

Looking for BFKL effects at the LHC with Mueller-Navelet jets

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Small x mini workshop

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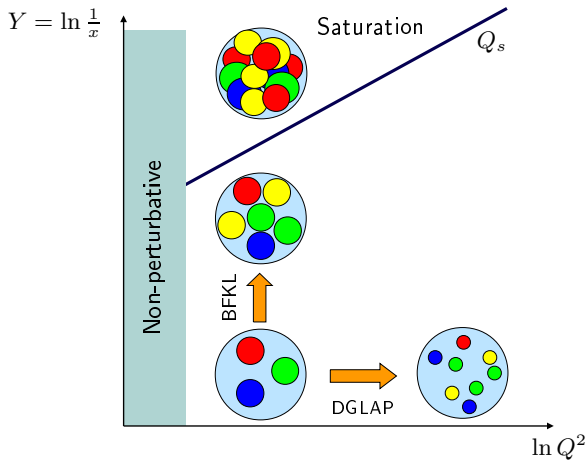
in collaboration with

L. Szymanowski (NCBJ Warsaw), S. Wallon (UPMC & LPT Orsay)

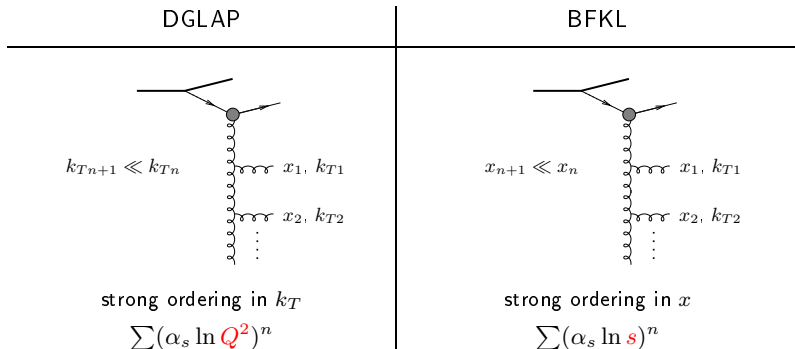
B. D, L. Szymanowski, S. Wallon, JHEP **1305** (2013) 096 [arXiv:1302.7012 [hep-ph]]

B. D, L. Szymanowski, S. Wallon, arXiv:1309.3229 [hep-ph] (to appear in PRL)

The different regimes of QCD



Small values of α_s (perturbation theory applies if there is a hard scale) can be compensated by large logarithmic enhancements.



When \sqrt{s} becomes very large, it is expected that a BFKL description is needed to get accurate predictions

QCD in the perturbative Regge limit

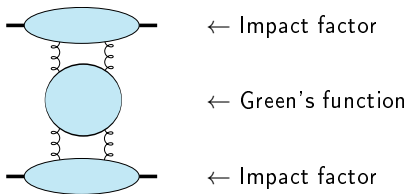
The amplitude can be written as:

$$\mathcal{A} = \underbrace{\text{Diagram 1}}_{\sim s} + \left(\underbrace{\text{Diagram 2}}_{\sim s} + \underbrace{\text{Diagram 3}}_{\sim s (\alpha_s \ln s)} + \dots \right) + \left(\underbrace{\text{Diagram 4}}_{\sim s (\alpha_s \ln s)^2} + \dots \right) + \dots$$

The diagrams represent different orders of perturbation theory in the Regge limit:

- Diagram 1: Two light blue ovals connected by two vertical wavy lines. $\sim s$
- Diagram 2: Two light blue ovals connected by two vertical wavy lines, with a horizontal wavy line connecting the two vertical lines. $\sim s$
- Diagram 3: Two light blue ovals connected by two vertical wavy lines, with a circular loop on the right vertical line. $\sim s (\alpha_s \ln s)$
- Diagram 4: Two light blue ovals connected by two vertical wavy lines, with two horizontal wavy lines connecting the two vertical lines. $\sim s (\alpha_s \ln s)^2$

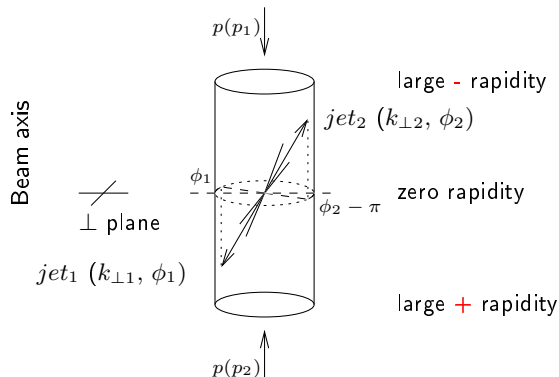
this can be put in the following form :



