Lattice aspects of Higgs boson physics

Karl Jansen

- **Introduction**
- **Realizing a Higgs-Yukawa model on the lattice**
- **Higgs boson mass bounds and their consequences**
- **Conclusions**

review article: P. Hegde et.al., arXiv:1310.5922

thesis of P. Gerhold, arXiv:1002.2569

thesis of J. Kallarackal, paperback, 79,-Euro

lattice plenary talk by J. Espinosa, arXiv:1311.1970

The Higgs sector of the standard model

- the Lagrangian of the scalar theory

$$L_\varphi[\varphi] = \tfrac{1}{2}\partial_\mu\varphi_x^\dagger\partial_\mu\varphi_x + \tfrac{1}{2}m_0^2\varphi_x^\dagger\varphi_x + \lambda_0\left(\varphi_x^\dagger\varphi_x\right)^2,$$

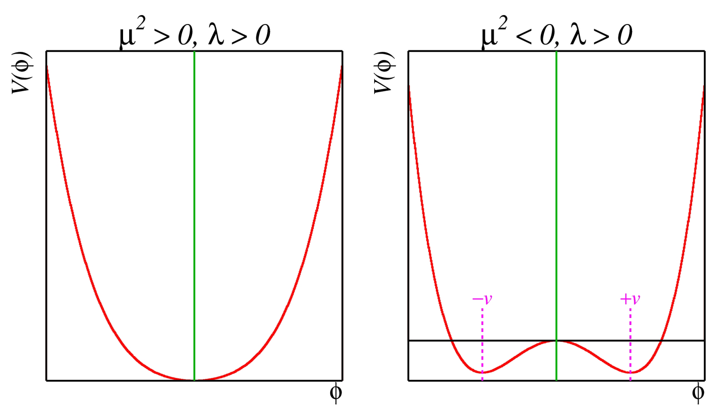
– m_0^2 bare Higgs boson mass, λ_0 bare quartic coupling

– φ 4-component real scalar field

– $O(N)$ invariance of Lagrangian

two phases $\langle\varphi\rangle \equiv v = 0$ symmetric phase

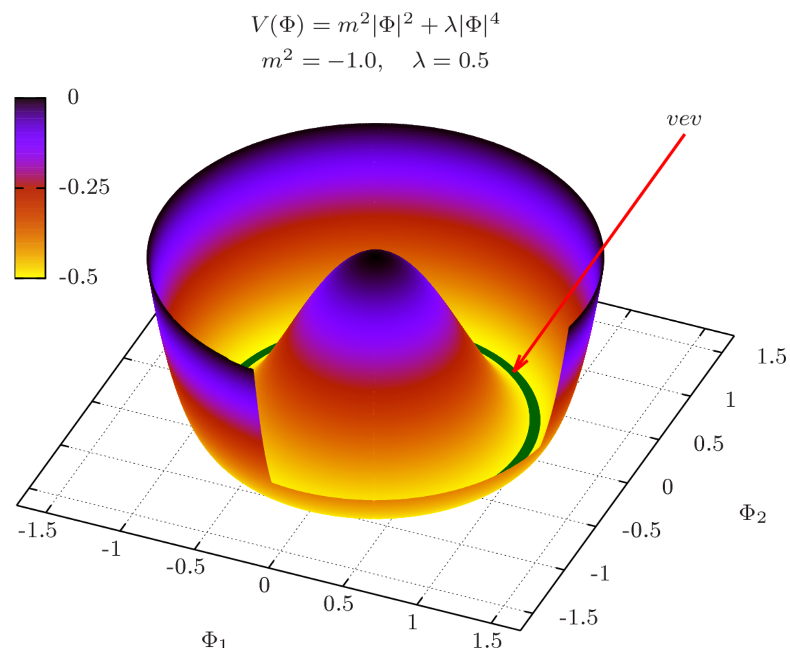
$v > 0$ spontaneously broken (Higgs) phase



$v = 0$ $\qquad$ $v > 0$; $m_0^2 = 2v^2 * \lambda_0$

The potential for $N = 2$

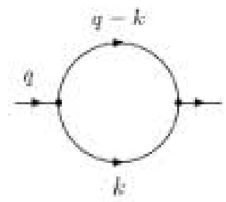
- massless Goldstone modes



(A. Nagy)

Renormalized quartic coupling constant

- 1-loop shift of quartic coupling



$$\delta\lambda_0(q) = \frac{(-i\lambda_0)^2}{2}\int \frac{d^4k}{(2\pi)^4}\frac{i}{k^2-m_0^2}\cdot\frac{i}{(k+q)^2-m_0^2} \propto \int dk\frac{k^3}{k^4}$$

- need to regulate theory

- introduce sharp momentum cut-off parameter Λ

- want to remove Λ eventually

The running quartic couping constant

- 1-loop analysis of broken phase of the scalar theory

$$\lambda^{\text{ren}} = \frac{\lambda_0}{1 - b_N \lambda_0 \, \log\left(\frac{\Lambda^2}{m_H^2}\right)} \; , b_N = \frac{N+8}{8\pi^2}$$

singularity (Landau pole): $\log\left(\frac{\Lambda^2}{m_H^2}\right) = \frac{1}{b_N \lambda_0}$

demanding $\Lambda \geq 2 m_H$(for fixed λ_0):

– avoid Landau pole

– suppression of cut-off effects e.g. in scattering amplitudes

$\Rightarrow$ upper bound for coupling (at $\lambda_0 = \infty$):

$$\lambda^{\text{ren}} < \frac{1}{b_N \, \log 2}$$

Consequences from 1-loop analysis

upper bound for renormalized quartic coupling: $\lambda^{\rm ren} < \frac{1}{b_N \; \log 2}$

Using relation $m_H^2 = 2\lambda^{\rm ren} v^2$, $v = 246\text{GeV}$

$\Rightarrow$ bound for Higgs boson mass (setting $N = 4$): $m_H^2 < \frac{4\pi^2 v^2}{3\log\frac{\Lambda}{m_H}}$

Interpretation of $\lambda^{\rm ren} = \lambda_0/(1 - b_N \lambda_0 \log\left(\Lambda^2/m_H^2\right))$

- for $\Lambda \to \infty : \lambda^{\rm ren} = 0 \leftarrow$ triviality of the φ^4-theory (and of the standard-model)

- cut-off cannot be removed from the theory
  $\Rightarrow$ standard model only effective theory, valid up to a certain cut-off value Λ

- intrinsic relation between cut-off and Higgs-boson mass

- interpretation of cut-off: energy scale of yet to be discovered
  physics beyond the standard model

The Higgs-Yukawa sector of the standard model

- the scalar theory

$$L_\varphi[\varphi] = \frac{1}{2}\partial_\mu\varphi_x^\dagger\partial_\mu\varphi_x + \frac{1}{2}m_0^2\varphi_x^\dagger\varphi_x + \lambda\left(\varphi_x^\dagger\varphi_x\right)^2$$

- the fermionic and Yukawa parts

$$(L_F + L_Y)[\bar\psi,\psi] = \bar\psi i\gamma_\mu\partial_\mu\psi + y_b\left(\bar t,\bar b\right)_L\varphi b_R + y_t\left(\bar t,\bar b\right)_L\tilde\varphi t_R + c.c.$$

exact $\mathrm{SU}(2)_L \times \mathrm{U}(1)_R$ chiral symmetry

$$\psi \to U_R P_+\psi + \Omega_L P_-\psi, \bar\psi \to \bar\psi P_+\Omega_L^\dagger + \bar\psi P_- U_R^\dagger,$$

$$\phi \to U_R\phi\Omega_L^\dagger, \phi^\dagger \to \Omega_L\phi^\dagger U_R^\dagger.$$

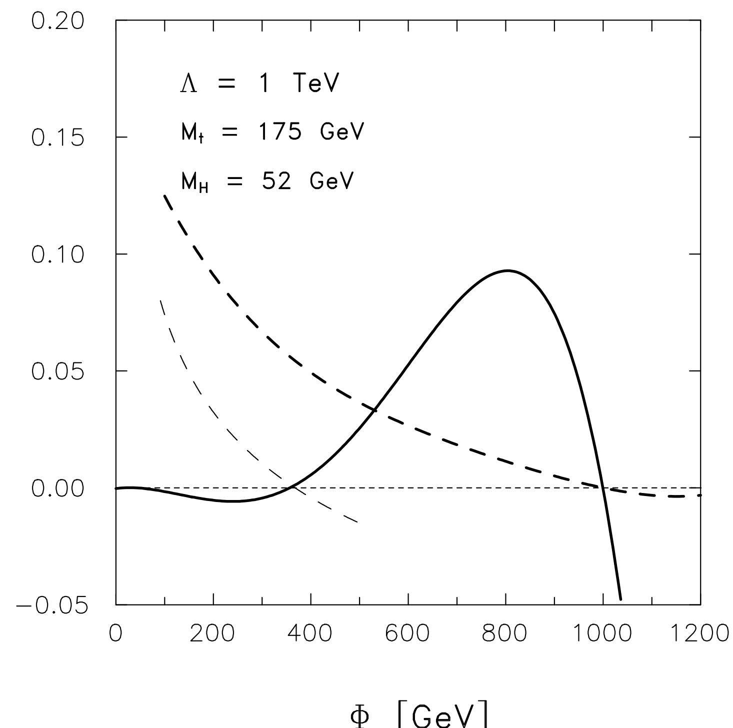
with $\Omega_L \in \mathrm{SU}(2)$, $U_R \in U(1)$ projectors: $P_\pm = \frac{1\pm\gamma_5}{2}$

Effects of adding fermions

- effective potential

$$U_{\text{eff}} = V + 1/2 \int_k \ln[k^2 + m^2] - 2N_{\text{F}} \int_k \ln[k^2 + y^2\varphi^2]$$

$\Rightarrow$ negative contribution for fermions $\rightarrow$ theory becomes unstable



- to avoid instability
  $\rightarrow$ (lower) bound on Higgs boson mass

lattice needed since instability maybe artefact of perturbation theory

remark, convexity of effective potential (L. O'Raifeartaigh, A. Wipf, H. Yoneyama)

Simulating Higgs-Yukawa sector of standard model on the lattice

- many attempts (≈ 1990), an incomplete list:
  - Smit-Swift model
    (Smit, Swift; Aoki; Bock, De, K.J., A. Hasenfratz, Jersak, Neuhaus), Shen, ...
  - Rome action (Rossi, Testa)
  - Mirror fermions (Montvay)
  - Domain wall fermions (Kaplan; Golterman, K.J. Vink, Petcher)

Unsuccessful:
not able to remove doublers from spectrum while maintaining chiral symmetry

Nevertheless, GW-relation, following Kaplan's idea opened the solution clash between *chiral symmetry* and *fermion proliferation*

$\rightarrow$ Nielsen-Ninomiya theorem:

For any lattice Dirac operator D the conditions

- D is local (bounded by $Ce^{-\gamma/a|x|}$)
- $\tilde{D}(p) = i\gamma_\mu p_\mu + \mathrm{O}(ap^2)$ for $p \ll \pi/a$
- $\tilde{D}(p)$ is invertible for all $p \neq 0$
- $\gamma_5 D + D\gamma_5 = 0$

can not be simultaneously fulfilled

The theorem simply states the fact that the Chern number is a cobordism invariant
(Friedan)

solution: **Ginsparg-Wilson relation**

$$\gamma_5 D + D\gamma_5 = 2aD\gamma_5 D$$

Ginsparg-Wilson relation implies an *exact lattice chiral symmetry* (Lüscher):

for any operator D which satisfies the Ginsparg-Wilson relation, the action

$$S = \bar{\psi} D \psi$$

is invariant under the transformations

$$\delta\psi = \gamma_5(1 - \tfrac{1}{2}aD)\psi \ , \qquad \delta\bar{\psi} = \bar{\psi}(1 - \tfrac{1}{2}aD)\gamma_5$$

$\Rightarrow$ almost continuum like behaviour of fermions

one local (Hernandez, Lüscher, K.J.) solution of GW relation:
overlap operator $D_{\rm ov}$ (Neuberger)

$$D_{\rm ov} = \left[1 - A(A^\dagger A)^{-1/2}\right]$$

with $A = 1 + s - D_{\rm w}(m_q = 0)$; s a tunable parameter, $0 < s < 1$

The scalar lattice action

- continuum action

$$S_\varphi[\varphi] = \sum_{x,\mu} \tfrac{1}{2}\partial_\mu \varphi_x^\dagger \partial_\mu \varphi_x + \sum_x \tfrac{1}{2} m_0^2 \varphi_x^\dagger \varphi_x + \sum_x \lambda \left(\varphi_x^\dagger \varphi_x\right)^2 ,$$

with $\partial_\mu \varphi(x) \to \nabla_{\rm latt}\varphi(x) = (\varphi(x+a\mu) - \varphi(x))/a$

and a rescaling $\Phi(x) = \sqrt{2\kappa}\varphi(x)$, $\lambda = \frac{\hat{\lambda}}{4\kappa^2}$, $m_0^2 = \frac{1-2\hat{\lambda}-8\kappa}{\hat{\kappa}}$

- lattice scalar action (setting lattice spacing $a = 1$)

$$S_\Phi = -\kappa \sum_{x,\mu} \Phi_x^\dagger \left[\Phi_{x+\hat{\mu}} + \Phi_{x-\hat{\mu}}\right] + \sum_x \Phi_x^\dagger \Phi_x + \hat{\lambda} \sum_x \left(\Phi_x^\dagger \Phi_x - 1\right)^2$$

Today: drastically changed situation

- have an *exact* lattice chiral symmetry
- computing power increased by $O(10^4)$
- substantially improved algorithms
- better theoretical understanding of lattice theory
  - finite size effects
  - lattice perturbative calculations
  - treating resonances on the lattice

Chiral invariant Higgs-Yukawa lattice action (Lüscher)

- the lattice fermionic and Yukawa parts

$$(L_F + L_Y)[\bar{\psi}, \psi] = \bar{\psi} D_{\text{ov}} \psi + y_b \left(\bar{t}, \bar{b}\right)_L \varphi b_R + y_t \left(\bar{t}, \bar{b}\right)_L \tilde{\varphi} t_R + c.c.$$

- has exactly the same form as in the continuum

- change from continuum:
  - $i\gamma_\mu \partial_\mu \to D_{\text{ov}}$
  - $P_\pm = \frac{1 \pm \gamma_5}{2} \to \hat{P}_\pm = \frac{1 \pm \hat{\gamma}_5}{2}, \hat{\gamma}_5 = \gamma_5 \left(1 - a D_{\text{ov}}\right)$

- exact *lattice* $\mathrm{SU}(2)_L \times \mathrm{U}(1)_R$ chiral symmetry

$$\psi \to U_R \hat{P}_+ \psi + \Omega_L \hat{P}_- \psi, \bar{\psi} \to \bar{\psi} P_+ \Omega_L^\dagger + \bar{\psi} P_- U_R^\dagger$$

$$\phi \to U_R \phi \Omega_L^\dagger, \phi^\dagger \to \Omega_L \phi^\dagger U_R^\dagger.$$

with $\Omega_L \in \mathrm{SU}(2)$, $U_R \in U(1)$

The original (old) motivation for a lattice study

- upper bound:
  $\rightarrow$ coupling becomes strong, unclear whether perturbation theory is valid

- lower bound:
  $\rightarrow$ is vacuum instability an artefact of perturbation theory?

