Approximate NNLO Corrections to Hadronic Jet Production

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Outline

- Approximate NNLO results *de Florian, Hinderer, Mukherjee, Ringer, Vogelsang '13*
- Mellin code and NNLL
- Conclusions

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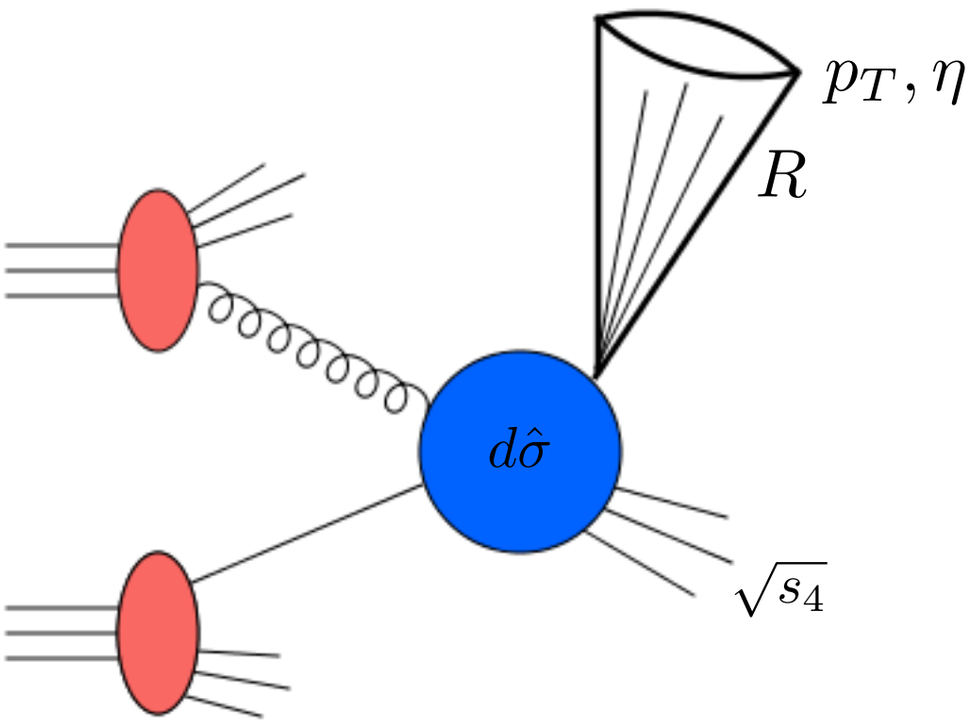
Single-inclusive jet production



$\text{anti}-k_T$ **algorithm** $\quad p=-1$

$$d_{jk} \equiv \min(k_{T_j}^{2p}, k_{T_k}^{2p}) \frac{R_{jk}^2}{R^2}$$

$$R_{jk}^2 \equiv (\eta_j - \eta_k)^2 + (\phi_j - \phi_k)^2$$

Cacciari, Salam, Soyez '08

cross section

$$\frac{p_T^2 d^2\sigma}{dp_T^2 d\eta} = \sum_{ab} \int_0^{V(1-W)} dz \int_{\frac{VW}{1-z}}^{1-\frac{1-V}{1-z}} dv\, x_a f_a(x_a,\mu_f)\, x_b f_b(x_b,\mu_f) \frac{d\hat{\sigma}_{ab}}{dv dz}(v,z,p_T,\mu_r,\mu_f,R)$$

where

$$v = \frac{u}{t+u}, \qquad z = \frac{s_4}{s}$$

$$V = 1 - x_T e^{-\eta}/2, \; VW = x_T e^{\eta}/2$$

$$t = (p_a - p_J)^2, \; u = (p_b - p_J)^2$$

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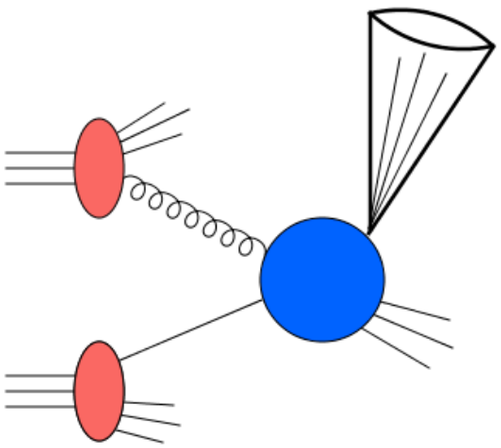

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$$\frac{s d\hat{\sigma}_{ab}}{dvdz} = \left(\frac{\alpha_s}{\pi}\right)^2 \left[\omega_{ab}^{(0)} + \frac{\alpha_s}{\pi}\omega_{ab}^{(1)} + \left(\frac{\alpha_s}{\pi}\right)^2 \omega_{ab}^{(2)} + \mathcal{O}(\alpha_s^3)\right]$$

threshold logarithms $\alpha_s^k \omega_{ab}^{(k)} \sim \alpha_s^k \left(\frac{\log^m(z)}{z}\right)_+ , \;\; \text{with} \;\; 0 \le m \le 2k-1$

threshold set by: $z = \frac{s_4}{s} \to 0$

Mellin space $\tilde{\omega}_{ab}(v,N) \equiv \int_0^1 dz (1-z)^{N-1} \frac{s d\hat{\sigma}_{ab}}{dvdz}$

