PHYSICS AT THE TERA SCALE

Helmholtz Alliance

Jets in Hadron Colliders at Higher Order

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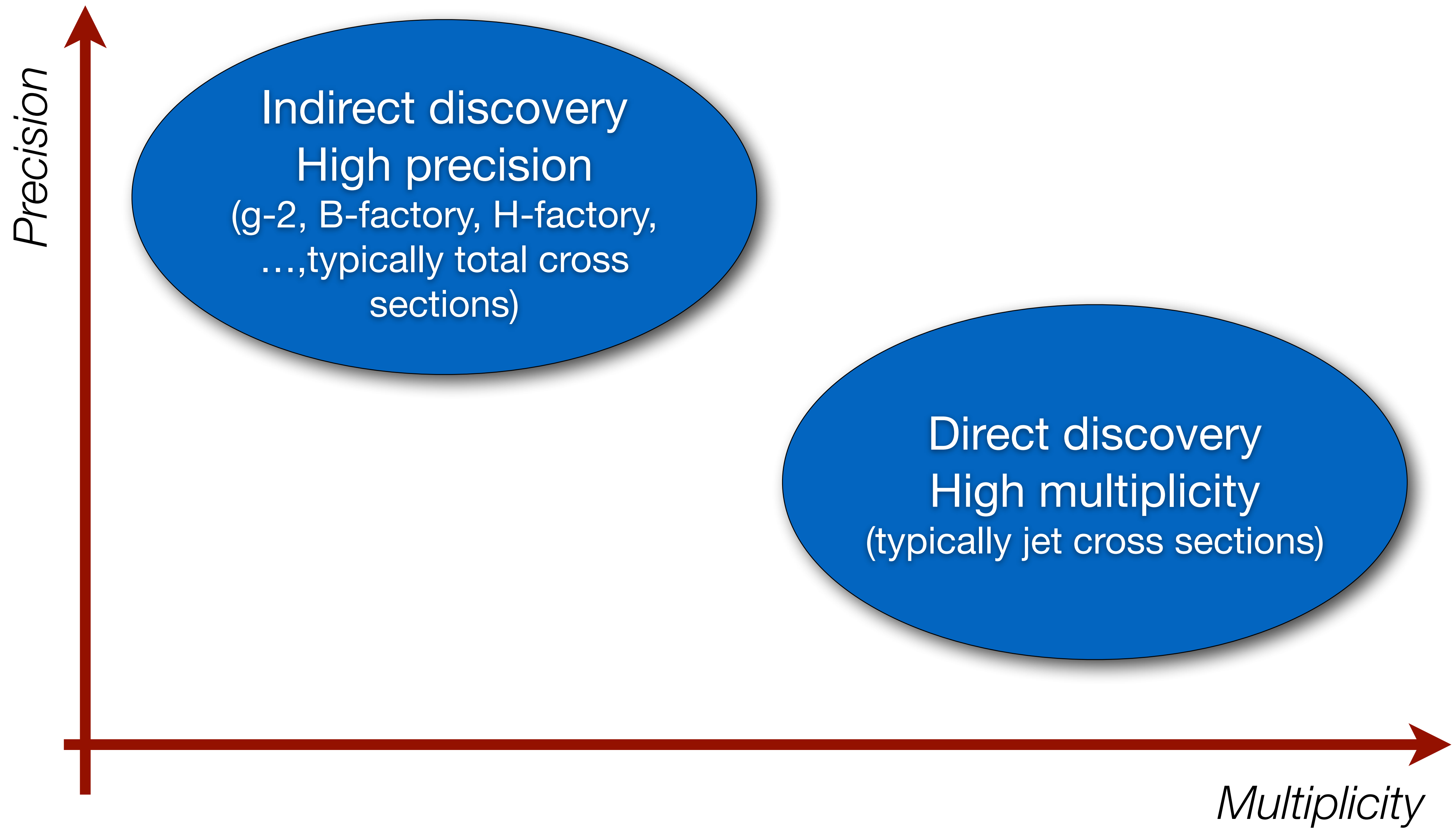
DESY

PDF School, September 29-30, 2014, DESY

HEP Arena (Experiment)

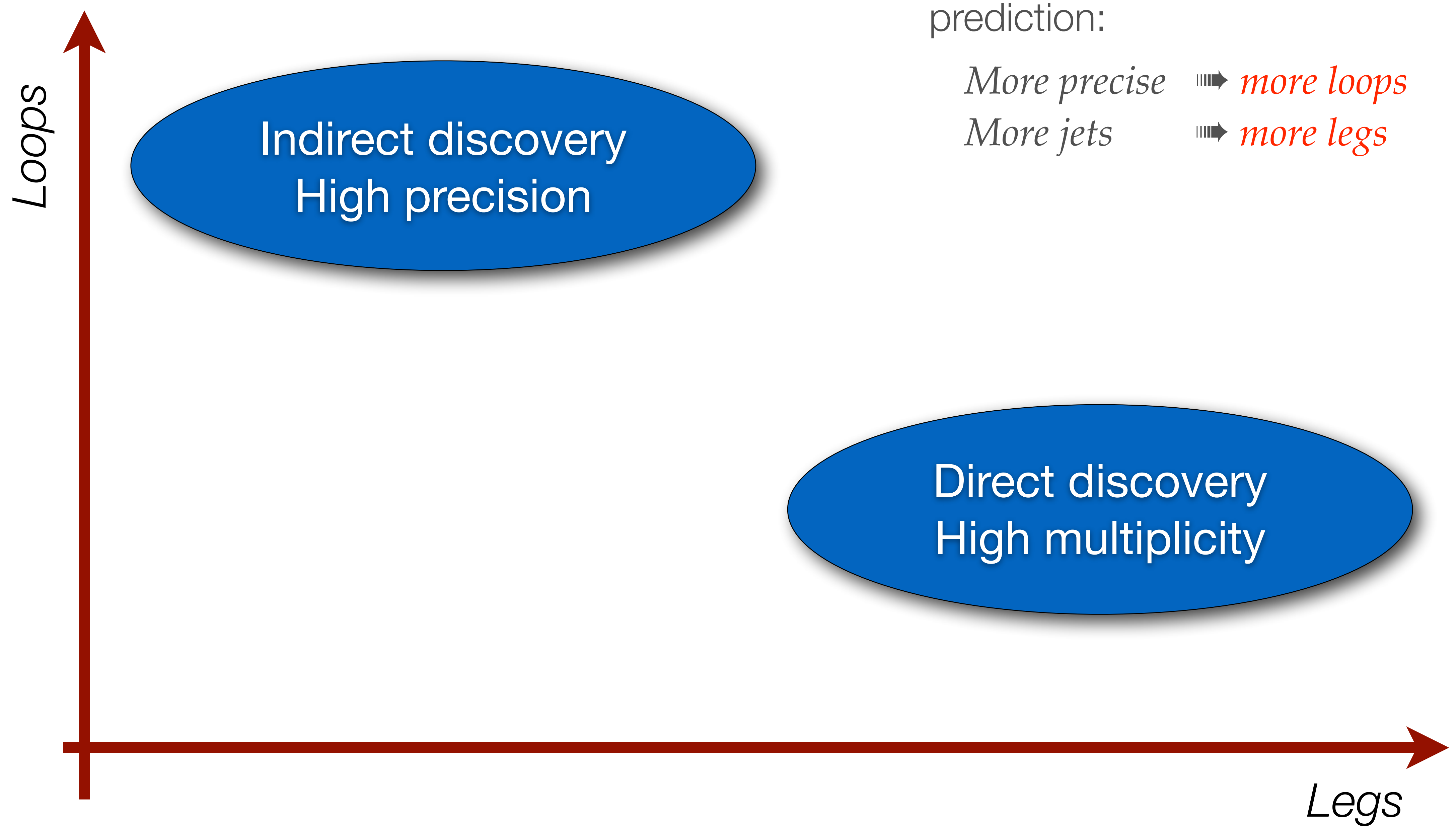


What does it mean from theory point of view?

HEP Arena (Theory)

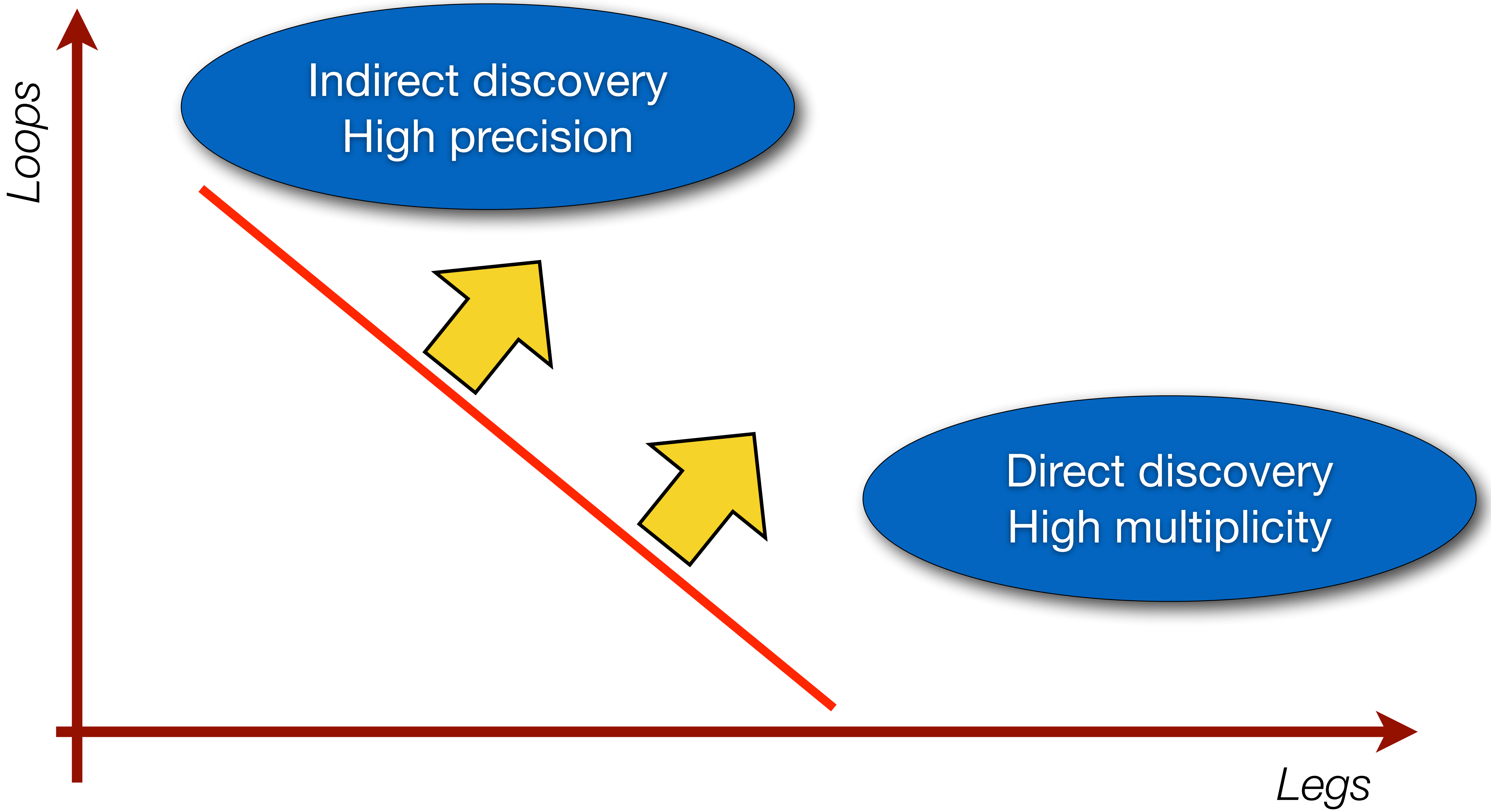
Perturbative field theory provides a systematic basis for making theory prediction:

More precise ➠ *more loops*
More jets ➠ *more legs*



HEP Arena (Theory)

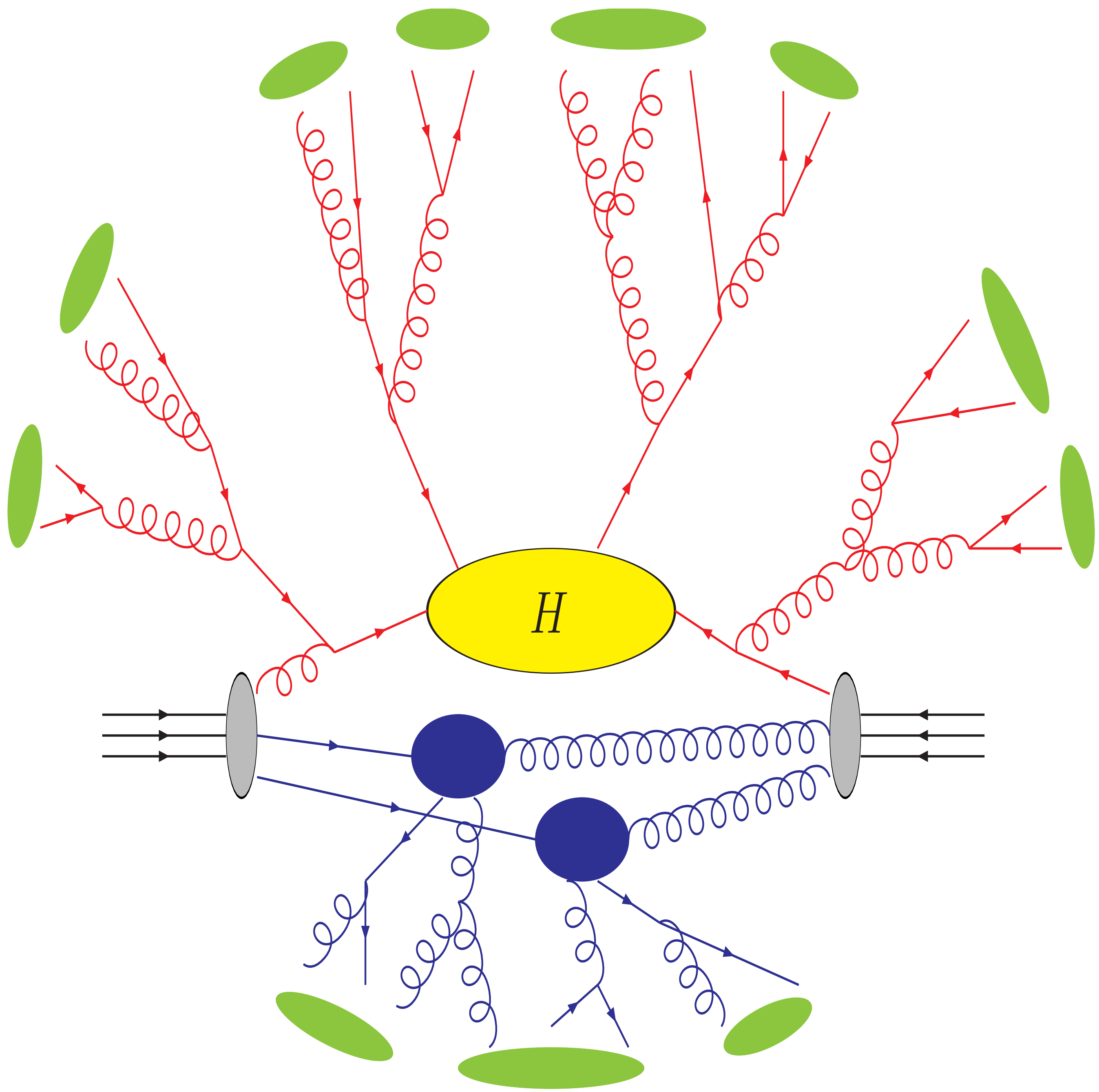
$$\text{Complexity} \sim \exp\left(\#\text{loops} + \#\text{legs}\right)$$