- if Higgs boson very heavy
  $\rightarrow$ non-perturbative width effects?

- effects of possible very heavy fermions

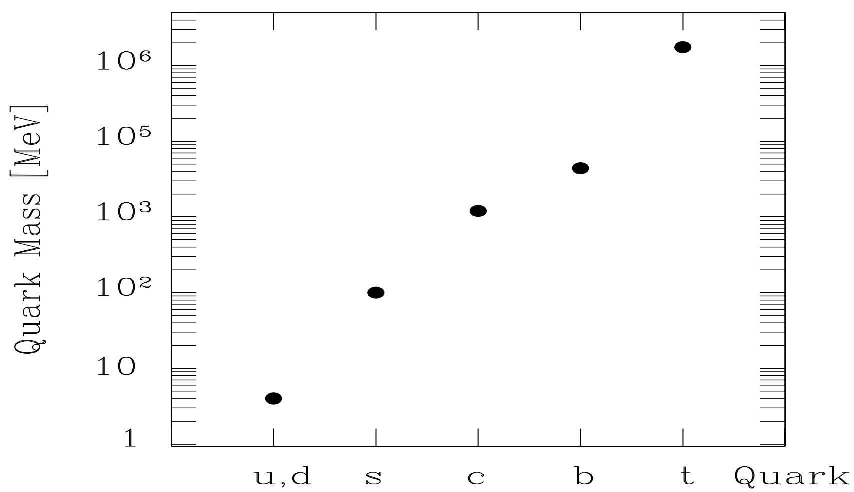
The Higgs boson and the standard model

- Higgs boson discovery Atlas animation

- With the Higgs boson discovery $\rightarrow$ standard model complete

- But, is our understanding of particle interaction also complete?

- Many open questions:
  - quark mass hierarchy:
  - dark matter
  - matter anti-matter asymmetry
  - insufficient amount of CP-violation



New motivation

- knowing Higgs boson mass
  $\Rightarrow$ clash with mass bounds $\rightarrow$ scale of new physics

- effects of higher dimensional operators
  $\rightarrow$ simple example of remnant of new physics

- possibility of non-trivial fixed points
  $\rightarrow$ exploration of phase diagram

want/need non-perturbative method

Higgs-Yukawa model phase diagram

(P. Gerhold, K.J.)

Let me describe a typical computer simulation:[...]
the first thing to do is to look for phase transitions (G. Parisi)

tools:

- analytical large N (number of fermion generations) calculations
  Monte Carlo Simulations

observables:

- magnetization $v_{\mathrm{mag}} = \langle \frac{1}{V} \sum_x \Phi_x \rangle$
  staggered magnetization: $v_{\mathrm{stagg}} = \langle \frac{1}{V} \sum_x (-1)^x \Phi_x \rangle$

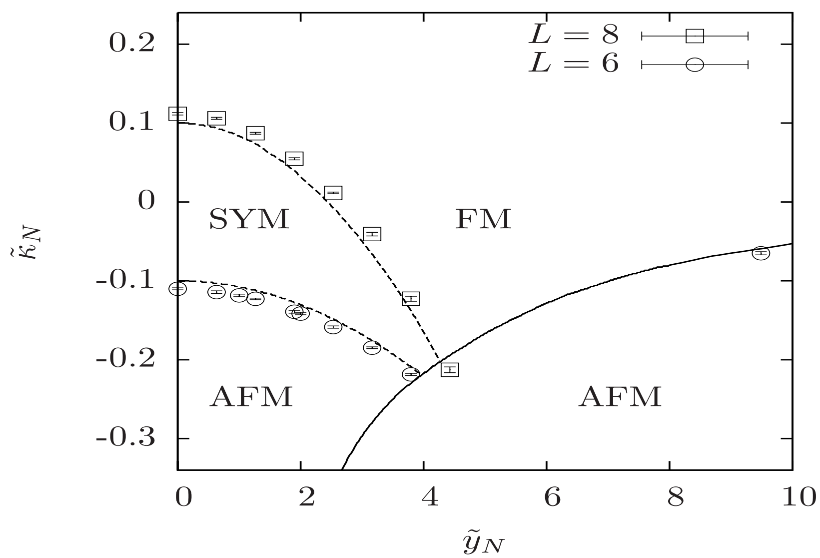
phases:

– ferromagnetic, $v_{\mathrm{mag}} > 0, v_{\mathrm{stagg}} = 0$ (broken phase)

– paramagnetic, $v_{\mathrm{mag}} = 0, v_{\mathrm{stagg}} = 0$ (symmetric)

– anti-ferromagnetic, $v_{\mathrm{mag}} = 0, v_{\mathrm{stagg}} > 0$

Comparing large-N and Monte Carlo: small Yukawa coupling



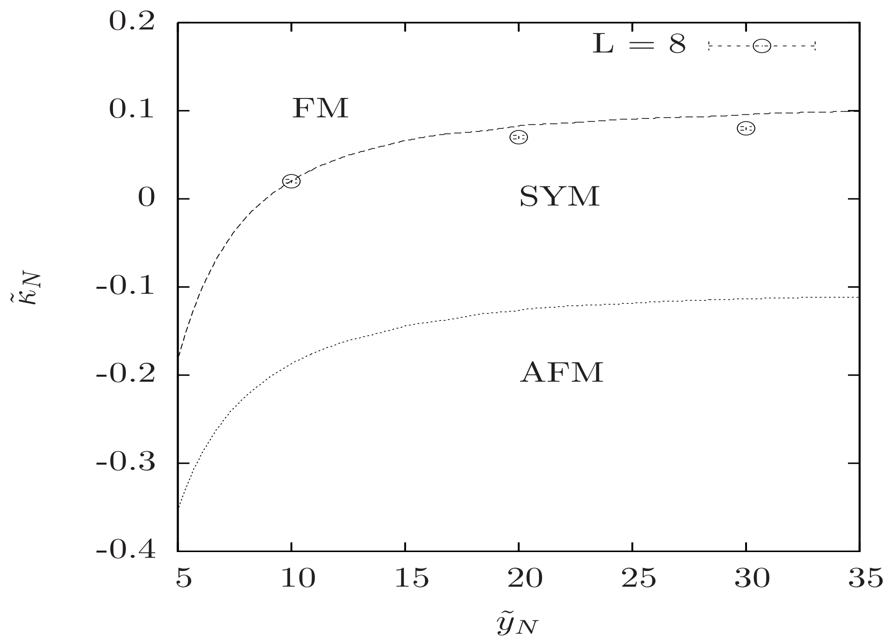
FM: ferromagnetic

SYM: symmetric

AFM: anti-ferromagnetic

Note that $N = 4$ here

Comparing large-N and Monte Carlo: large Yukawa coupling



FM: ferromagnetic

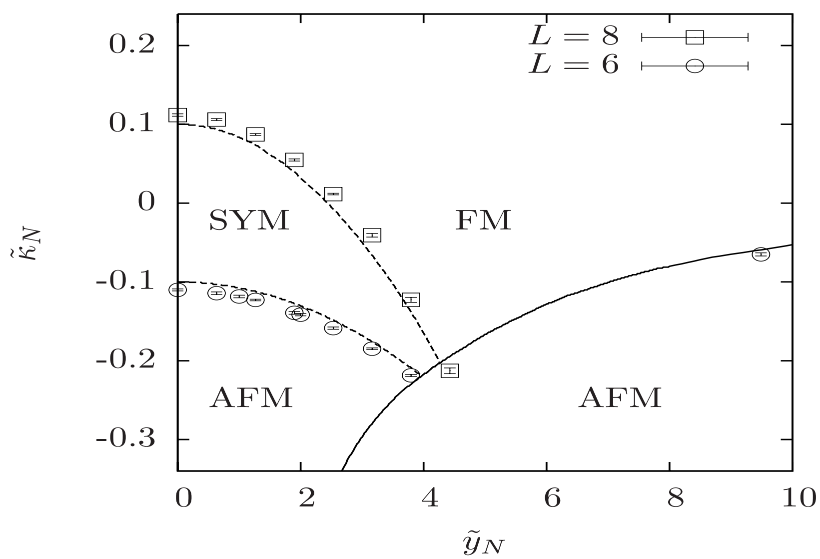
SYM: symmetric

AFM: anti-ferromagnetic

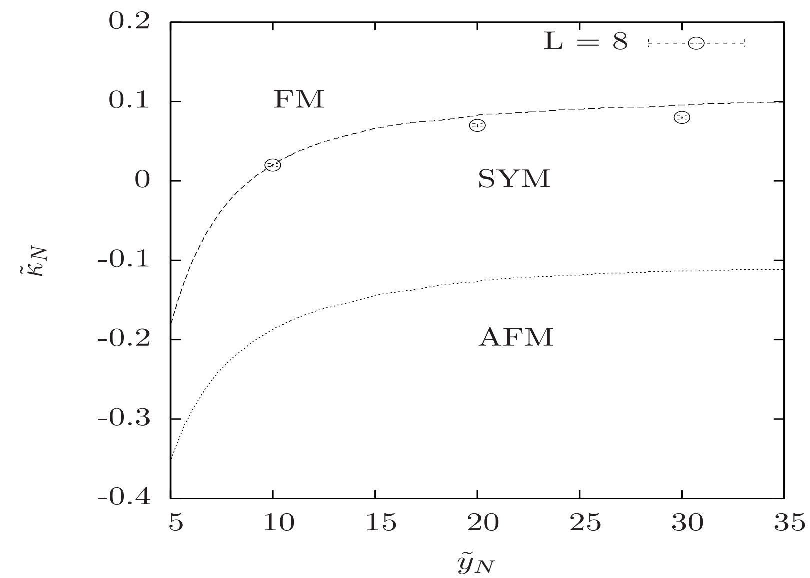
special large-N expansion for large y
for $y \to \infty$: recover pure O(4) non-linear σ-model

Higgs-Yukawa model phase diagram

(P. Hedge, G. Hou, D. Lin, K. Nagai, A. Nagy, K. Ogawa, K.J.)



standard model region



large Yukawa-coupling region

FM: ferromagnetic (AFM: anti-ferromagnetic); SYM: symmetric

large Yukawa coupling: non-trivial fixed points?

Critical phenomena

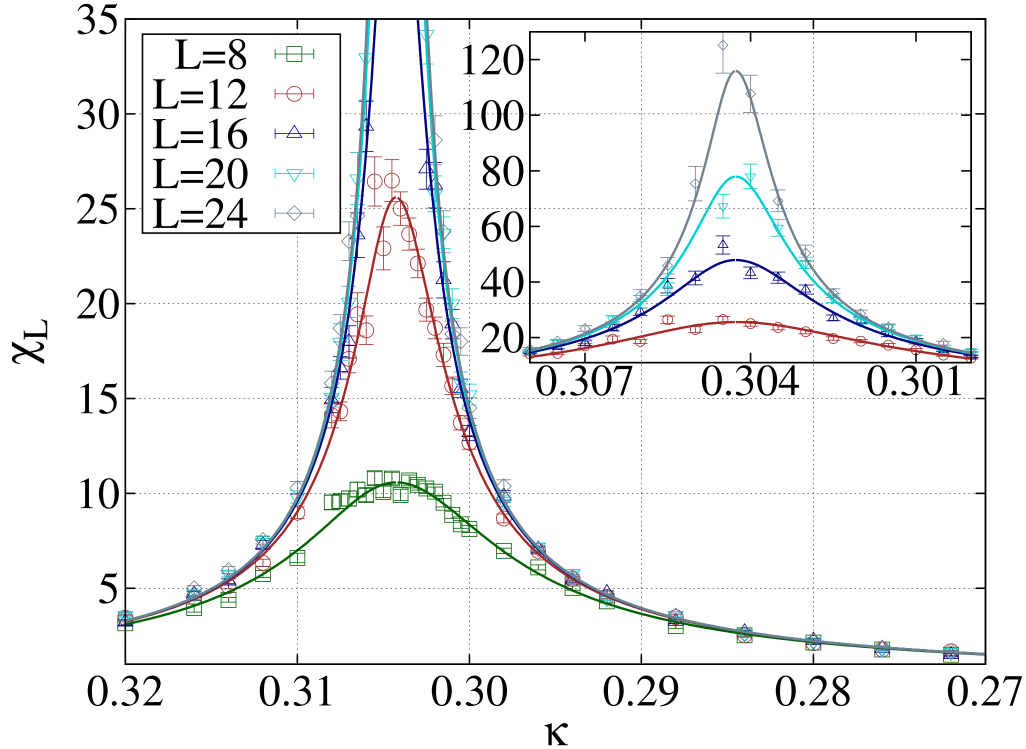
- Critical exponents of model define universality class
- Use finite volume to compute critical exponents
- Investigation of susceptibility: $\chi_L = V\left[\langle v_{\mathrm{mag}}^2(L)\rangle - \langle v_{\mathrm{mag}}\rangle^2\right]$
- Scales like:

$$\begin{aligned}\chi_L\left(|T-T_c^L| \gg 1\right) &\sim \left|T-T_c^L\right|^{-\gamma}\\ \chi_L\left(|T-T_c^L| \to 0\right) &\sim L^{1/\nu}\\ T_c^L - T_c^\infty &\sim L^{-1/\nu}\end{aligned}$$

  for Gaussian fixed point: $\nu = 1/2$ and $\gamma = 1$

- T represents either κ in O(4)-model or y in Higgs-Yukawa model
- $\log$-corrections in case of triviality: $L^{1/\nu} \to L^2\left(\log L\right)^{1/2}$

