Master Integrals

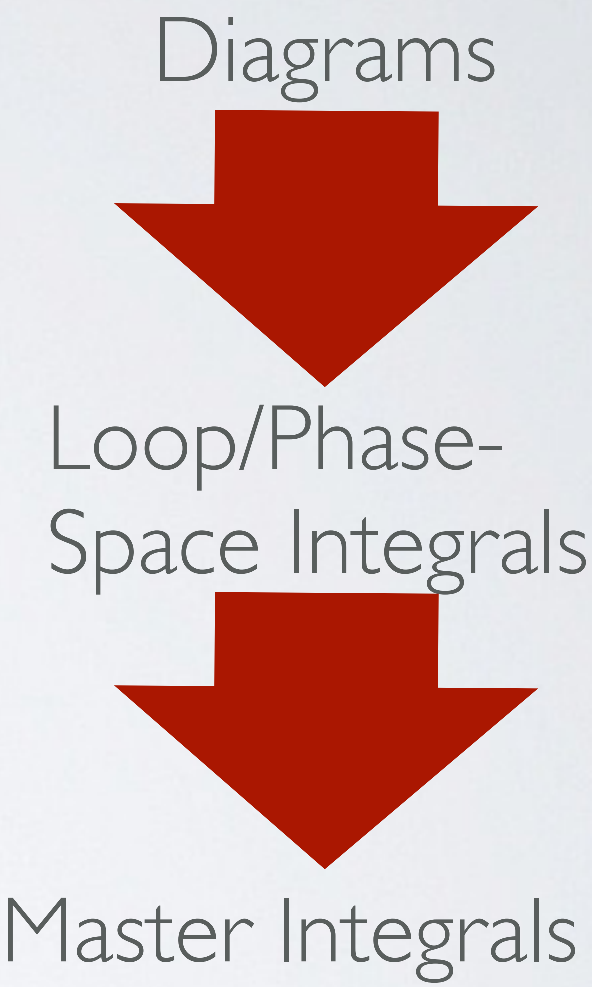
Diagrams

Loop/Phase-Space Integrals

Master Integrals

REDUCING COMPLEXITY

- Master integrals are complicated
- Again huge jump in complexity

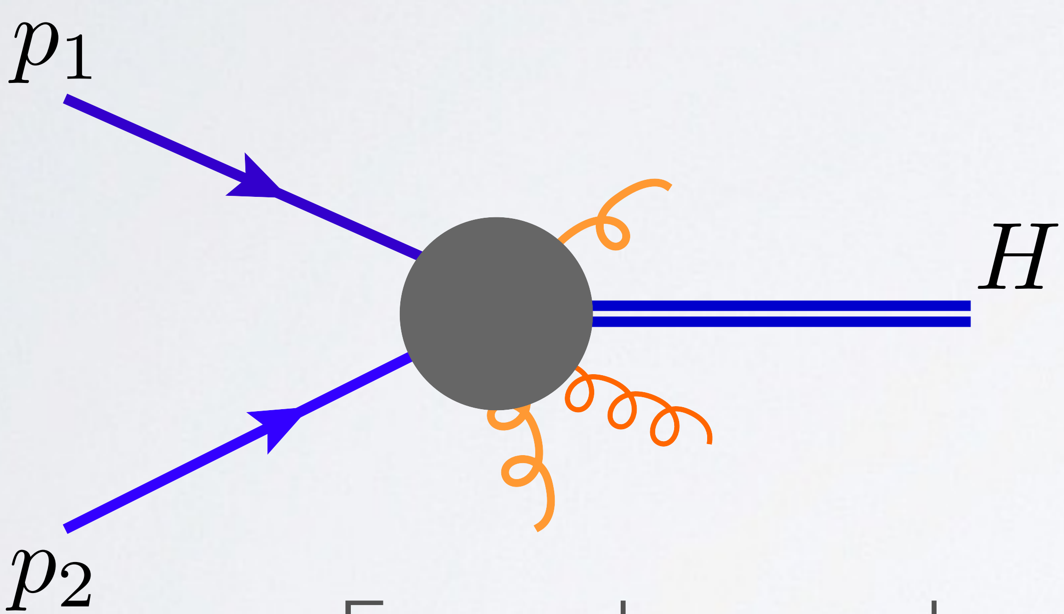


NNLO - double Real:	**N3LO** - Double Real Virtual:
18 Master Integrals	~**350** Master Integrals

Let's do something else first!

THRESHOLD EXPANSION

- Systematic expansion of matrix elements in an automated way (Expansion by Regions)