- Higher order corrections to BFKL kernel are known at NLL order (Lipatov Fadin; Camici, Ciafaloni), now for arbitrary impact parameter $\alpha_S \sum_n (\alpha_S \ln s)^n$ resummation
- impact factors are known in some cases at NLL
 - $\gamma^* \rightarrow \gamma^*$ at $t = 0$ (Bartels, Colferai, Gieseke, Kyrielleis, Qiao; Balitski, Chirilli)
 - forward jet production (Bartels, Colferai, Vacca; Caporale, Ivanov, Murdaca, Papa, Perri; Chachamis, Hentschinski, Madrigal, Sabio Vera)
 - inclusive production of a pair of hadrons separated by a large interval of rapidity (Ivanov, Papa)
 - $\gamma_L^* \rightarrow \rho_L$ in the forward limit (Ivanov, Kotsky, Papa)

Mueller-Navelet jets

- Consider two jets (hadrons flying within a narrow cone) **separated by a large rapidity**, i.e. each of them almost fly in the direction of the hadron “close” to it, and with very similar transverse momenta
- in a pure LO collinear treatment, these two jets should be emitted **back to back** at leading order: $\Delta\phi - \pi = 0$ ($\Delta\phi = \phi_1 - \phi_2 =$ relative azimuthal angle) and $k_{\perp 1} = k_{\perp 2}$. There is no phase space for (untagged) emission between them



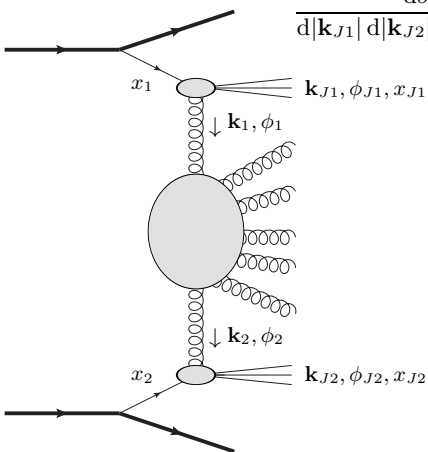
k_T -factorized differential cross section

$$\frac{d\sigma}{d|\mathbf{k}_{J1}| d|\mathbf{k}_{J2}| dy_{J1} dy_{J2}} = \int d\phi_{J1} d\phi_{J2} \int d^2\mathbf{k}_1 d^2\mathbf{k}_2$$

$$\times \Phi(\mathbf{k}_{J1}, x_{J1}, -\mathbf{k}_1)$$

$$\times G(\mathbf{k}_1, \mathbf{k}_2, \hat{s})$$

$$\times \Phi(\mathbf{k}_{J2}, x_{J2}, \mathbf{k}_2)$$



with $\Phi(\mathbf{k}_{J2}, x_{J2}, \mathbf{k}_2) = \int dx_2 f(x_2) V(\mathbf{k}_2, x_2)$ $f \equiv$ PDF $x_J = \frac{|\mathbf{k}_J|}{\sqrt{s}} e^{y_J}$

It is useful to define the coefficients \mathcal{C}_n as

$$\mathcal{C}_n \equiv \int d\phi_{J1} d\phi_{J2} \cos(n(\phi_{J1} - \phi_{J2} - \pi)) \\ \times \int d^2\mathbf{k}_1 d^2\mathbf{k}_2 \Phi(\mathbf{k}_{J1}, x_{J1}, -\mathbf{k}_1) G(\mathbf{k}_1, \mathbf{k}_2, \hat{s}) \Phi(\mathbf{k}_{J2}, x_{J2}, \mathbf{k}_2)$$

- $n = 0 \implies$ differential cross-section

$$\mathcal{C}_0 = \frac{d\sigma}{d|\mathbf{k}_{J1}| d|\mathbf{k}_{J2}| dy_{J1} dy_{J2}}$$

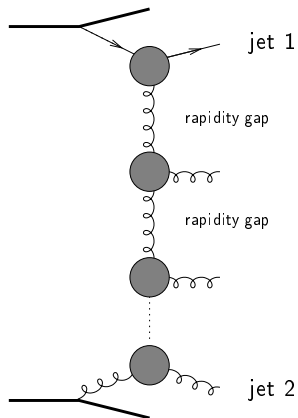
- $n > 0 \implies$ azimuthal decorrelation

$$\frac{\mathcal{C}_n}{\mathcal{C}_0} = \langle \cos(n(\phi_{J,1} - \phi_{J,2} - \pi)) \rangle \equiv \langle \cos(n\varphi) \rangle$$

- sum over $n \implies$ azimuthal distribution

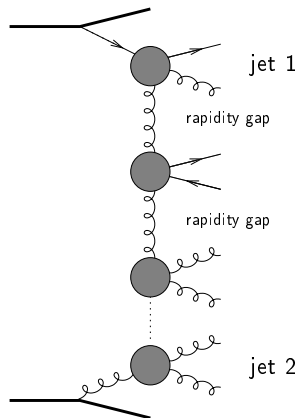
$$\frac{1}{\sigma} \frac{d\sigma}{d\varphi} = \frac{1}{2\pi} \left\{ 1 + 2 \sum_{n=1}^{\infty} \cos(n\varphi) \langle \cos(n\varphi) \rangle \right\}$$