Representative study of O(4)-model



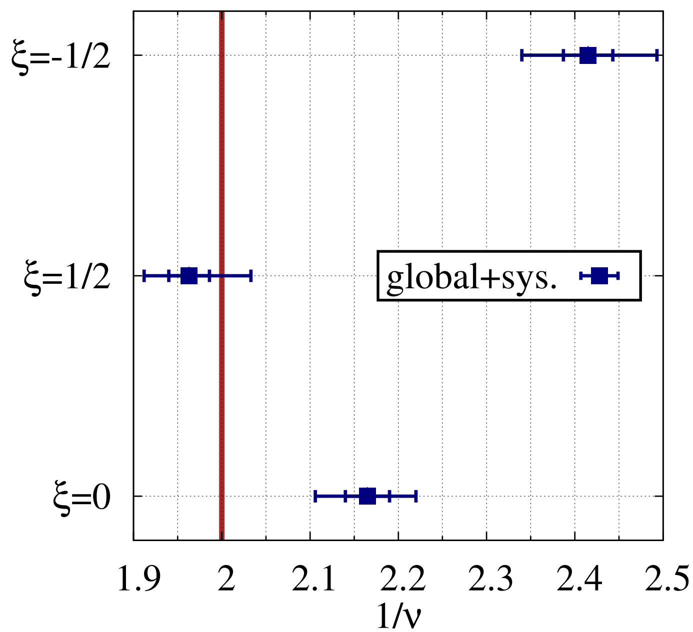
- observe divergence of susceptibility with volume

Finite size scaling

- use finite size scaling of peak height

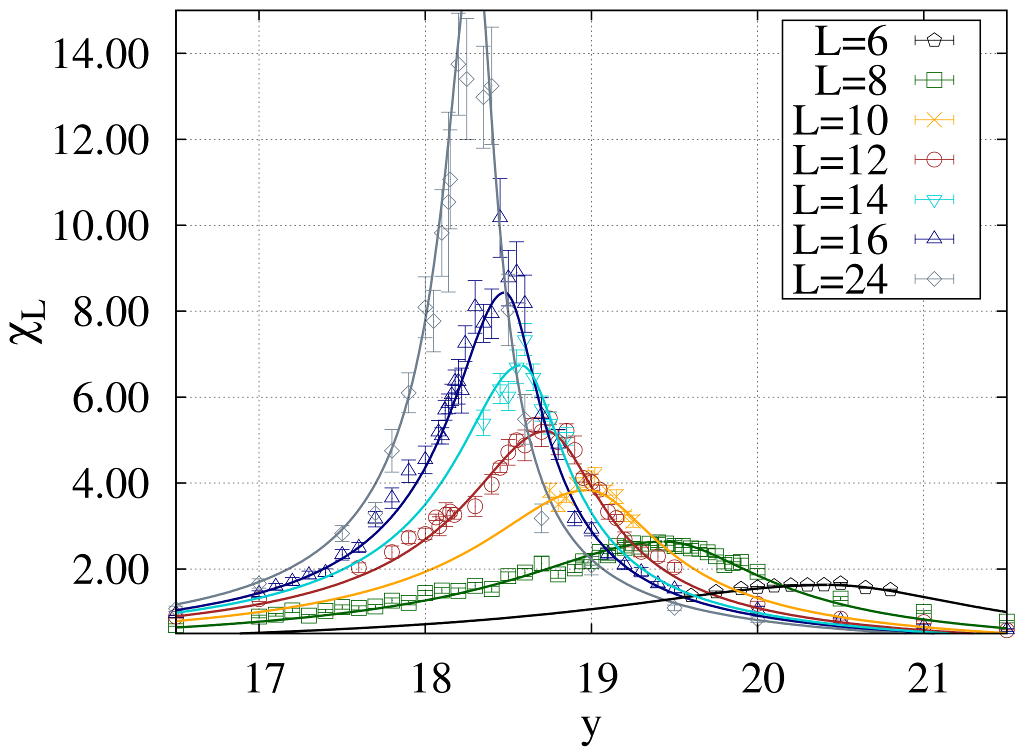
$$\chi_{\max}(L;\xi) = \left[A_1 \cdot (L[\log L]^{\xi})^{1/\nu}\right]$$

- play game: use different values of ξ



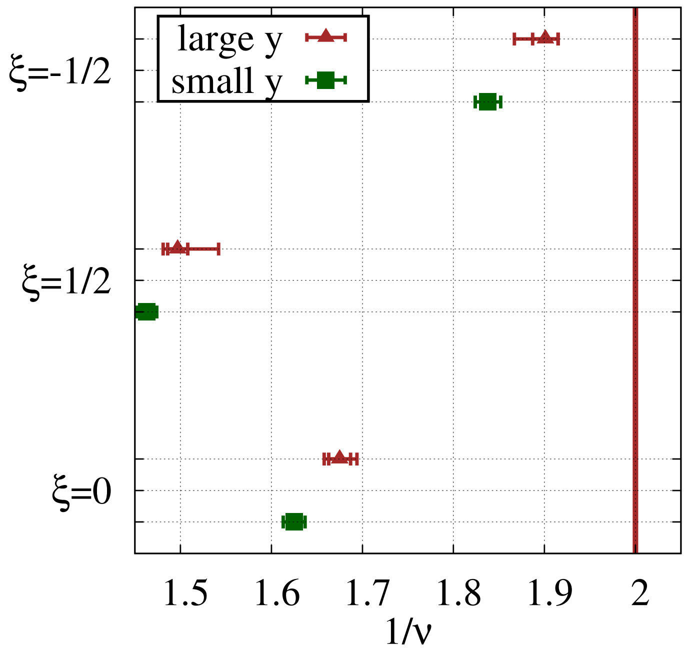
- only value of $\xi = 1/2$ gives back correct value of $\nu = 1/2$

Repeat analysis for large Yukawa coupling region

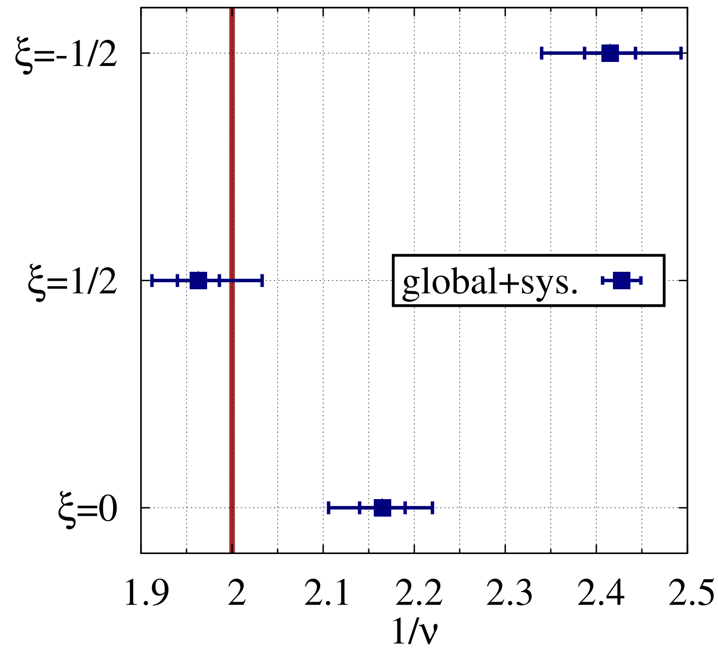


- divergence of susceptibility $\longrightarrow$ 2nd order phase transition
  $\Longrightarrow$ continuum limit can be taken

Playing the ξ-game for the Higgs-Yukawa model



large Yukawa coupling



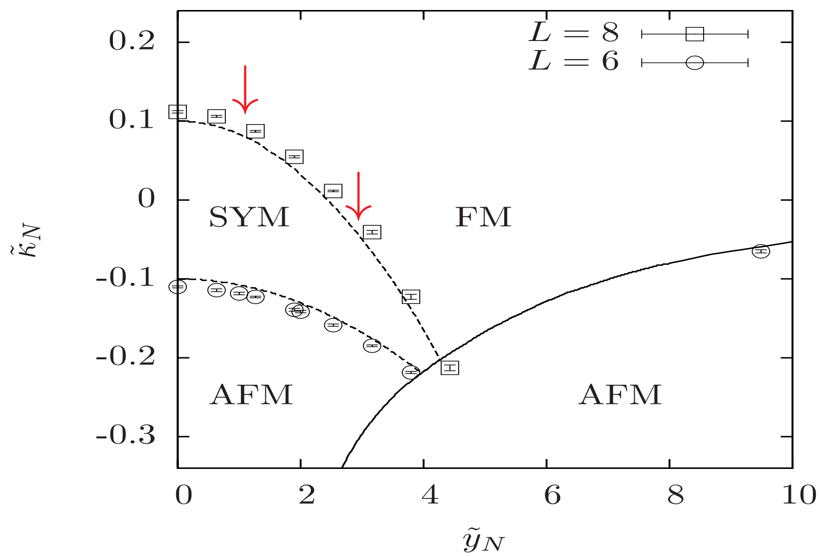
O(4) model

- theoretical value of $\nu = 1/2$ not known
- small Yukawa couling region corresponds to Gaussian fixed point
- large Yukawa coupling region $\approx$ small Yukawa coupling region
  $\rightarrow$ evidence that also for $y \gg 1$ Gaussian fixed point
- theoretical determination of log-correction for Higgs-Yukawa theory desirable

Returning to small Yukawa-coupling region: mass bounds

(P. Gerhold, J. Kallarackal, K.J.)

- small Yukawa-coupling: SM scenario
- simulations: approaching critical line from broken phase



The algorithm

Usage of Polynomial Hybrid Monte Carlo Algorithm (Frezzotti, K.J.)

improvements (Gerhold):

- special preconditioning techniques for fermion matrix:
  $\rightarrow$ factors of O(10)-O(100) improvement for condition number

- Fourier acceleration

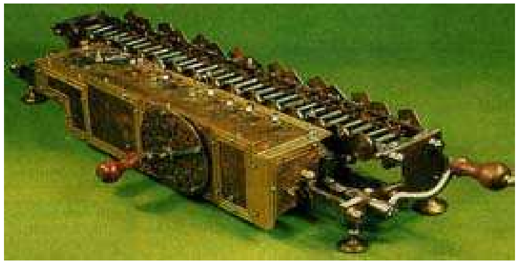
	FACC	traLength	Nconf	ACtime	cost
$\kappa = 0.12313$	No	2.0	2020	132.1 ± 6.4	2662 ± 129
$\kappa = 0.12313$	Yes	2.0	21780	1.1 ± 0.1	37 ± 1
$\kappa = 0.30400$	No	1.0	2580	34.9 ± 2.1	450 ± 28
$\kappa = 0.30400$	Yes	1.0	22360	3.8 ± 0.2	171 ± 8

- exact Krylow space reweighting

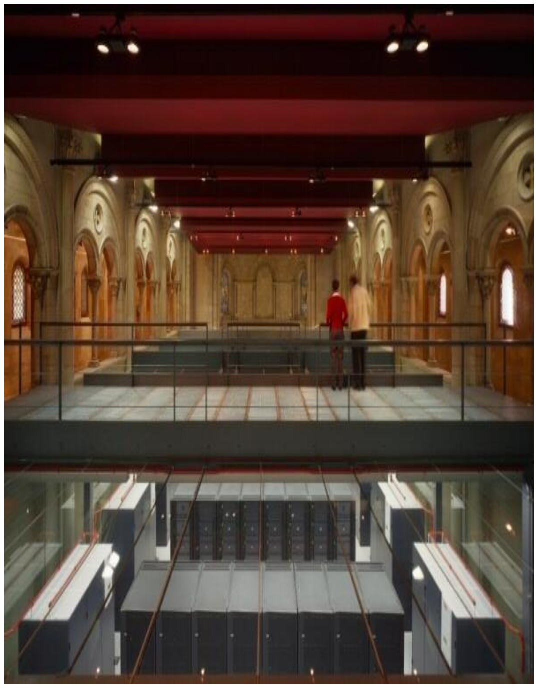
- multiple time scale integrators

Supercomputer

- Leibniz Computer



- MareNostrum, IBM, Barcelona
  40 Teraflops peak performance
- BlueGeneQ, NIC, FZ-Jülich
  5 Petaflop peak performance



Determination of physical scale

- physical input Higgs boson expectation value $v_r/a = 246\,\text{GeV}$
  top and bottom quark masses: $m_t/a \approx 175\,\text{GeV}$, $m_b/a \approx 4.2\,\text{GeV}$

- renormalized quartic coupling: $\lambda = \frac{m_H^2}{v_r^2}$

- renormalized Yukawa couplings: $y_{t,b} = \frac{m_{t,b}}{v_r}$

- setting the value of the lattice spacing $246\,\text{GeV} = \frac{v_r}{a} \equiv \frac{v}{\sqrt{Z_G \cdot a}}$, $\Lambda = a^{-1}$

- renormalization constant from Goldstone propagator $\left[\tilde{G}_G(\hat{p}^2)\right]^{-1} = \frac{\hat{p}^2 + m_{Gp}^2}{Z_G}$

Lattice observables to extract masses

Goldstone propagator:

$$\tilde{G}_G(p) = \frac{1}{3}\sum_{\alpha=1}^{3}\langle \tilde{g}_p^\alpha \tilde{g}_{-p}^\alpha \rangle, \quad \tilde{g}_p^\alpha = \frac{1}{\sqrt{L_s^3 \cdot L_t}}\sum_x e^{-ipx} g_x^\alpha$$

with discrete lattice momenta $p_\mu = 2\pi n_\mu / L_{s,t}$, $n_\mu = 0, \ldots, L_{s,t} - 1$

Higgs boson propagator

$$\tilde{G}_H(p) = \langle \tilde{h}_p \tilde{h}_{-p} \rangle, \quad \tilde{h}_p = \frac{1}{\sqrt{L_s^3 \cdot L_t}}\sum_x e^{-ipx} h_x$$

Fermion correlator

$$C_f(\Delta t) = \frac{1}{L_t \cdot L_s^6}\sum_{t=0}^{L_t - 1}\sum_{\vec{x},\vec{y}} \left\langle \mathrm{Re}\,\mathrm{Tr}\,\left(f_{L,t+\Delta t,\vec{x}} \cdot \bar{f}_{R,t,\vec{y}} \right) \right\rangle$$

$\Rightarrow$ fits to standard propagator forms $(Z/(p^2 + m^2)$ or to exponential decay in Euclidean time

Vaccum expectation value on the lattice

lattice simulations $\rightarrow$ *finite volume* $\rightarrow$ vanishing vacuum expectation value

- computing scalar expectation value:
  - introduce an external source J
  - take double limit

$$v = \lim_{J\to 0} \lim_{V\to\infty} \langle \Phi \rangle_{J,V}$$

alternative approach: global transformation on each field configuration at $J = 0$

$$\Phi_x^{rot} = U[\Phi]\Phi_x$$

with $U[\varphi] \in \mathrm{SU}(2)$ such that

$$\sum_x \Phi_x^{rot} = \begin{pmatrix} 0 \\ \left|\sum_x \Phi_x\right| \end{pmatrix}$$