$$z = \frac{m_H^2}{\hat{s}} \sim 1$$

- Expand around production threshold of the Higgs boson

$$\delta = 1 - z$$

Diagrams

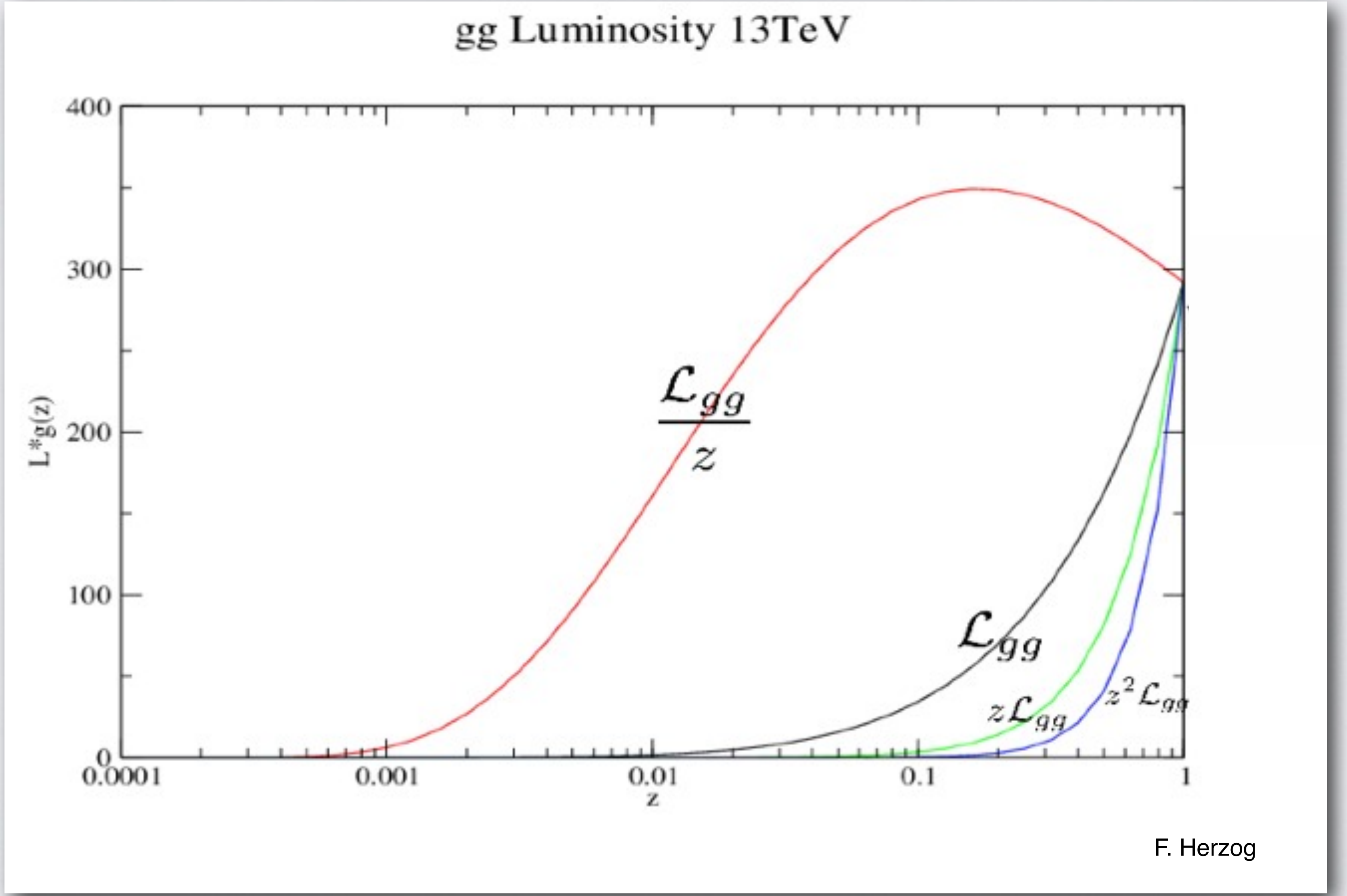
Loop/Phase-Space Integrals

Threshold Expansion

Master Integrals

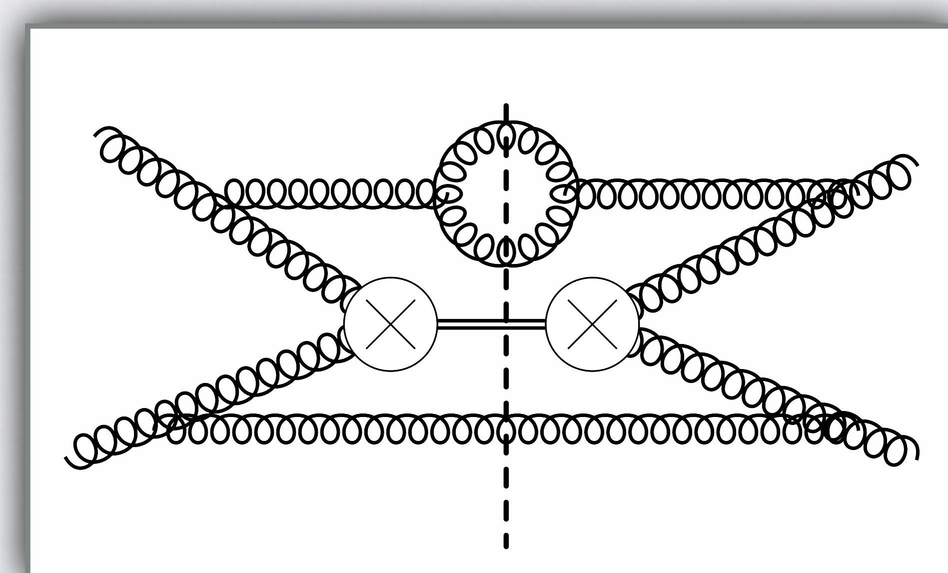
GG-LUMINOSITY

$$\sigma = \sum \int \frac{dz}{z} \mathcal{L}_{12}(\tau/z)\hat{\sigma}(z)$$



THRESHOLD EXPANSION

Phase-Space Integrals:



- Re-parametrise outgoing momenta

$$p_f \to \delta p_f$$

- Taylor expand the integrand

Diagrams

Loop/Phase-Space Integrals

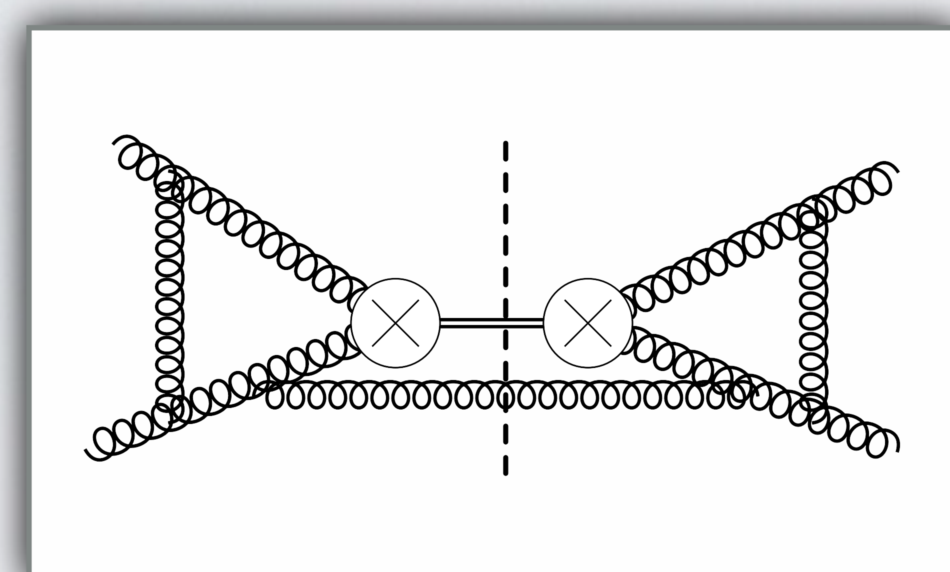
Threshold Expansion

Master Integrals

THRESHOLD EXPANSION

Loop-Integrals

$$\int \frac{d^d k}{(2\pi)^d}$$



- Loop momentum is not fixed
- Follow the method of expansion by regions

Soft	Coll 1	Coll 2	Hard
$k \to \delta k$	$k \to k \vert\vert p_1$	$k \to k \vert\vert p_2$	k

- Parametrise and expand systematically in every region
- Sum of regions yields the full result

Diagrams

Loop/Phase-Space Integrals

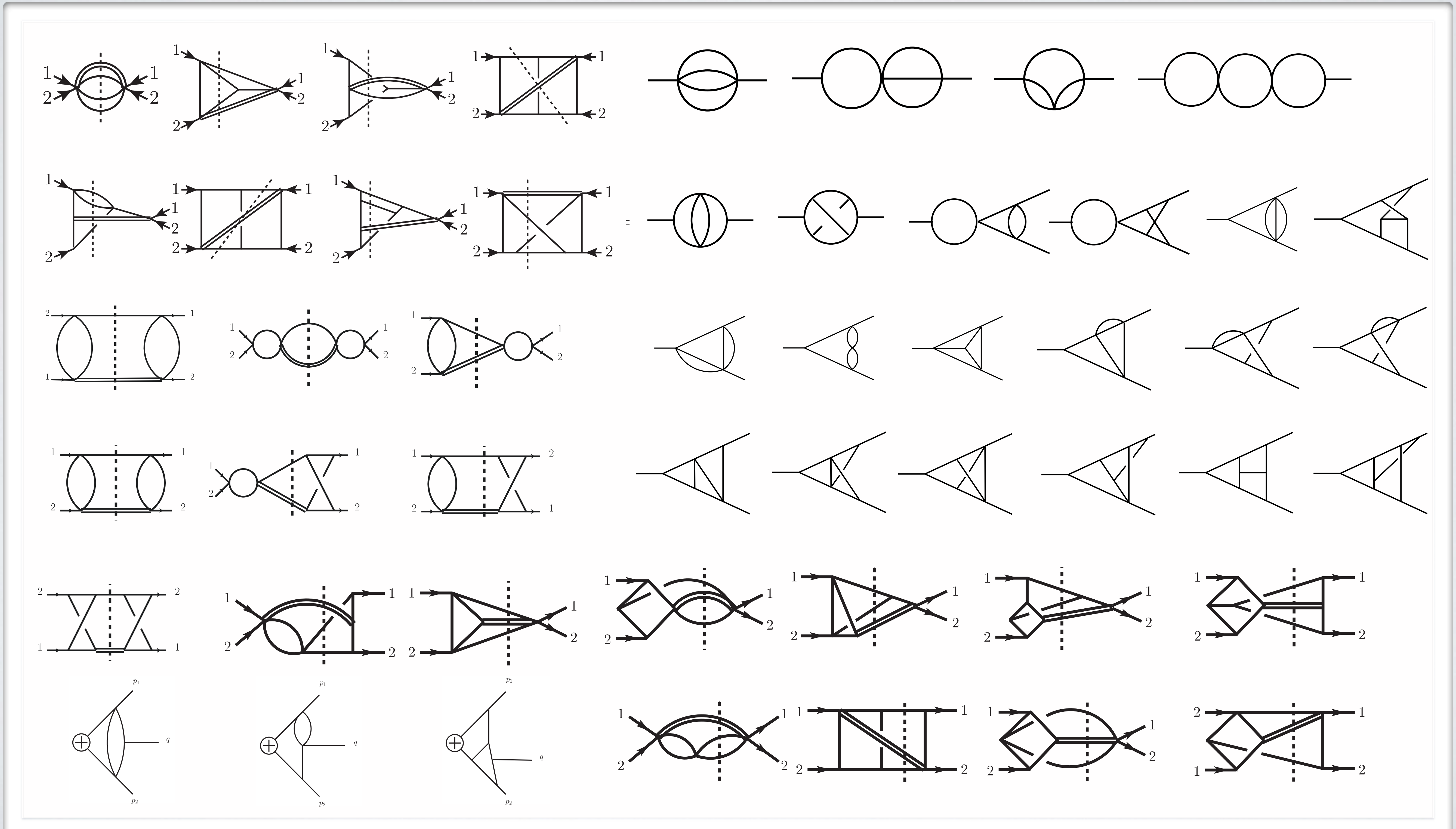
Threshold Expansion

Master Integrals

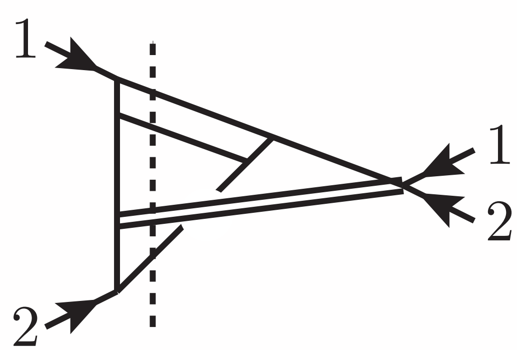
THRESHOLD EXPANSION

- Systematic expansion for loop and phase-space integrals
- Key-feature: Integration-By-Part identities can be used after expansion to relate to master integrals.
- Sub-leading terms via the same method
- Reduced number of master integrals
- The same master integrals appear in every order in δ