LL BFKL



$$\sum (\alpha_s \ln s)^n$$

NLL BFKL



$$\sum (\alpha_s \ln s)^n + \alpha_s \sum (\alpha_s \ln s)^n$$

Results for a symmetric configuration

In the following we show results for

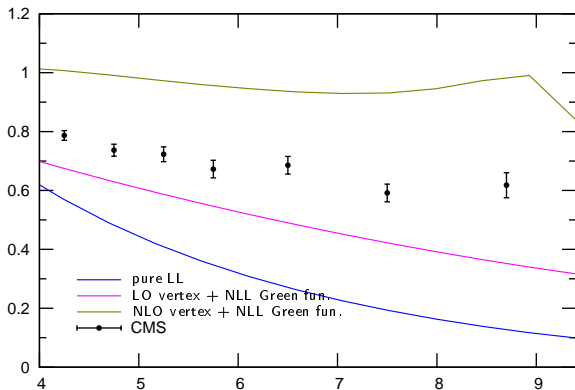
- $\sqrt{s} = 7 \text{ TeV}$
- $35 \text{ GeV} < |\mathbf{k}_{J1}|, |\mathbf{k}_{J2}| < 60 \text{ GeV}$
- $0 < y_1, y_2 < 4.7$

These cuts allow us to compare our predictions with the first experimental data on azimuthal correlations of Mueller-Navelet jets at the LHC presented by the CMS collaboration (CMS-PAS-FSQ-12-002)

note: unlike experiments we have to set an upper cut on $|\mathbf{k}_{J1}|$ and $|\mathbf{k}_{J2}|$. We have checked that our results don't depend on this cut significantly.

Azimuthal correlation $\langle \cos \varphi \rangle$

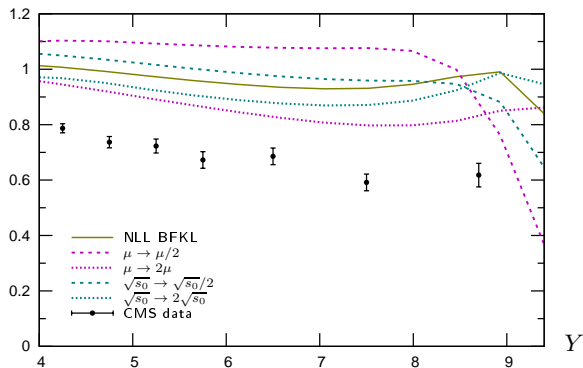
$$\frac{c_1}{c_0} = \langle \cos \varphi \rangle \equiv \langle \cos(\phi_{J_1} - \phi_{J_2} - \pi) \rangle$$


 $35 \text{ GeV} < |\mathbf{k}_{J_1}| < 60 \text{ GeV}$
 $35 \text{ GeV} < |\mathbf{k}_{J_2}| < 60 \text{ GeV}$
 $0 < y_1 < 4.7$
 $0 < y_2 < 4.7$
 $Y \equiv y_1 + y_2$

The NLO corrections to the jet vertex lead to a large increase of the correlation

Azimuthal correlation $\langle \cos \varphi \rangle$

$$\langle \cos \varphi \rangle \equiv \langle \cos(\phi_{J1} - \phi_{J2} - \pi) \rangle$$



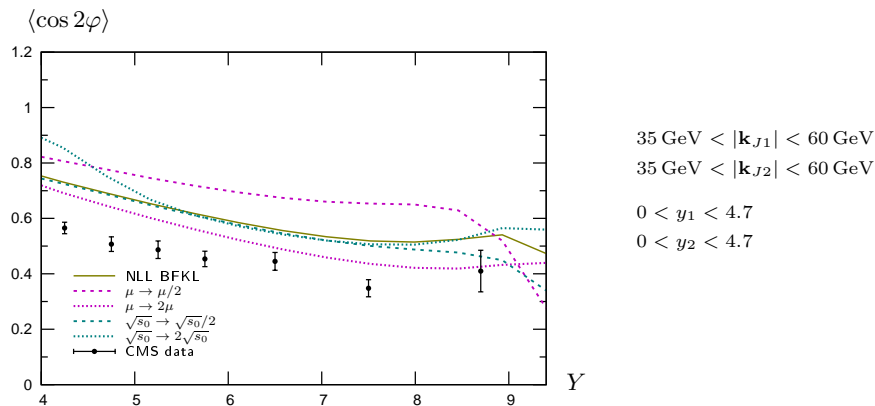
$$35 \text{ GeV} < |\mathbf{k}_{J1}| < 60 \text{ GeV}$$