$\rightarrow$ establishes an equivalent definition in the thermodynamic limit

$$v = \langle \Phi^{rot} \rangle$$

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  top and bottom quark masses: $m_t/a \approx 175\,\text{GeV}$, $m_b/a \approx 4.2\,\text{GeV}$

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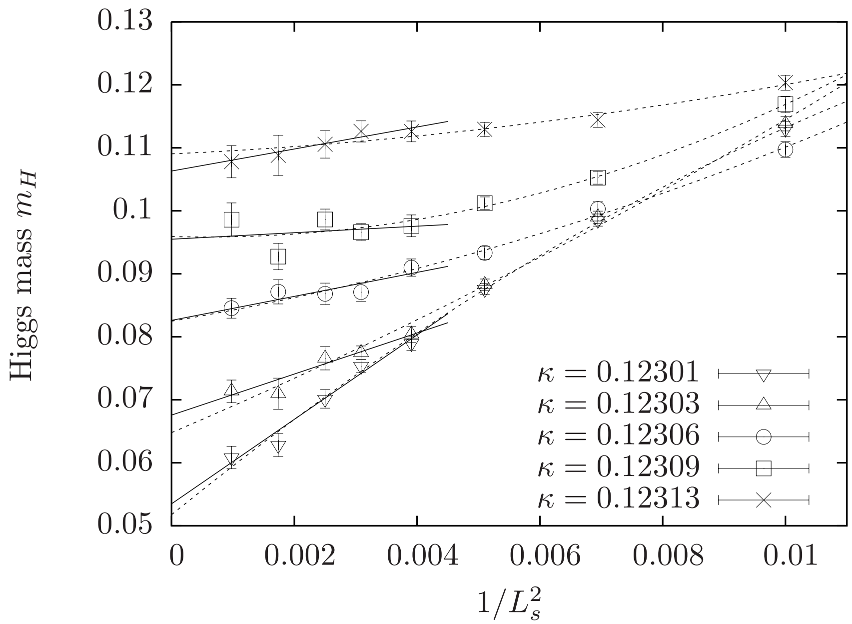
- renormalization constant from Goldstone propagator $\left[\tilde{G}_G(\hat{p}^2)\right]^{-1} = \frac{\hat{p}^2 + m_{Gp}^2}{Z_G}$

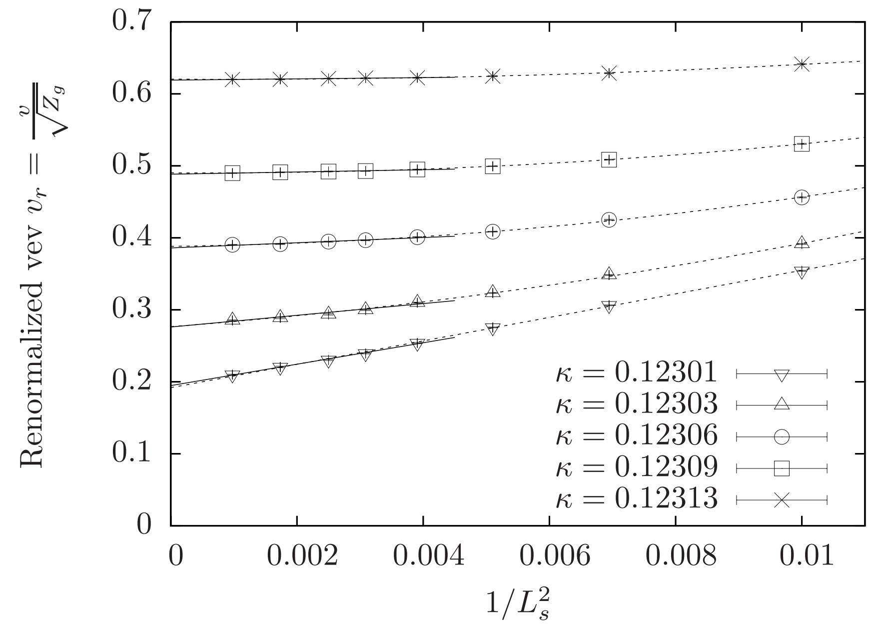
Finite size effects

Goldstone bosons induce significant finite size effects of the form

$$f_{v,m}^{(p)}(L_s^{-2}) = A_{v,m}^{(p)} + B_{v,m}^{(p)} \cdot L_s^{-2} + C_{v,m}^{(p)} \cdot L_s^{-4}$$

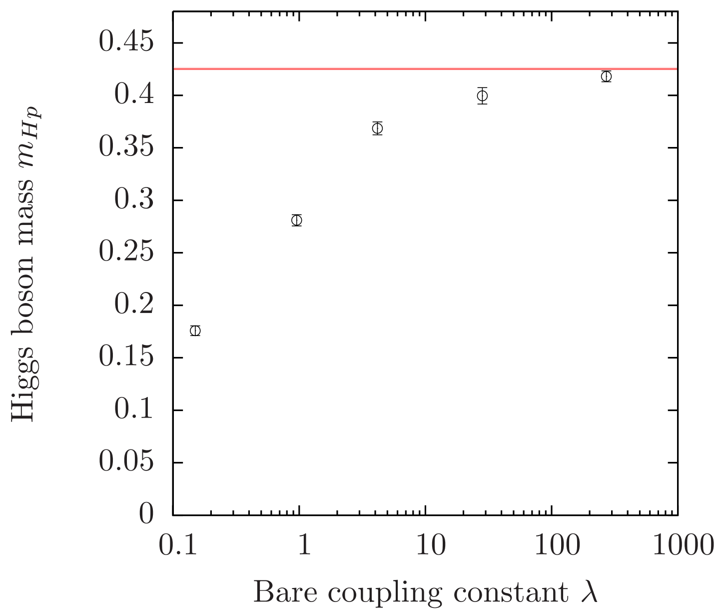
- data are well described by theoretical expectation
  (but had to go to lattices of size 40^4)
- allows infinite volume extrapolation
- use difference of only $1/L^2$ and combined $1/L^2 + 1/L^4$ fits as systematic errors





Largest Higgs boson mass at $\lambda = \infty$

- largest Higgs boson mass obtained at $\lambda = \infty$
- however, at large λ no analytical control anymore

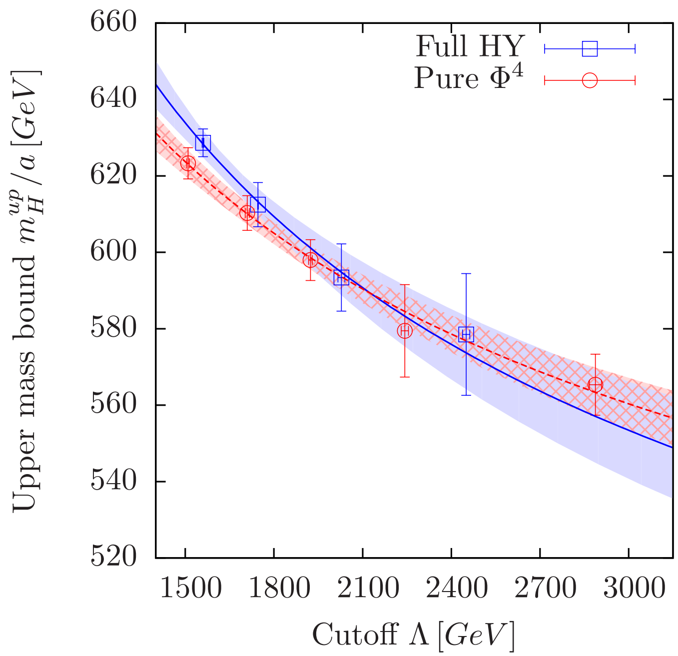


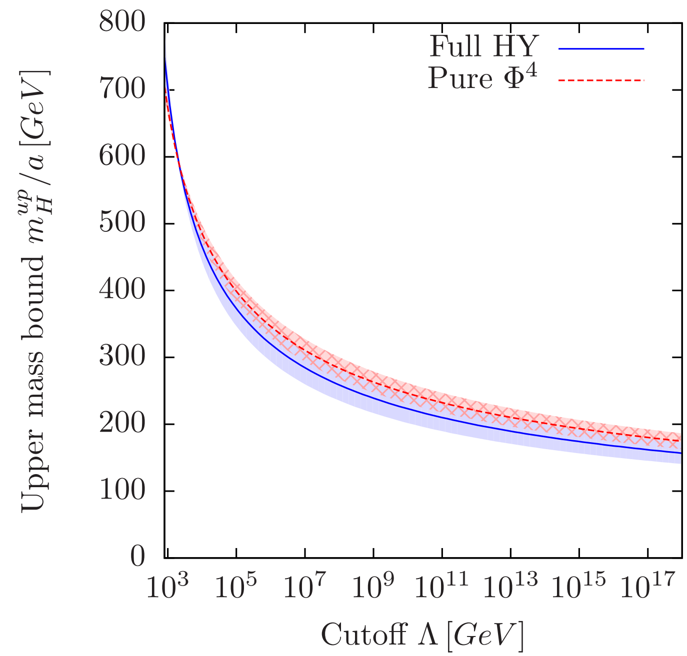
Upper Higgs boson mass bounds

fit data to expected theoretical dependence on cut-off Λ

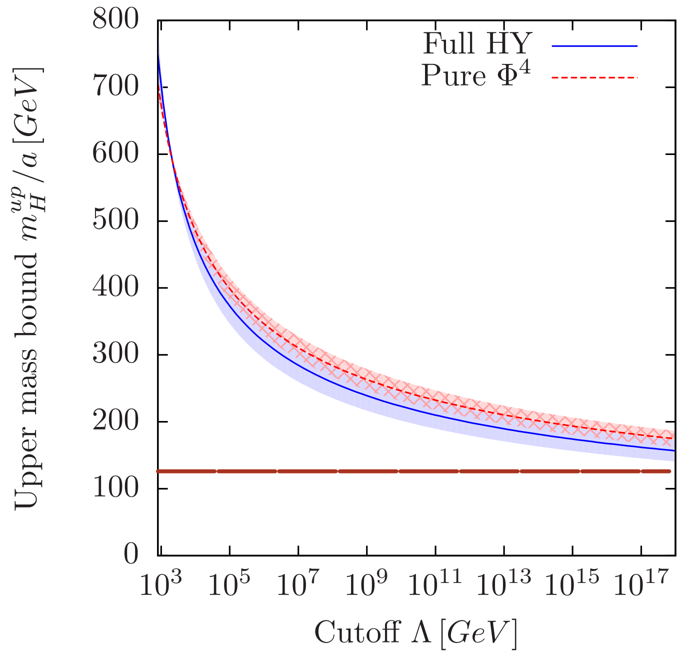
$$\frac{m_{Hp}}{a} = A_m \cdot \left[\log(\Lambda^2/\mu^2) + B_m\right]^{-1/2}$$

- *infinite volume* data are well described by theoretical expectation
  $\rightarrow$ consistent with triviality of Higgs-Yukawa model
- compare pure Φ^4 theory and Higgs-Yukawa:
  $\rightarrow$ see no significant effect





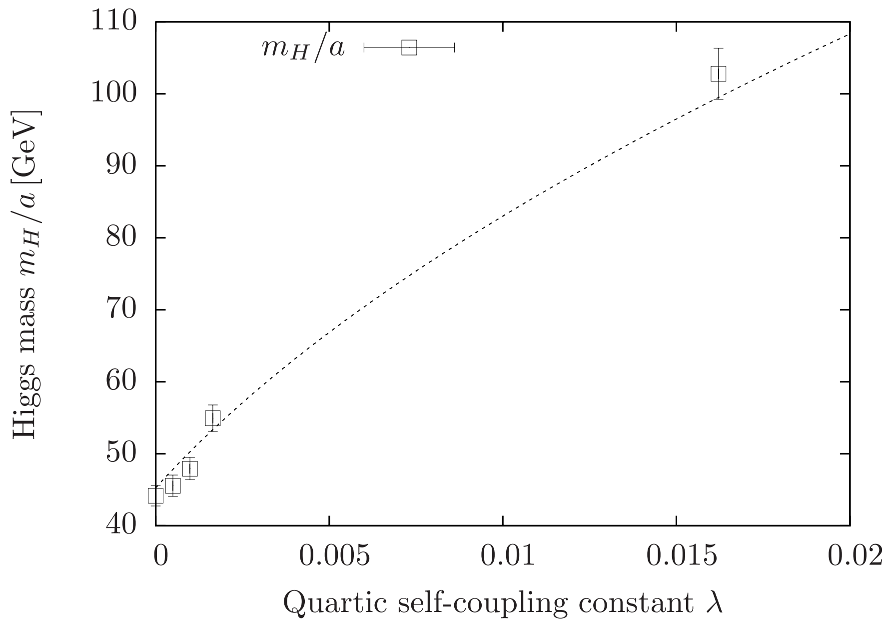
What if only upper Higgs boson mass bound counts



- standard model can be valid up to very high energy
- would have to include quantum gravity
  - theoretical challenge, no consistent formulation exists
  - experimental challenge: how to probe such scales?