MASTER INTEGRALS



MASTER INTEGRALS



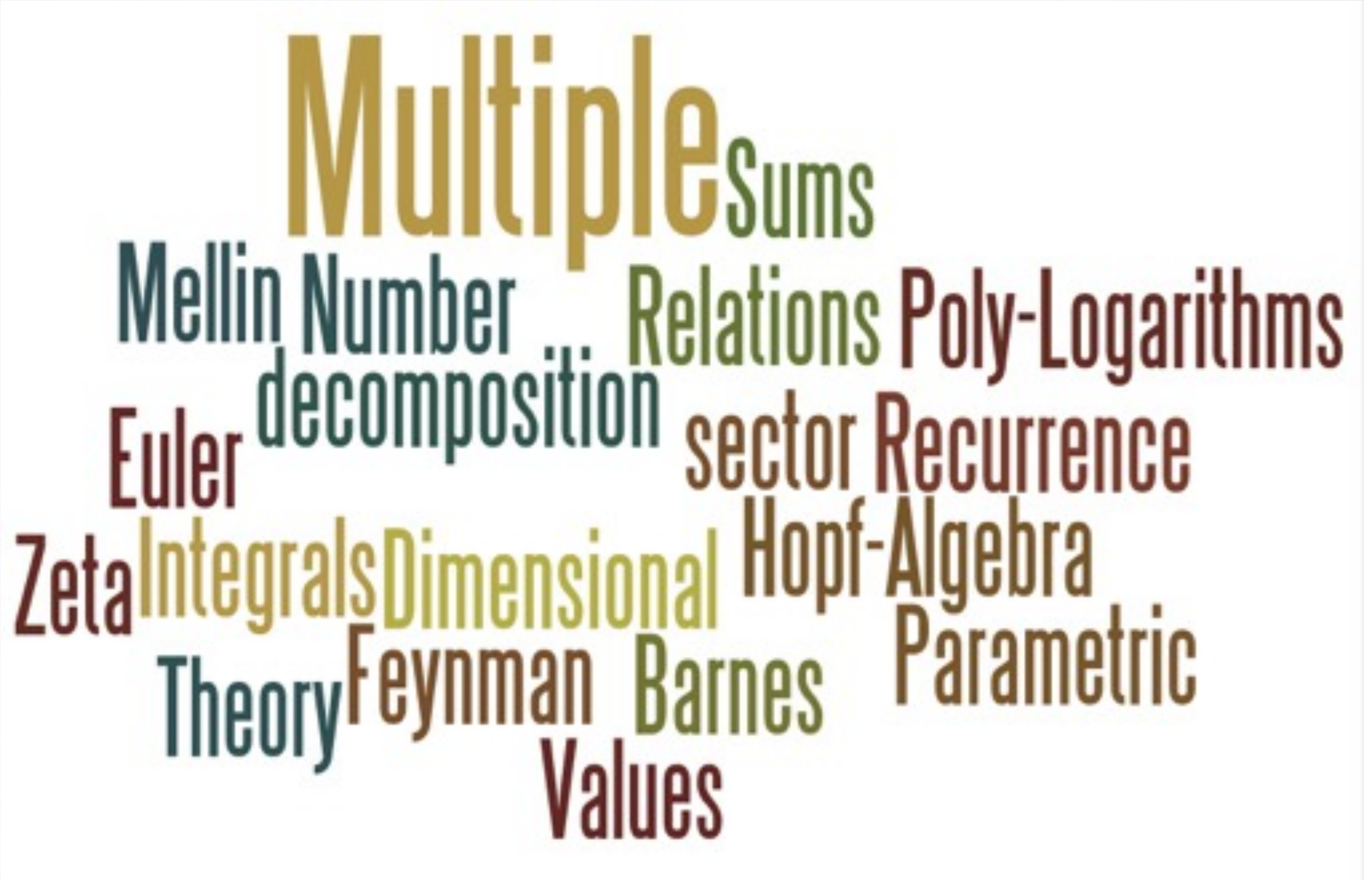
$$= \frac{\Gamma(12-6\epsilon)\Gamma(3-3\epsilon)\Gamma(1-\epsilon)}{\Gamma(5-6\epsilon)\Gamma(2-\epsilon)^4}\Big[\mathcal{I}_{9,1}(\epsilon)+\mathcal{I}_{9,2}(\epsilon)\Big]$$

$$\begin{aligned}
\mathcal{I}_{9,1}(\epsilon) = -\int_0^\infty dt_1\, dt_2 \int_0^1 dx_1\, dx_2\, dx_3\, & t_1^{2-4\epsilon}\,(1+t_1)^{\epsilon-1}\, t_2^{1-2\epsilon} \\
& \times x_1^{-\epsilon}\,(1-x_1)^{2-4\epsilon}\, x_2^{1-3\epsilon}\,(1-x_2)^{-\epsilon}\, x_3^{-\epsilon}\,(1+t_2x_3)^{1-3\epsilon}\,(1+t_2x_2x_3)^{\epsilon} \\
& \times \left(t_1t_2^2x_1x_2x_3+t_2^2x_2x_3+t_1t_2x_1x_2+t_1t_2x_3+t_2x_2x_3+t_2+t_1+1\right)^{3\epsilon-3}\,,
\end{aligned}$$