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Threshold resummation

$$\tilde{\Omega}^{\rm res}_{ab}(v,N) = \sum_{c,d} \Delta_a(N_a)\,\Delta_b(N_b)\,J_c^{(\rm jet)}(N,R)\,J_d^{(\rm recoil)}(N) \times \Delta^{(\rm int)}_{ab\to cd}(N,v)\,\Delta_c^{(\rm ng)}(N)$$

Kidonakis, Sterman '97

initial state: soft-collinear

$$\Delta_i(N_i) = -\int_0^1 dz \frac{z^{N_i-1}-1}{1-z}\left[\int_{(1-z)^2}^1 \frac{d\lambda}{\lambda} A^{(f_i)}[\alpha_s(\lambda(2p_i\cdot\zeta)^2)] + \frac{1}{2}\nu^{(f_i)}[\alpha_s((1-z)^2(2p_i\cdot\zeta)^2)]\right]$$

$$N_a = N(-u/s) \qquad N_b = N(-t/s)$$

final state: unobserved jet

$$J_d^{(\rm recoil)}(N) = \int_0^1 dz \frac{z^{N-1}-1}{1-z}\left[\int_{(1-z)^2}^{(1-z)} \frac{d\lambda}{\lambda} A^{(f_2)}[\alpha_s(\lambda s)] + B_2'[\alpha_s((1-z)s)] + B_2''[\alpha_s((1-z)^2 s)]\right]$$

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Kidonakis, Sterman '97

soft, large angle gluon emission

$$\Delta^{(\rm int)}_{ab\to cd}(N,v) = {\rm tr}\left\{ H(\alpha_s(\mu_R^2))\,\bar{\mathcal{P}}\exp\left[\frac{1}{2}\int_s^{s/\bar{N}^2}\frac{d\mu^2}{\mu^2}\Gamma_S^{(f)\dagger}(\alpha_s(\mu^2))\right] S(\alpha_s(s/\bar{N}^2))\,\mathcal{P}\exp\left[\frac{1}{2}\int_s^{s/\bar{N}^2}\frac{d\mu^2}{\mu^2}\Gamma_S^{(f)}(\alpha_s(\mu^2))\right]\right\}$$

trace in color space

hard scattering matrix $H = H^{(0)} + \frac{\alpha_s}{\pi}H^{(1)}$

soft matrix $S = S^{(0)} + \frac{\alpha_s}{\pi}S^{(1)}$

$$(\Gamma_S^{(f)})_{KL} = (\Gamma_{S'}^{(f)})_{KL} + \delta_{KL}\frac{\alpha_s}{\pi}\sum_{i=a,b,1,2} C_{(f_i)}\frac{1}{2}[-\ln(2\nu_i)+1-i\pi]$$

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Threshold resummation

?

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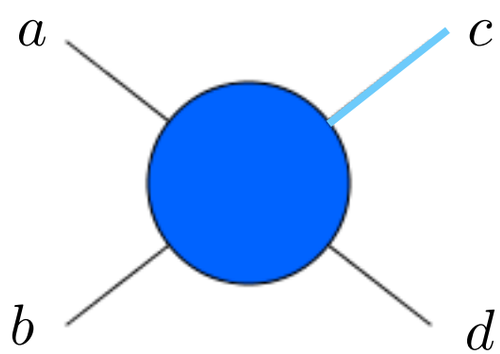
Jet mass at threshold

LO:



→ jet massless

Kidonakis, Oderda, Sterman '98

scheme 1: keep jet massless at threshold

$$\exp\left[\frac{\alpha_s}{\pi}\ln^2\bar{N}\left(C_a+C_b-\frac{1}{2}C_c-\frac{1}{2}C_d\right)\right]$$

no dependence on R

Kidonakis, Owens '00; Moch, Kumar '13

scheme 2: jet allowed to be massive at threshold

$$\exp\left[\frac{\alpha_s}{\pi}\ln^2\bar{N}\left(C_a+C_b-\frac{1}{2}C_d\right)+\frac{\alpha_s}{\pi}C_c\,\ln(R)\ln(\bar{N})\right]$$