Lower bound

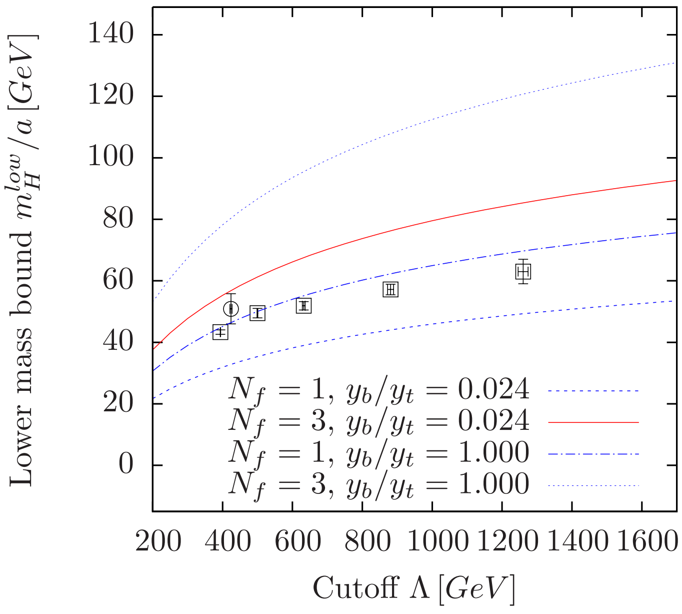
- analytic: effective potential from lattice perturbation theory
  - self-consistent determination of vacuum expectation value
$$0 = dU_{\text{eff}}/dv = -m^2 v - 4\lambda v^3 - \frac{\mathsf{d}}{\mathsf{d}v}(U_{\text{impr}}[v] + U_F[v])$$
  - Higgs boson mass
$$m_{Hp}^2 = 12\lambda v^2 + \frac{\mathsf{d}^2}{\mathsf{d}v^2}(U_{\text{impr}}[v] + U_F[v])$$
- confront with numerical simulation



- fixed cut-off $\Lambda = 1/a$
- lower bound reached at $\lambda = 0$ (accordance with expectation from P.T.)
- agreement with lattice effective potential

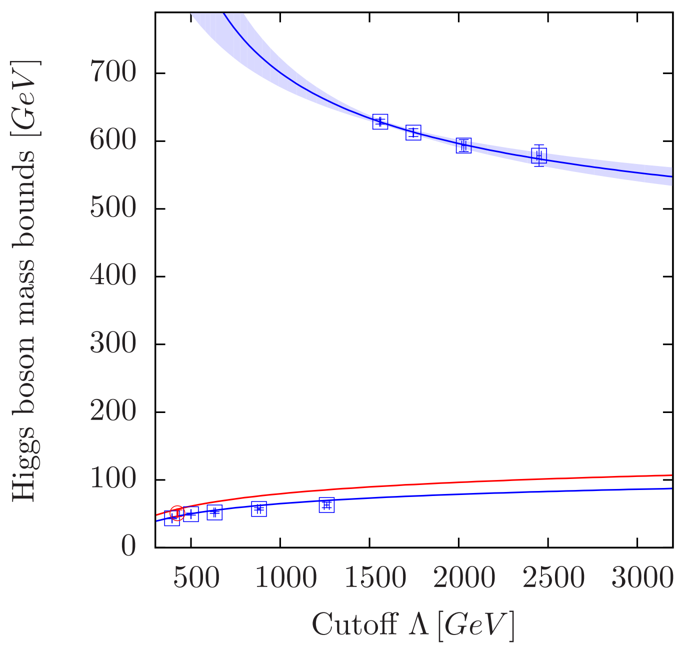
Result for lower Higgs boson mass bound

- data in infinite volume limit
- reliable description from effective potential
- most realistic: $N_f = 3, y_b/y_t = 0.024$ (circle in graph)



Final lower and upper Higgs boson mass bounds

- cut-off depence of lower and upper bounds
- allowed range of Higgs boson mass:
  $50\text{GeV} < m_H < 650\text{GeV}$ at cut-off $\Lambda = 1.5\text{TeV}$



Resonance parameters of Higgs boson from the lattice

(P. Gerhold, J. Kallarackal, K.J.)

Finite volume energy levels:

- measure two-particle Goldstone energy in center of mass frame

$$W = 2\sqrt{m^2 + k^2}$$

$\Rightarrow$ value of k

$\Rightarrow$ infinite volume scattering phase δ_0 (Lüscher)

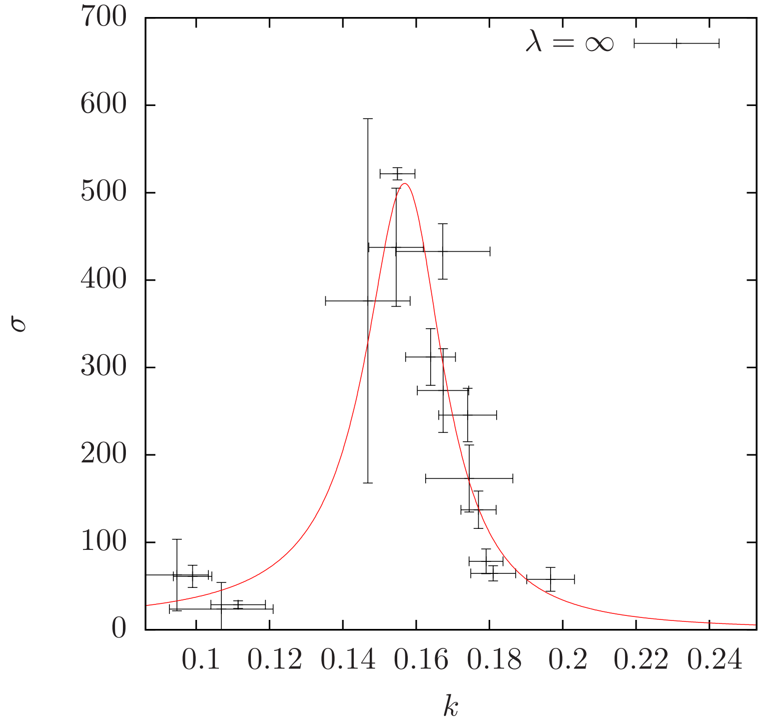
$$\tan \delta_0(k) = \frac{\pi^{\frac{3}{2}} q}{\mathcal{Z}_{00}(q^2)}, \quad q = \frac{kL}{2\pi}$$

$$\mathcal{Z}_{00}(q^2) = \sum_{\vec{n} \in \mathbb{Z}^3} \frac{1}{\sqrt{4\pi}} \frac{1}{n^2 - q^2}.$$

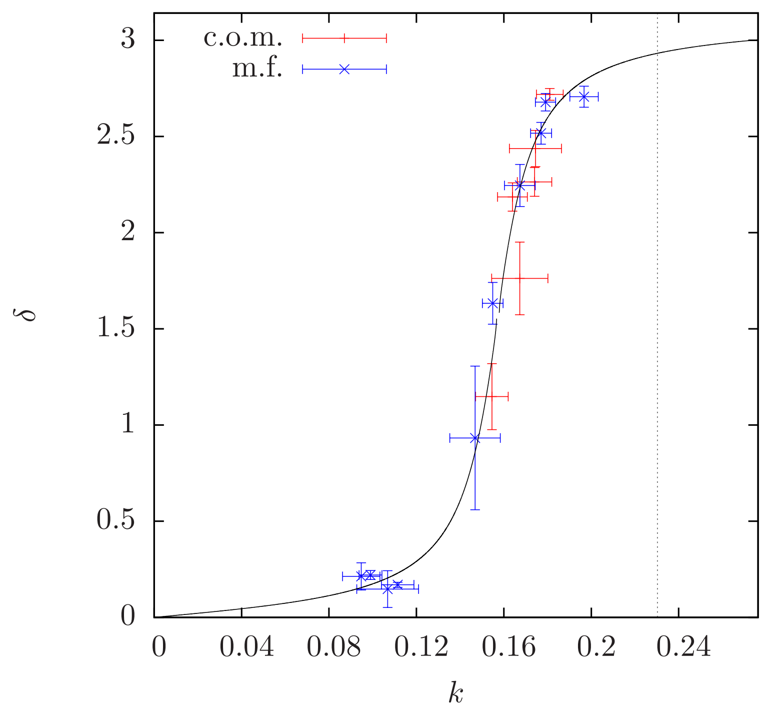
Generalization to moving frames (Gottlieb, Rummukainen; Feng, Renner, K.J.)

$\rightarrow$ many more finite volume enegy levels

Scattering phase and cross section

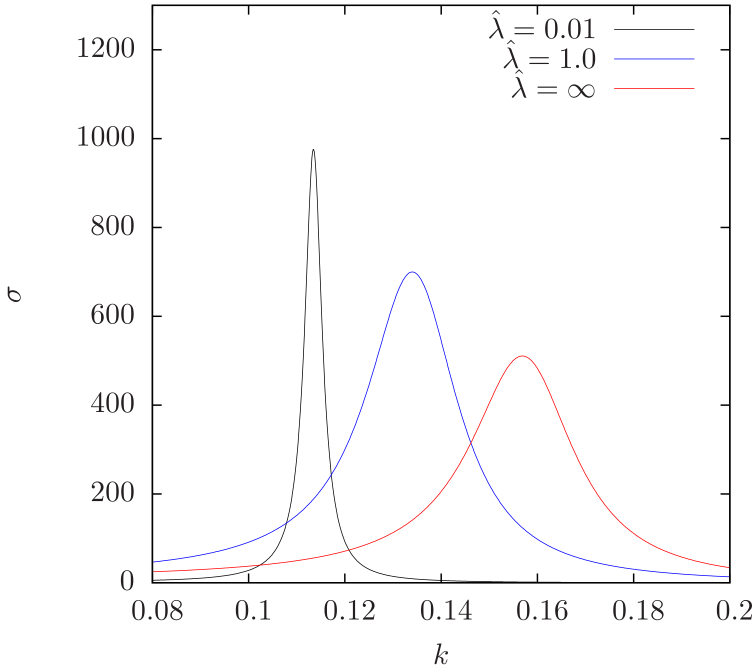


cross section



scattering phase

Coupling dependence of Higgs boson width



Breit-Wigner fit

$$f(k) = 16\pi \frac{M_H^2 \Gamma_H^2}{(M_H^2 - 4m_G^2)((W_k^2 - M_H^2)^2 + M_H^2 \Gamma_H^2)}$$

λ	aM_H	$a\Gamma_H$	$a\Gamma_H^{\rm pert}$	$aM_H^{\rm stable}$
0.01	0.2811(6)	0.007(1)	0.0054(1)	0.278(2)
1.0	0.374(4)	0.033(4)	0.036(8)	0.386(28)
∞	0.411(3)	0.040(4)	0.052(2)	0.405(4)

Extension to a fourth fermion generation

(Hou; Holdom, Hou, Hurth, Mangano, Sultansoy, Ünel)

Motivation:

- offers potential to generate sufficient amount of CP violation (Hou)

- heavy fermion mass
  $\rightarrow$ large Yukawa couplings
  $\rightarrow$ need of non-perturbative study

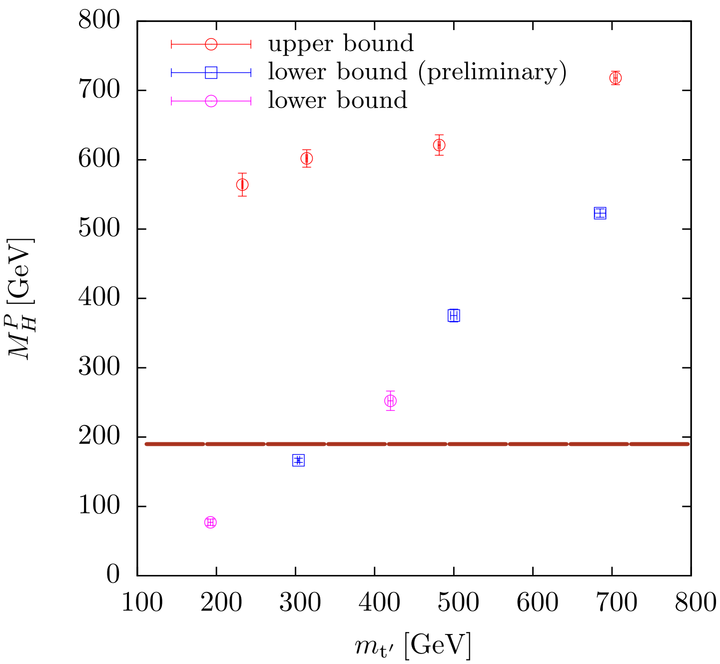
- here: effect of 4th fermion generation on Higgs boson mass bounds

- strong dynamics due to large Yukawa coupling?

Fermion mass dependence of Higgs boson mass bounds

(J. Bulava, P. Gerhold, J. Kallarackal, A. Nagy, K.J.)

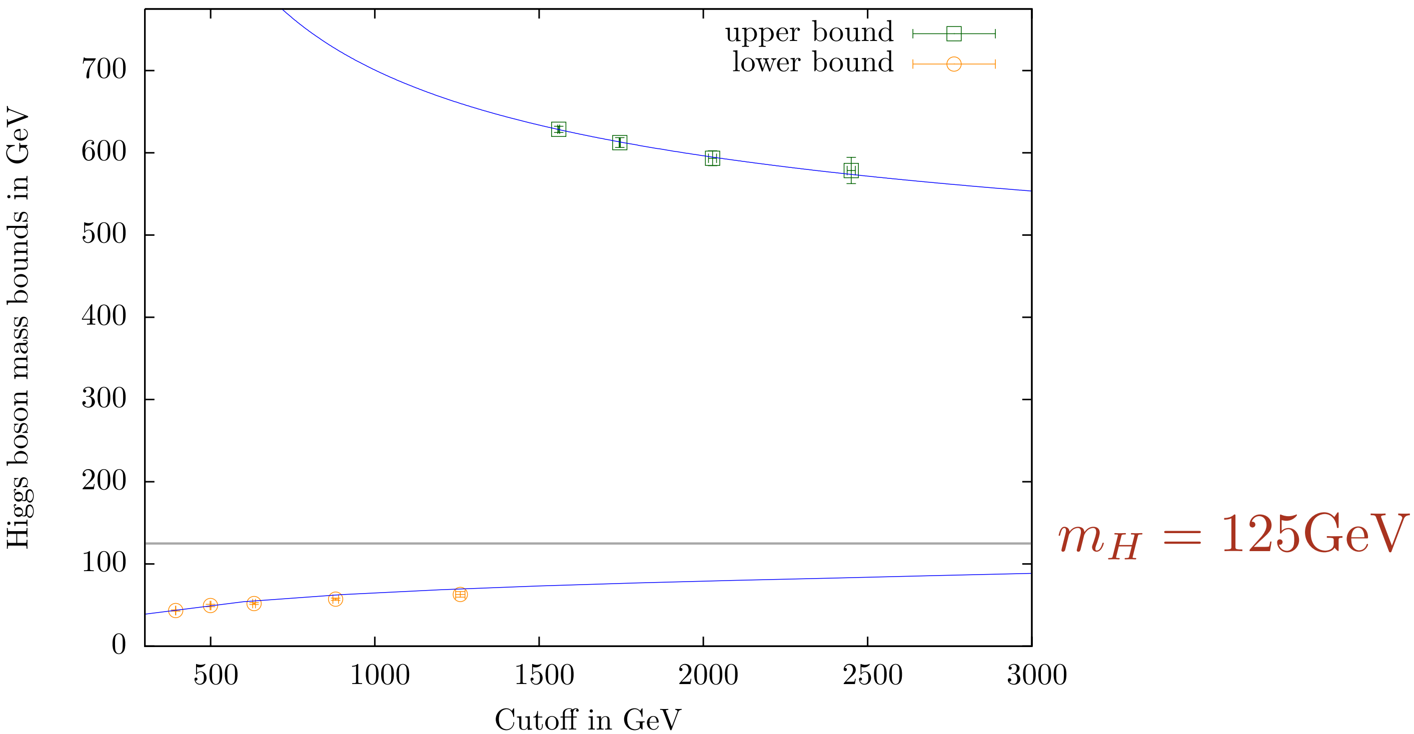
- motivated by 4th fermion generation scenario