$$\begin{aligned}
\mathcal{I}_{9,2}(\epsilon) = \int_0^\infty dt_1\, dt_2 \int_0^1 dx_1\, dx_2\, dx_3\, & t_1^{2-4\epsilon}\,(1+t_1)^{\epsilon-1}\, t_2^{1-2\epsilon} \\
& \times x_1^{1-\epsilon}\,(1-x_1)^{2-4\epsilon}\, x_2^{1-3\epsilon}\,(1-x_2)^{-\epsilon}\, x_3^{-\epsilon}\,(1+t_2x_3)^{1-3\epsilon}\,(1+t_2x_2x_3)^{\epsilon} \\
& \times \left(t_1t_2^2x_1x_2x_3+t_2^2x_1x_2x_3+t_2x_1+t_1t_2x_1x_2+t_1t_2x_3+t_2x_1x_2x_3+t_1+x_1\right)^{3\epsilon-3}
\end{aligned}$$

MASTER INTEGRALS

- One of the key challenges
- We apply a large variety of the most modern methods in integral-calculation



- We even use methods from number theory
- Sometimes even this is not enough - Challenge to develop **new** technology

Let’s ask 2 questions

?

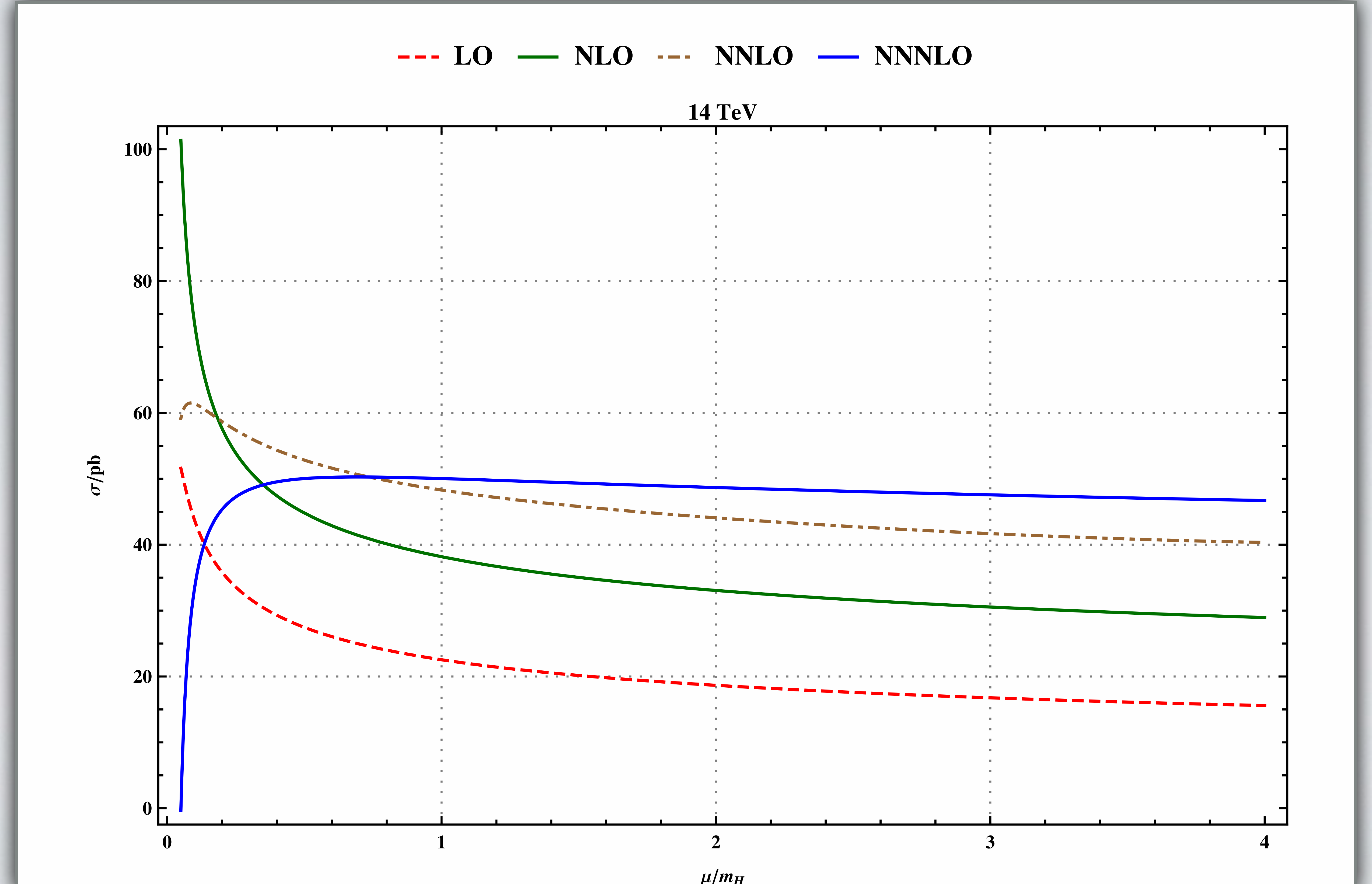
1. What is the method ✓

   Analytic Advancements

2. What is the benefit

   Phenomenological Implications

SOFT-VIRTUAL CROSS-SECTION



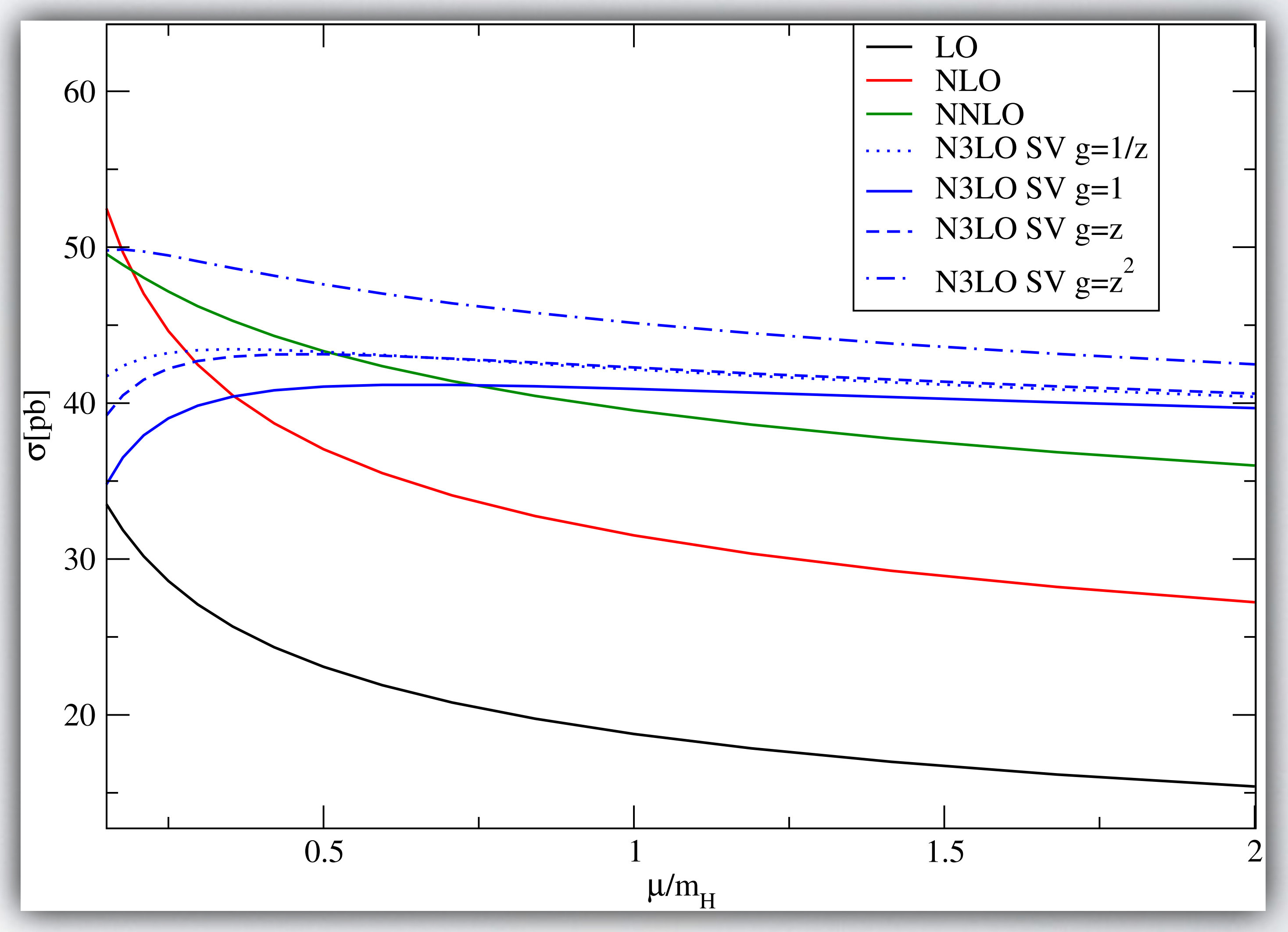
CAVEAT

- Soft-virtual term is **ambiguous**

$$\sigma = \int dx_1 dx_2 f(x_1) f(x_2) \left[z g(z) \right] \left[\frac{\hat{\sigma}(z)}{z g(z)} \right]_{\text{threshold}}$$