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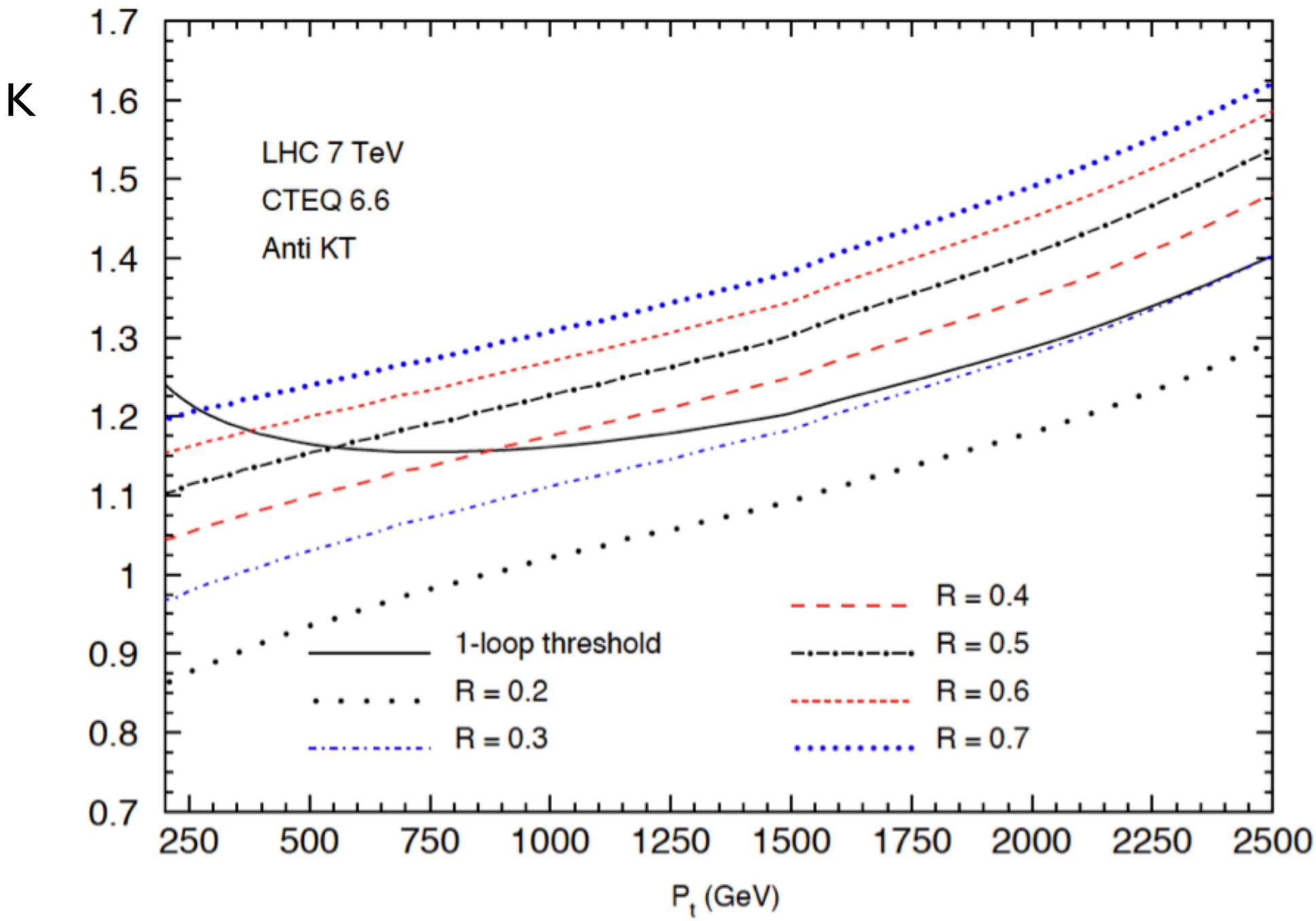

scheme 1

Moch, Kumar '13

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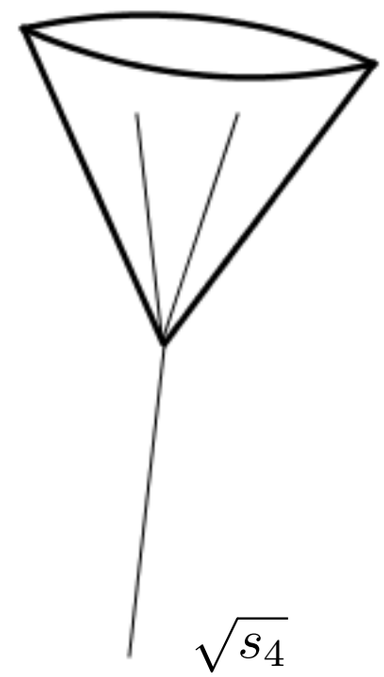
Jet massive at threshold

possible final state at NLO

threshold is set by $\sqrt{s_4} = 0$



$$\log J_c^{(\mathrm{jet})} = \int_s^{s/\bar{N}^2} \frac{dq^2}{q^2} \alpha_s(q^2) \left(-\frac{C_c}{2\pi} \log\left(\frac{p_T^2 R^2}{s} \right) \right)$$

dependence on R

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Analytical results at NLO

Full analytical NLO calculation in the "Narrow Jet Approximation"

- $\mathcal{A}\log R + \mathcal{B} + \mathcal{O}(R^2)$
- $\mathrm{anti}-k_T$ algorithm
- accurate to better than 2-3% even for large R

Jäger, Stratmann, Vogelsang '04; Mukherjee, Vogelsang '13

matching coefficent for NLL accuarcy

$$\tilde{\omega}^{\text{1st order}}_{qq'\to qq'}(N) = \mathrm{Tr}\left\{H^{(0)}S^{(0)}\right\}\left[1+\frac{\alpha_s}{\pi}C_F\left(\frac{5}{2}\ln^2\bar{N}+\frac{3}{4}\ln\bar{N}+2\ln\bar{N}\log(v(1-v))+3\ln\bar{N}\log\left(\frac{\mu_F^2}{s}\right)\right)\right]$$
$$+\frac{\alpha_s}{\pi}\mathrm{Tr}\left\{-\left[H^{(0)}(\Gamma^{(1)})^\dagger S^{(0)}+H^{(0)}S^{(0)}\Gamma^{(1)}\right]\ln\bar{N}\right.$$
$$\left.+H^{(1)}S^{(0)}+H^{(0)}S^{(1)}\right\}$$