$m_{\text{Higgs}} = 125\text{GeV}$

- strong dependence on fermion mass
- our conclusion for fourth fermion generation:
  - phenomenologically: $m_{\text{top}'} \gtrsim 350\text{GeV}$
  - fourth fermion generation ruled out by our lattice calculations
    (also phenomenologically ruled out in SM framework)

Consequences of a $125\mathrm{GeV}$ Higgs boson mass from lattice bounds



- Higgs boson mass right in the funnel of mass bounds

A more detailed look

- relation of couplings at 1-loop order, β-functions
  - running of quartic coupling

    $$\frac{d\lambda}{d\ln\mu} = \frac{1}{16\pi^2}\left\{-\mathbf{6y_t^4} + 12y_t^2\lambda + \frac{3}{8}\left[2g^4 + (g^2 + g'^2)^2\right] - 3\lambda(3g^2 + g'^2) + 24\lambda^2\right\}$$

  - running of Yukawa coupling

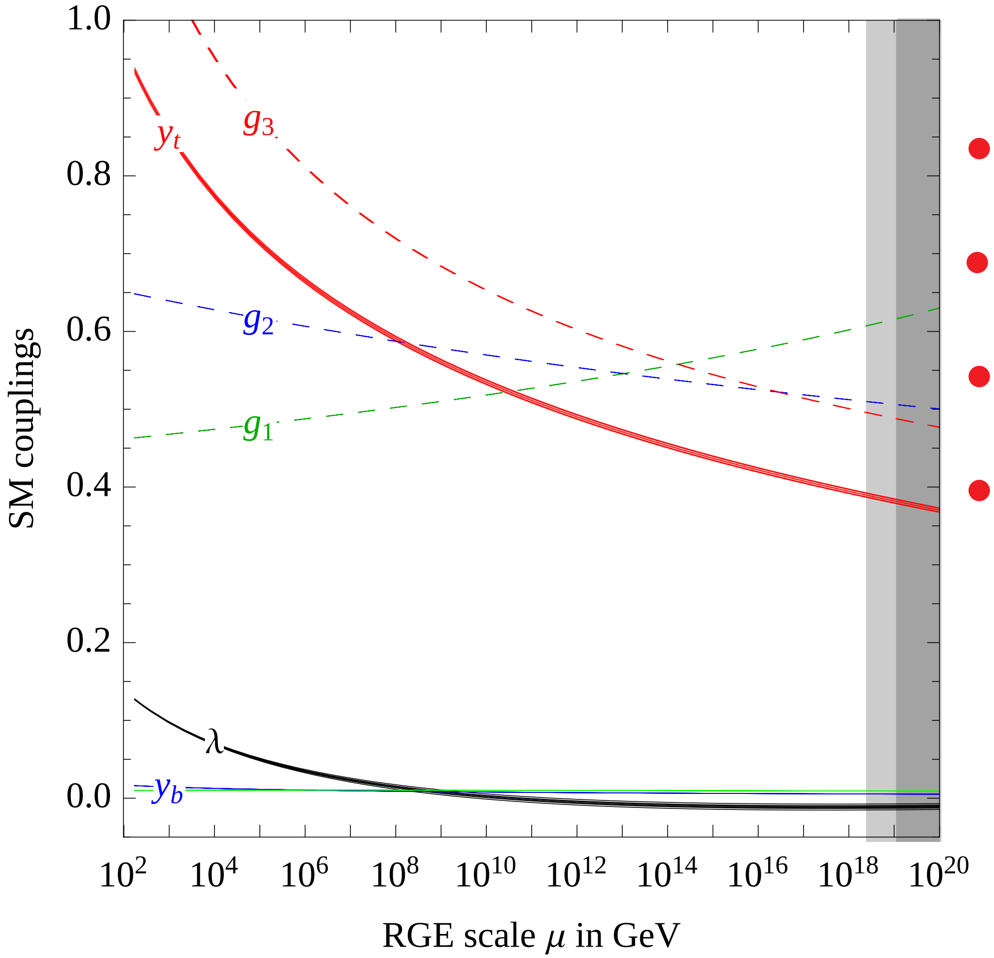
    $$\frac{dy_t}{d\ln\mu} = \frac{y_t}{16\pi^2}\left[\frac{9}{2}y_t^2 - \mathbf{8g_s^2} - \frac{9}{4}g^2 - \frac{17}{12}g'^2\right]$$

- g_s strong coupling, g, g' electroweak couplings

- inter-relation between all couplings

- effect of strong coupling indirect through Yukawa coupling

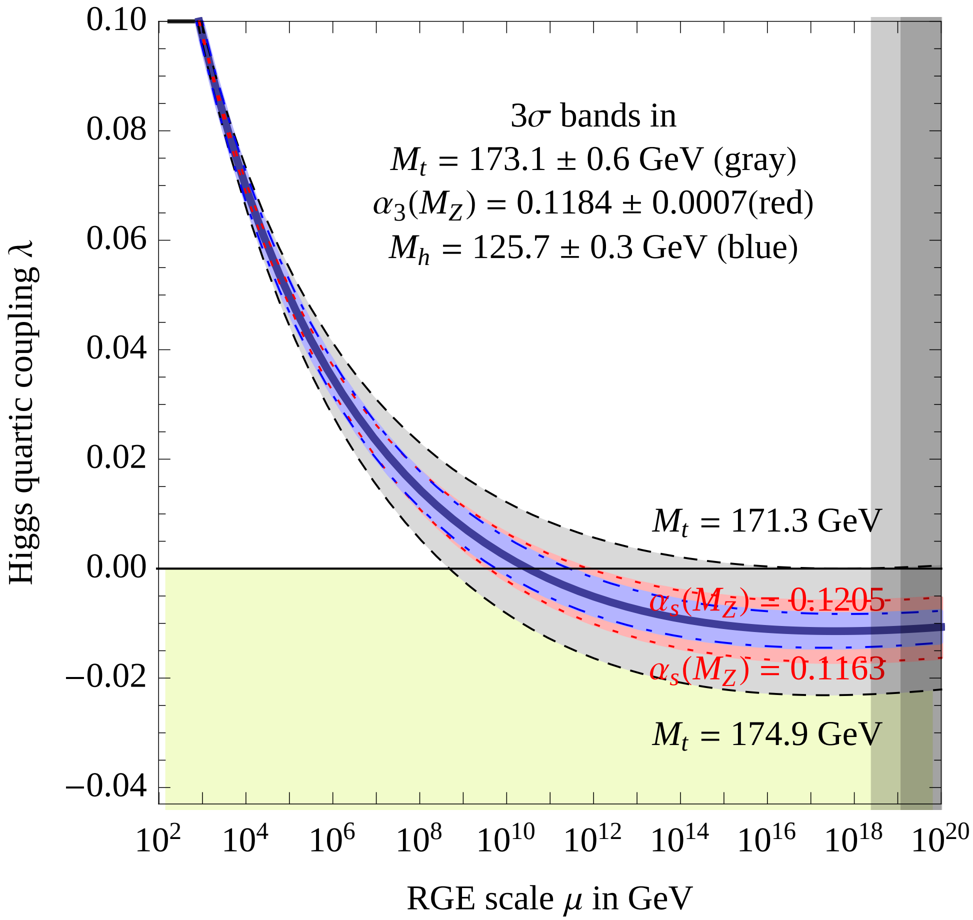
Running of all couplings

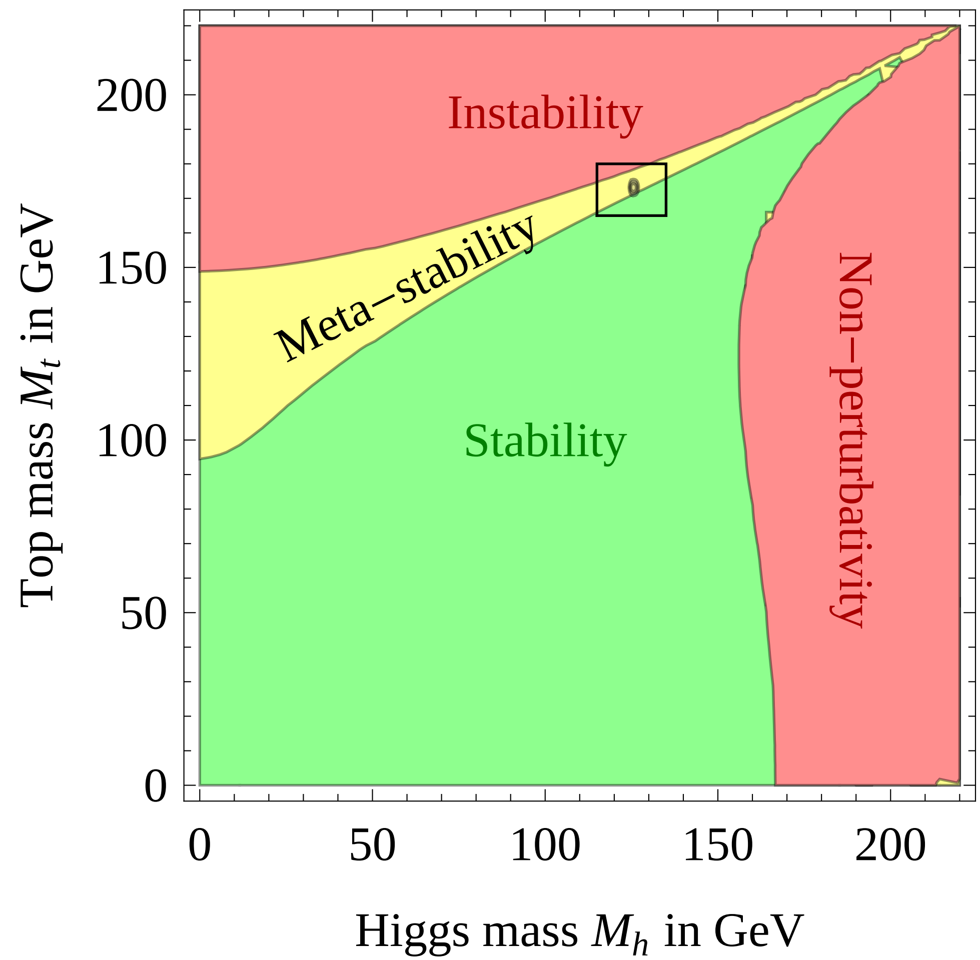


(J. Espinosa, 1311.1970)

- quartic coupling $\lambda < 0$
- metastable minimum of effective potential
- SM vacuum not lowest minimum
- tunneling to new minimum

When the vacuum becomes metastable





subtle parameter dependence
is it there at all? (F. Jegerlehner)

why do we live in this corner?

Bound from NNLO continuum perturbation theory

$$M_H[\mathrm{GeV}] > 129.6 + 2\frac{m_{\mathrm{top}}[\mathrm{GeV}]-173.35}{1} - 0.5\frac{\alpha_s(M_Z)-0.1184}{0.0007} \pm 0.3_{\mathrm{th}}$$

(Buttazzo et.al., Espinosa, plenary talk at Lattice 2013)

- predicts metastability at very high cut-off $\Lambda \approx 10^{10}\mathrm{GeV}$
  $\rightarrow$ in-accessible for lattice calculations

- lifetime of standard model vacuum longer than age of universe

- scenario depends subtle on parameters: can bound be changed?

Lattice: effect of higher dimensional operator: $\lambda_6\phi^6$ term

- $\lambda_6\phi^6$ term proxy for new physics
- analysis by lattice perturbation theory of effective potential
- goal: confront with simulation results
- effective potential

$$U(\breve{v}) = \tfrac{1}{2}m_0^2\breve{v}^2 + \lambda\breve{v}^4 + \lambda_6\breve{v}^6 + U_F(\breve{v}) + 6\lambda\breve{v}^2(P_H + P_G)$$
$$+\lambda_6\breve{v}^4(15P_H + 9P_G) + \lambda_6\breve{v}^2(45P_H^2 + 54P_HP_G + 45P_G^2)$$
$$P_G = \sum_{p\neq 0}\tfrac{1}{\hat{p}^2}, \qquad P_H = \sum_{p\neq 0}\tfrac{1}{\hat{p}^2+m_H^2}$$

fermionic contribution

$$U_F(\breve{v}) = -\frac{2\,N_f}{V}\left[\sum_p \log\left|\nu(p) + y_t\cdot\breve{v}\cdot\left(1-\tfrac{1}{2\rho}\right)\nu(p)\right|^2\right.$$
$$\left.+\sum_p \log\left|\nu(p) + y_b\cdot\breve{v}\cdot\left(1-\tfrac{1}{2\rho}\right)\nu(p)\right|^2\right]$$

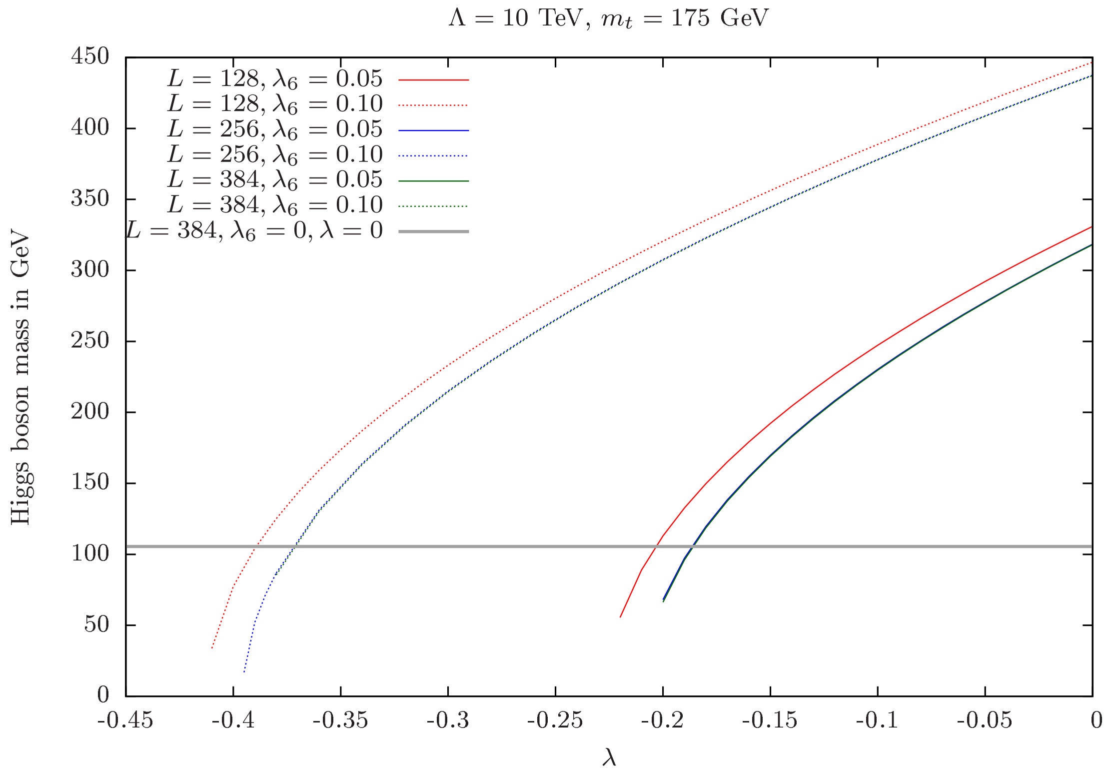
The cutoff, vev and Higgs boson mass are given by:

$$\Lambda = \frac{246\mathrm{GeV}}{vev}, \qquad U'(vev) = 0, \qquad U''(vev) = m_H^2$$

Lowering the Higgs boson mass (apparantly)

(D. Lin, B. Knippschild, K. Nagai, A. Nagy, K.J.)