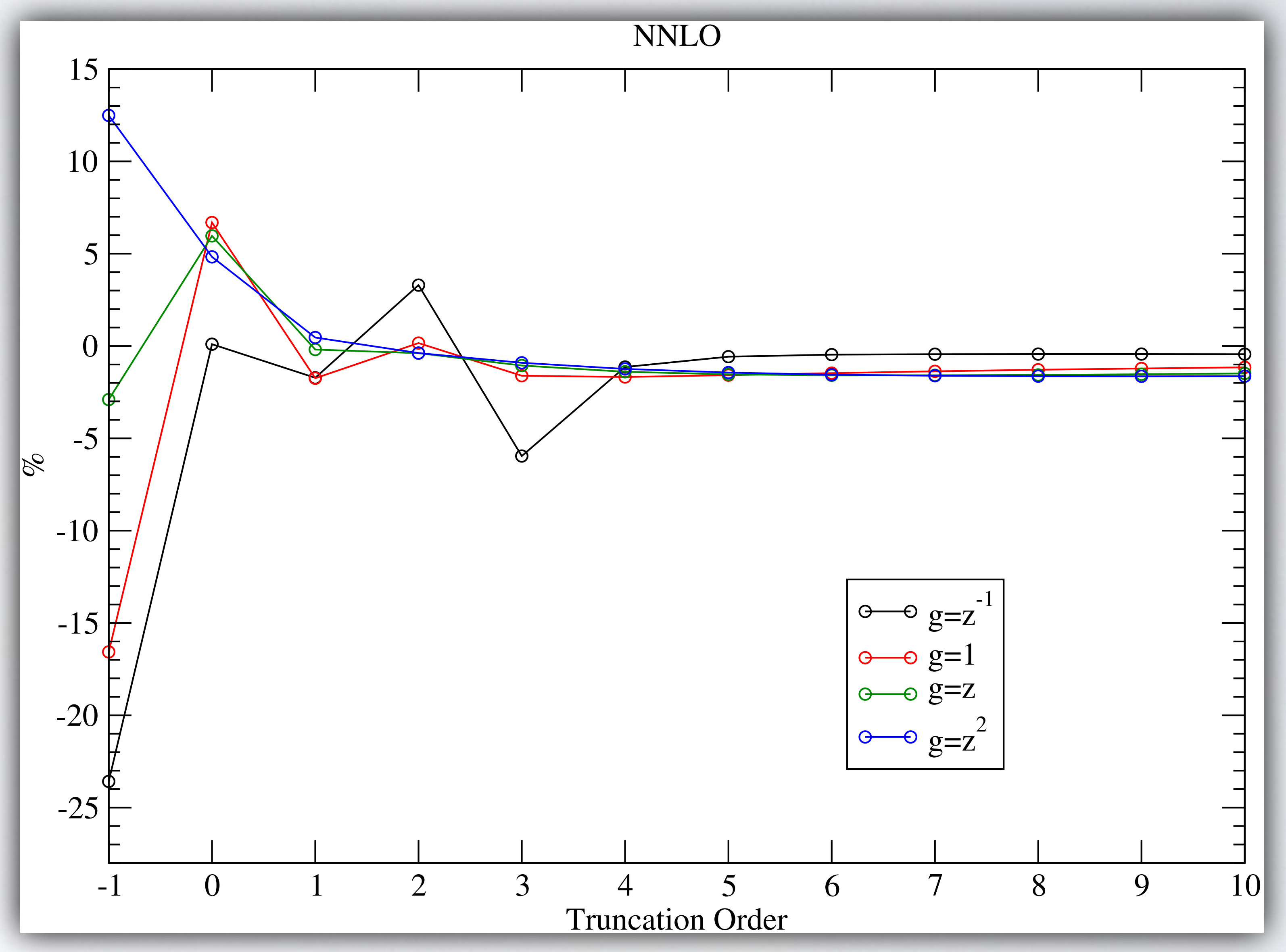
- We can choose $g(z)$ as long as

$$\lim_{z \to 1} g(z) = 1$$



CONVERGENCE @ NNLO

Sub-leading terms will cure the problem

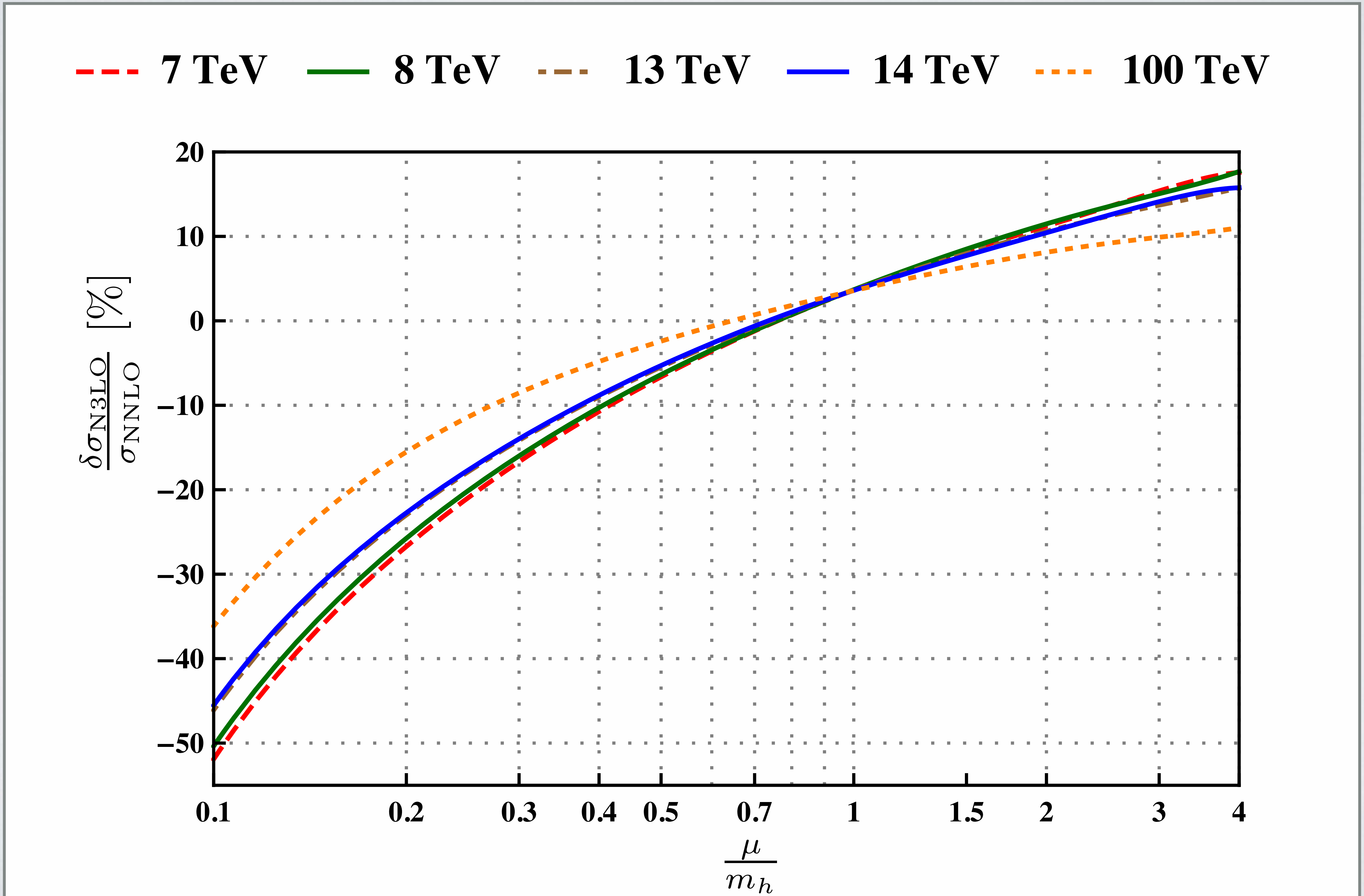


Stay tuned

CONCLUSION/OUTLOOK

- We broke the N3LO barrier
- We presented the Soft-Virtual Term at N3LO
  A classical result in perturbative QCD
- Further terms in the threshold expansion are essential for phenomenology
- Full kinematic cross-section at N3LO
  for the Higgs boson and more