Increasing complexity by 1unit takes about 10 years…

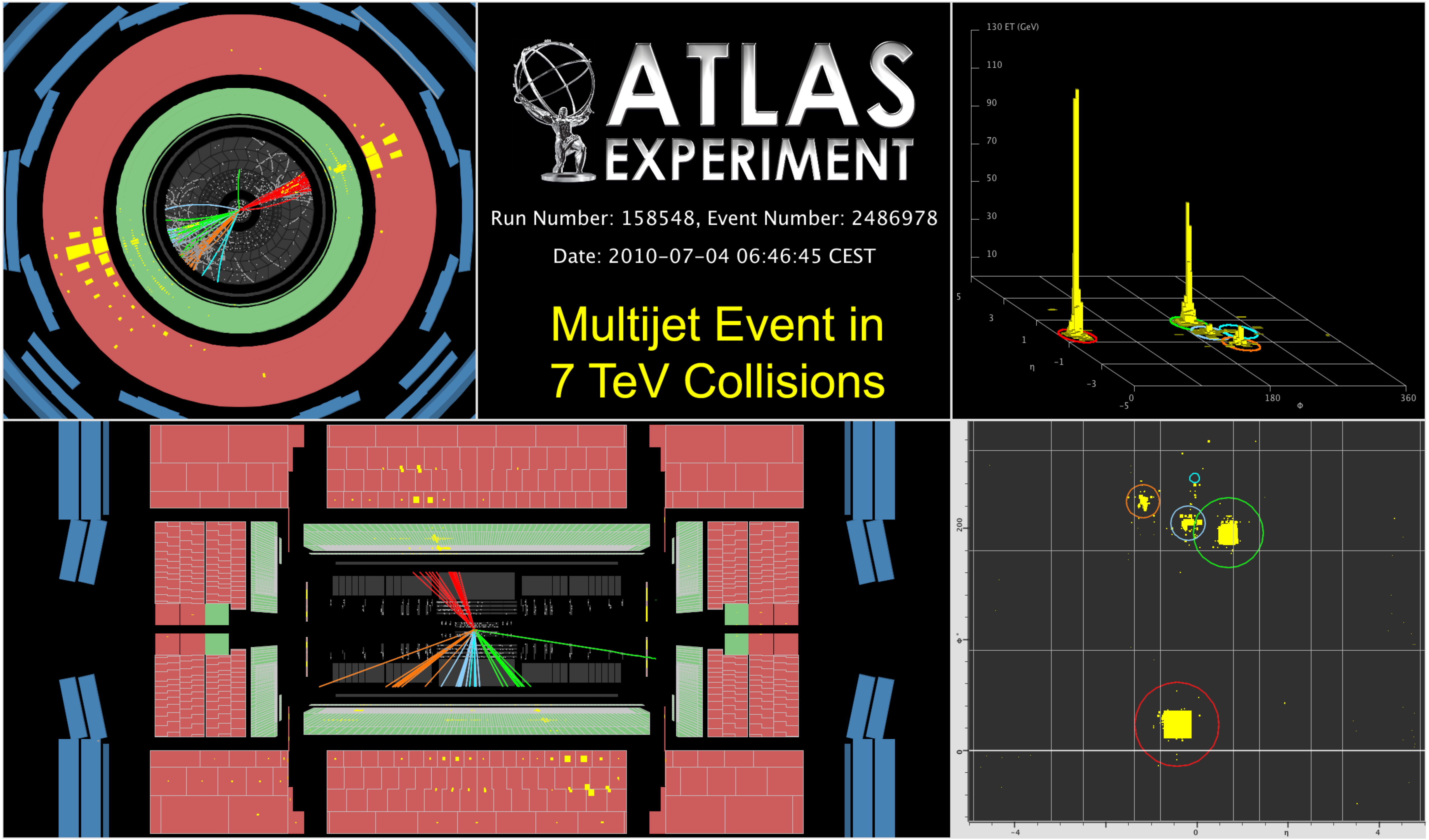
Structure of the theory predictions

From theory point of view an event at the LHC looks very complicated



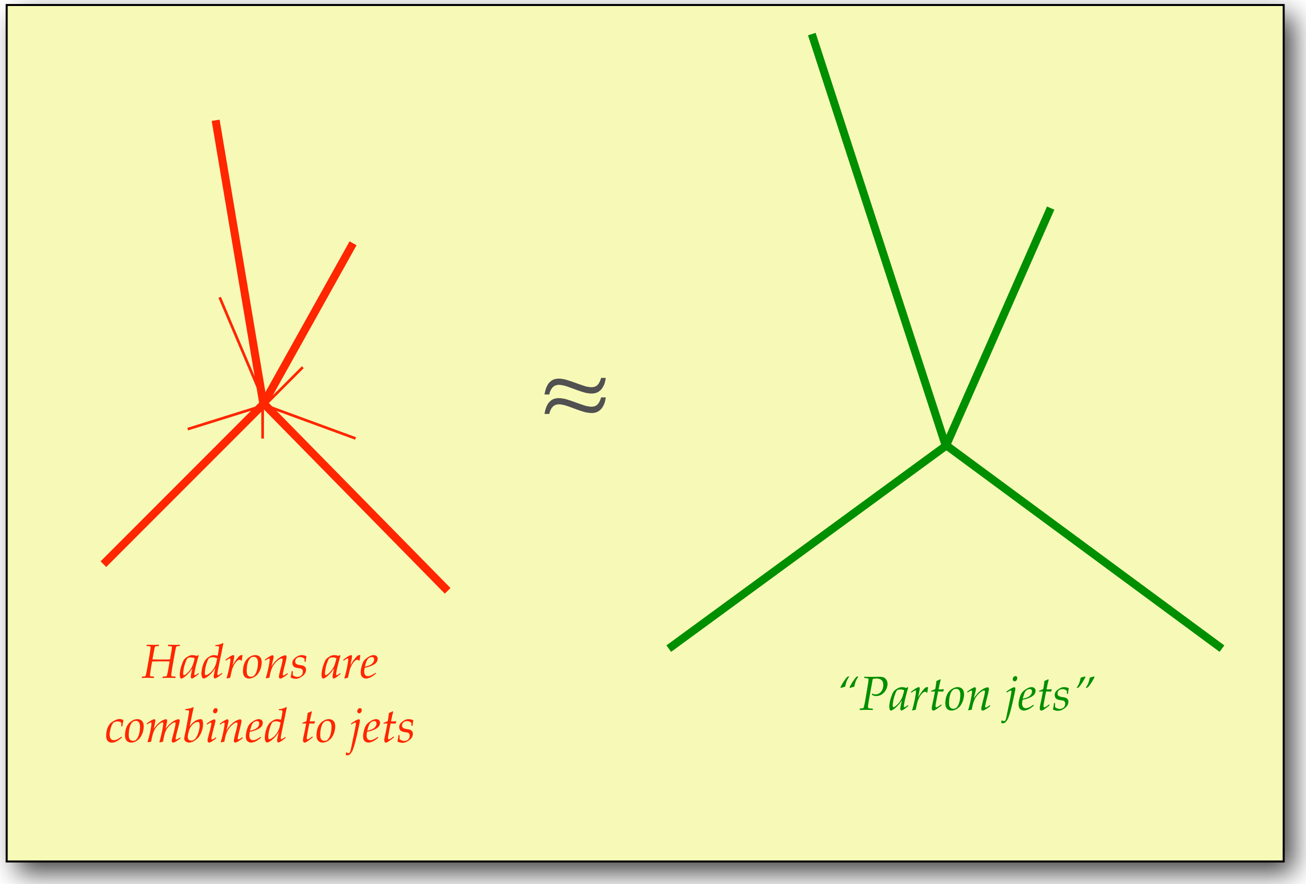
1. Incoming hadron (gray bubbles)
   - Parton distribution function
   - Multi parton distribution functions
2. Hard part of the process (yellow bubble)
   - Matrix element calculation, cross sections at LO, NLO, NNLO level
3. Radiation (red graphs)
   - Parton shower calculation
   - Partonic decay
   - Matching to NLO, NNLO
4. Underlying event (blue graphs)
   - Models based on multiple interaction
   - Diffraction
5. Hardonization (green bubbles)
   - Universal models
   - Hadronic decay
   - ....

Jet Event


ATLAS
EXPERIMENT
Run Number: 158548, Event Number: 2486978
Date: 2010-07-04 06:46:45 CEST
Multijet Event in
7 TeV Collisions
130 ET (GeV)
110
90
70
50
30
10
5
3
1
η
-1
-3
-5
0
180
360
Φ
200
φ°
0
-4
0
η
4

Jet Observable

- In general with jet observables we try to measure some properties (geometrical) of the hadronic final states and describe them with few variables.
- Perturbative calculations are not reliable for predicting long distance physics effects, but the detector is far away from the interaction point.
- The observables has to be infrared safe. It means the observables are insensitive for long distant effect (soft and collinear radiation).



For every $0 < \lambda < 1$

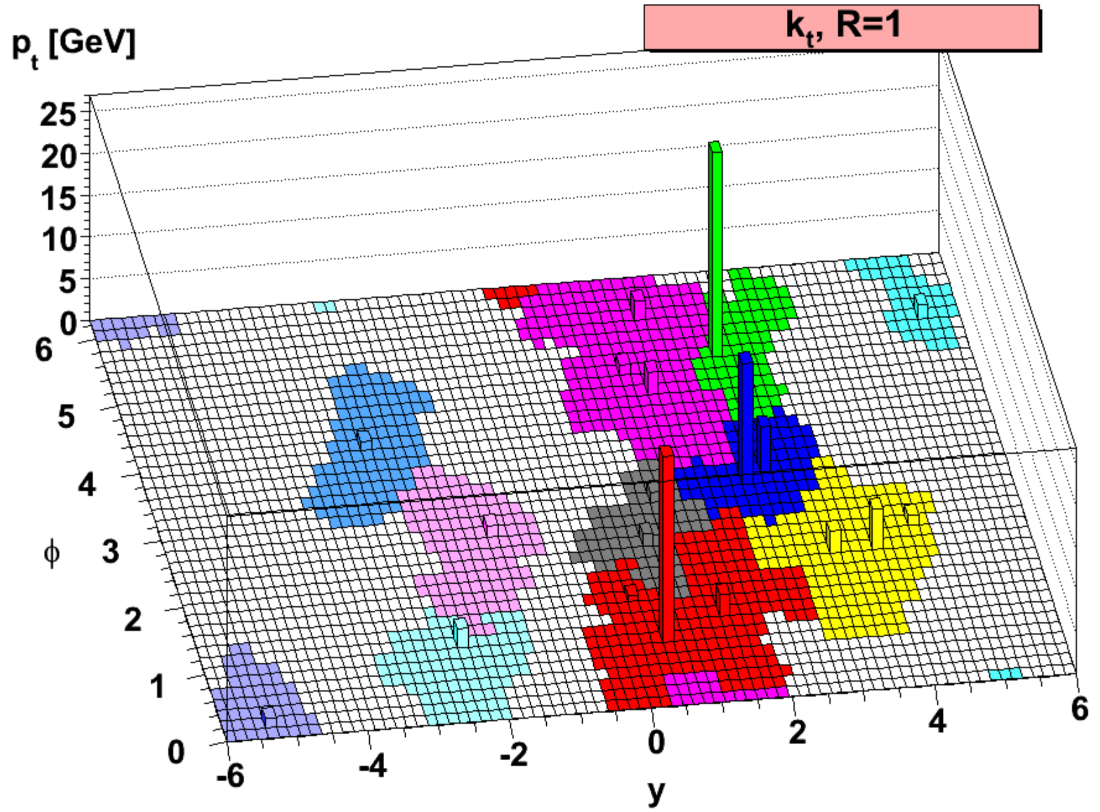
$$O_J(p_1, \ldots, (1-\lambda)p_n, \lambda p_n) = O_J(p_1, \ldots, p_n)$$

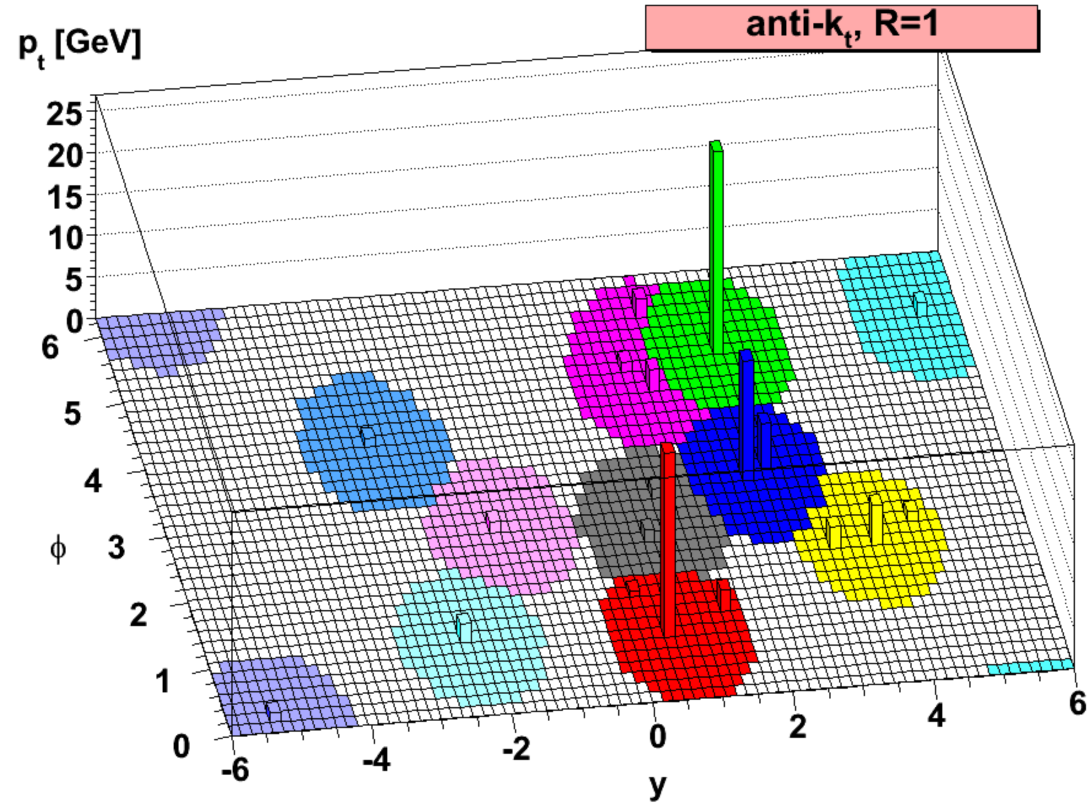
Simplest observable is the total cross section:

$$O_J(p_1, \ldots, p_n) = 1$$

Jet Observables

- In hadron collision we usually use the so called sequential jet algorithms such as kT, anti-kT,…