- fixing $\lambda_6 > 0$ allows to change λ to negative values



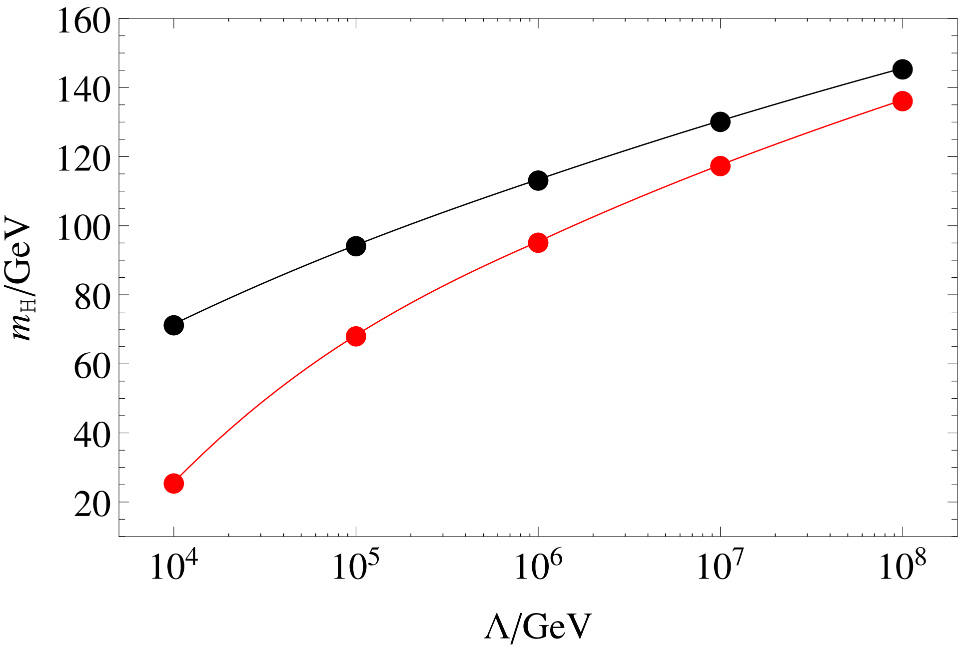
$\lambda_6 = 0.1$

$\lambda_6 = 0.05$

Influence of $\lambda_6\phi^6$-term from RG-flow

(H. Gies, C. Gneiting, R. Sondenheimer)

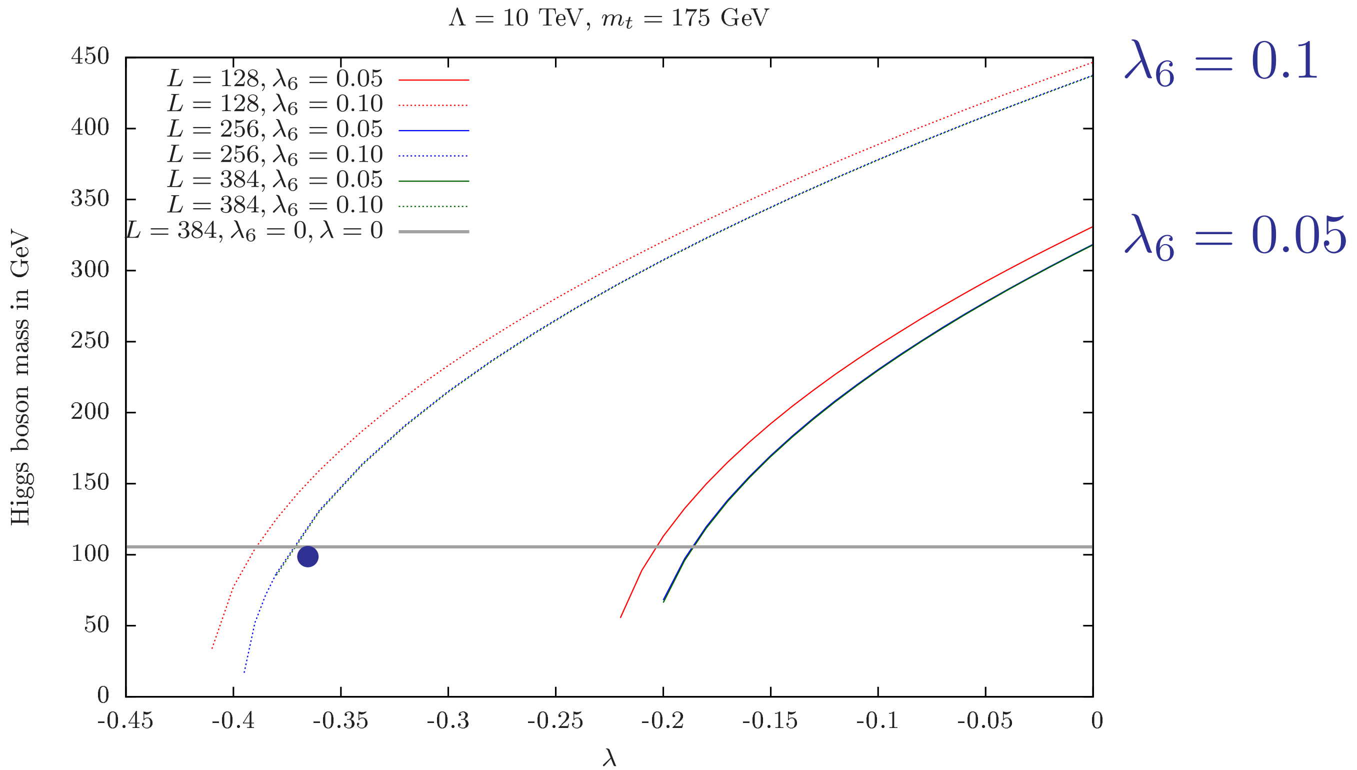
- framework of effective average action approach (Wetterich)



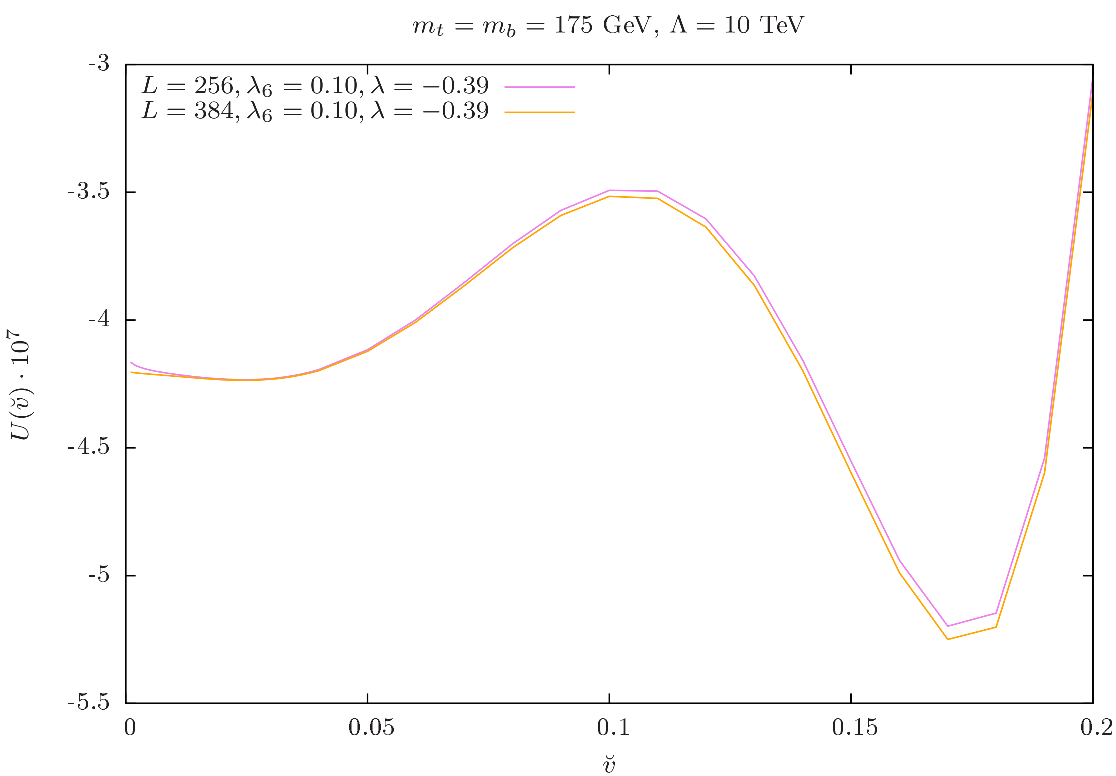
- upper curve $\lambda = \lambda_6 = 0$
- lower curve $\lambda = -0.2, \ \lambda_6 = 3.3$

Lowering the Higgs boson mass (apparantly)

- let's have a close look here: ●

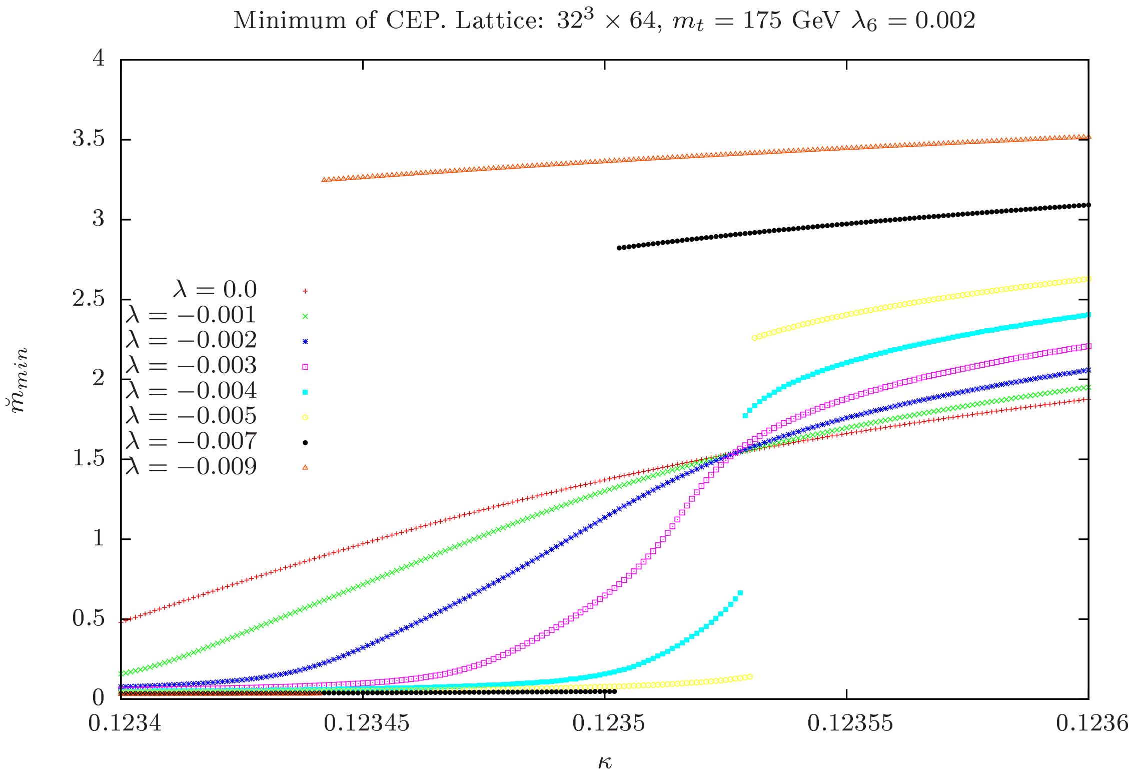


Appearance of second minimum



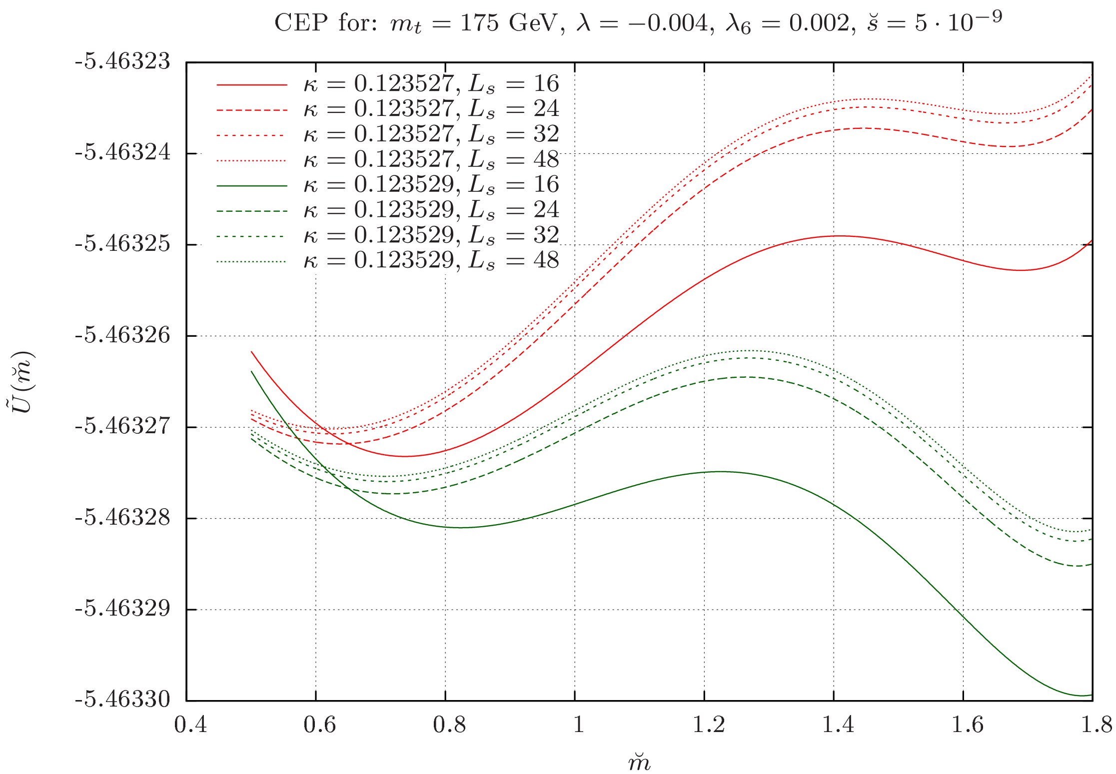
- second minimum at $v = 0.2\Lambda$ $\Rightarrow$ physical
- metastability at low value of cut-off $\Rightarrow$ lattice computations possible