$$35 \text{ GeV} < |\mathbf{k}_{J2}| < 60 \text{ GeV}$$

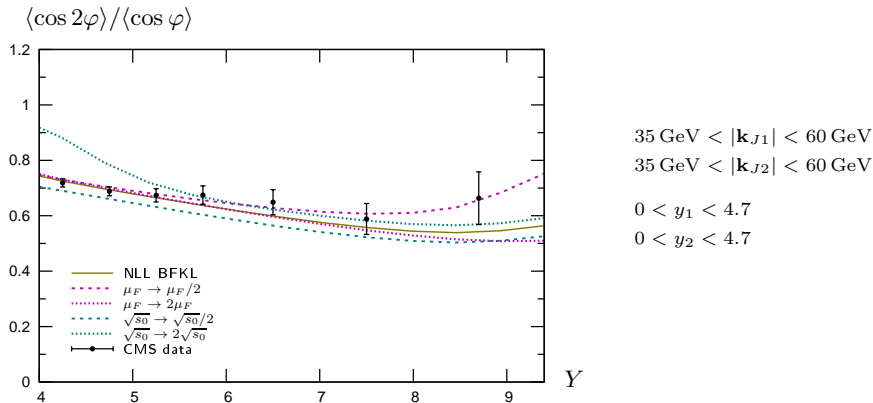
$$0 < y_1 < 4.7$$

$$0 < y_2 < 4.7$$

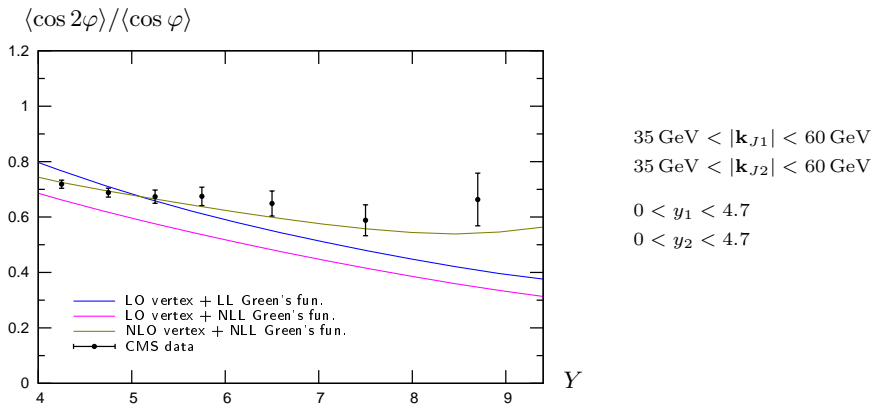
- NLL BFKL predicts a too small decorrelation
- The NLL BFKL calculation is still rather dependent on the scales, especially the renormalization / factorization scale

Azimuthal correlation $\langle \cos 2\varphi \rangle$ 

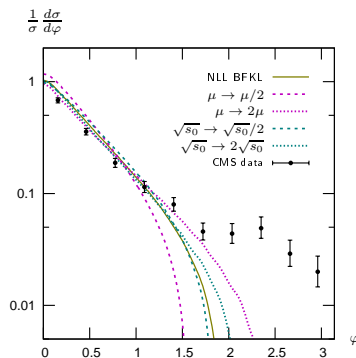
- The agreement with data is a little better for $\langle \cos 2\varphi \rangle$ but still not very good
- This observable is also very sensitive to the scales

Azimuthal correlation $\langle \cos 2\varphi \rangle / \langle \cos \varphi \rangle$ 

- This observable is more stable with respect to the scales than the previous ones
- The agreement with data is good across the full Y range

Azimuthal correlation $\langle \cos 2\varphi \rangle / \langle \cos \varphi \rangle$ 

It is necessary to include the NLO corrections to the jet vertex to reproduce the behavior of the data at large Y

Azimuthal distribution (integrated over $6 < Y < 9.4$)

- Our calculation predicts a too large value of $\frac{1}{\sigma} \frac{d\sigma}{d\varphi}$ for $\varphi \lesssim \frac{\pi}{2}$ and a too small value for $\varphi \gtrsim \frac{\pi}{2}$
- It is not possible to describe the data even when varying the scales by a factor of 2

- The agreement of our calculation with the data for $\langle \cos 2\varphi \rangle / \langle \cos \varphi \rangle$ is good and quite stable with respect to the scales
- The agreement for $\langle \cos n\varphi \rangle$ and $\frac{1}{\sigma} \frac{d\sigma}{d\varphi}$ is not very good and very sensitive to the choice of the renormalization scale μ_R
- An all-order calculation would be independent of the choice of μ_R . This feature is lost if we truncate the perturbative series
⇒ How to choose the renormalization scale?
'Natural scale': sometimes the typical momenta in a loop diagram are different from the natural scale of the process