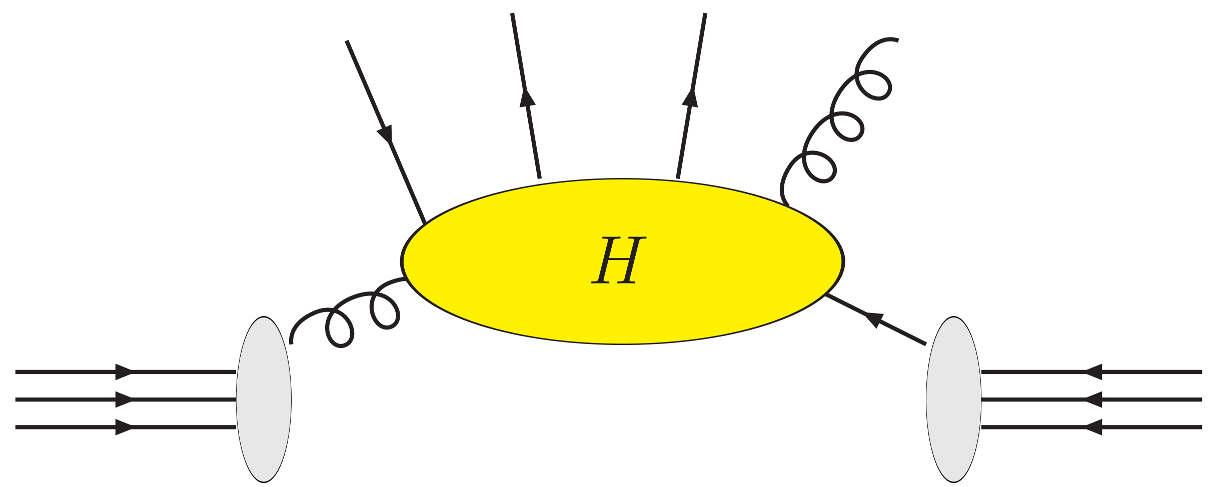




- The other type of jet observables are the event shape variables, such beam thrust, or N-jettiness,…

Perturbative Framework

FIXED ORDER



Let us calculate an N-jet cross section at “all order”. This would be

$$\sigma[O_J] = \sum_{m=N}^{\infty} \int \overbrace{d\{p,f\}_m}^{\textit{phase space integral}} \overbrace{f_{a/A}^{\mathrm{bare}}(\eta_{\mathrm{a}})\, f_{b/B}^{\mathrm{bare}}(\eta_{\mathrm{b}})}^{\textit{parton distributions}}$$

$$\times \sum_{n=0}^{\infty} \underbrace{\left|M^{(n)}(\{p,f\}_m;\mu^2,\alpha_{\mathrm{s}}(\mu^2))\right|^2}_{\textit{matrix element}} \underbrace{O_J(\{p,f\}_m)}_{\textit{observable}}$$

$$\sim \left[\frac{\alpha_{\mathrm{s}}(\mu^2)}{2\pi}\right]^{m+n}$$

Legs and Loops

$m+n = N$ ➠ *LO*

$m+n = N+1$ ➠ *NLO*

$m+n = N+2$ ➠ *NNLO*

……

Perturbative Framework

FIXED ORDER

$$\sigma[O_J] = \sum_{\substack{m=N \\ \text{LEGS}}}^{\infty} \sum_{\substack{n=0 \\ \text{LOOPS}}}^{\infty} \int_m d\hat{\sigma}_m^{(n)}\left(\mu^2, \alpha_s(\mu^2)\right) \otimes F^{\text{bare}}$$

Let us calculate an N-jet cross section at "all order". This would be

$$\sigma[O_J] = \sum_{m=N}^{\infty} \int \overbrace{d\{p,f\}_m}^{\text{phase space integral}} \overbrace{f_{a/A}^{\text{bare}}(\eta_a) f_{b/B}^{\text{bare}}(\eta_b)}^{\text{parton distributions}}$$

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Legs and Loops

$m+n = N$ ➠ *LO*

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$m+n = N+2$ ➠ *NNLO*

……

PDF

The bare pdf is defined in $d = 4 - 2\epsilon$ and it is singular

$$f_{a/A}^{\text{bare}}(\eta) = \left[Z_{a,b}(\mu^2, 1/\epsilon) \otimes K(\mu^2) \otimes f_{b/A}(\mu^2) \right](\eta)$$

Renormalized PDF

The renormalization factor removes the collinear singularities of the hard interaction

$$Z_{a,b}(\mu^2, 1/\epsilon, z) = 1 + \frac{\alpha_s(\mu^2)}{2\pi} \frac{1}{\epsilon} P_{a,b}^{(0)}(z) + \cdots$$

DGLAP kernels

This removes only the singularities and it defines the MSbar scheme. But we can subtract some finite contributions, too.

$$K_{a,b}(\mu^2, z) = 1 + \frac{\alpha_s(\mu^2)}{2\pi} K_{a,b}^{(1)}(z) + \cdots$$

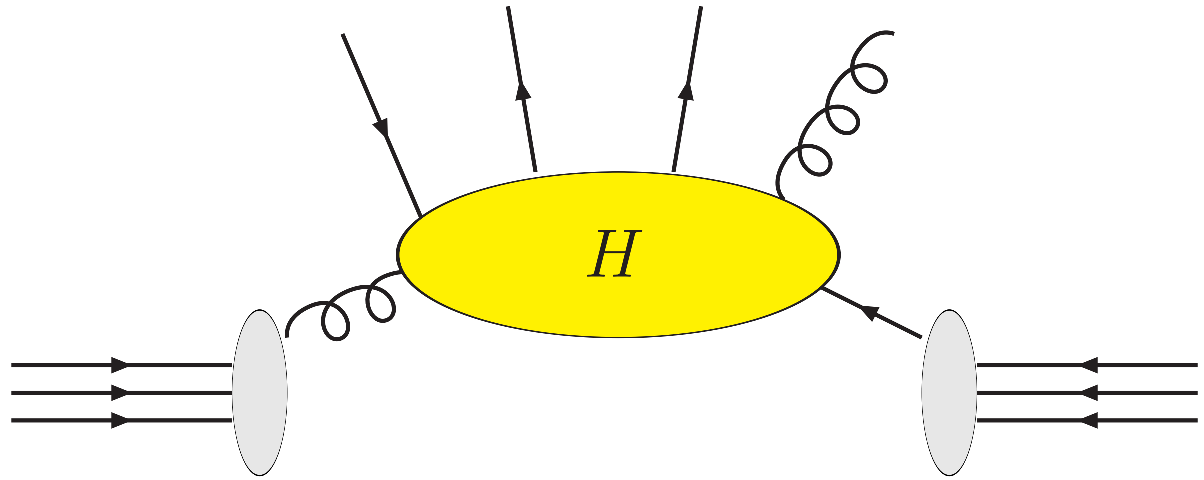
In MSbar it is always

$$K_{a,b}(\mu^2, z) = 1$$

The renormalized PDFS obey the DGLAP evolution equation:

$$\mu^2 \frac{d}{d\mu^2} f_{a/A}(\eta, \mu^2) = \left[P_{a,b}(\mu^2) \otimes f_{a/A}(\mu^2) \right](\eta)$$

LO level



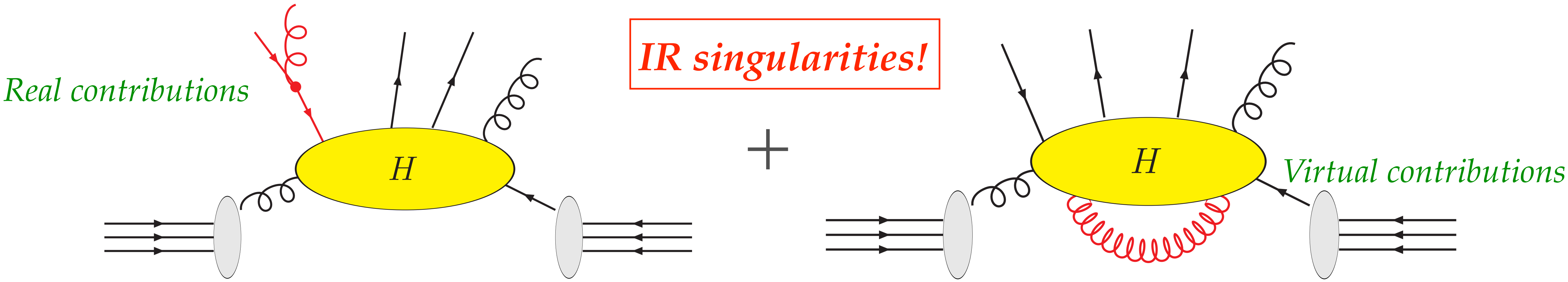
$$\sigma^{LO}[O_J] = \int d\{p,f\}_N \, f_{a/A}(\eta_{\rm a},\mu^2) f_{b/B}(\eta_{\rm b},\mu^2)$$ *Renormalized PDFs*

$$\times \left|M^{(0)}(\{p,f\}_N;\mu^2,\alpha_{\rm s}(\mu^2))\right|^2 O_J(\{p,f\}_m)$$

Only tree level matrix element

- ✓ Easy to calculate, no IR singularities. Several matrix element generators are available (ALPGEN, HELAC, MADGRAPH, SHERPA)
- ✓ It is well defined with LO PDF running.
- ✗ Strong dependence on the unphysical scales (renormalization and factorization scales)
- ✗ Every jet is represented by a single parton
- ✗ No quantum corrections

NLO cross section

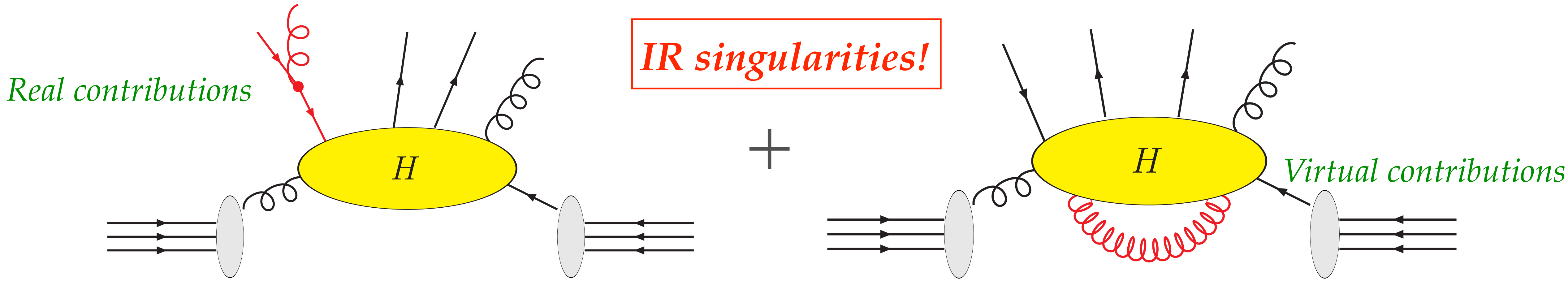


At NLO level we have only singular contributions and we have to rearrange the cross section in such a way to be able to calculate it in d=4 dimension.

$$\begin{aligned}\sigma[O_J] = \int_m d\hat{\sigma}_N^{(0)} \otimes & \left[1 + Z^{(1)}(\mu^2, 1/\epsilon) \right] \otimes F(\mu^2) \\ + \int_{N+1} & \left[d\hat{\sigma}_{N+1}^{(0)} \right] \otimes F(\mu^2) \\ + \int_N & \left[d\hat{\sigma}_N^{(1)} \right] \otimes F(\mu^2)\end{aligned}$$

Nowadays this is a standard and straightforward procedure and we have automated tools for this.