Scan of phase diagram from perturbative effective potential



- strategy:
  - fix value of λ_6
  - vary λ until system becomes metastable $\rightarrow \lambda_{\text{min}}$
  - determine Higgs boson mass at λ, λ_6, Λ
  - repeat for increasing values of λ_6

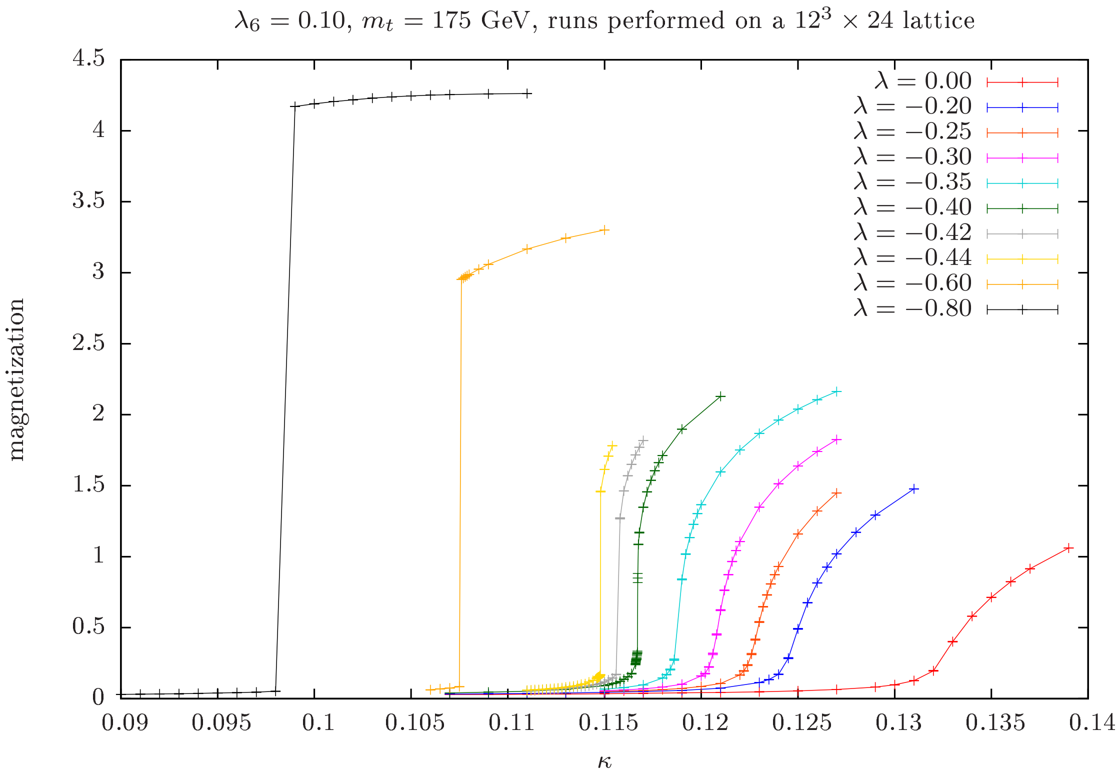
Scan of phase diagram from perturbative CEP



$\lambda_6 = 0.1$

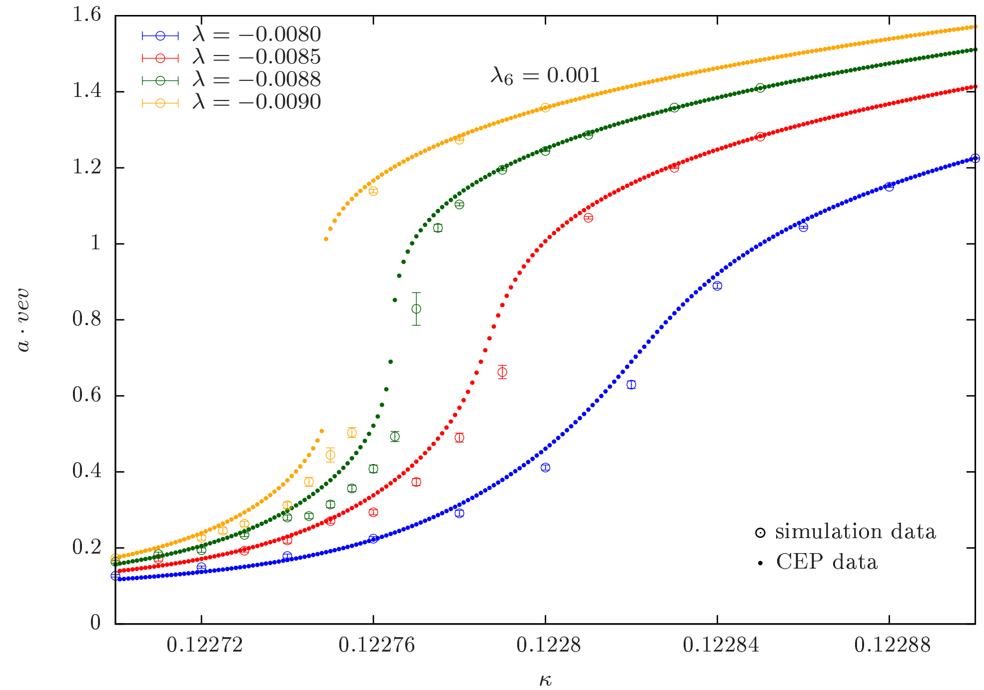
$\lambda_6 = 0.05$

Scan of phase diagram from lattice calculations



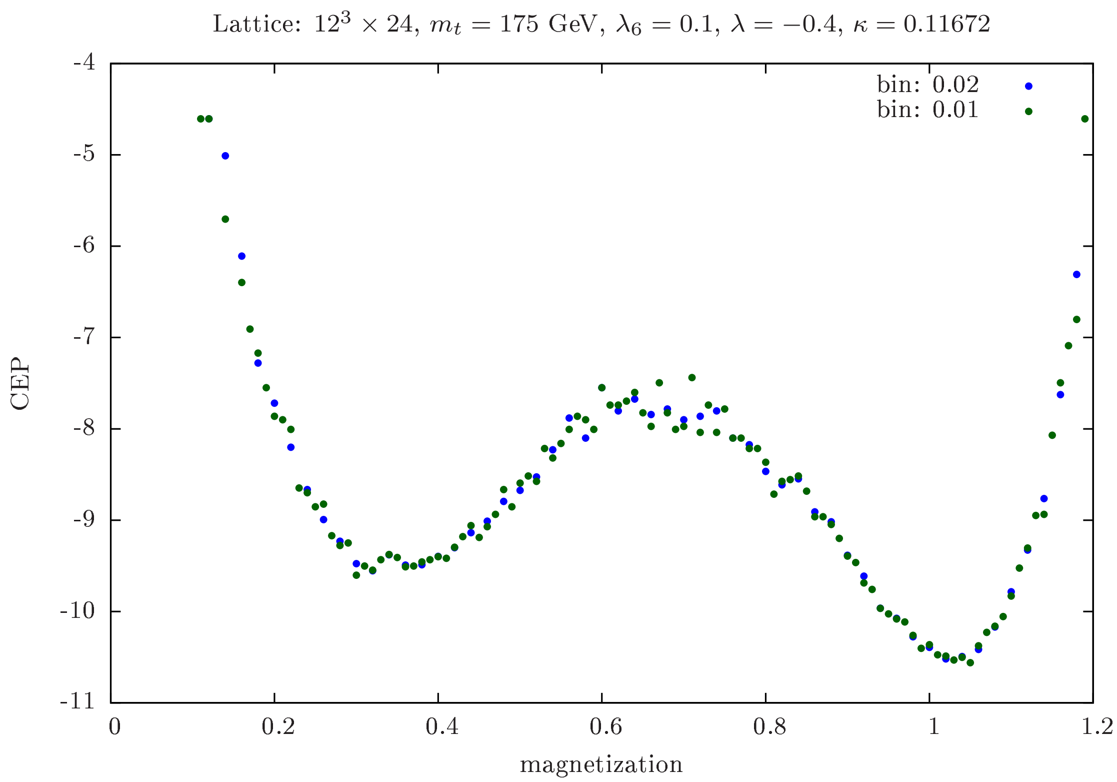
- see also jumps in vacuum expectation value
- connected to metastability of perturbative effective potential?

Comparison to effective potential



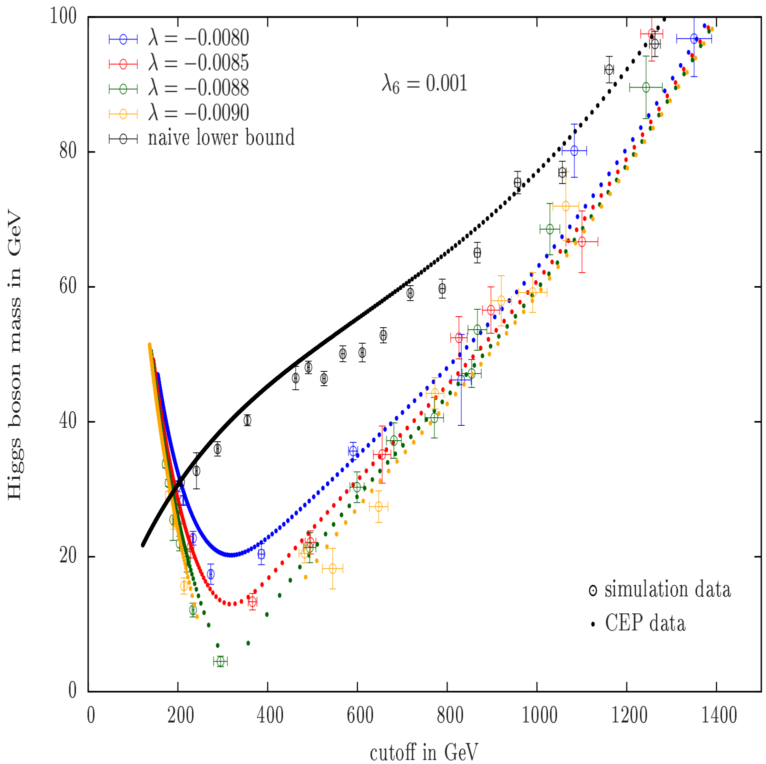
- nice agreement
  → effective potential can be used to analyze the model
  (at least for $\lambda_6 \ll 1$)

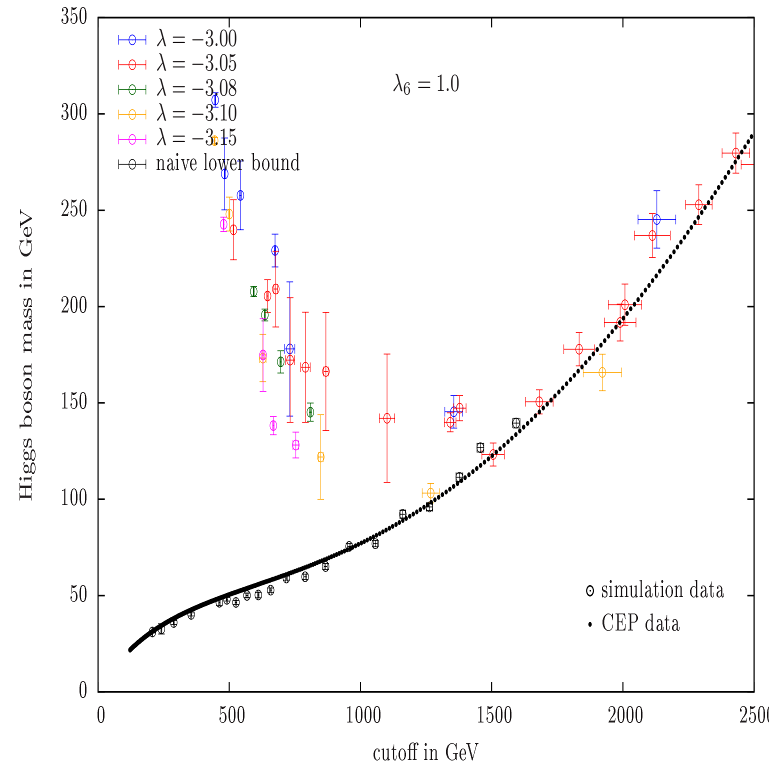
Effective potential from lattice computations



- compute probability distribution (histogram)
  $\rightarrow$ effective potential
- observe metastable potential
- Higgs boson mass?
- staying in scaling region?

A first look at the Higgs boson mass with active Φ^6-term





lowering the mass

increasing mass for larger λ_6

Warning:

- small volume so far
- only relative comparison

Summary

- study of a chiral invariant lattice Higgs-Yukawa model
- analysis of phase structure
  $\rightarrow$ no evidence for non-trivial fixed points
- established cut-off dependence of lower and upper Higgs boson mass bounds
- for $M_H \approx 124 - 126\mathrm{GeV}$
  $\Rightarrow$ standard model could be valid for very large cut-offs
- metastable vacuum at very large cut-off $O(10^{16}\mathrm{GeV})$
- ruled out 4th generation within SM framework
- addition of higher dimensional $\lambda_6\phi^6$-term
  - Higgs boson mass lowered when $\lambda < 0$
  - appearance of metastability
- what happens at large value of λ_6?