DEDUCTOR:
a parton shower generator

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DESY

Work with Dave Soper

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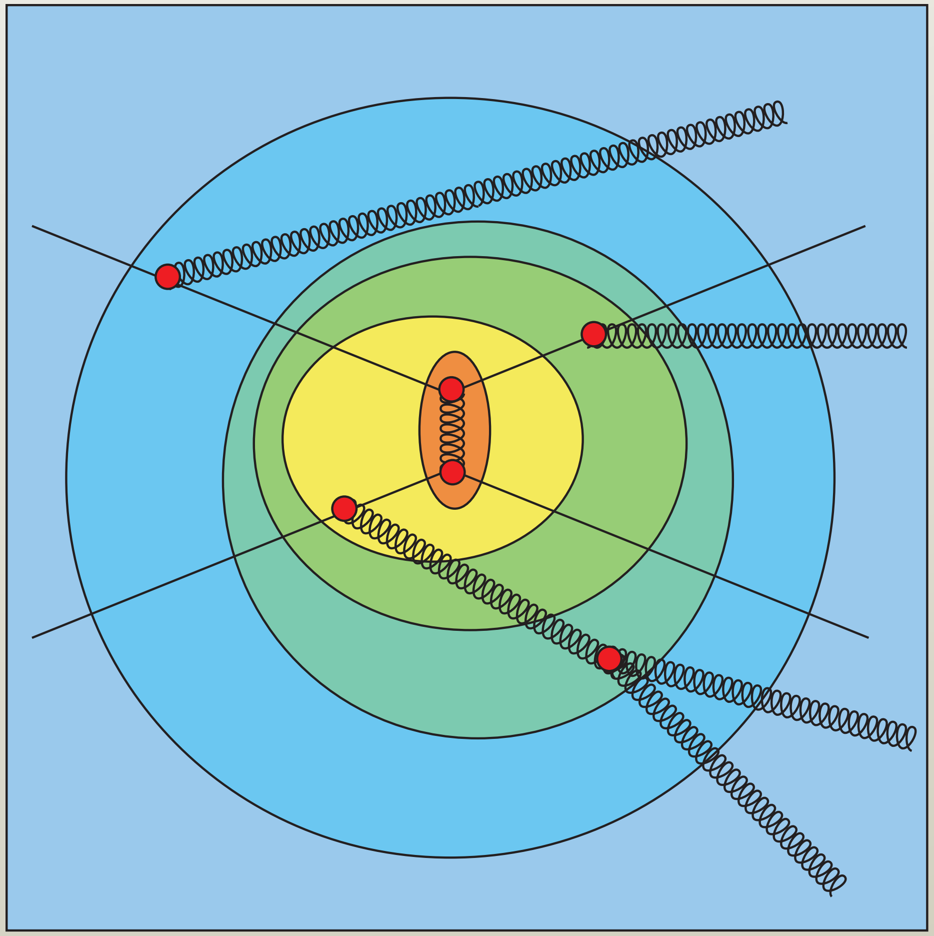
DEDUCTOR is hardness ordered

- Parton shower evolves with "shower time" t.
- Small $t \Rightarrow$ hard.
- Large $t \Rightarrow$ soft.
- For initial state interactions, evolution is backwards in physical time.



- This is similar to PYTHIA and SHERPA.

Design goal

- The main goal is to have a structure that is adapted to a better treatment of color and spin.
- This treatment is only partly implemented for now.

The shower state

- The fundamental object is the quantum density matrix in color and spin space, with basis vectors

$$\big|\{c,s\}_m\big\rangle\big\langle\{c',s'\}_m\big|$$

- For two initial state partons plus m final state partons, let

$$\rho(\{p,f,c',c,s',s\}_m,t)$$

be probability to have momenta $\{p\}_m$ and flavors $\{f\}_m$ and be in this color-spin state.

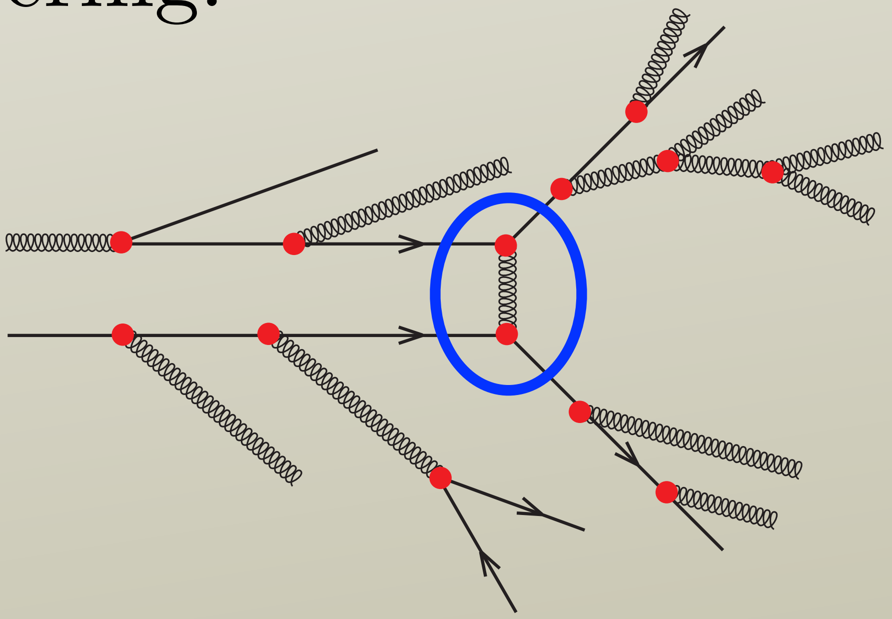
- Consider the function $\rho(\{p,f,c',c,s',s\}_m,t)$ at fixed t as a vector $\big|\rho(t)\big)$.

Standard generators

- No spin, just average over spins.
- Diagonal color states only, $\big|\{c\}_m\big\rangle\big\langle\{c\}_m\big|$
- Use the “leading color” approximation.

Current DEDUCTOR

- No spin, just average over spins.
- Off diagonal color states, $\big|\{c\}_m\big\rangle\big\langle\{c'\}_m\big|$.
- Use an approximate version of color "LC+."
- With color states $\big|\{c\}_m\big\rangle\big\langle\{c'\}_m\big|$, we can start the shower with color-ordered amplitudes for the hard scattering.



Other differences

- Even with spin averaging and leading color, DEDUCTOR is not the same as PYTHIA or SHERPA.

- The splitting functions are more complicated.

- Initial state b and c quarks have masses.

- The shower ordering variable is not k_T.

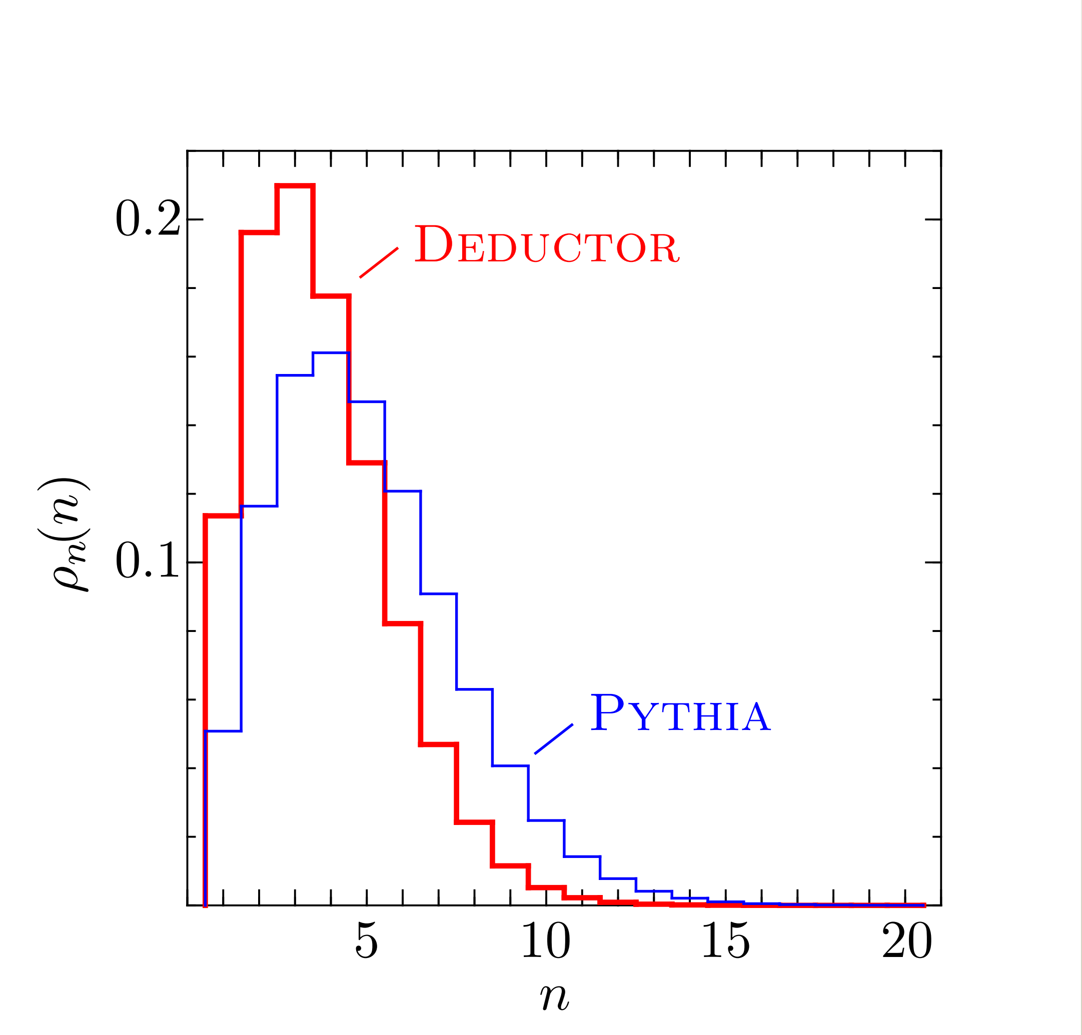
- I will return to these last two points later.

Comparisons to PYTHIA

- A parton shower generator is quite complicated.
- Thus we need some sanity checks.
- For this, we compare to PYTHIA at the parton level for an 8 TeV LHC.
- In DEDUCTOR, we use just leading color.
- We do not expect exact agreement. Our parton distributions are different and default PYTHIA has a larger strong coupling.

Number of partons in a jet

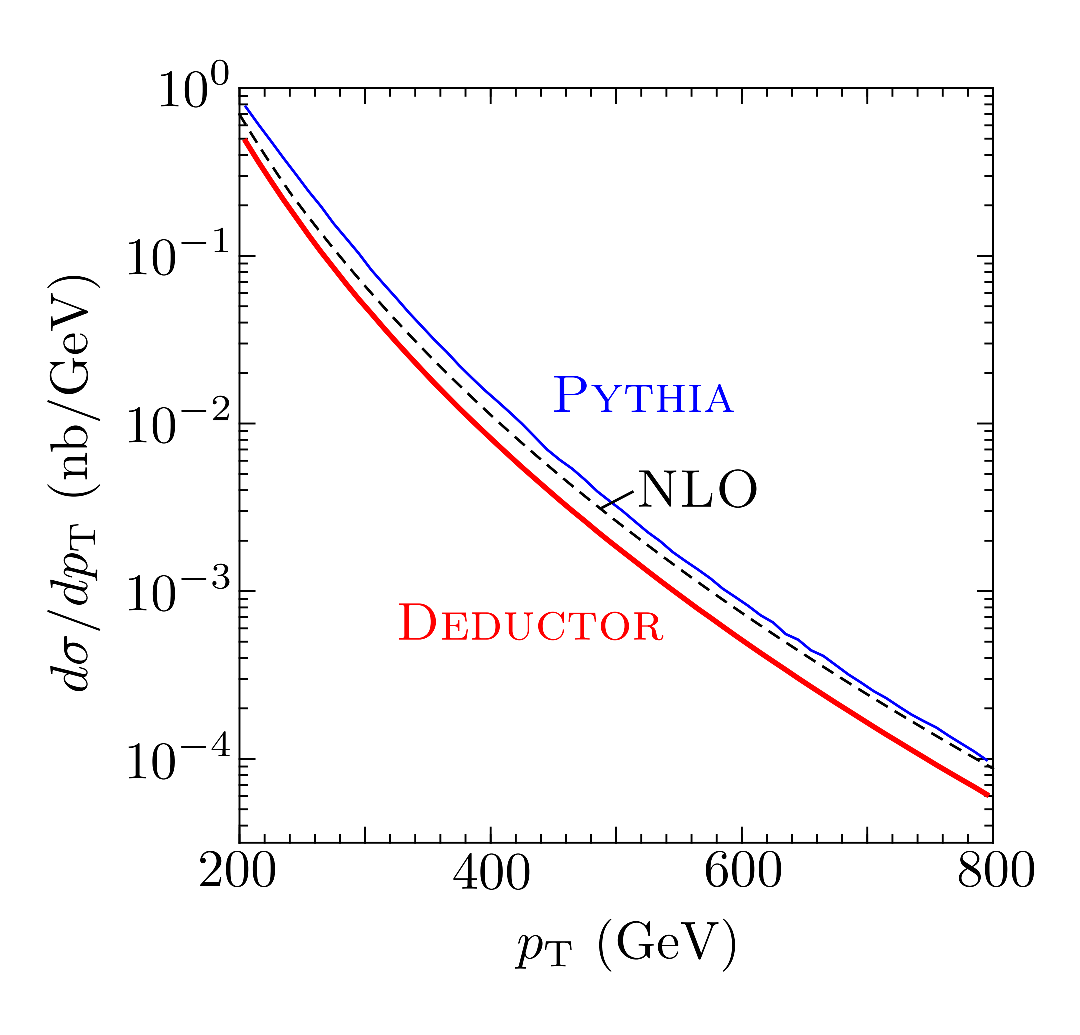
- Construct jets with the k_T algorithm with $\mathrm{R} = 0.4$.
- Look at jets with $P_\mathrm{T} > 200$ GeV, $|y| < 2$.
- PYTHIA jets are somewhat more evolved.
- But the distributions are pretty similar.