NLO cross section

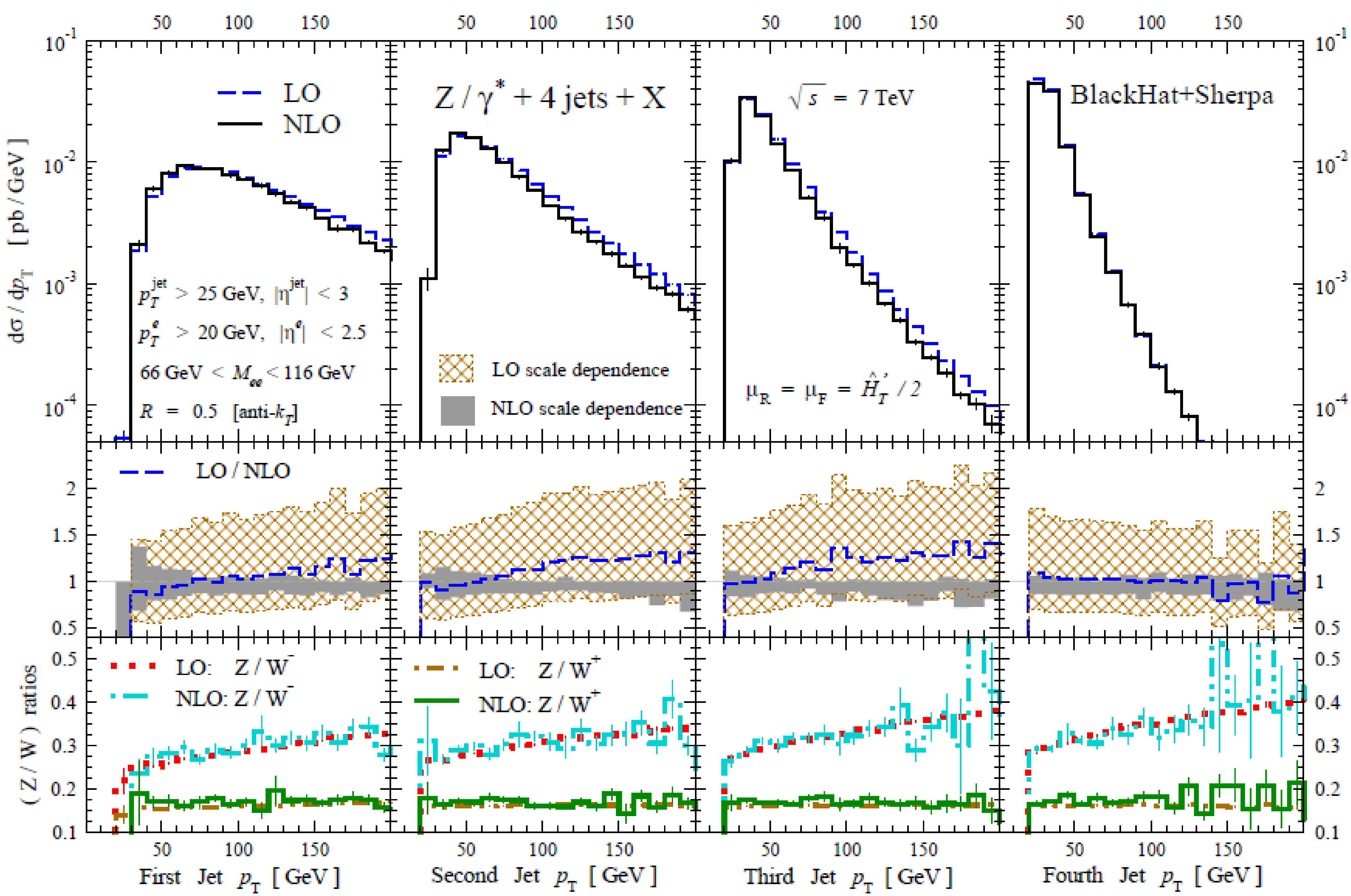


At NLO level we have only singular contributions and we have to rearrange the cross section in such a way to be able to calculate it in d=4 dimension.

$$\sigma[O_J] = \int_m d\hat{\sigma}_N^{(0)} \otimes \left[1 + Z^{(1)}(\mu^2, 1/\epsilon) + \int_1 d\bar{R}(\mu^2) + V(\mu^2)\right] \otimes F(\mu^2)$$
$$+ \int_{N+1} \left[d\hat{\sigma}_{N+1}^{(0)} - d\hat{\sigma}_N^{(0)} \otimes R(\mu^2)\right] \otimes F(\mu^2)$$
$$+ \int_N \left[d\hat{\sigma}_N^{(1)} - d\hat{\sigma}_N^{(0)} \otimes V(\mu^2, 1/\epsilon)\right] \otimes F(\mu^2)$$

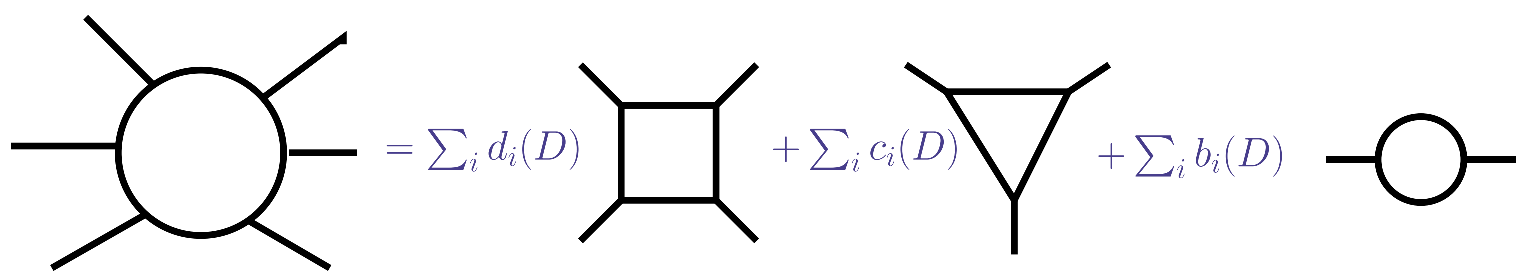
Nowadays this is a standard and straightforward procedure and we have automated tools for this.

NLO Cross Sections



NLO is the New Standard

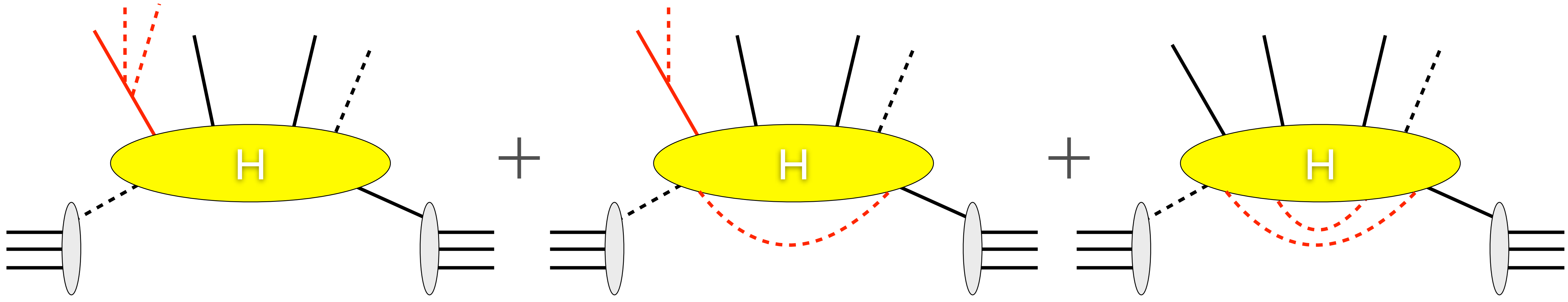
- ✓ It is not that easy to calculate but the NLO calculations are automated.
- ✓ Several public and non-public codes are available HELAC, MADGRAPH, SHERPA+BLACKHAT, AUTODIPOLE, TEVJET, aMC@NLO…
- ✓ The bottleneck was the automation of the 1-loop matrix elements. There are two approaches:
  - ➡ Algebraic method, based on unitary cut



$$\mathcal{M} = \sum d(D)\mathrm{boxes(D)} + \sum c(D)\mathrm{triangles(D)} + \sum b(D)\mathrm{bubbles(D)}$$

  - ➡ Numerical method, the IR and UV singularities of the 1-loop graph are subtracted at integrand level, and the loop integration is performed in d = 4 dimension numerically. pp —> W+5jets, e+e- —> 7jets.
- ✗ The scale dependence can be still big in some processes.
- ✗ Jets are still represented by max. two partons. It is still very poor.