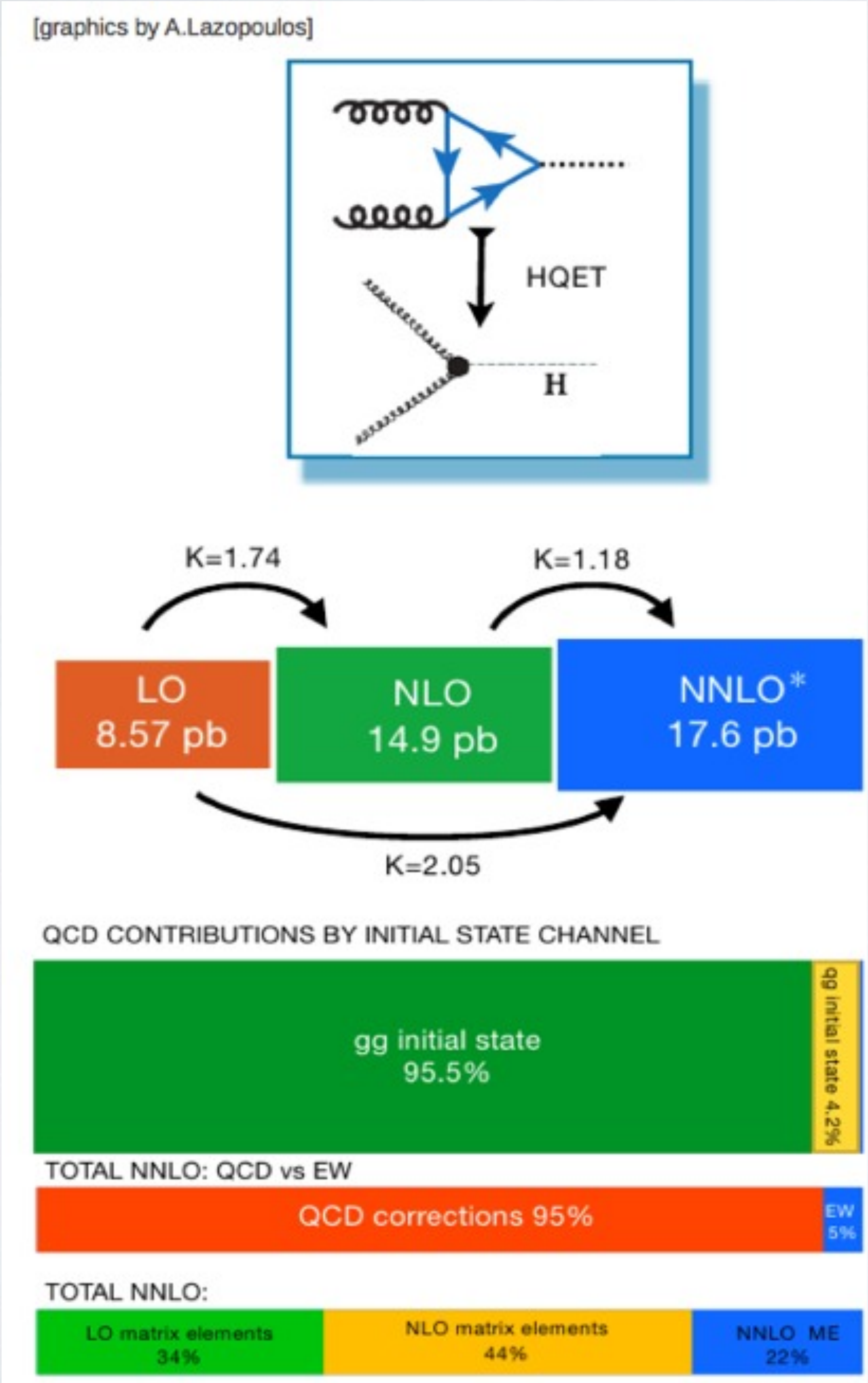
SOFT-VIRTUAL CROSS-SECTION



THRESHOLD LOGARITHMS

$$\begin{aligned}
\hat{\eta}^{(3)}(z) \simeq\ & \delta(1-z)\, 1124.308887\ldots && (\to 5.1\%) \\
& + \left[\frac{1}{1-z}\right]_+ 1466.478272\ldots && (\to -5.85\%) \\
& - \left[\frac{\log(1-z)}{1-z}\right]_+ 6062.086738\ldots && (\to -22.88\%) \\
& + \left[\frac{\log^2(1-z)}{1-z}\right]_+ 7116.015302\ldots && (\to -52.45\%) \\
& - \left[\frac{\log^3(1-z)}{1-z}\right]_+ 1824.362531\ldots && (\to -39.90\%) \\
& - \left[\frac{\log^4(1-z)}{1-z}\right]_+ 230 && (\to 20.01\%) \\
& + \left[\frac{\log^5(1-z)}{1-z}\right]_+ 216\,. && (\to 93.72\%)
\end{aligned}$$

GG - FUSION



- QCD dominates
- K-Factor is large
- Large corrections at NNLO
- gg-initial-state largest

SOFT-VIRTUAL -CHECKS

How can we be sure we got it right?

- We canceled 6 non-trivial IR / UV poles!
- Plus - distributions agree with prediction by Moch/Vogt/Vermaseren
- Multiple independent calculations of every step
  (internally and for some contributions also by other groups)