Jet cross section

- Look at one jet inclusive cross section $d\sigma/dp_{\mathrm{T}}$ with k_{T} jet algorithm with $R = 0.4$.

- Plot Pythia, Deductor, and NLO.

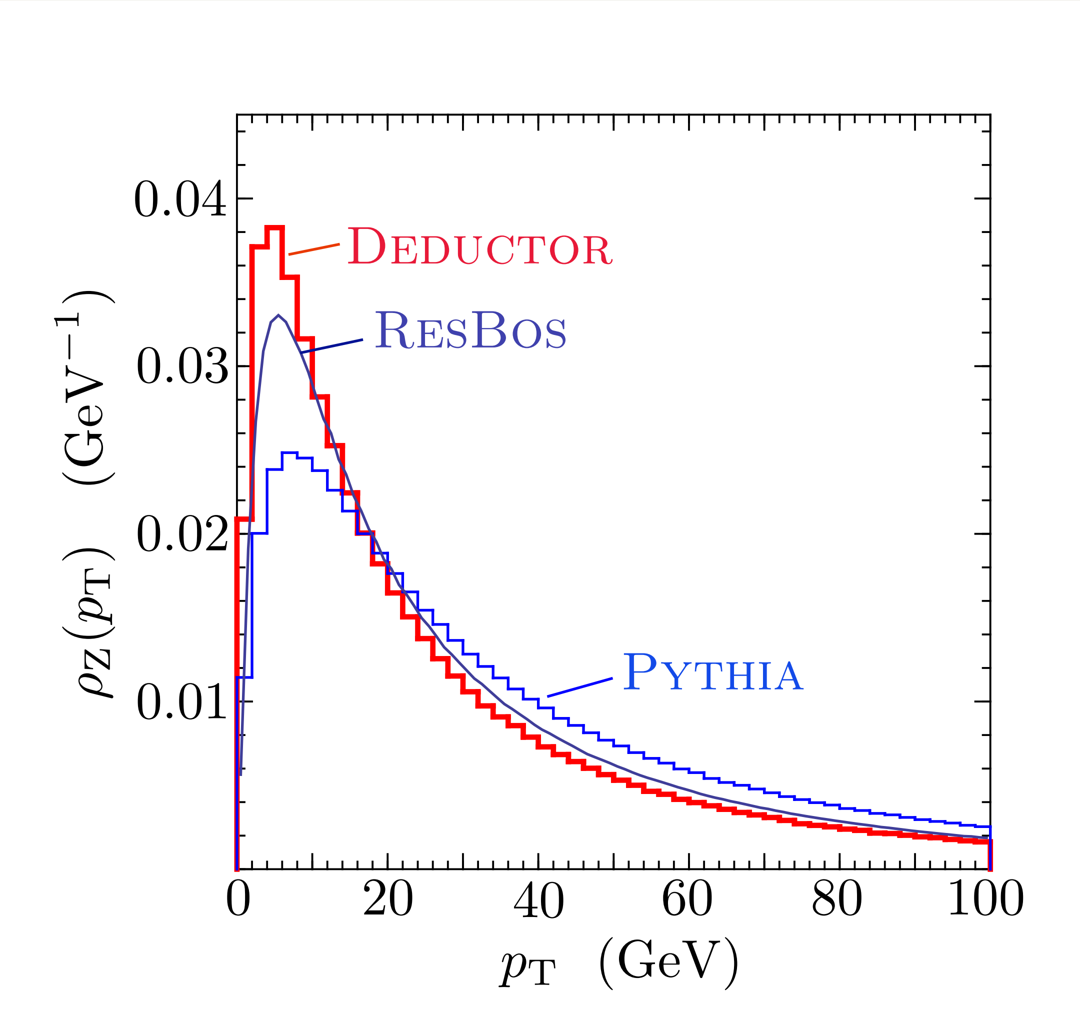


- The jet cross section is highly sensitive to showering effects.

- Both Pythia and Deductor are within about 30% of NLO so we judged the agreement to be satisfactory.

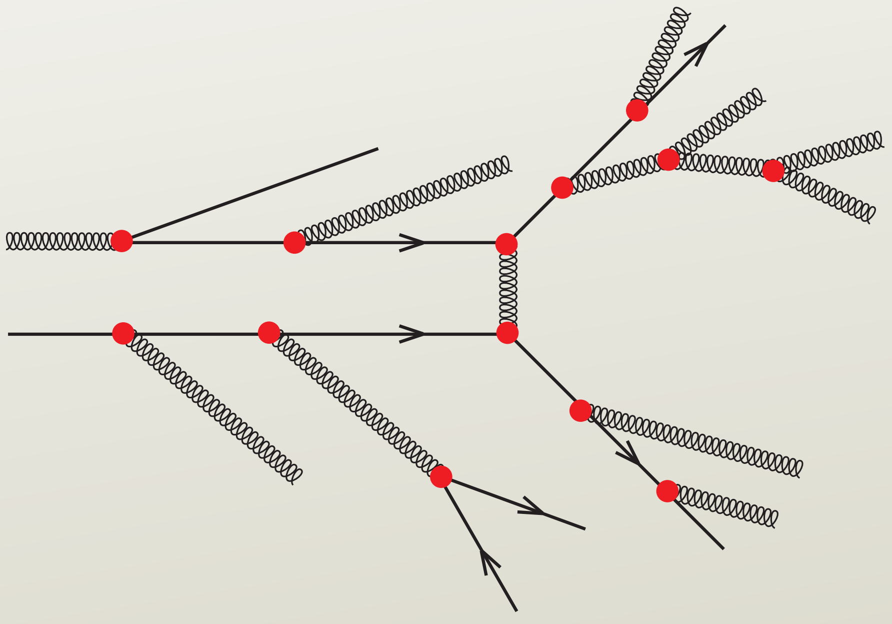
Drell-Yan P_T distribution

- Look at distribution of $P_{\rm T}$ of e^+e^- pairs with $M > 400$ GeV.
- $\int_0^{100\ \rm GeV} dp_{\rm T}\,\rho(p_{\rm T}) = 1.$
- A parton shower should get this right except for soft effects at $P_{\rm T} < 10$ GeV.



- We compare DEDUCTOR, PYTHIA, and the analytic log summation in RESBOS.
- DEDUCTOR appears to do well.

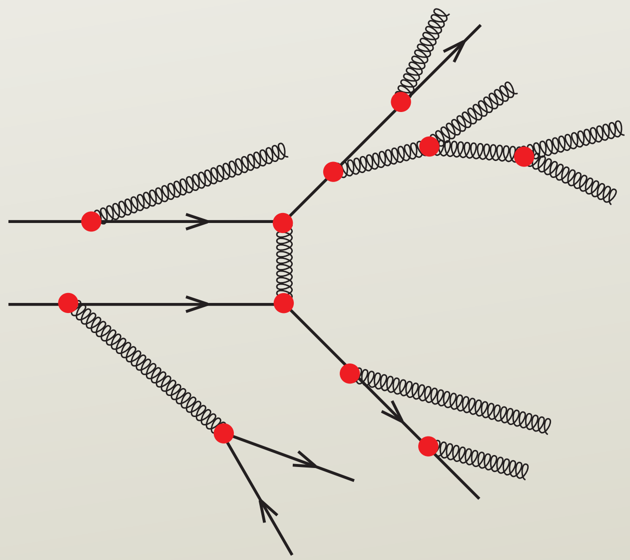
Masses of initial state partons



- The masses of b and c quarks are not zero.
- When the virtuality scale of the shower reaches a few GeV, the b and c masses matter.
- Therefore, DEDUCTOR keeps $m(\mathrm{b}) \neq 0$ and $m(\mathrm{c}) \neq 0$ even for initial state quarks.
- This required a little work ...

Evolution of parton distribution functions

- A parton shower needs parton distribution functions.



- At hard scattering, need $f_{a/A}(\eta_{\rm a},\mu^2)f_{b/B}(\eta_{\rm b},\mu^2)$.

- At a splitting on line "a," need a factor

$$\frac{f_{\hat{a}/A}(\hat{\eta}_{\rm a},\mu^2)}{f_{a/A}(\eta_{\rm a},\mu^2)}$$

where $\hat{\eta}_{\rm a} = \eta_{\rm a}/z$ is the new momentum fraction.

- $\overline{\mathrm{MS}}$ distributions $f_{a/A}(\eta,\mu^2)$ obey an evolution equation,

$$\mu^2 \frac{d}{d\mu^2} f_{a/A}(\eta,\mu^2) = \sum_{\hat a} \int_0^1 \frac{dz}{z}\, \frac{\alpha_s(\mu^2)}{2\pi} P_{a\hat a}\!\left(z, z\frac{m^2}{\mu^2}\right) f_{\hat a/A}(\eta/z,\mu^2)$$

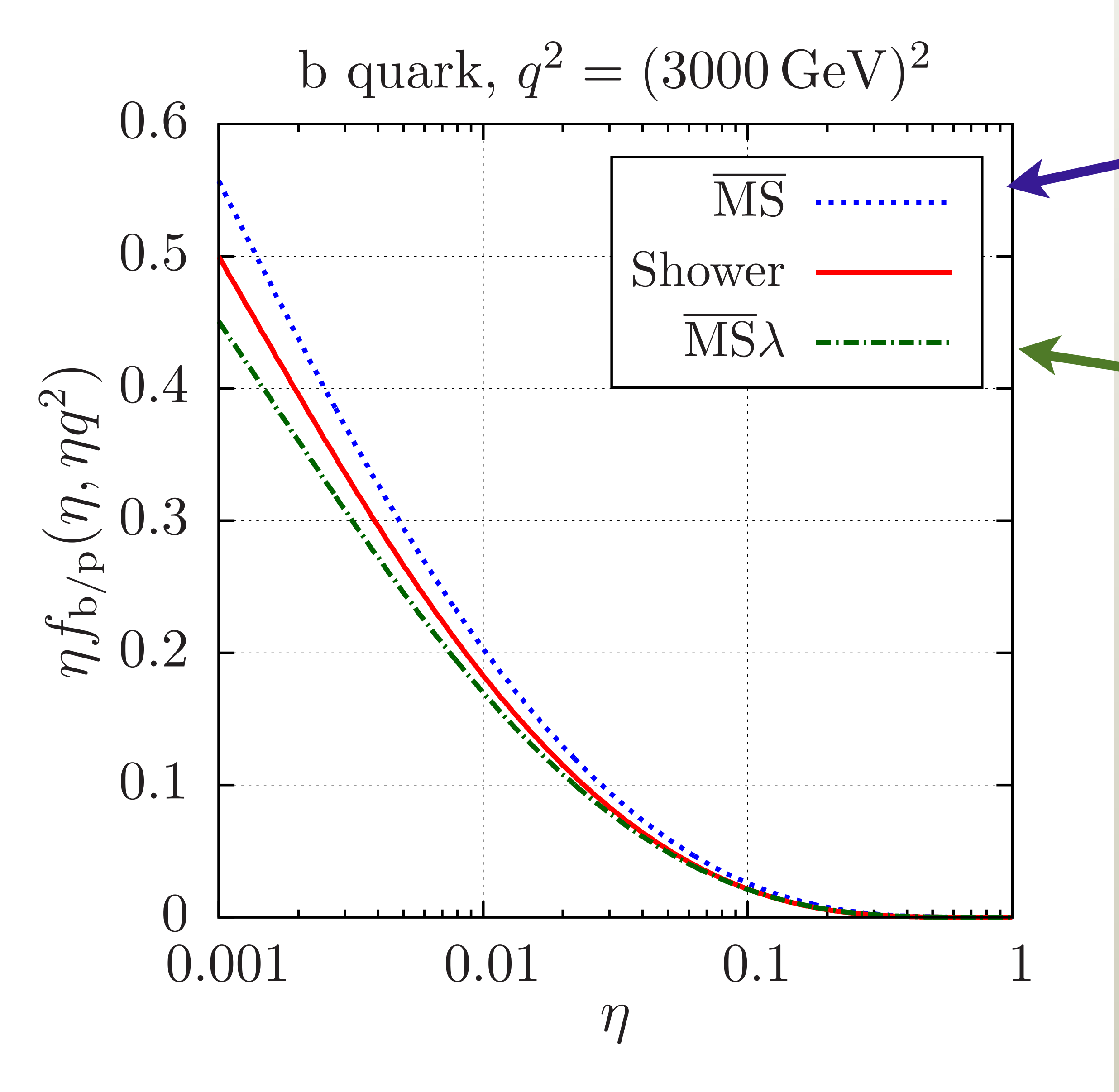
- The initial state shower contains splitting functions.