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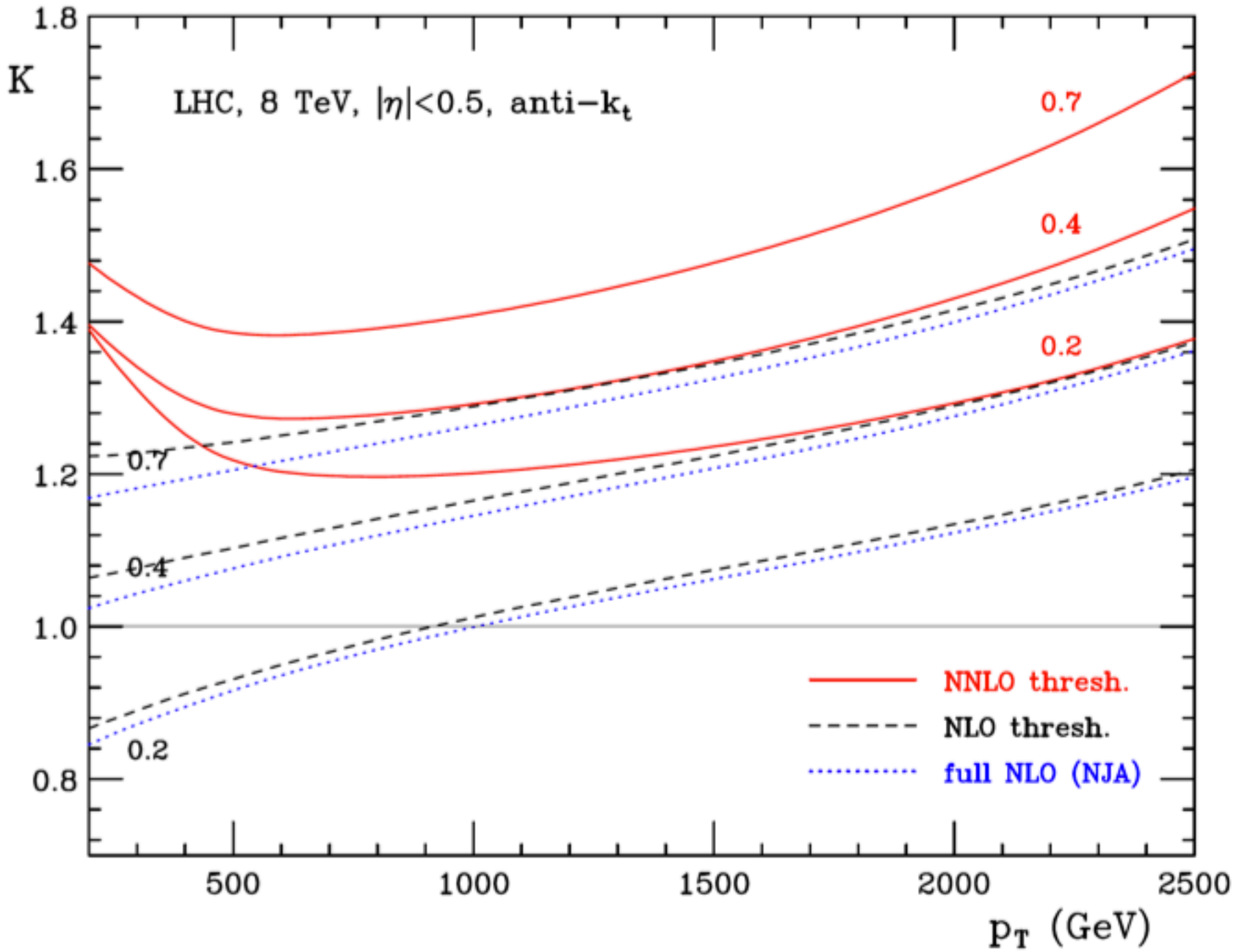

using MSTW-nnlo PDFs

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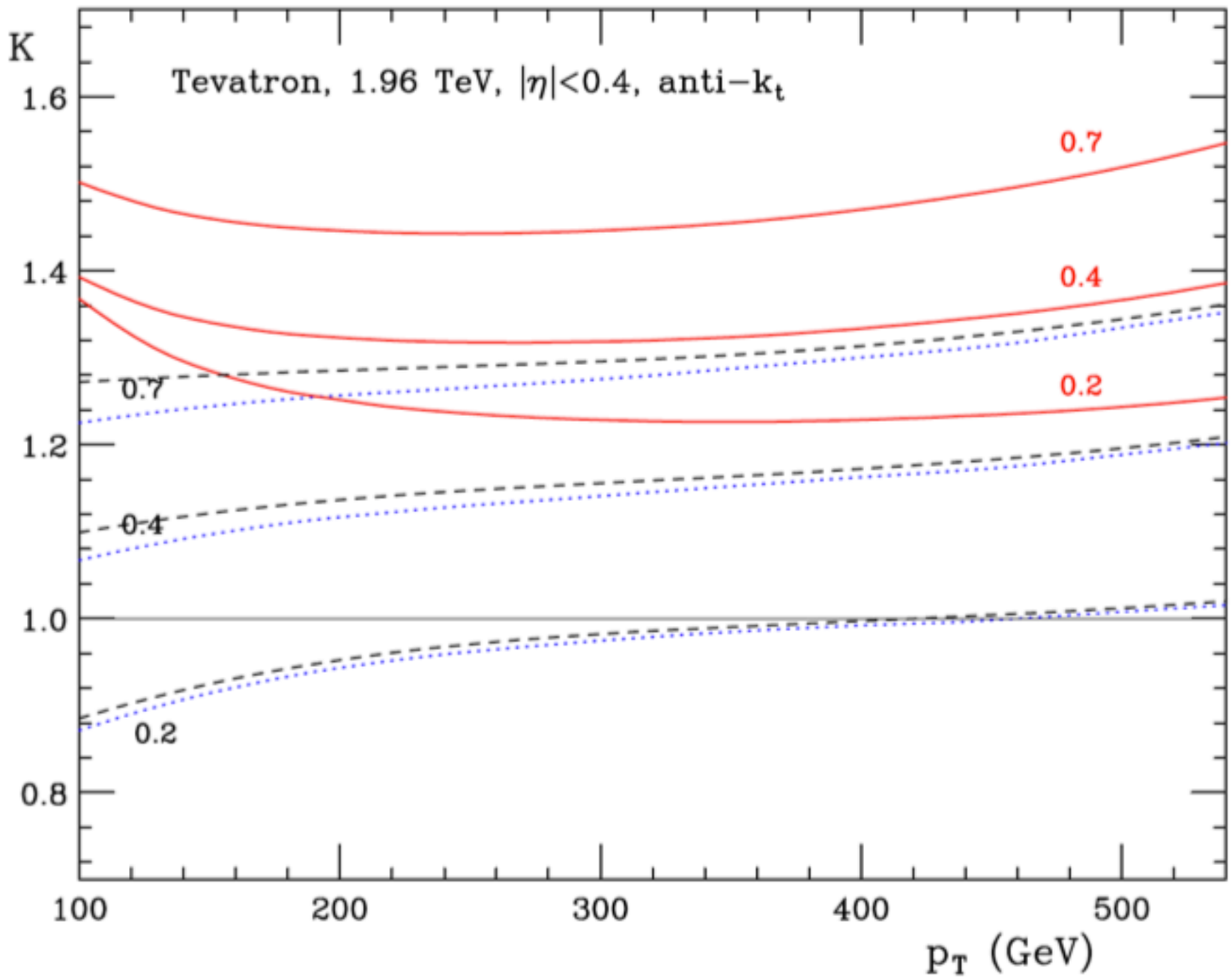