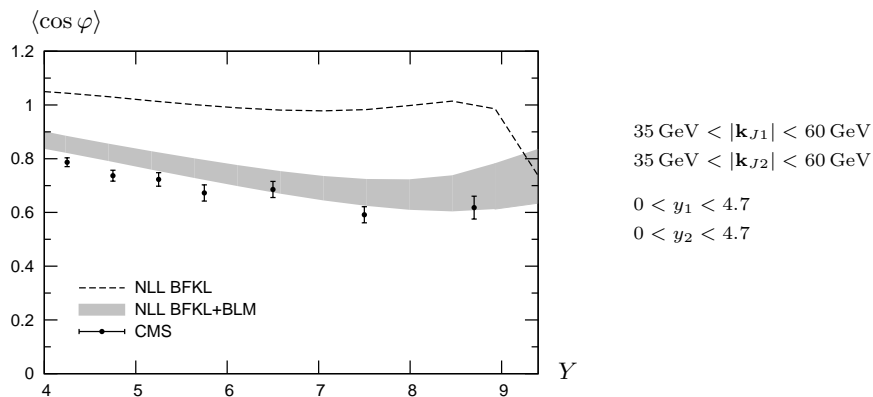
We decided to use the **Brodsky-Lepage-Mackenzie** (BLM) procedure to fix the renormalization scale

The **Brodsky-Lepage-Mackenzie** (BLM) procedure resums the self-energy corrections to the gluon propagator at one loop into the running coupling. These contributions are formally of higher-order but they are proportional to

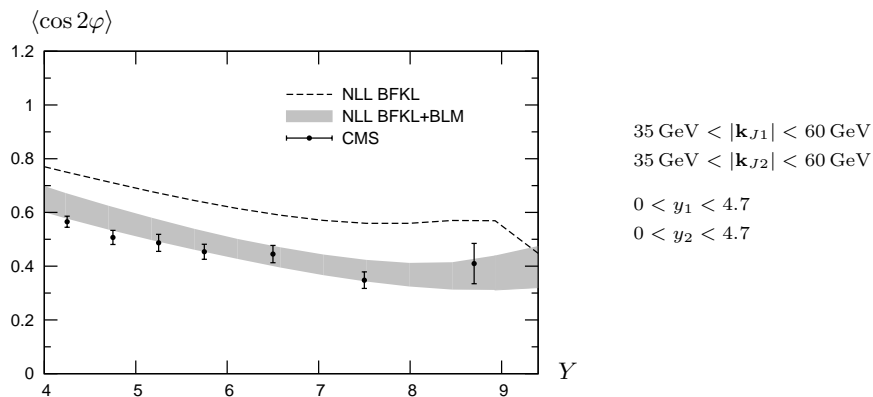
$$\beta_0 = \frac{11N_c - 2N_f}{3} \simeq 7.67$$

First attempts to apply BLM scale fixing to BFKL processes lead to problematic results. **Brodsky, Fadin, Kim, Lipatov and Pivovarov** suggested that one should first go to a physical renormalization scheme like MOM and then apply the 'traditional' BLM procedure, i.e. identify the β_0 dependent part and choose μ_R such that it vanishes.

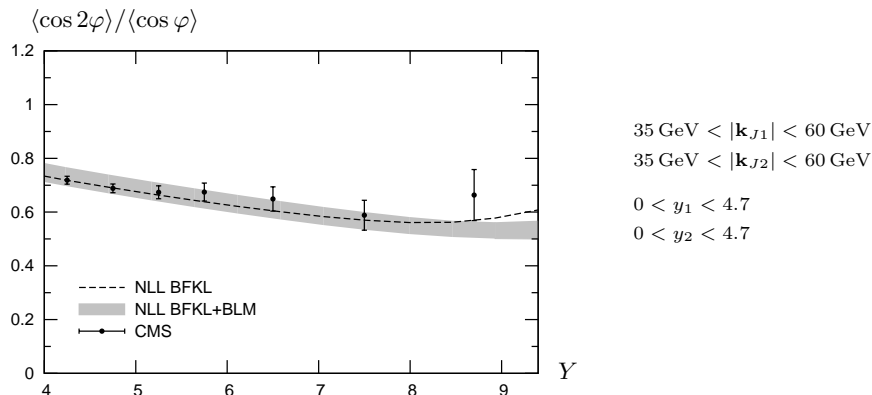
We find a scale which is typically about 5 times larger than $\sqrt{|\mathbf{k}_{J1}| \cdot |\mathbf{k}_{J2}|}$

Azimuthal correlation $\langle \cos \varphi \rangle$ 

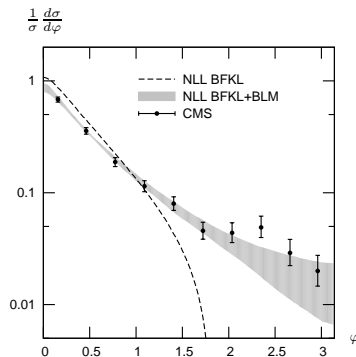
Using the BLM scale setting, the agreement with data becomes much better

Azimuthal correlation $\langle \cos 2\varphi \rangle$ 

Using the BLM scale setting, the agreement with data becomes much better

Azimuthal correlation $\langle \cos 2\varphi \rangle / \langle \cos \varphi \rangle$ 

Because it is much less dependent on the scales, the observable $\langle \cos 2\varphi \rangle / \langle \cos \varphi \rangle$ is almost not affected by the BLM procedure and is still in good agreement with the data

Azimuthal distribution (integrated over $6 < Y < 9.4$)

With the BLM scale setting the azimuthal distribution is in good agreement with the data across the full φ range.