- $P_{a\hat a}(z)$ needs to match the shower splitting functions.

- The standard $P_{a\hat a}(z)$ does not, because the shower splitting functions depend on quark masses.

- Therefore we need revised parton distribution functions with revised evolution.

Effect of modified evolution

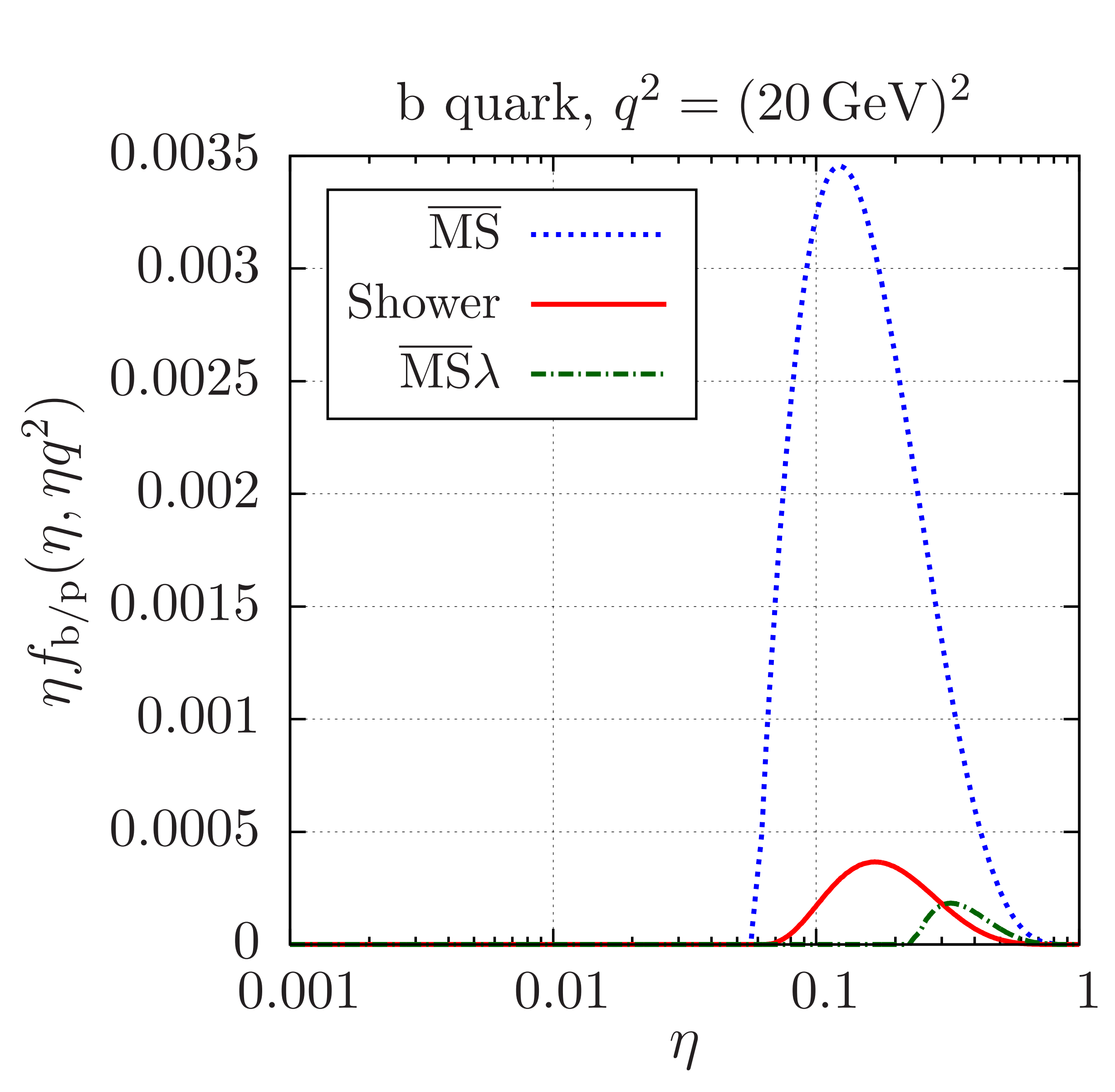


Normal $\overline{\mathrm{MS}}$.

$\overline{\mathrm{MS}}$ but starting at $\mu^2 = 4m(\mathrm{b})^2$

The b-quark distribution as a function of η at fixed shower time: $q^2 = \mu^2/\eta$ is fixed.

- Near the threshold.

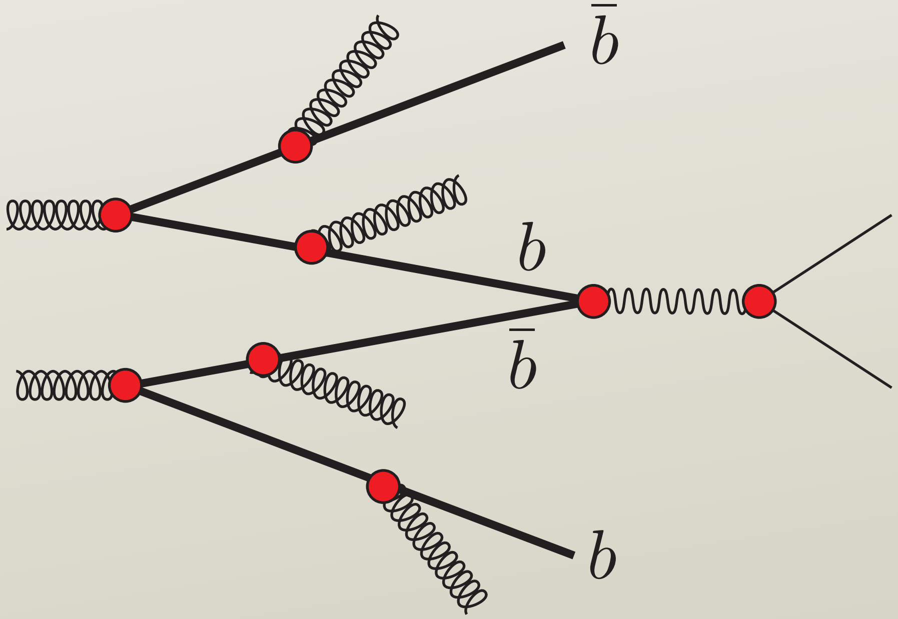


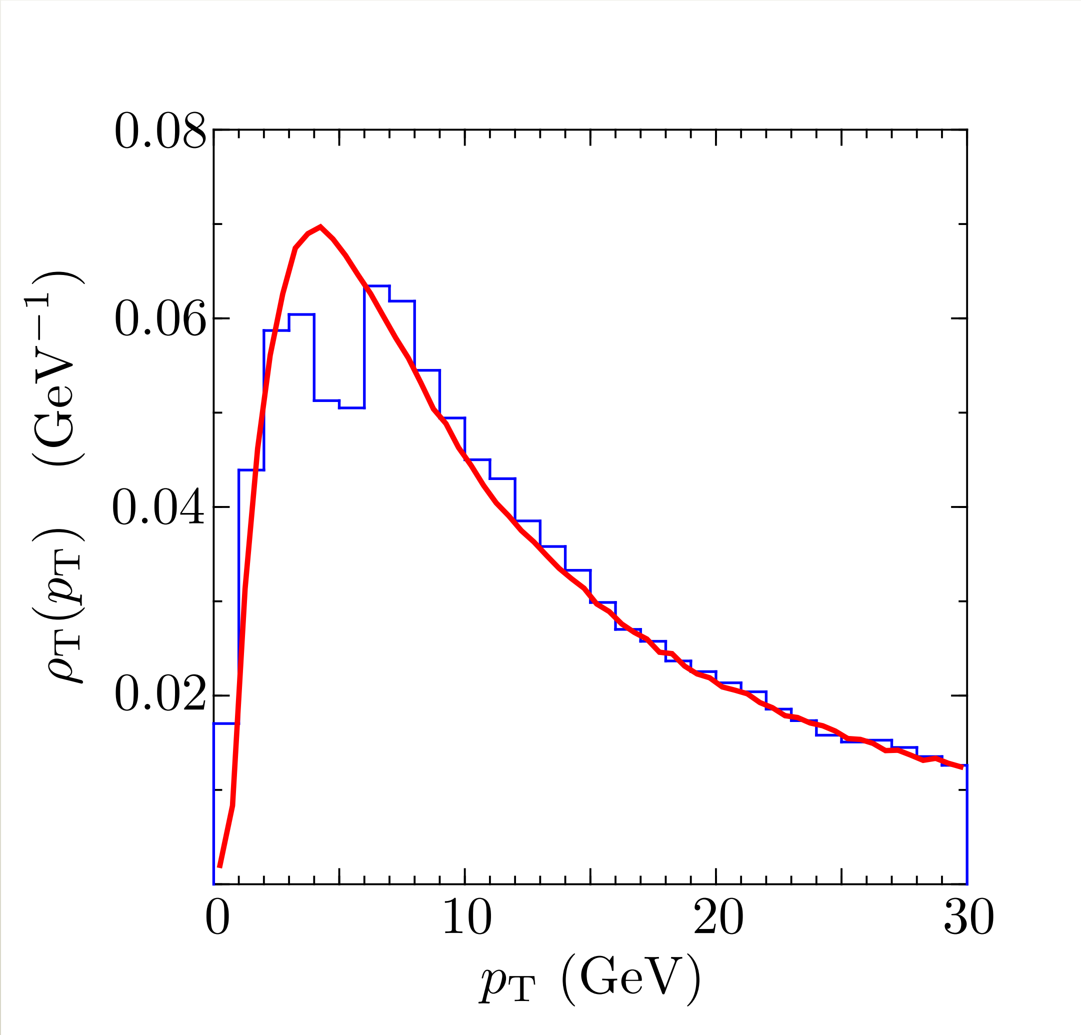
With $\overline{\mathrm{MS}}$, too many b-quarks.

With mass effects, hardly any b-quarks.

b-quark P_T

- Look at Drell-Yan production of e^+e^- pairs as earlier.
- This time, look at events in which the annihilating quarks were b$\bar{\text{b}}$.





- Look at the $p_{\rm T}$ distribution of the associated $\bar{\text{b}}$.
- Deductor (red curve) has a sensible result, while the Pythia distribution (blue histogram) has a strange dip.

Shower ordering variable

- Originally, Pythia used virtuality to order splittings.
- Now, Pythia and Sherpa use "$k_{\rm T}$."
- Deductor uses Λ,

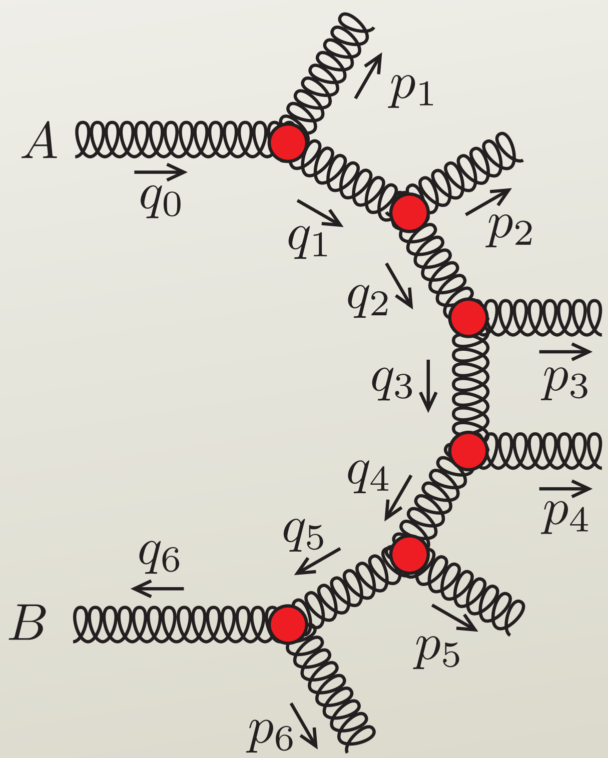
$$\Lambda_i^2 = \frac{p_i^2 - m_i^2}{2\,p_i\cdot Q_0}\,Q_0^2 \qquad \text{(final state)}$$

$$\Lambda_i^2 = \frac{|p_i^2 - m_i^2|}{2\eta_i\,p_{\rm A}\cdot Q_0}\,Q_0^2 \qquad \text{(initial state)}$$

where
Q_0 is a fixed timelike vector;
$p_{\rm A}$ is the incoming hadron momentum;
η_i is the parton momentum fraction.