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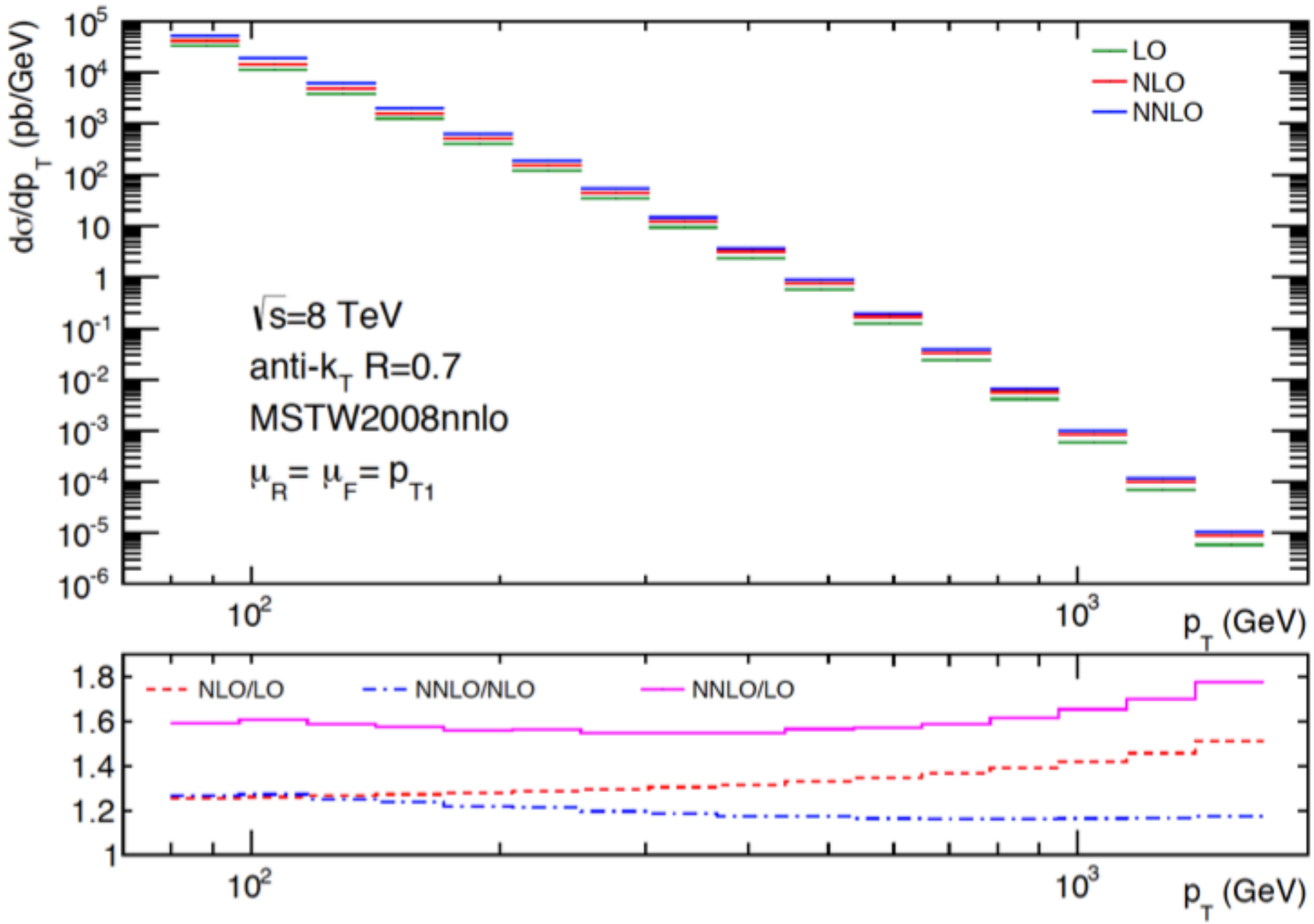