NNLO & N^{k}LO



Double real contribution *Real-virtual contribution* *2-loop contribution*

At higher order there are more and more terms and singular regions and we need more and more subtraction term. In the simple NLO style even the bookkeeping of these terms is challenging.

$$\sigma[O_J] = \sum_{R=0}^{\infty} \sum_{L=0}^{\infty} \int_{N+R} \left[d\sigma_{N+R}^{(L)}[O_J] - \sum_{\substack{r=0 \\ r+l>0}}^{R} \sum_{l=0}^{L} d\sigma_{N+R-r}^{(L-l)}[O_J] \otimes \widetilde{H}^{(r,l)}(\mu^2) \right]$$

$$\otimes \left[1 + \sum_{\substack{r,l=0 \\ r+l>0}}^{\infty} \int_r dH^{(r,l)}(\mu^2) \right] \otimes \left[1 + \sum_{k=0}^{\infty} Z^{(k)}(\mu^2, 1/\epsilon) \right] \otimes F(\mu^2)$$

The $\widetilde{H}^{(r,l)}(\mu^2)$ singular functions can be expressed in terms of $H^{(r,l)}(\mu^2)$

What is available in Code?

- In the recent years (last 10-15 years) progress has been made in jet physics on NNLO computation.
  Czakon, Fiedler, Mitov, J.Currie,T.Gehrmann, N..Glover,A.Gehrmann-deRidder, J.Pires; G.Abeloff, P. Maierhofer, S. Pozzorini, R. Boughezal, F. Caola, K. Melnikov, F. Petriello, M. Schulze, M. Bruscherseifer, F. Caola, F. Cascioli,T. Gehrmann, M. Grazzini, C.Anastasiou, A. Lazopoulos, F. Herzog, R.Mueller, P. Bolzoni,V. Del Duca, G. Somogyi, Z.Trocsanyi,…

- There are several approaches to deal with the real singularities.
  - Antenna subtraction method (generalization of the NLO antenna method)
  - Residue subtraction method (generalized FKS method)
  - (Sadly there is no generalization of the dipole method…)
- With antenna method the single and dijet cross section is available
- With residue method: top pair production, H+jet

- Small number of two loop matrix elements are known for jet calculations:
  2 → 2: massless parton scattering, e.g. gg → gg, qq → gg, etc
  2 → 2: processes with one offshell leg, e.g. qq → V +jet, gg → H+jet
  2 → 2: qq → tt, gg → tt known numerically

??? Automation

- The complexity is enormous and the increasing computer power doesn't help. There are some outgoing work:
  Gluza, Kajda, Kosower; Mastrolia, Ossola; Kosower, Larsen; Badger, Frellesvig, Zhang; Larsen; Caron-Huet, Larsen; Zhang; Mastrolia, Mirabella, Ossola, Peraro; Kleiss, Malamos, Papadopoulos, Verheyn; Johansson, Kosower, Larsen; Feng, Huang

More to the List..

- ✓ IR subtraction schemes
  - ✓ sector decomposition Heinrich; Anastasiou, Melnokov, Petriello; Binoth, Heinrich
    - – $pp \to H$, $pp \to V$

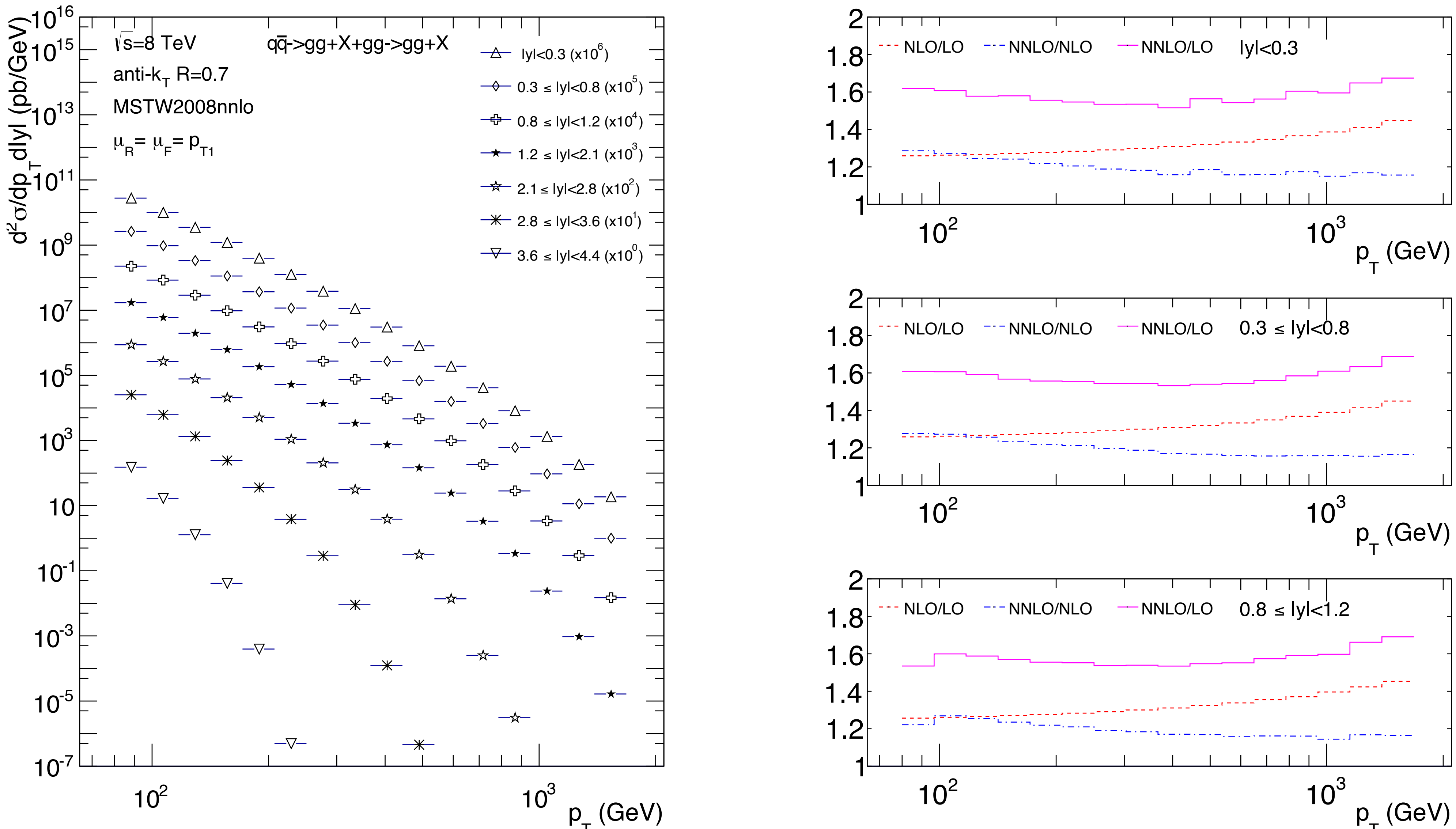
      Anastasiou, Melnikov, Petriello; Melnikov, Petriello; Anastastiou, Dissertori, Stockli; Anastasiou, Herzog, Lazopoulos
  - ✓ q_T subtraction Catani, Grazzini
    - – $pp \to H$, $pp \to V$, $pp \to VH$, $pp \to \gamma\gamma$

      Grazzini; Catani, Cieri, Ferrera, de Florian, Grazzini; Catani, Ferrera, Grazzini; Fererra, Grazzini, Tramontano; Catani, Cieri, de Florian, Ferrera, Grazzini
  - ✓ STRIPPER - sector improved residue subtraction Czakon
    - – $pp \to t\bar{t}$ Czakon; Czakon, Mitov
  - ✓ Antenna subtraction Gehrmann, Gehrmann-De Ridder, NG
    - – $e^+e^- \to 3$ jet Gehrmann, Gehrmann-De Ridder, NG, Heinrich; Weinzierl
    - – $pp \to 2$ jet

      Pires, NG; Gehrmann-De Ridder, Pires, NG; Gehrmann, Gehrmann-De Ridder, Pires, NG

Numerical Results

J.Currie,T.Gehrmann, N..Glover,A.Gehrmann-deRidder, J.Pires

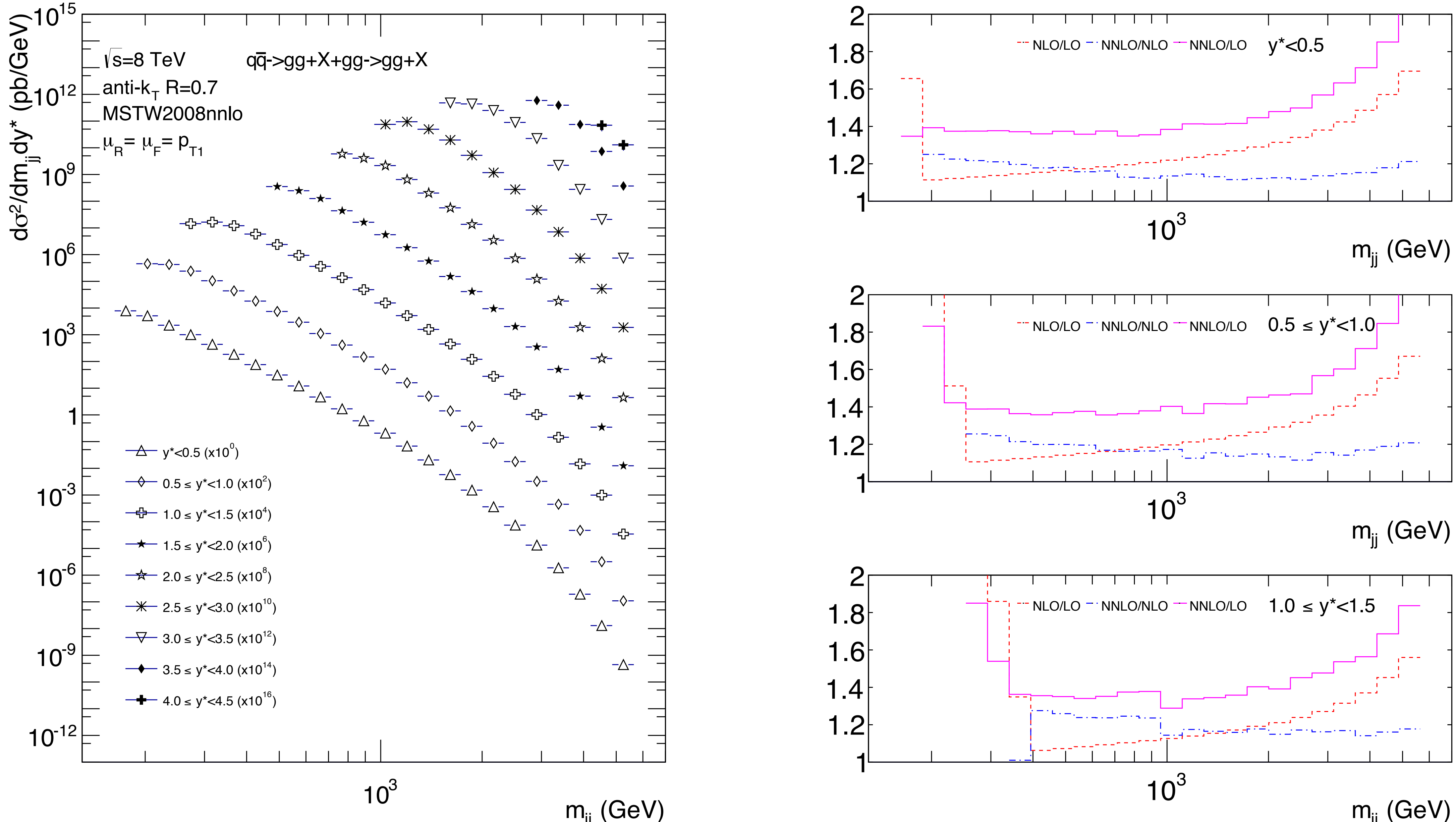


(a) (b)