A consequence

- Consider an initial state shower.
- Take $p_i^2 = 0$.
- The hard scattering is $i = h$ somewhere in the middle.



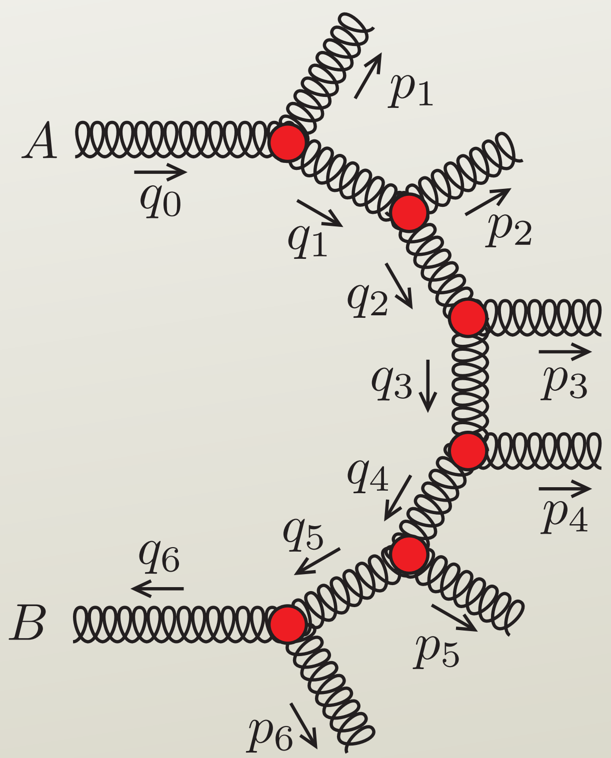
- Consider the case that $z_i = \eta_i/\eta_{i+1} \ll 1$ for all i as we move toward hadron A.

- Similarly $z_i \ll 1$ as we move toward hadron B.

- Denote the transverse part of q_i by $\boldsymbol{q}_i$.
- As we move toward hadron A, shower ordering requires

$$\boldsymbol{q}_{i-1}^2 < \boldsymbol{q}_i^2 \qquad \text{for } k_{\rm T} \text{ ordering}$$

$$z_i \boldsymbol{q}_{i-1}^2 < \boldsymbol{q}_i^2 \qquad \text{for } \Lambda \text{ ordering}$$



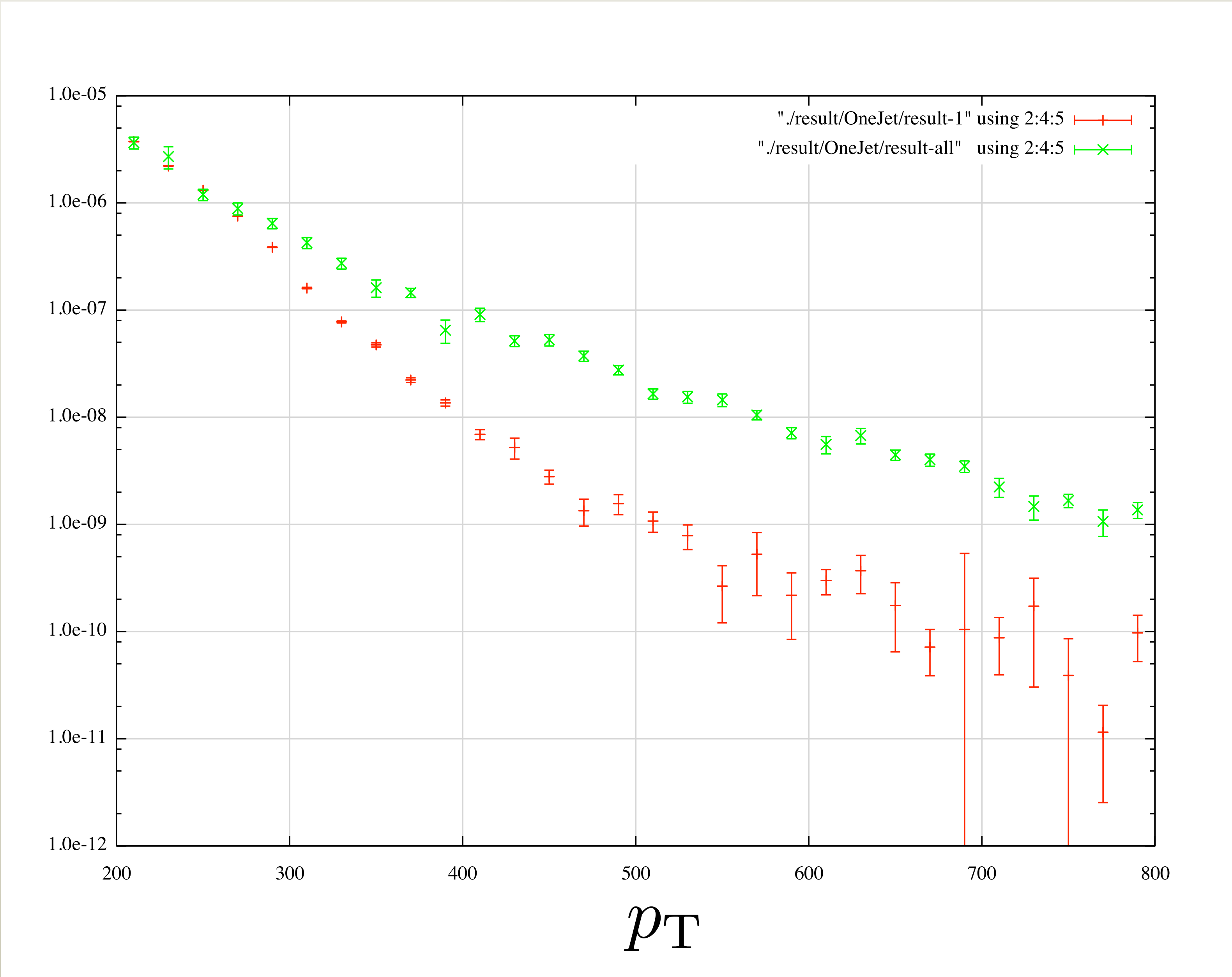
- We impose $\boldsymbol{q}_i^2 < \boldsymbol{q}_h^2$ in order to distinguish the hardest momentum transfer.
- Evidently, Λ ordering allows a wider phase space for gluon emissions.
- Get the phase space of "cut pomeron" exchange.

Example of unordered k_Ts

- $z_i \boldsymbol{q}_{i-1}^2 < \boldsymbol{q}_i^2$ for Λ ordering
- For this study, *drop* the requirement that $\boldsymbol{q}_i^2 < \boldsymbol{q}_h^2$.

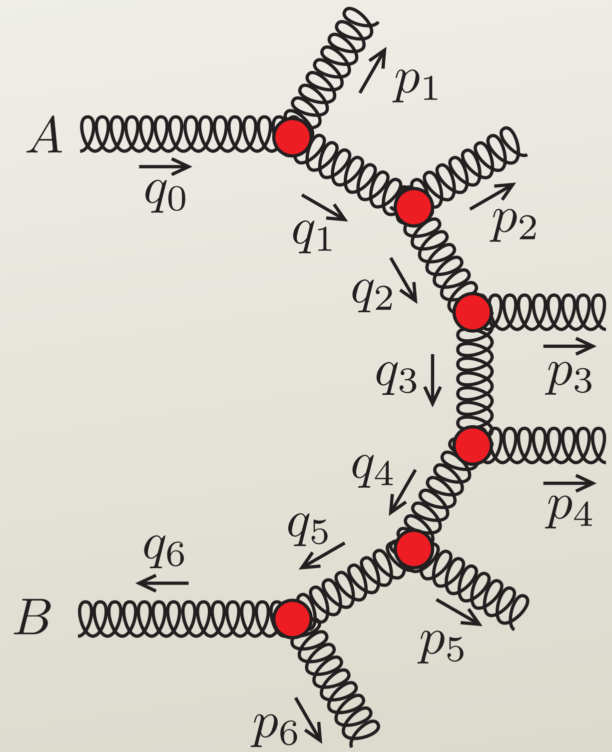
- Plot p_T for jet events in which $p_T < 300$ GeV at the Born level.

- Compare to normal jet p_T distribution.



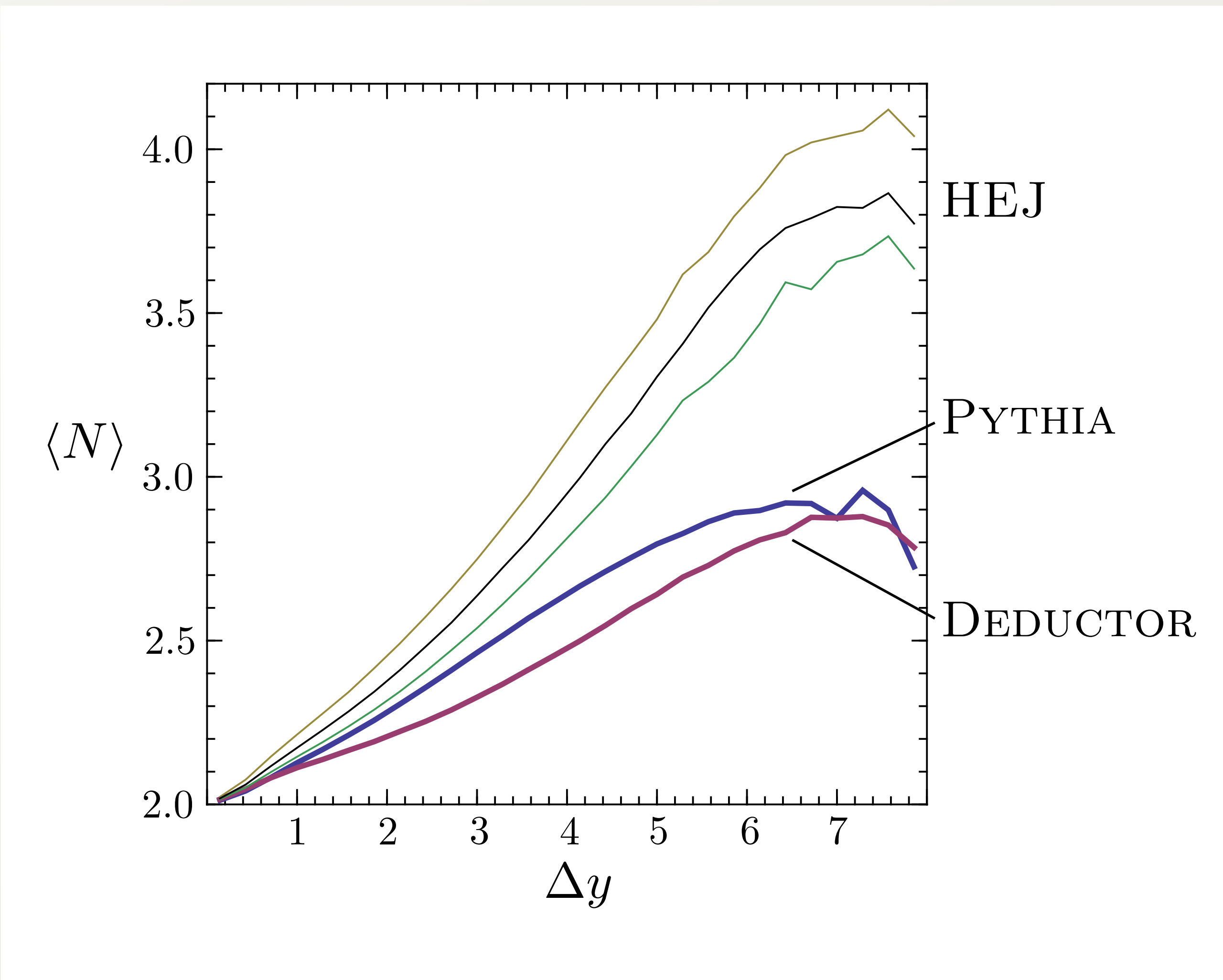
A cut pomeron example

- Examine jets i with $|\boldsymbol{p}_{i\perp}| > 30$ GeV.
- Select the two jets j with the largest y and the largest $-y$.
- Demand that these have $\sum_{j=1}^{2} |p_{j\perp}| > 120$ GeV.