→ see talk by Joao Pires

Currie, Gehrmann-De-Ridder, Glover, Pires '13

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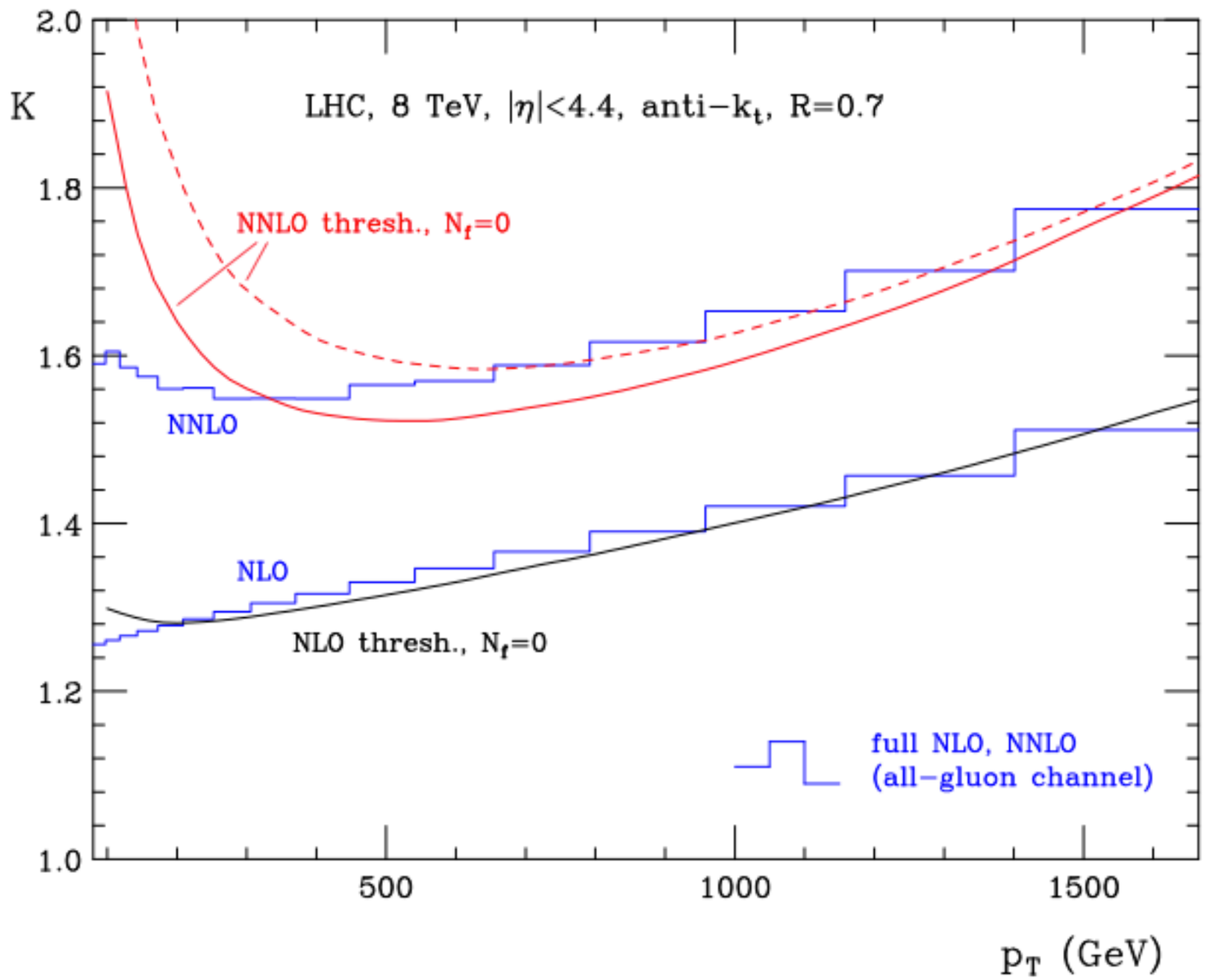
K-factors in the gluon-only channel