Figure 3: (a) The doubly differential inclusive jet transverse energy distribution, $d^2\sigma/dp_T d|y|$, at $\sqrt{s} = 8$ TeV for the anti-k_T algorithm with $R = 0.7$ and for $E_T > 80$ GeV and various $|y|$ slices and (b) double differential k-factors for $p_T > 80$ GeV and three $|y|$ slices: $|y| < 0.3$, $0.3 < |y| < 0.8$ and $0.8 < |y| < 1.2$.

Numerical Results

J.Currie,T.Gehrmann, N..Glover,A.Gehrmann-deRidder, J.Pires



(a) (b)

Figure 4: (a) Exclusive dijet invariant mass distribution, $d\sigma/dm_{jj}dy^*$, at $\sqrt{s} = 8$ TeV for $y^* < 0.5$ with $p_{T1} > 80$ GeV, $p_{T2} > 60$ GeV and $|y_1|$, $|y_2| < 4.4$ at NNLO (blue), NLO (red) and LO (dark-green) and (b) the ratios of different perturbative orders, NLO/LO, NNLO/LO and NNLO/NLO.

Predictive power in pQCD

Observable computed in pQCD:

$$\sigma[O_J] = \sum_{n=0}^{N} c_n(\mu^2) \left[\frac{\alpha_s(\mu^2)}{2\pi}\right]^n + R_N(\mu^2)$$

Fixed order: only take few terms of the sum and ignore the rest of it. It can be done if

1. α_s is small
2. $c_n(\mu^2)$ doesn't grow fast with n
3. $R_N(\mu^2)$ is small enough

Otherwise we have to do something else. A typical observable is

$$\sigma[O_J] = 1 + \alpha_s(L^2 + L + 1) + \alpha_s^2(L^4 + L^3 + L^2 + L + 1) + \cdots$$

$$1 \quad \Rightarrow \quad \pi^2, 2, etc$$

Effective expansion parameter: $\alpha_s L^2$

Reorganize the perturbative sum!!!

Resummation

Usually we reorganize the perturbative in exponentiated form:

$$\sigma[O_J] = \exp\Bigg(\underbrace{\underbrace{\underbrace{\underbrace{L g_1(\alpha_s L)}_{LL} + g_2(\alpha_s L)}_{NLL} + \alpha_s g_3(\alpha_s L)}_{NNLL} + \alpha_s^2 g_4(\alpha_s L)}_{NNNLL} + \cdots \Bigg) C(\alpha_s)$$

What is L the log of? It depends on the observable. If we do analytic resummation, usually we have to do the resummation procedure for every observable separately.