- Let Δy be the rapidity difference between the extremal jets.
- Count the total number N of (> 30 GeV) jets.
- Plot $\langle N \rangle$ versus Δy.
- Expect $\langle N \rangle > 2$ for large Δy.

Result for $\langle N \rangle$ versus Δy



- Andersen and Smillie program for cut pomeron physics HIGH ENERGY JETS,

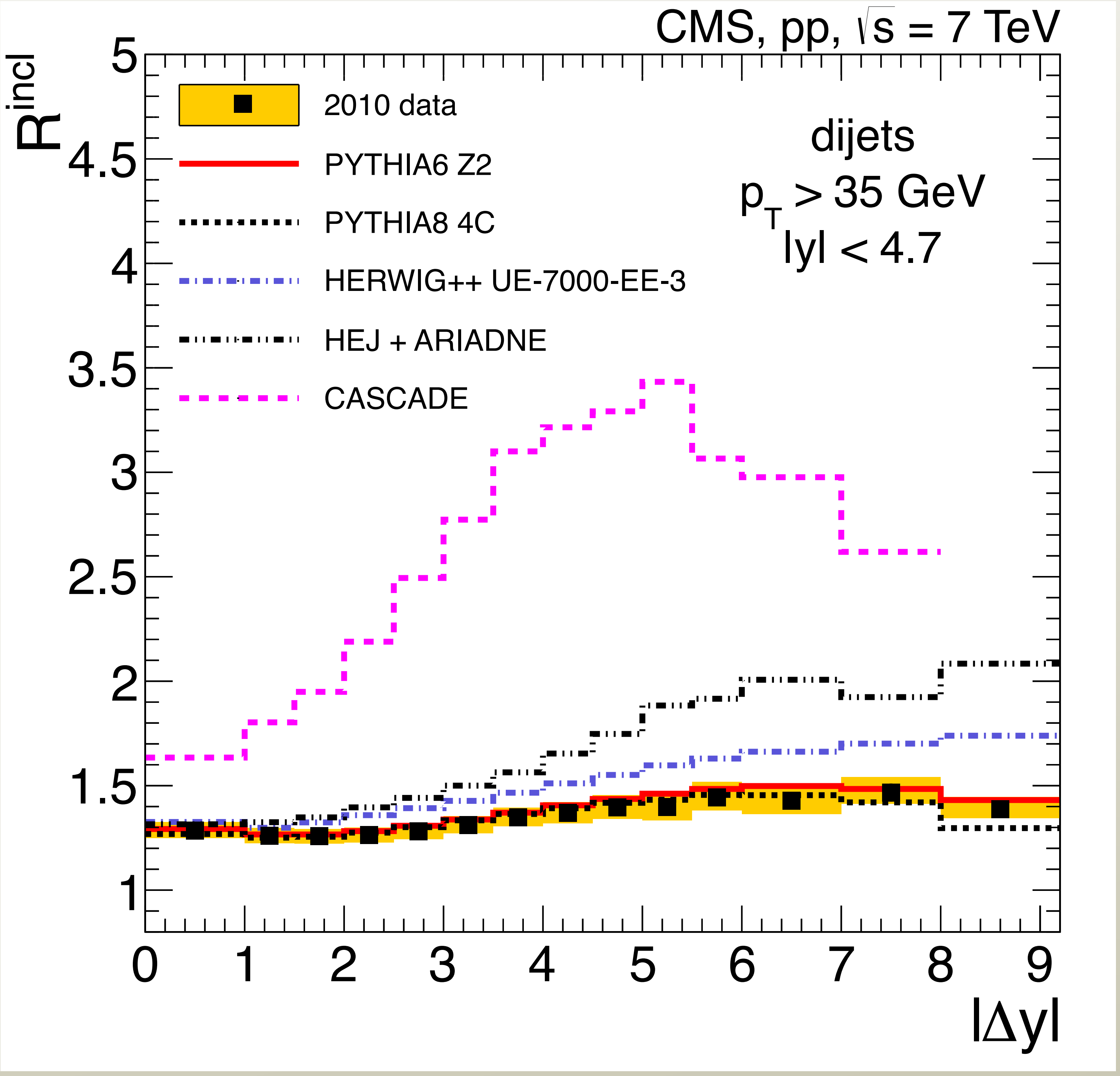
- DEDUCTOR generates what I think is a sensible amount of extra jet activity

- I do not understand why PYTHIA generates more.

Comparison to data

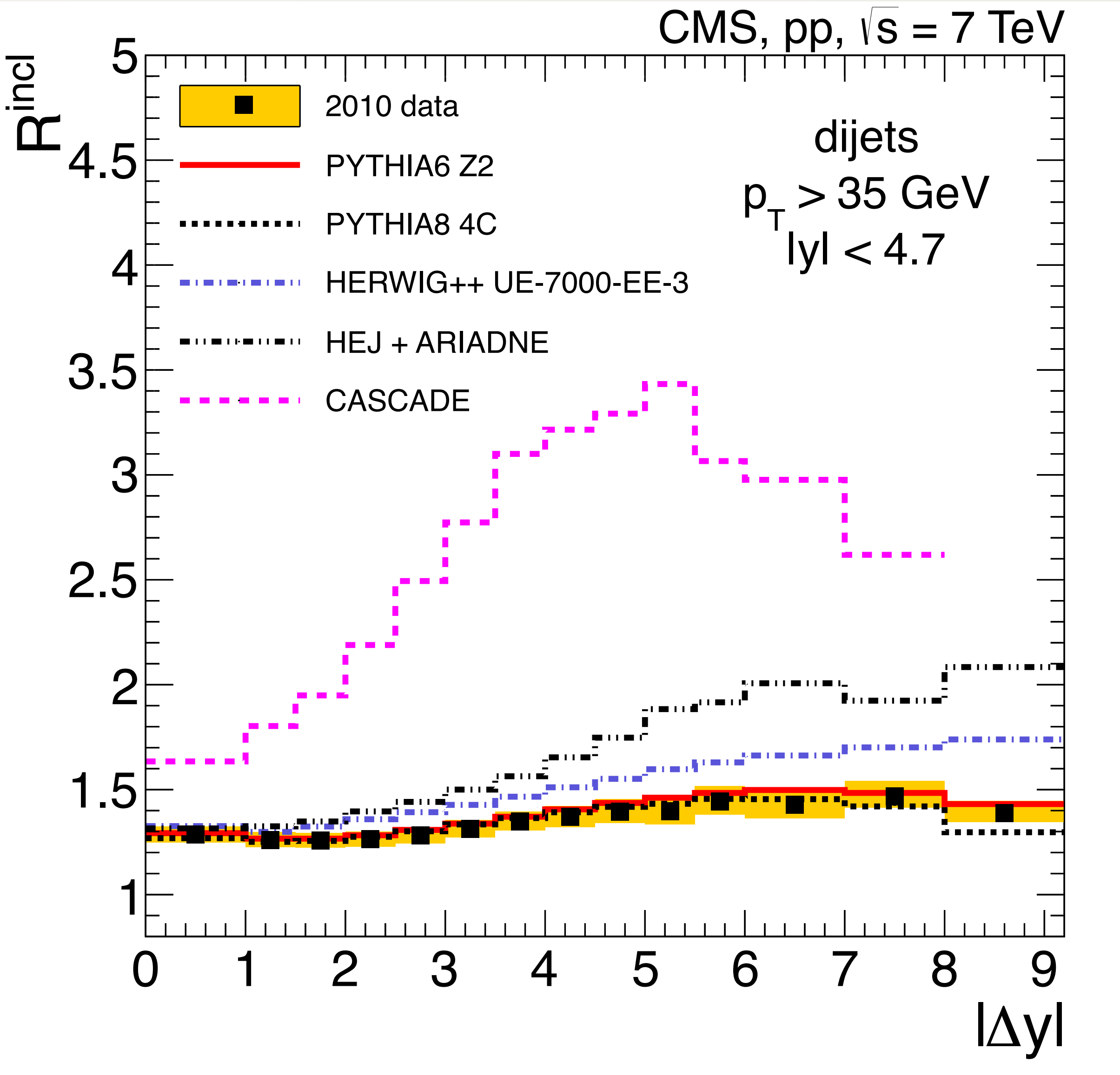
- CMS looked at jet events with $\sqrt{s} = 7$ TeV.
- Jets with $p_{\mathrm{T}} > 35$ GeV and $|y| < 4.7$ were selected.
- σ^{incl} = inclusive cross section to have two jets with rapidity difference Δy.
- σ^{excl} = cross section to have two jets and no more, with rapidity difference Δy.
- $R^{\mathrm{incl}} = \sigma^{\mathrm{incl}}/\sigma^{\mathrm{excl}}$.

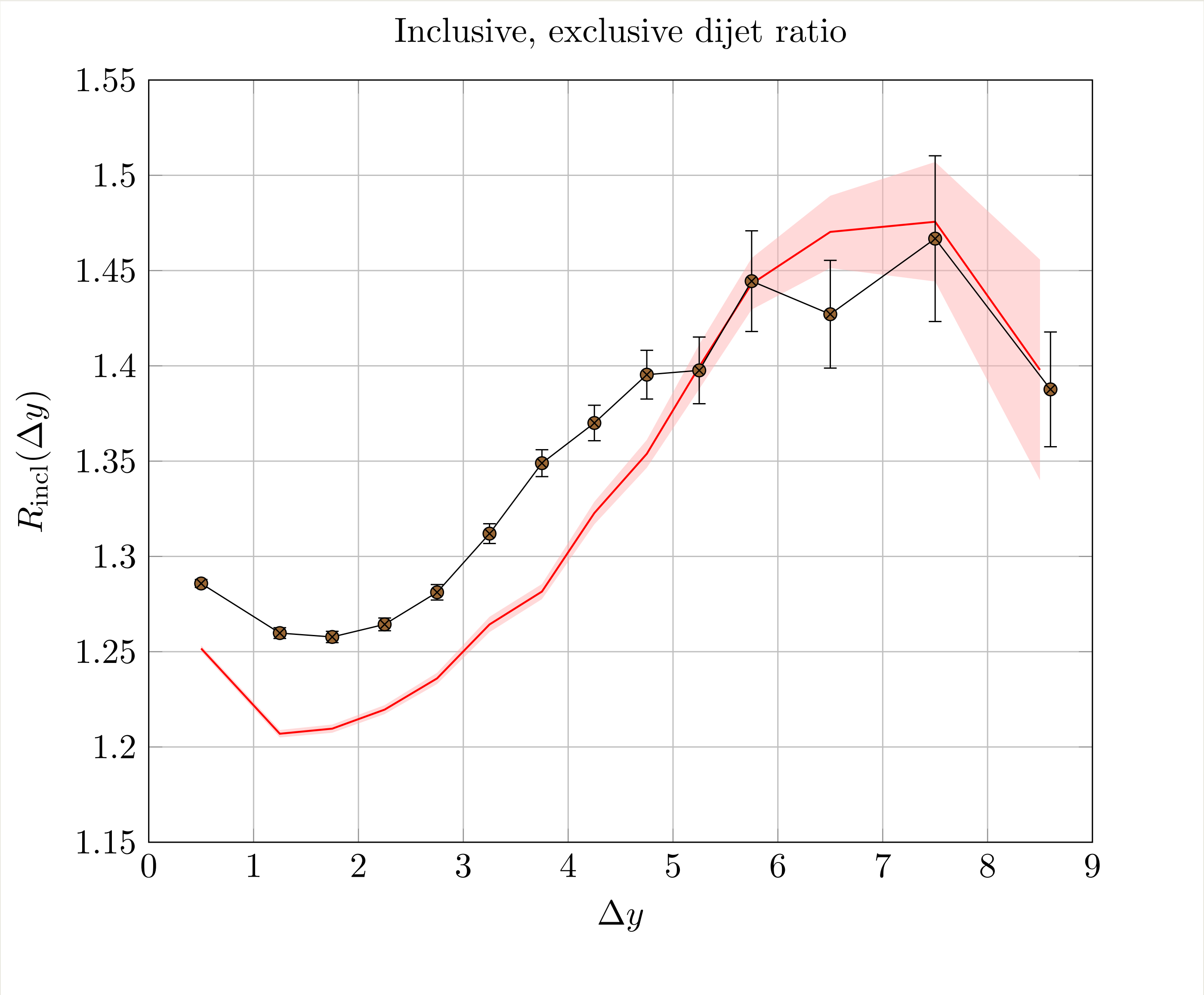
CMS result



CMS, pp, √s = 7 TeV
R^{incl}
2010 data
PYTHIA6 Z2
PYTHIA8 4C
HERWIG++ UE-7000-EE-3
HEJ + ARIADNE
CASCADE
dijets
p_T > 35 GeV
|y| < 4.7
|Δy|

Comparison to DEDUCTOR





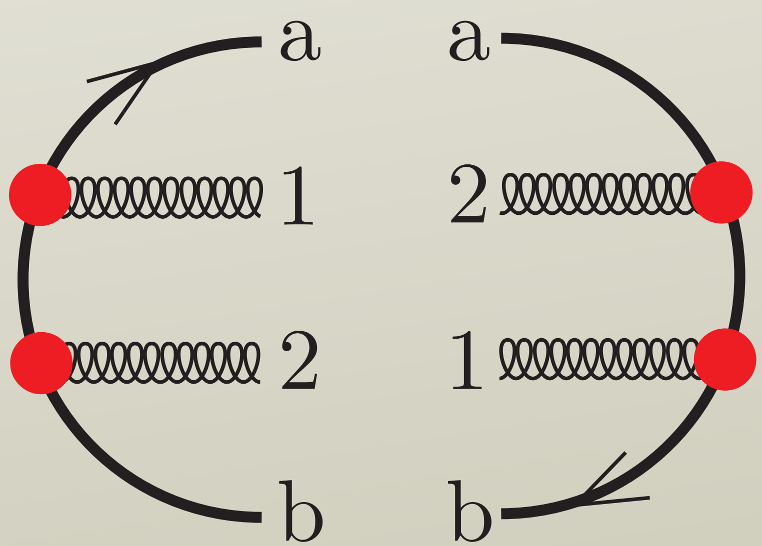
Color

- Forget spin.