$$v = \frac{u}{t+u}$$

$$v' = \frac{t+s}{s}$$

scale choice

$$p_{T,1} \neq p_T$$



Currie, Gehrmann-De-Ridder, Glover, Pires '13

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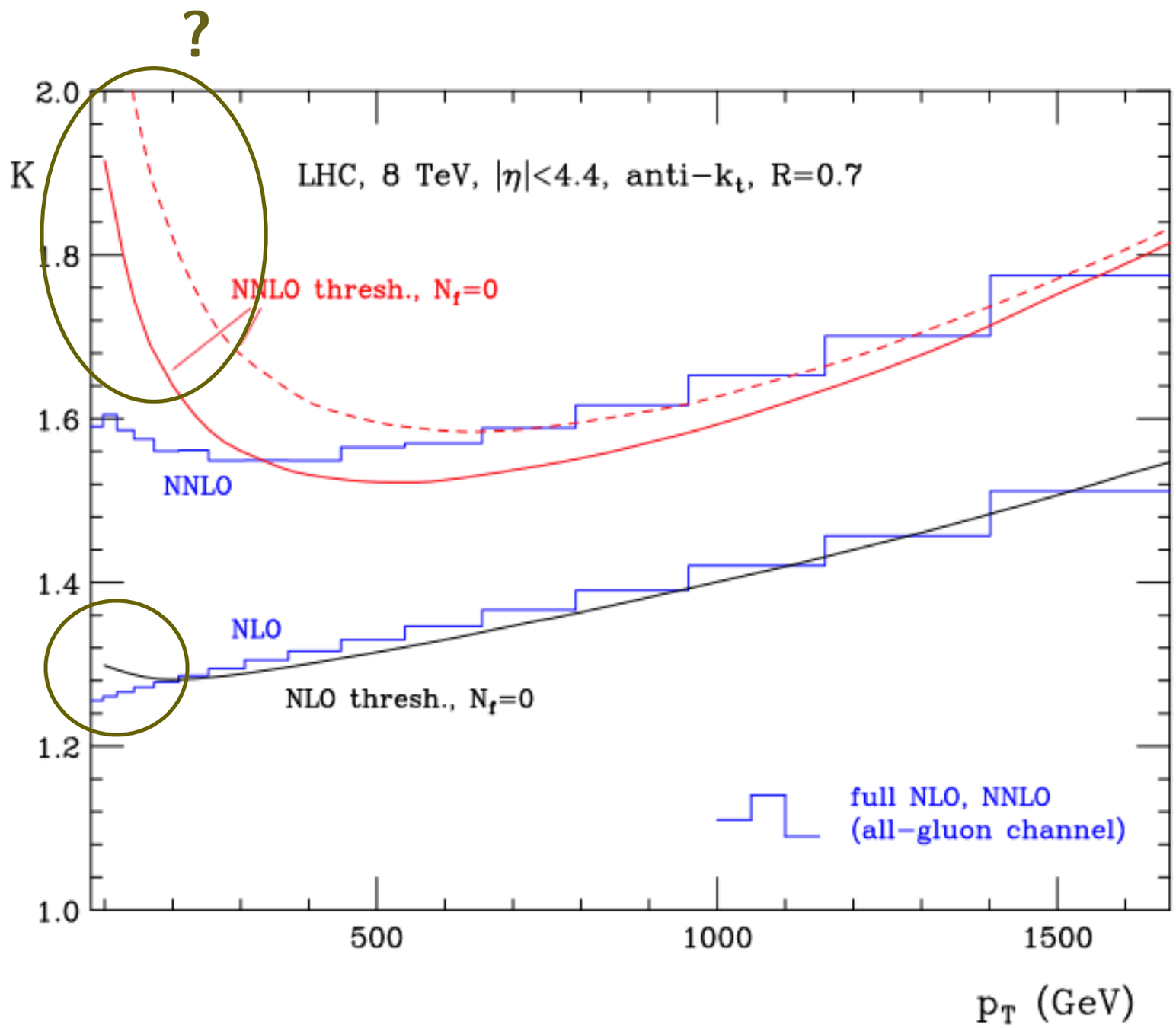

Currie, Gehrmann-De-Ridder, Glover, Pires '13

using MSTW-nnlo PDFs

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de Florian, Hinderer, Mukherjee, Ringer, Vogelsang '13

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Mellin space code

Expanded resummed function in Mellin space

$$\tilde{\Omega}(N) = f_{\rm Born}(v) + \alpha_s \left(f_2(v) L^2 + f_1(v) L + f_0(v) \right) + \alpha_s^2 \left(g_4(v) L^4 + g_3(v) L^3 + g_2(v) L^2 + \ldots \right) + \ldots$$

$$L = \log(\bar{N})$$

Exact Mellin moments of full set of distributions in z-space

$$\tilde{\omega}(N) \equiv \int_0^1 dz\, z^{N-1} \omega(z)$$

Compute Mellin inverse of $\tilde{\Omega}(N)$ via

$$\Omega(z) = \omega(z) + \frac{1}{2\pi i} \int dN\, z^{-N} \left(\tilde{\Omega}(N) - \tilde{\omega}(N) \right)$$