Using the BLM scale setting:

- The agreement $\langle \cos n\varphi \rangle$ with the data becomes much better
- The agreement for $\langle \cos 2\varphi \rangle / \langle \cos \varphi \rangle$ is still good and unchanged as this observable is weakly dependent on μ_R
- The azimuthal distribution is in much better agreement with the data

But the configuration chosen by CMS with $\mathbf{k}_{J_{\min 1}} = \mathbf{k}_{J_{\min 2}}$ does not allow us to compare with a fixed-order $\mathcal{O}(\alpha_s^3)$ treatment (i.e. without resummation)

These calculations are unstable when $\mathbf{k}_{J_{\min 1}} = \mathbf{k}_{J_{\min 2}}$ because the cancellation of some divergencies is difficult to obtain numerically

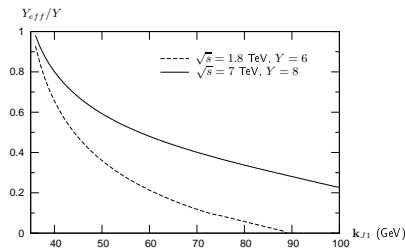
An asymmetric configuration is necessary for fixed order but can be problematic for BFKL: a BFKL calculation does not preserve energy-momentum conservation.

This was studied at LL by [Del Duca and Schmidt](#). They introduced an effective rapidity Y_{eff} defined as

$$Y_{eff} \equiv Y \frac{\int d\phi \cos(n\phi) \frac{d\sigma^{\mathcal{O}(\alpha^3)}}{dy_1 dy_2 d\mathbf{k}_{J1} d\mathbf{k}_{J2}}}{\int d\phi \cos(n\phi) \frac{d\sigma^{\text{BFKL}}}{dy_1 dy_2 d\mathbf{k}_{J1} d\mathbf{k}_{J2}}}$$

When one replaces Y by Y_{eff} in the expression of \mathcal{C}_n and truncates to $\mathcal{O}(\alpha^3)$, the exact $2 \rightarrow 3$ result is obtained

Variation of Y_{eff}/Y as a function of \mathbf{k}_{J_2} for fixed $\mathbf{k}_{J_1} = 35$ GeV:



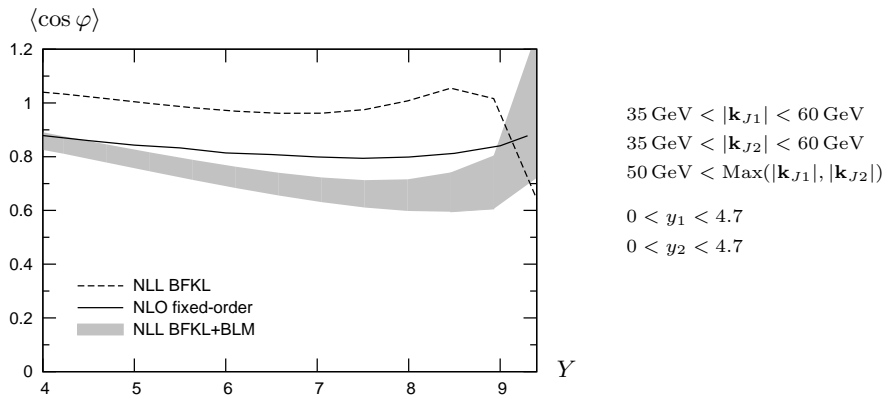
- When \mathbf{k}_{J_1} and \mathbf{k}_{J_2} are close, Y_{eff} is close to 1 \rightarrow this effect should not be very important when $\mathbf{k}_{J_{min1}} = \mathbf{k}_{J_{min2}}$
- Y_{eff} decreases quickly when the difference between \mathbf{k}_{J_1} and \mathbf{k}_{J_2} increases
- Y_{eff} is closer to Y when going to higher energies and/or rapidities
- One can hope that the effect is less severe at NLL (work in progress)

Results for an asymmetric configuration

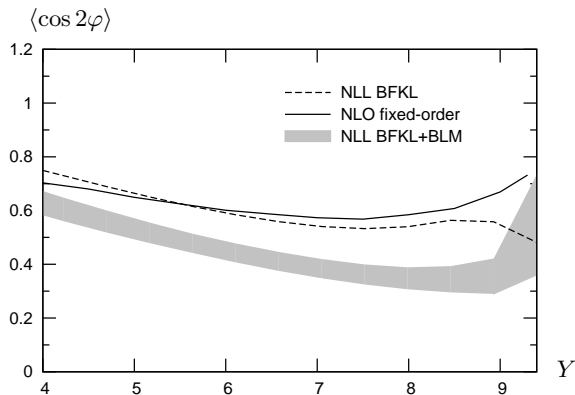
In this section we choose the cuts as

- $35 \text{ GeV} < |\mathbf{k}_{J1}|, |\mathbf{k}_{J2}| < 60 \text{ GeV}$
- $50 \text{ GeV} < \text{Max}(|\mathbf{k}_{J1}|, |\mathbf{k}_{J2}|)$
- $0 < y_1, y_2 < 4.7$