- The fundamental object is the quantum density matrix in color, with basis vectors

$$\big|\{c\}_m\big\rangle\big\langle\{c'\}_m\big|$$

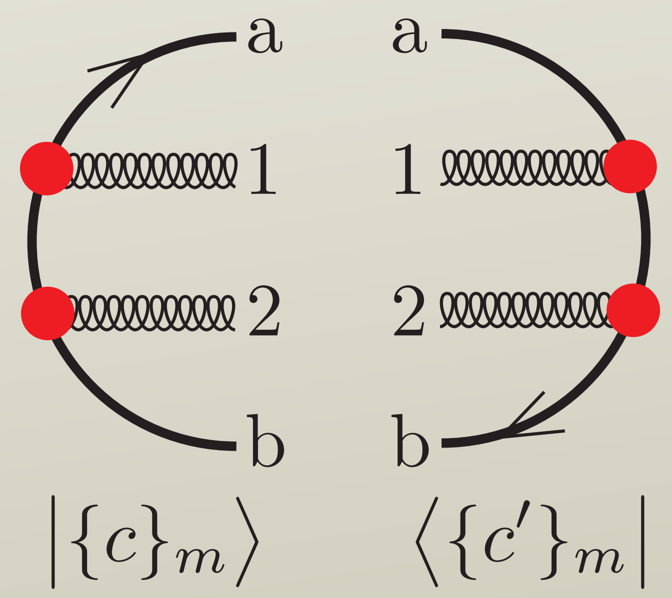
- Picture for this:



$\big|\{c\}_m\big\rangle$ $\big\langle\{c'\}_m\big|$

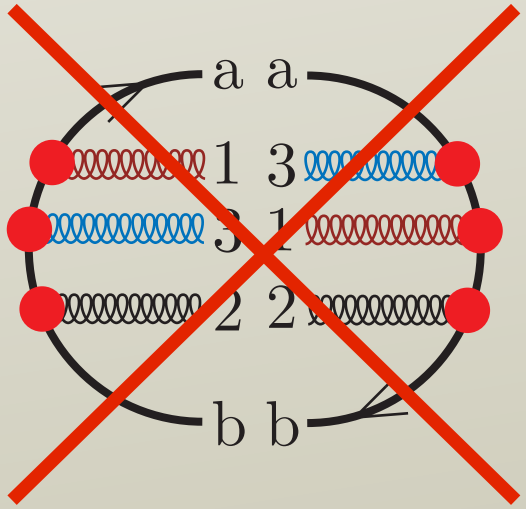
The leading color (LC) approximation

- Only states with $\{c'\}_m = \{c\}_m$ are allowed.



The color suppression index

- At each shower step, calculate a "color suppression index" I.
- $I = 0$ with the leading color approximation.
- At the end of the shower, a cross section is proportional to $1/N_\mathrm{c}^N$ with $N \geq I$.
- At each step of the shower, $I_\mathrm{new} \geq I_\mathrm{old}$.
- The neglected states have $I = 2$.



- Thus the neglected states give $1/N_\mathrm{c}^2$ contributions to cross sections.
- Are these contributions are unimportant?

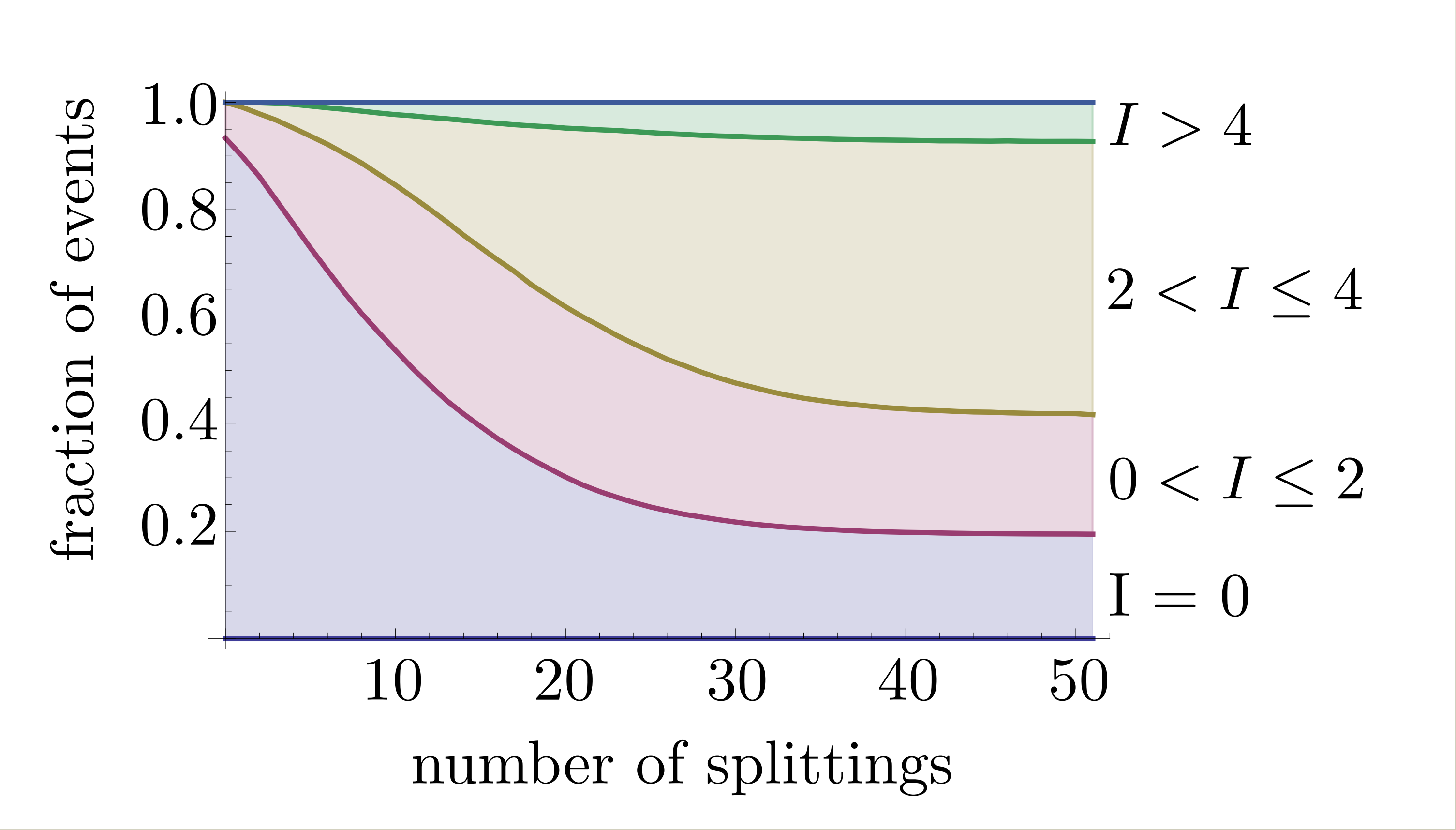
Practical application

- At each splitting, I can grow.
- We can set a maximum value, $I_{\max}$.
- When $I = I_{\max}$, we stop I from growing.
- This amounts to using $U(3)$ instead of $SU(3)$ as the color group.

Is $I > 0$ rare?

- Look at jet events.

- Trace fraction of events with different values of I as a function of number of splittings.



- $I > 0$ is not rare.

Does $I > 0$ matter?

- We are finding that typical observables are not highly sensitive to the color state after just a few splittings.
- Thus the LC approximation gives approximately the same result as the LC+ approximation.
- This conclusion may be observable dependent.

Gap fraction

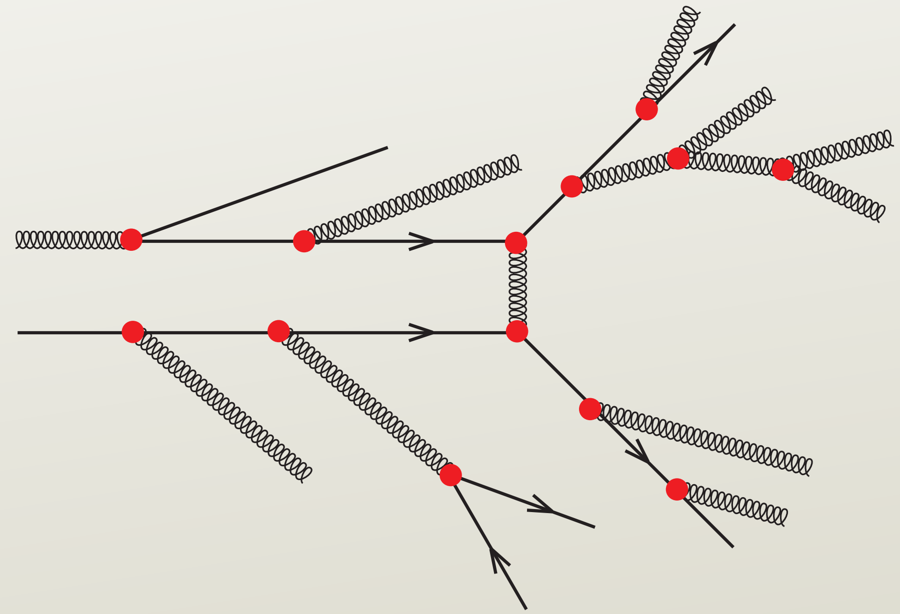
- Consider events with two jets with $p_\mathrm{T} > 200$ GeV.
- One jet has $y > 2$ and one has $y < -2$.
- Look for events with a gap:

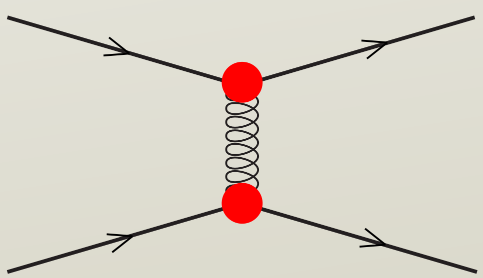
  No jets with $-2 < y < 2$ with $p_\mathrm{T} > p_\mathrm{T}^{\max}$.

- If we make $p_\mathrm{T}^{\max}$ small, the fraction of events with such a gap is small.

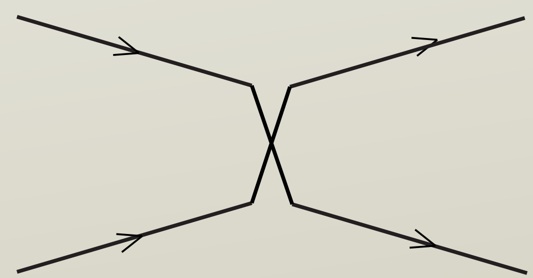
The gap fraction and color

- Consider small angle quark-quark scattering followed by shower.