previous results for cross section differential in p_T only

de Florian, Vogelsang '13

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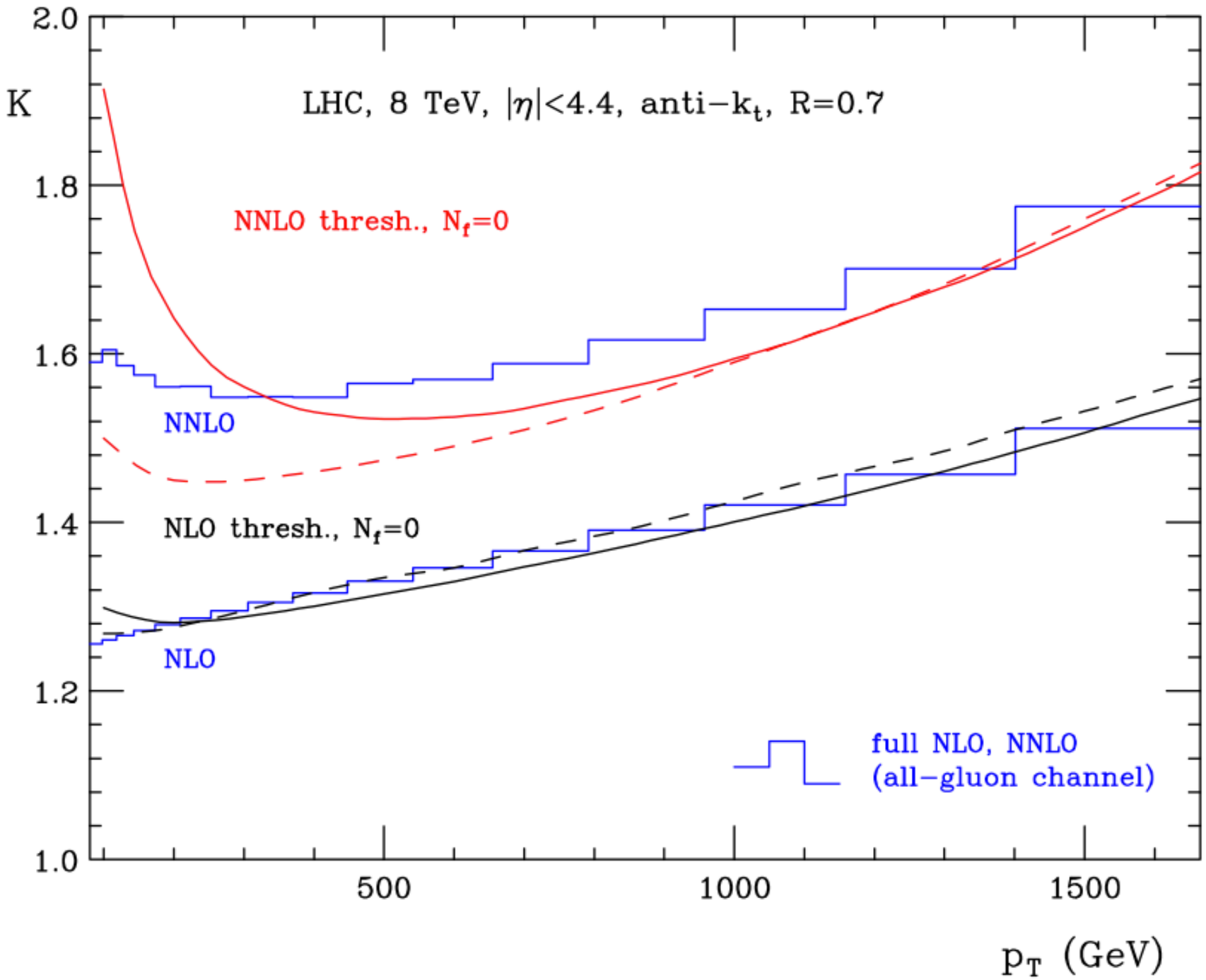

solid : $z-$space
dashed : Mellin $-$ space

(work in progress)

- numerical results are very sensitive to the 4th tower of threshold logarithms
- in particular the μ_R scale dependence is relevant
- even more problematic for higher $\mathrm{N^kLO}$ expansions

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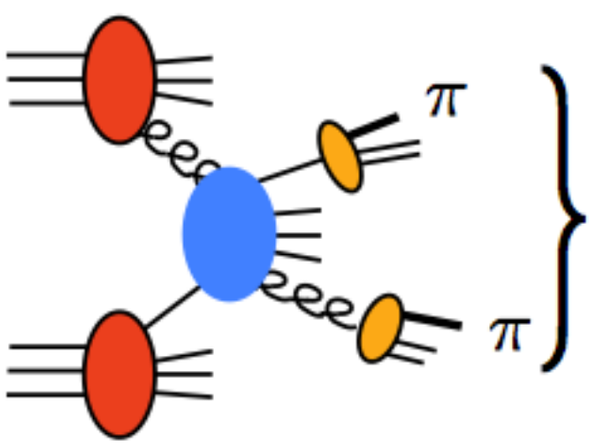
towards NNLL accuracy

- resummed exponents $\Delta(N_a),\ J_c^{\rm jet}(N,R), J_d^{\rm recoil}(N)$
- Gamma matrix $\Gamma^{(2)}$ *Aybat, Dixon, Sterman '06*
- decomposition in color space for $H^{(1)},\ S^{(1)}$

cf. Di-Hadron production in hadronic collisions



pair mass^2

$$M^2 = (p_\pi + p'_\pi)^2$$

"like" Drell-Yan

Hinderer, Ringer, Sterman, Vogelsang (work in progress)

Kelley, Schwartz '10

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Accuracy of Resummation

$$\mathcal{O}(\alpha_s^k): \qquad C_{kn} \times \alpha_s^k \ln^n \bar{N}, \quad \text{where } n \leq 2k$$

Fixed Order

Resummation

LO	1					
NLO	$\alpha_s L^2$	$\alpha_s L$	α_s			
NNLO	$\alpha_s^2 L^4$	$\alpha_s^2 L^3$	$\alpha_s^2 L^2$	$\alpha_s^2 L$	α_s^2	
...	...	...	...	...	...	
N^{k}LO	$\alpha_s^k L^{2k}$	$\alpha_s^k L^{2k-1}$	$\alpha_s^k L^{2k-2}$	$\alpha_s^k L^{2k-3}$	$\alpha_s^k L^{2k-4}$	...

LL NLL NNLL

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Conclusions

- Approximate results for hadronic jet production at NNLO
- Promising results from the Mellin space code
- Extension to NNLL is crucial
- Proper treatment of non-global logarithms required at NNLO and beyond

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Back-up slides

Renormalization scale dependence at NNLO

general structure

$$d\sigma = \sum_{n=0}^{N} A_n(\mu_R)\alpha_s^{n+2}(\mu_R)$$

up to NNLO

$$\begin{aligned} d\sigma = \alpha_s^2\ A_0 \\ + \alpha_s^3\ (A_1' + 2b_0\, L\, A_0) \\ + \alpha_s^4\ \left(A_2' + 3b_0\, L\, A_1' + (3b_0^2\, L^2 + 2b_1\, L)A_0\right) \end{aligned}$$

see for example Glover '02

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