And we compare our results with the NLO fixed-order code Dijet ([Aurenche, Basu, Fontannaz](#)) in the same configuration

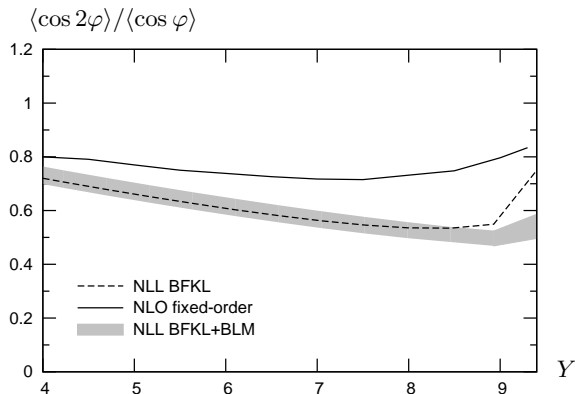
Azimuthal correlation $\langle \cos \varphi \rangle$ 

- As in the symmetric case, the BLM procedure strongly modifies the result of our BFKL calculation
- The NLO fixed-order and NLL BFKL+BLM calculations are very close

Azimuthal correlation $\langle \cos 2\varphi \rangle$ 

$35 \text{ GeV} < |\mathbf{k}_{J1}| < 60 \text{ GeV}$
 $35 \text{ GeV} < |\mathbf{k}_{J2}| < 60 \text{ GeV}$
 $50 \text{ GeV} < \text{Max}(|\mathbf{k}_{J1}|, |\mathbf{k}_{J2}|)$
 $0 < y_1 < 4.7$
 $0 < y_2 < 4.7$

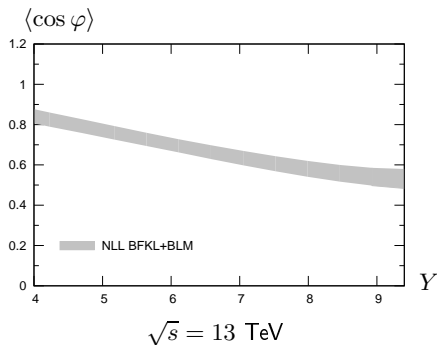
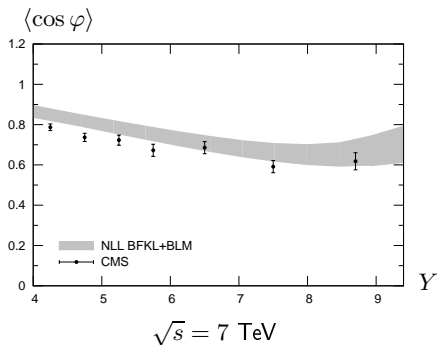
- As in the symmetric case, the BLM procedure strongly modifies the result of our BFKL calculation
- The BLM procedure leads to a larger difference between NLO fixed-order and NLL BFKL+BLM

Azimuthal correlation $\langle \cos 2\varphi \rangle / \langle \cos \varphi \rangle$ 

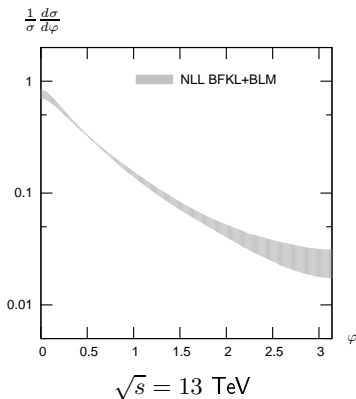
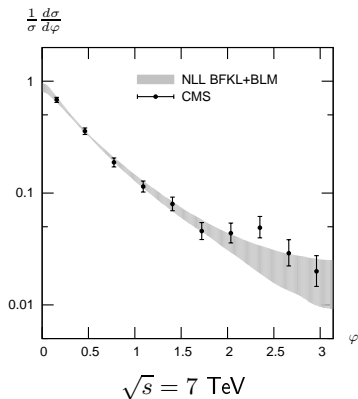
$35 \text{ GeV} < |\mathbf{k}_{J1}| < 60 \text{ GeV}$
 $35 \text{ GeV} < |\mathbf{k}_{J2}| < 60 \text{ GeV}$
 $50 \text{ GeV} < \text{Max}(|\mathbf{k}_{J1}|, |\mathbf{k}_{J2}|)$
 $0 < y_1 < 4.7$
 $0 < y_2 < 4.7$