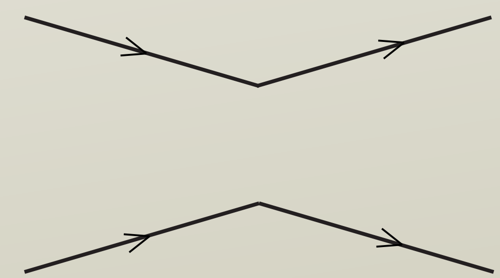




$= \frac{1}{2}$

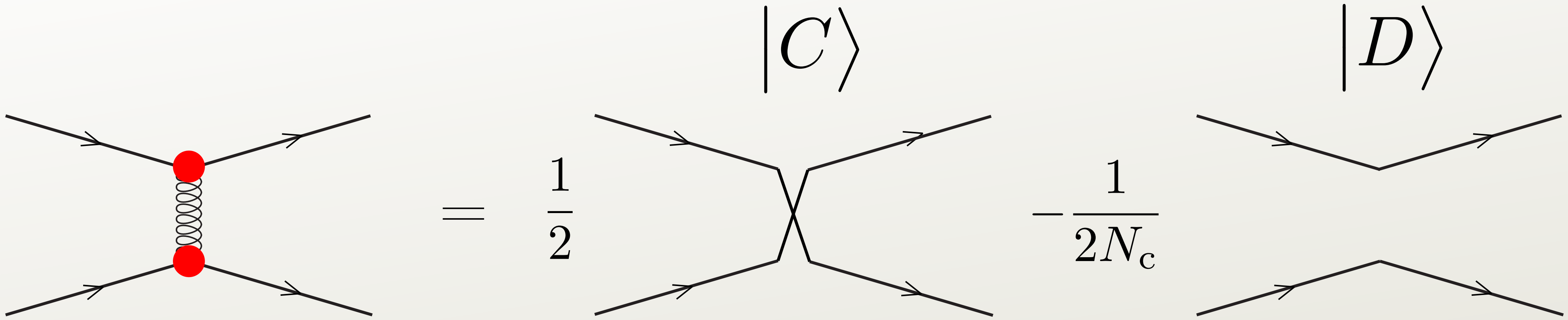


$- \frac{1}{2N_{\rm c}}$



Forward partons color connected to backwards partons

Forward partons color disconnected from backwards partons



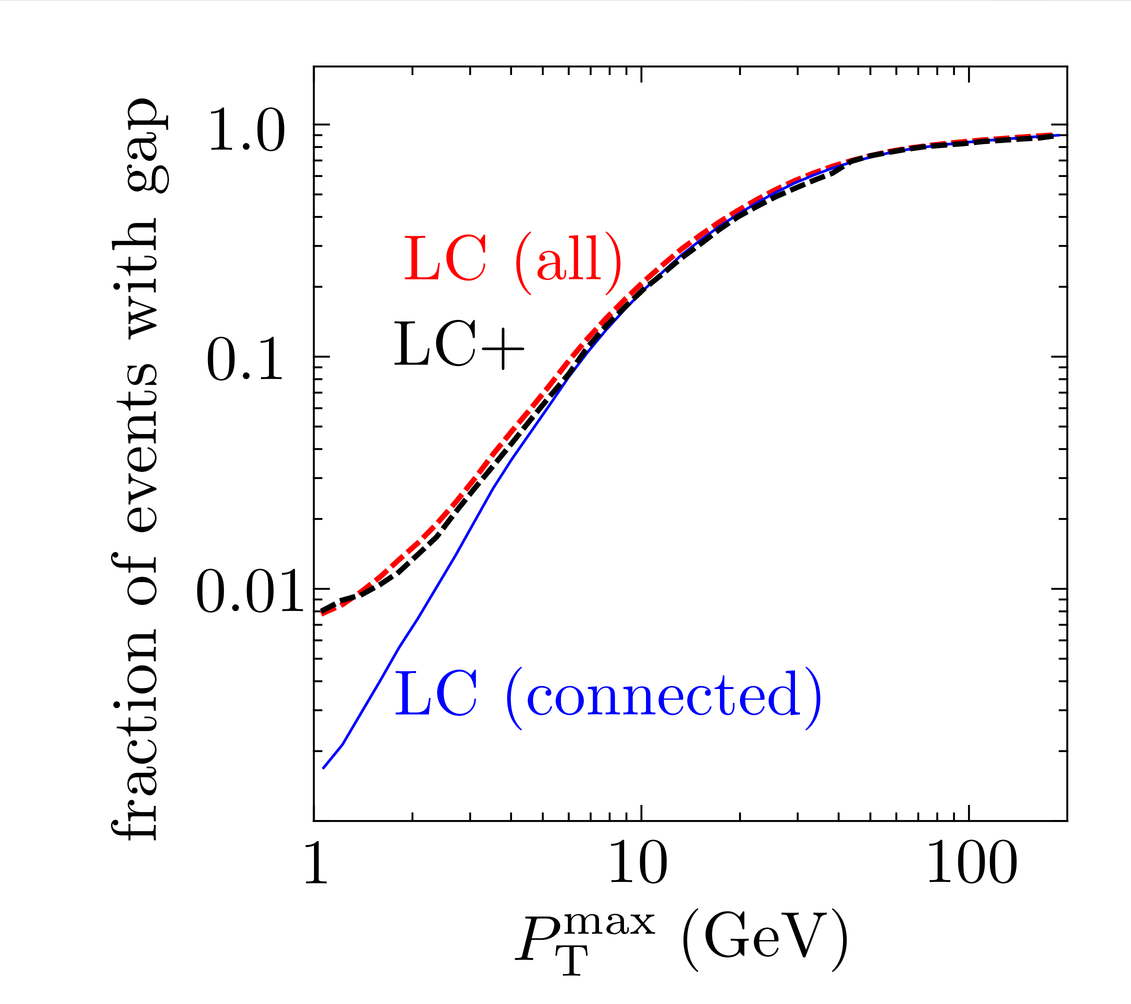
$$\rho = \frac{1}{4}|C\rangle\langle C| - \frac{1}{4N_c}|C\rangle\langle D| - \frac{1}{4N_c}|D\rangle\langle C| + \frac{1}{4N_c^2}|D\rangle\langle D|$$

LC — Needs LC+ — can be LC

gap survival probability low

gap survival probability high

Gap fractions



- Color structure is important for gap survival probability.

Conclusion

- DEDUCTOR is designed to do a better job with color and spin compared to other shower generators.
- Even with leading color and no spin, it has some novel features.
- It appears to produce sensible results.
- We are working on exploiting it.
- It is available at

  http://www.desy.de/~znagy/deductor

  http://pages.uoregon.edu/soper/deductor/

Conclusion, Outlook

This is *just a design of the parton shower* and it make sense at LO level. As far as I know there is no formal definition even at leading order level.

$$\sigma[F_J] = \sum_{m=2}^{\infty} \left(\rho_m\middle|\mathcal{F}_J\middle|1\right) = \sum_m \left[d\{p,f\}_m\right] \mathrm{Tr}\{\rho(\{p,f\}_m) F_J(\{p,f\}_m)\}$$

We need a formal proof that the perturbative sum of the cross section can be rearranged as a product.

$$\sigma[F_J] = \left(1\middle|\mathcal{F}_J \left[\mathcal{W}^{LO}(t_f) + \mathcal{W}^{NLO}(t_f) + \cdots\right]\right.$$

Finite corrections

$$\mathbb{T}\exp\left\{\int_0^{t_f} d\tau \left[\mathcal{H}^{LO}(\tau) + \mathcal{H}^{NLO}(\tau) + \cdots\right]\right\}$$

Parton shower

$$\left[\left|\rho^{LO}\right) + \left|\rho^{NLO}\right) + \cdots\right]$$

Hard state

Bonus slides

Operator formalism

The shower state

- The fundamental object is the quantum density matrix in color and spin space, with basis vectors

$$\big|\{c,s\}_m\big\rangle\big\langle\{c',s'\}_m\big|$$

- For two initial state partons plus m final state partons, let

$$\rho(\{p,f,c',c,s',s\}_m,t)$$

  be probability to have momenta $\{p\}_m$ and flavors $\{f\}_m$ and be in this color-spin state.

- Consider the function $\rho(\{p,f,c',c,s',s\}_m,t)$ at fixed t as a vector $\big|\rho(t)\big)$.

Evolution

- Evolution with shower time t: $\big|\rho(t)\big) = \mathcal{U}(t,0)\big|\rho(0)\big)$

$$\frac{d}{dt}\mathcal{U}(t,t') = [\mathcal{H}_{\rm I}(t) - \mathcal{V}(t)]\,\mathcal{U}(t,t')$$

splitting

no splitting

$$\frac{d}{dt}\mathcal{U}(t,t') = [\mathcal{H}_{\rm I}(t) - \mathcal{V}(t)]\,\mathcal{U}(t,t')$$

- Since $\mathcal{V}(t)$ is simple, rewrite as

split

$$\mathcal{U}(t,t') = \mathcal{N}(t,t') + \int_{t'}^{t} d\tau\, \mathcal{U}(t,\tau)\,\mathcal{H}_{\rm I}(\tau)\,\mathcal{N}(\tau,t')$$

exponentiate the probability of not splitting

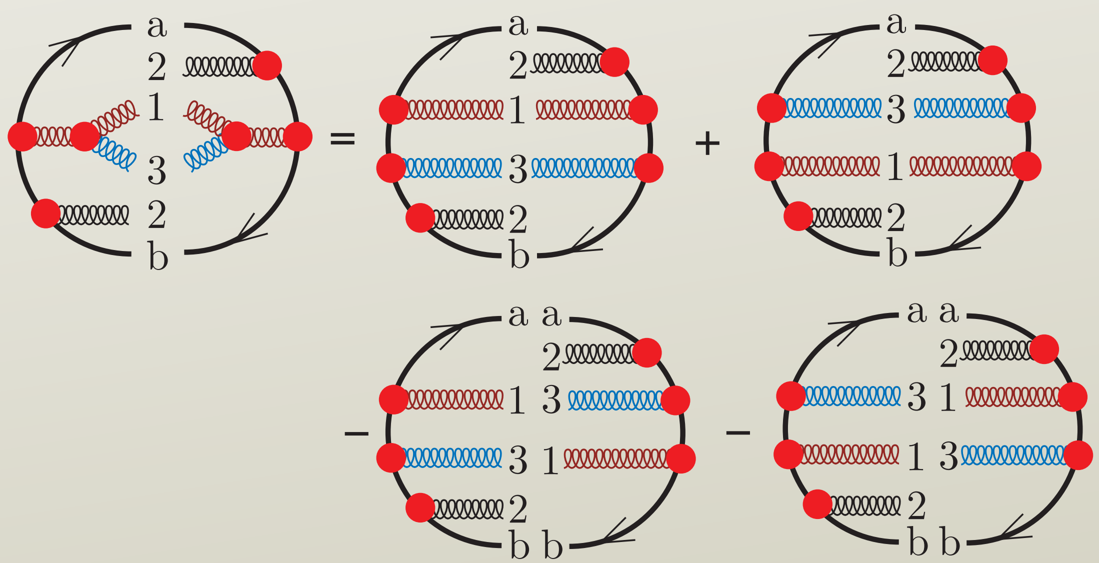
$$\mathcal{N}(t,t') = \mathbb{T}\exp\left\{-\int_{t'}^{t} d\tau\,\mathcal{V}(\tau)\right\}$$

this is the Sudakov factor

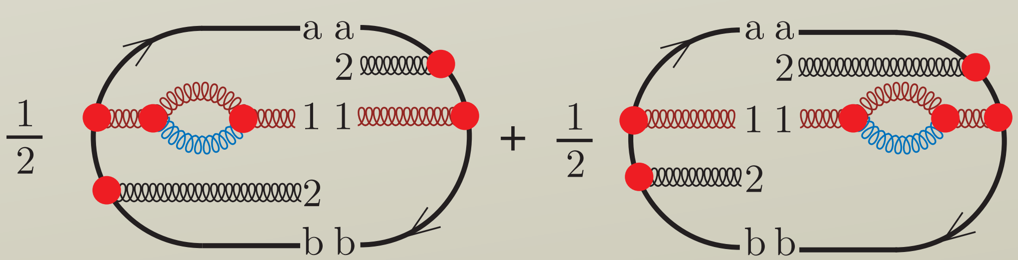
How is this possible?

- For terms kept, the Sudakov exponent needs to be a number not a matrix in the color space.

- For this splitting, keep all terms.