$$\log(1-T)$$

$$\log(1/x)$$

$$\log(1-m^2/\hat{s})$$

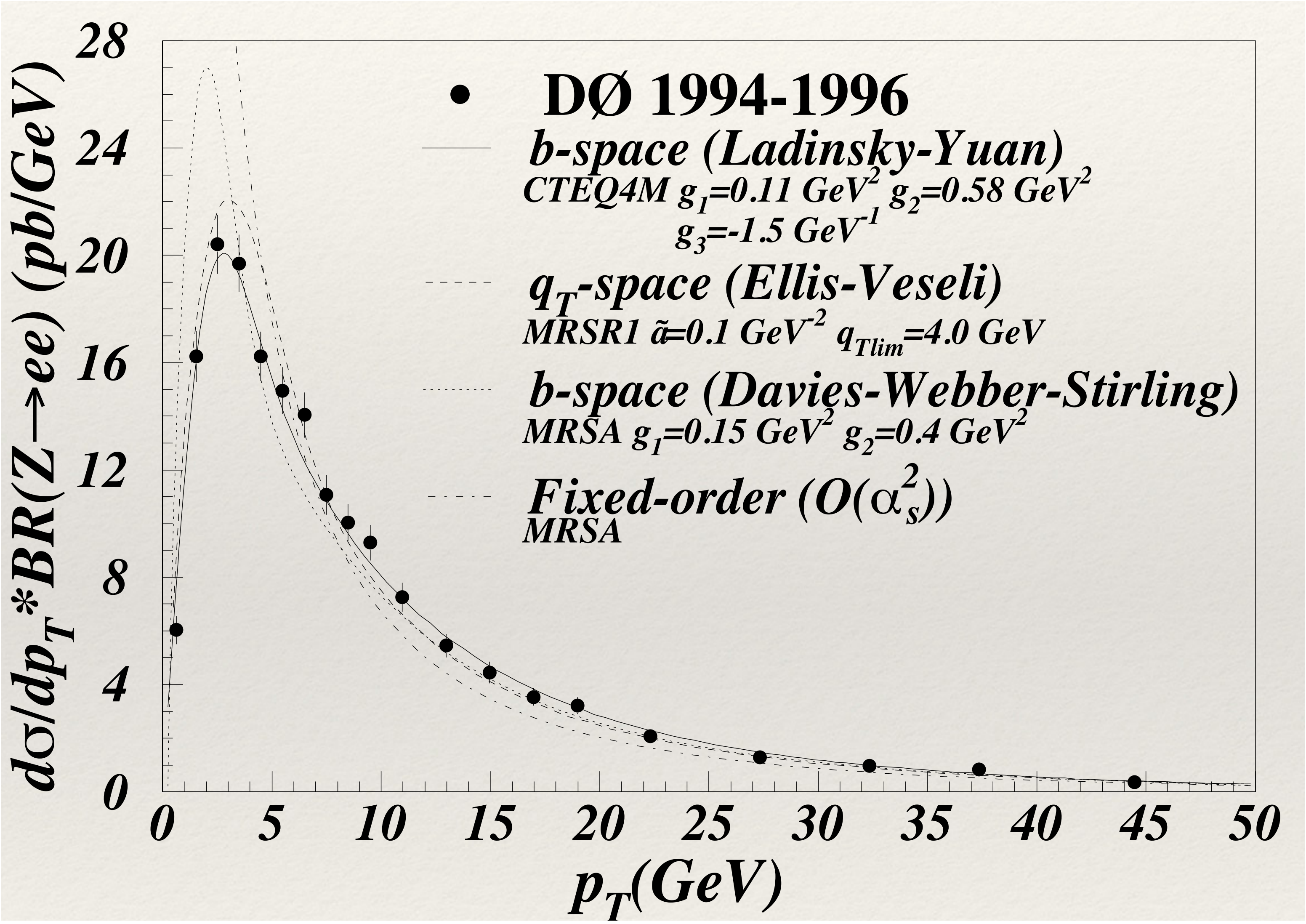
$$\log(k_T^2/M^2)$$

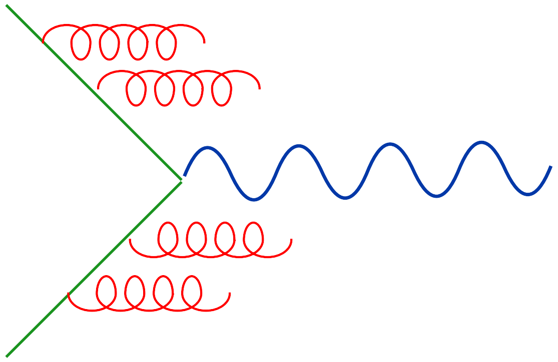
$$\log(1-x)$$

Visible Logs

"Visible logs", like the Drell-Yan transverse momentum

$$\log(k_T^2/M^2)$$

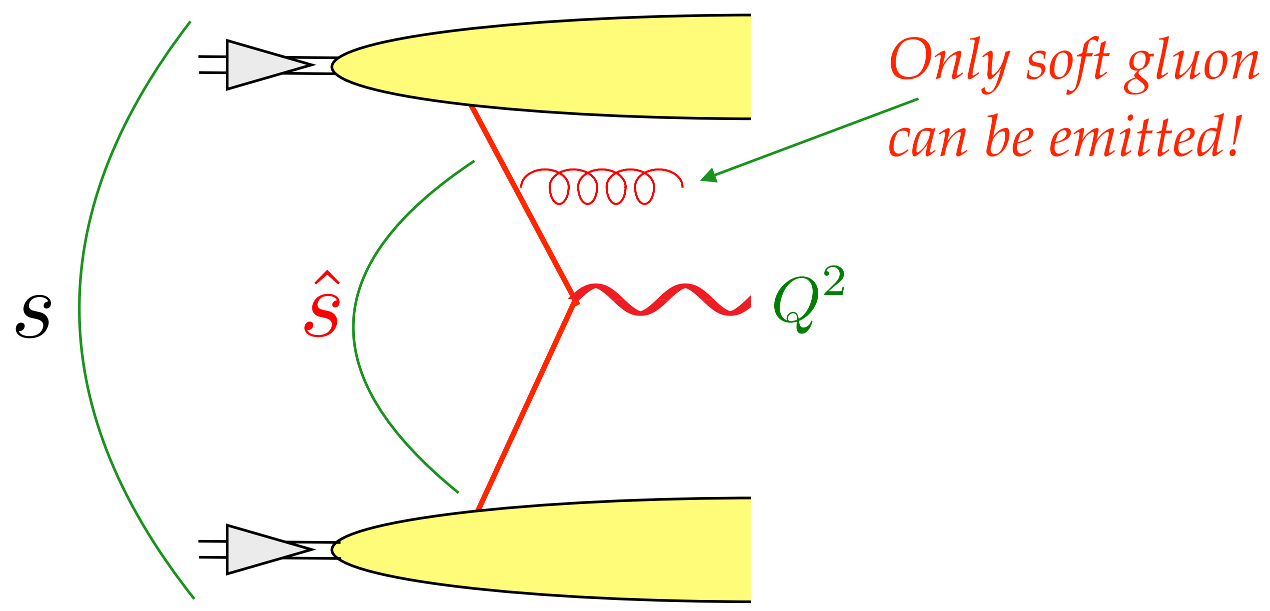




Z boson gets recoil from soft gluon emission

Invisible Logs

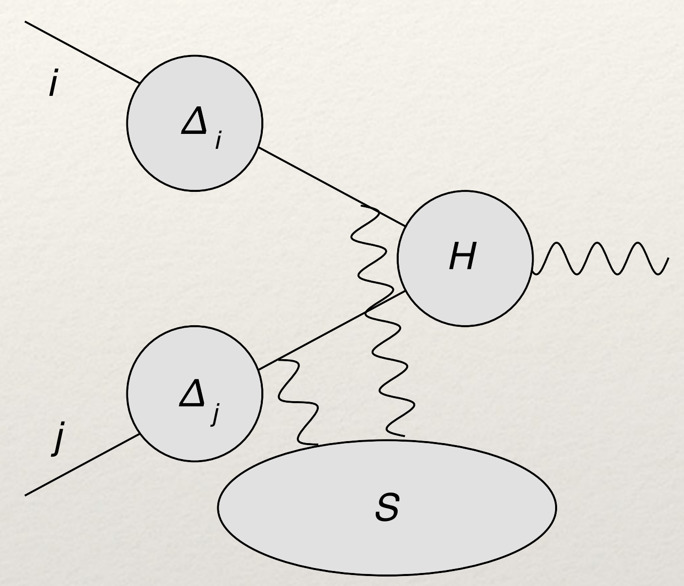
"Invisible logs", live under the integral. They are the so called threshold logs.



$$s \approx Q^2 \quad \Longrightarrow \quad \frac{Q^2}{\hat{s}} \approx 1 \quad \Longrightarrow \quad L = \log\left(1 - \frac{Q^2}{\hat{s}}\right)$$

How to do Resummation?

- There are many ways to do. It is depending on
  - the observable
  - the logarithm
  - and the resummer
- Every resummation method relies on
  - factorization (separating the degree of freedoms)
  - approximation in the kinematics



Contopanagos, EL, Sterman

Simple example: Drell-Yan threshold log resummation:

$$\sigma(N) = \Delta(N, \xi_1, \mu^2)\Delta(N, \xi_2, \mu^2)S(N, \xi_1, \xi_2, \mu^2)H(\mu^2)$$

$$\mu^2 \frac{d\sigma(N)}{d\mu^2} = \xi_1 \frac{d\sigma(N)}{d\xi_1} = \xi_2 \frac{d\sigma(N)}{d\xi_2} = 0$$

Solve these equation!

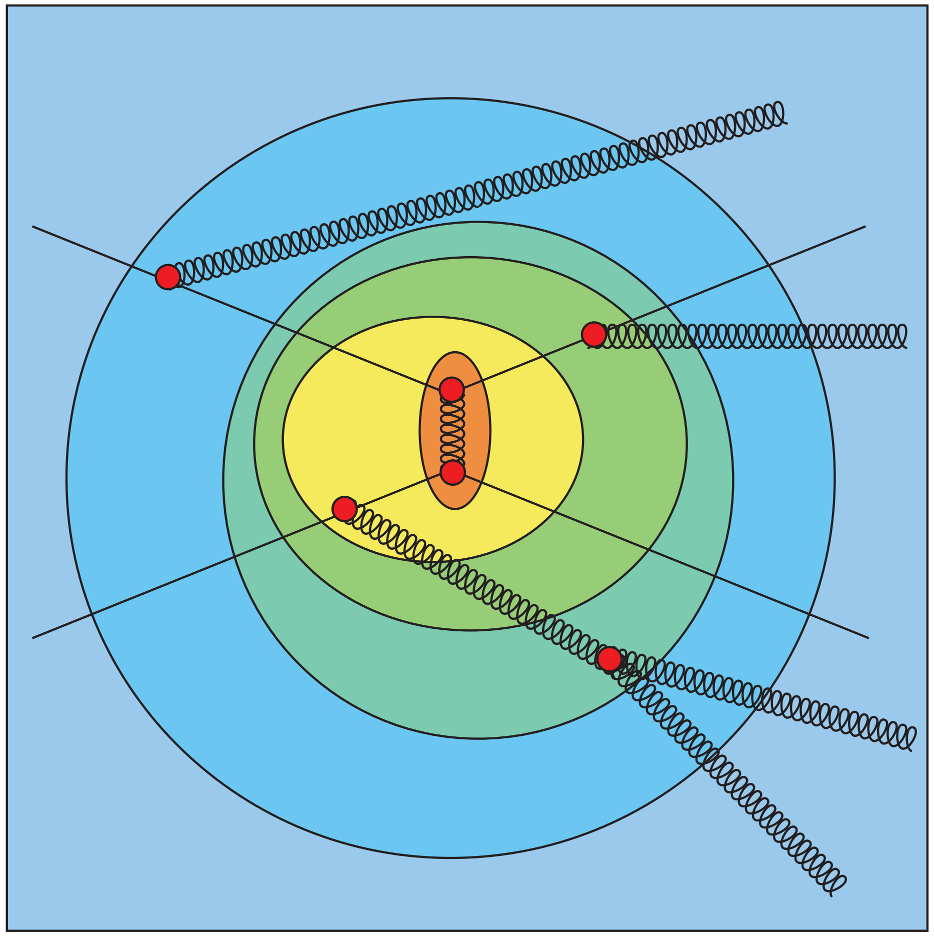
Parton Showers

- Parton shower evolves with "shower time" t.

- Small $t \Rightarrow$ hard.

- Large $t \Rightarrow$ soft.

- For initial state interactions, evolution is backwards in physical time.



- This is similar to PYTHIA and SHERPA.

Parton Shower

What is parton shower??? How to define it? Is it pQCD or black art?
If it is pQCD then we need good PDF for parton shower. Otherwise we can live with LO* and LO**.
Let us try to define from pQCD!

STARTING POINT:
Parton shower generates all the possible partonic final states in a fully exclusive way (momentum, flavor, color spin).

Start with the formal definition of the all order cross section:

$$\sigma[O_J] = \sum_{\substack{m=N \\ \text{LEGS}}}^{\infty} \sum_{\substack{n=0 \\ \text{LOOPS}}}^{\infty} \int_m d\hat{\sigma}_m^{(n)}\left(\mu^2, \alpha_s(\mu^2)\right) \otimes F^{\text{bare}}$$

We reorganized this sum by adding and subtracting singular contributions in order to be able to calculate anything in d=4 dimension.

Parton Showers

I am pretty sure that everybody ignored this:

$$\sigma[O_J] = \sum_{R=0}^{\infty}\sum_{L=0}^{\infty}\int_{N+R}\left[d\sigma_{N+R}^{(L)}[O_J] - \sum_{\substack{r=0\\r+l>0}}^{R}\sum_{l=0}^{L} d\sigma_{N+R-r}^{(L-l)}[O_J]\otimes\widetilde{H}^{(r,l)}(\mu^2)\right]$$

$$\otimes\left[1+\sum_{\substack{r,l=0\\r+l>0}}^{\infty}\int_r dH^{(r,l)}(\mu^2)\right]\otimes\left[1+\sum_{k=0}^{\infty}Z^{(k)}(\mu^2,1/\epsilon)\right]\otimes F(\mu^2)$$

We have to focus on the total cross section: $O_J = 1$ and $N = 0$

Now this is well defined in d=4 dimension but at fixed order it suffers on the large logarithms (all order it is not a convergent sum, but that is an different issue).

We should reorganize this sum to be able to resum these effects.

Can we do this?

Parton Showers

Yes, we can do this:

$$\sigma[O_J] = \big(1\big|\mathcal{O}_J\Big[\mathcal{W}^{LO}(\mu_{\mathrm{f}}^2) + \mathcal{W}^{NLO}(\mu_{\mathrm{f}}^2) + \cdots\Big]$$
$$\mathbb{T}\exp\left\{\int_{\mu_{\mathrm{f}}^2}^{\mu_0^2}\frac{d\mu^2}{\mu^2}\Big[\mathcal{S}^{LO}(\mu^2) + \mathcal{S}^{NLO}(\mu^2) + \cdots\Big]\right\}$$
$$\mathcal{F}(\mu_0^2)\Big[\big|\hat{\rho}^{LO}(\mu_0^2)\big) + \big|\hat{\rho}^{NLO}(\mu_0^2)\big) + \big|\hat{\rho}^{NNLO}(\mu_0^2)\big) + \cdots\Big]$$

Little bit of notation:

$$\big|\hat{\rho}^{N^kLO}(\mu_0^2)\big) \quad \Longleftrightarrow \quad \sum_{R=0}^{k}\Big[d\hat{\sigma}_R^{(k-R)}[1] - \sum_{r=0}^{R}\sum_{\substack{l=0 \\ r+l>0}}^{k-R} d\hat{\sigma}_{R-r}^{(k-R-l)}[1]\otimes\widetilde{H}^{(r,l)}(\mu^2)\Big]$$

The fixed order notation is not suitable for representing fully exclusive final states…

Parton showers resum partonic exclusive partonic splittings.

Summary

- I won't give you any deadline for NNLO or there higher order…
- The higher order calculations are not easy at all. There are two approaches to the problem:
  - *Pragmatic way*: Look at the problem as a mathematical puzzle that you have to crack. We have integrals with singularities, regularize them, extract the singularities, cancelled them and you have the finite result.
  - *Purist way: Try to understand the problem from first principles. Understand the structure and the physical meaning of the singularities and use them to be able to make calculation.*
- *Of course the best strategy is the combination of these two.*
- *Understandig higher order at fixed order level can help us to improve resummation thechniques and parton shower algorithms.*