Using BLM or not, there is a sizable difference between BFKL and fixed-order

- With upcoming LHC data at higher energies, we could look for BFKL effects comparing with results at 7 TeV
- We reproduced our analysis with $\sqrt{s} = 13$ TeV with the same cuts:
 - $35 \text{ GeV} < |\mathbf{k}_{J1}|, |\mathbf{k}_{J2}| < 60 \text{ GeV}$
 - $50 \text{ GeV} < \text{Max}(|\mathbf{k}_{J1}|, |\mathbf{k}_{J2}|)$ (for asymmetric configuration)
 - $0 < y_1, y_2 < 4.7$
- Energy-momentum conservation should be less problematic because of the larger energy

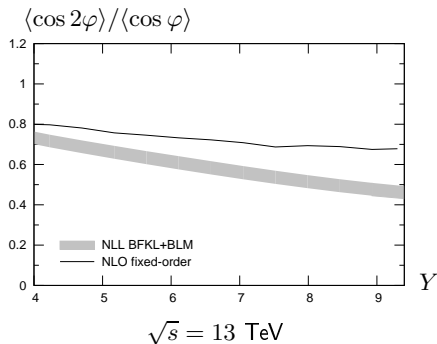
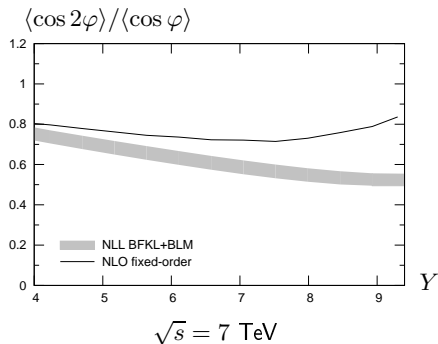
Azimuthal correlation $\langle \cos \varphi \rangle$ 

The behavior is similar at 13 TeV and at 7 TeV

Azimuthal distribution (integrated over $6 < Y < 9.4$)

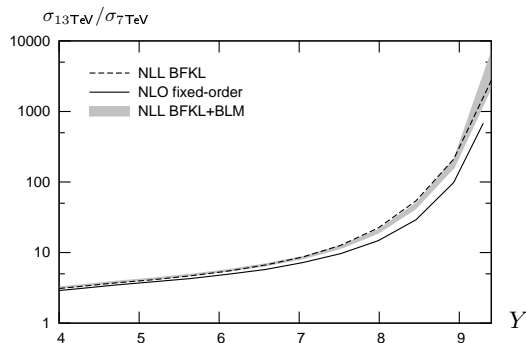
The behavior is similar at 13 TeV and at 7 TeV

Azimuthal correlation $\langle \cos 2\varphi \rangle / \langle \cos \varphi \rangle$ (asymmetric configuration)



The difference between BFKL and fixed-order is smaller at 13 TeV than at 7 TeV

Cross section



$$35 \text{ GeV} < |\mathbf{k}_{J1}| < 60 \text{ GeV}$$

$$35 \text{ GeV} < |\mathbf{k}_{J2}| < 60 \text{ GeV}$$

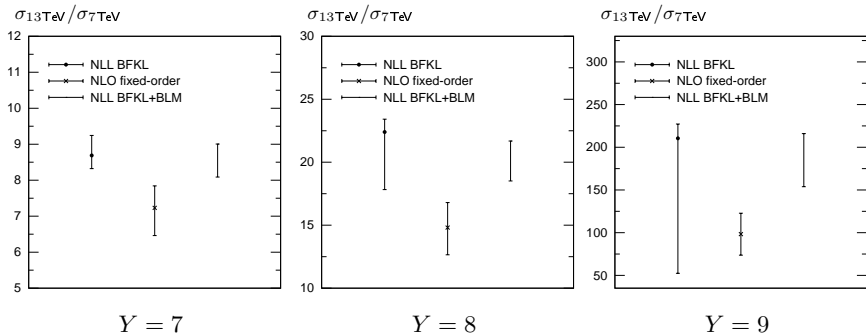
$$50 \text{ GeV} < \text{Max}(|\mathbf{k}_{J1}|, |\mathbf{k}_{J2}|)$$

$$0 < y_1 < 4.7$$

$$0 < y_2 < 4.7$$

- In a BFKL treatment, a strong rise of the cross section with increasing energy is expected.
- This rise is faster than in a fixed-order treatment

Cross section



Taking into account the scales and PDFs uncertainties, the difference between BFKL and fixed order is quite small

- We studied Mueller-Navelet jets at full (vertex + Green's function) **NLL** accuracy and compared our results with the first data from the **LHC**
- The agreement with **CMS** data at 7 TeV is greatly improved by using the **BLM** scale fixing procedure
- $\langle \cos 2\varphi \rangle / \langle \cos \varphi \rangle$ is almost not affected by BLM and shows a clear difference between **NLO fixed-order** and **NLL BFKL** in an asymmetric configuration but one should be careful because of energy-momentum conservation issues
- We did the same analysis at 13 TeV:
 - Azimuthal decorrelations don't show a very different behavior at 13 TeV compared to 7 TeV
 - **NLL BFKL** predicts a stronger rise of the cross section with increasing energy than a **NLO fixed-order** calculation