- The corresponding contribution to $\mathcal{V}(t)$ has the color structure

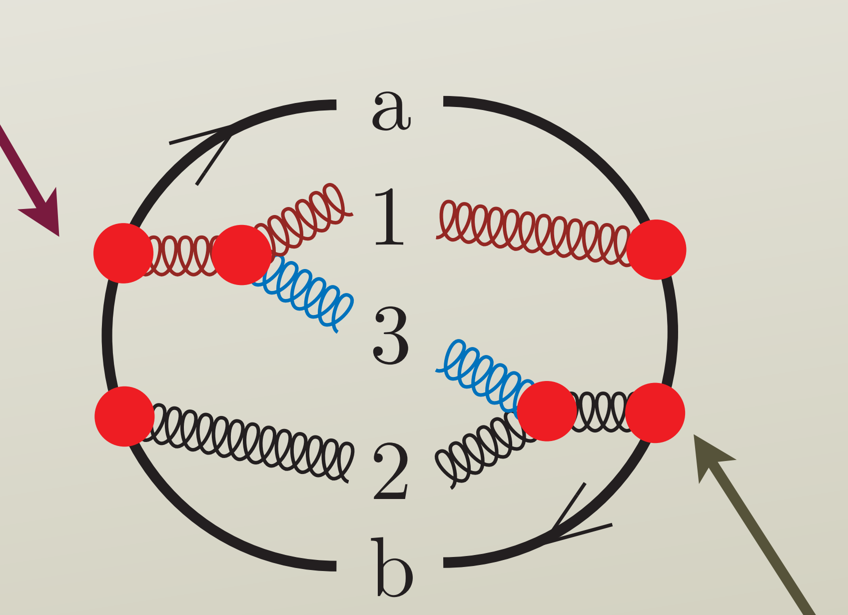


- The color loops simply give a factor C_{A}.

Interference graphs

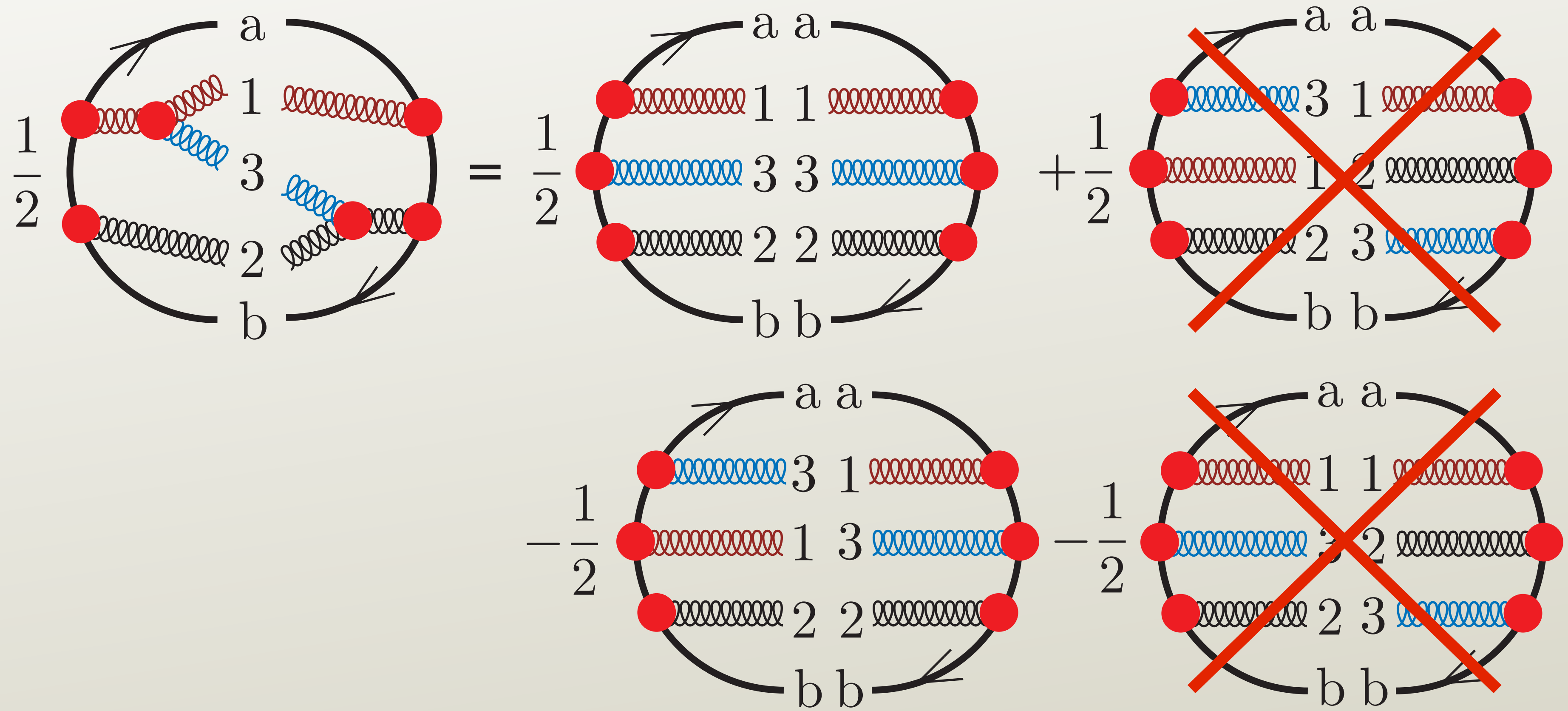
- Interference graphs are important for soft gluon emission.

One parton is the "emitter."

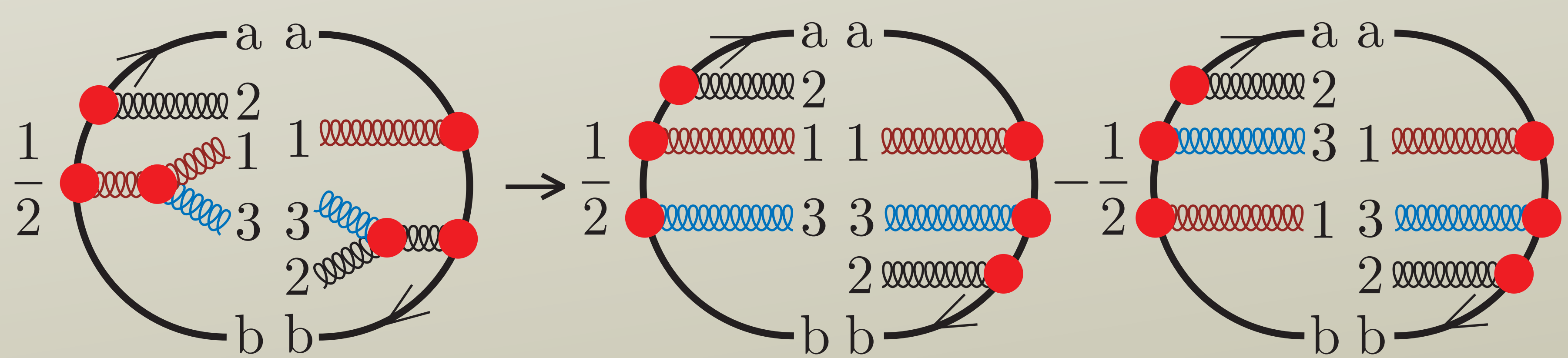


The other is the "helper."

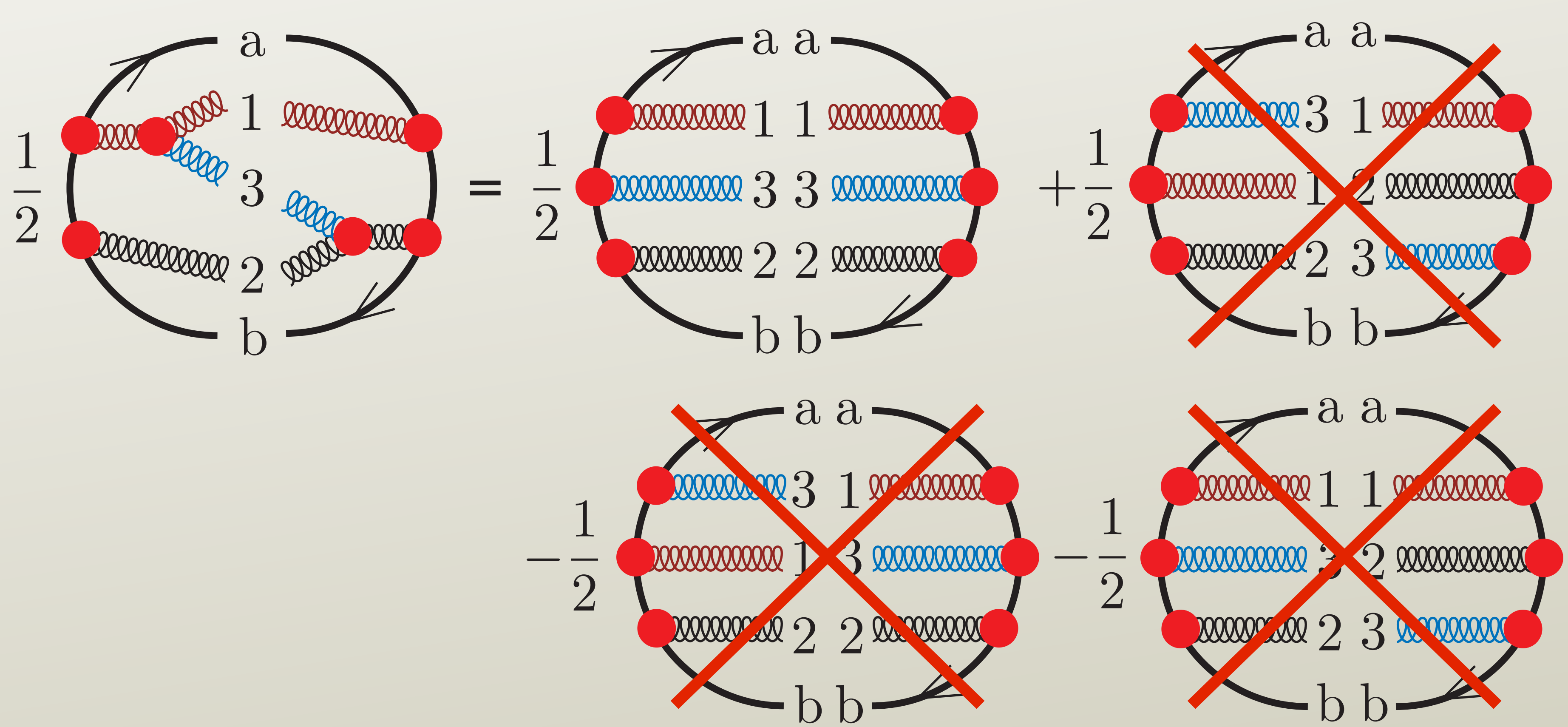
- The LC+ approximation keeps two contributions.



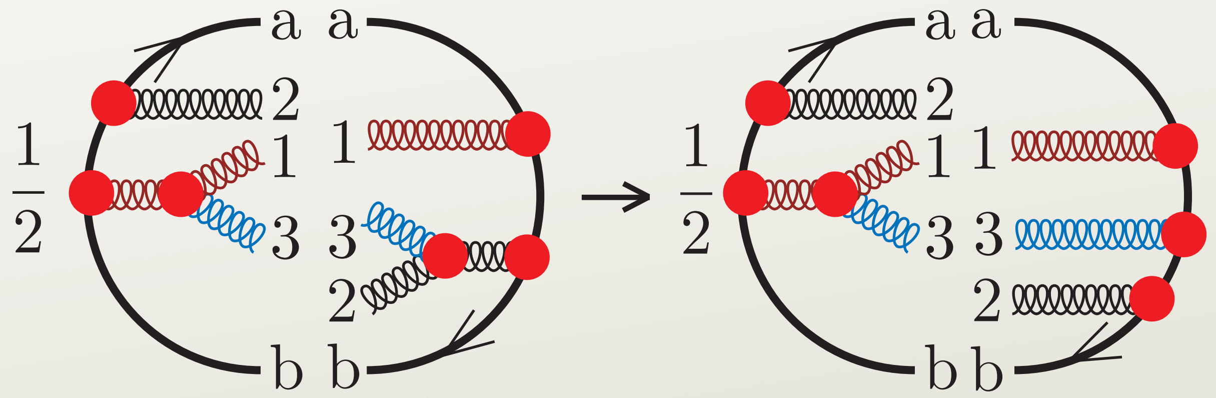
- Another example, starting from a non-diagonal state:



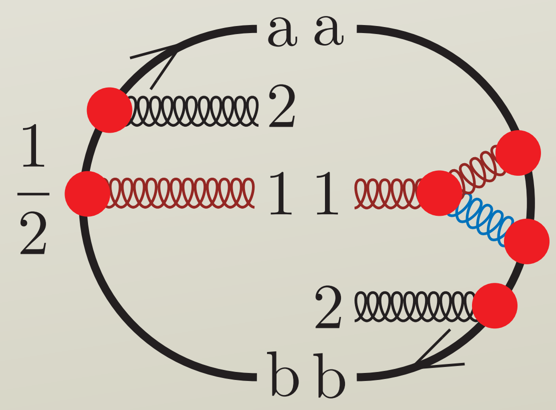
- The LC approximation keeps just one contribution.



- This amounts to



- The corresponding contribution to $\mathcal{V}(t)$:



- There is no color matrix. Just a factor $C_{\mathrm{A}}/4$.