Gauss–Bonnet Boson Stars in AdS

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in collaboration with

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Physics Letters B (2013) (arXiv:1308.3391) and arXiv:1310.7223

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Models of Gravity

NORDIC STRING MEETING 2014
Golm, February 3rd, 2014

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Motivation

- Set up a **model to support boson star solutions in Gauss-Bonnet gravity** in $4+1$ dimensions
- Gauss-Bonnet theory which appears naturally in the **low energy effective action of quantum gravity models**
- We are interested in the **effect of Gauss-Bonnet gravity** and will study these objects in **the minimal number of dimensions** in which the term does not become a **total derivative.**
- Higher dimensions appear in attempts to find a **quantum description of gravity** as well as in unified models.
- For black holes many of their properties in $(3+1)$ dimensions **do not extend to higher dimensions.**
- Discovery of the **Higgs Boson** in 2012: **fundamental scalar fields do exist in nature.**

Solitons in non-linear field theories

General properties of soliton solutions

- **Localized, finite energy, stable, regular solutions** of non-linear field equations
- Can be viewed as **models of elementary particles**

Examples

- **Topological solitons**: Skyrme model of hadrons in high energy physics one of first models and magnetic monopoles, domain walls etc.
- **Non-topological solitons**: Q-balls (named after Noether charge Q) (flat space-time) and boson stars (generalisation in curved space-time)

Non-topolocial solitons

Properties of non-topological solitons

- Solutions possess the same **boundary conditions at infinity as the physical vacuum state**
- Degenerate vacuum states **do not necessarily exist**
- Require an **additive conservation law**, e.g. **gauge invariance** under an **arbitrary global phase transformation**

S. R. Coleman, Nucl. Phys. B **262** (1985), 263, R. Friedberg, T. D. Lee and A. Sirlin, Phys. Rev. D **13** (1976) 2739), D. J. Kaup, Phys. Rev. **172** (1968), 1331, R. Friedberg, T. D. Lee and Y. Pang, Phys. Rev. D **35** (1987), 3658, P. Jetzer, Phys. Rept. **220** (1992), 163, F. E. Schunck and E. Mielke, Class. Quant. Grav. **20** (2003) R31, F. E. Schunck and E. Mielke, Phys. Lett. A **249** (1998), 389.

Why study Q-balls and boson stars?

Q-balls

- Scalar fields **prevented from collapse** by Heisenberg's uncertainty principle and repulsive self-interaction
- Supersymmetric Q-balls have been considered as **possible candidates for baryonic dark matter**

Boson stars

- Described by **relatively simple equations**
- Simple **toy models for a wide range of objects** such as particles, compact stars, e.g. neutron stars and even centres of galaxies
- **Gauss-Bonnet gravity**: its spectrum does **not include new propagating degrees of freedom besides gravitation**
- Toy models for **AdS/CFT correspondence.** Planar **boson stars** in AdS have been interpreted as Bose-Einstein condensates of glueballs

Model for Gauss–Bonnet Boson Stars

- **Action**

$$S = \frac{1}{16\pi G_5}\int d^5x\sqrt{-g}\,(R - 2\Lambda + \alpha\mathcal{L}_{GB} + 16\pi G_5\mathcal{L}_{\text{matter}})$$
$$\mathcal{L}_{GB} = \left(R^{MNKL}R_{MNKL} - 4R^{MN}R_{MN} + R^2\right) \qquad (1)$$

- **Matter Lagrangian** $\mathcal{L}_{matter} = -\left(\partial_\mu\psi\right)^*\partial^\mu\psi - U(\psi)$
- **Gauge mediated potential**

$$U_{\text{SUSY}}(|\psi|) = m^2\eta_{\text{susy}}^2\left(1 - \exp\left(-\frac{|\psi|^2}{\eta_{\text{susy}}^2}\right)\right) \qquad (2)$$

$$U_{\text{SUSY}}(|\psi|) = m^2|\psi|^2 - \frac{m^2|\psi|^4}{2\eta_{\text{susy}}^2} + \frac{m^2|\psi|^6}{6\eta_{\text{susy}}^4} + O\left(|\psi|^8\right) \qquad (3)$$

A. Kusenko, Phys. Lett. B **404** (1997), 285; Phys. Lett. B **405** (1997), 108, L. Campanelli and M. Ruggieri, Phys. Rev. D **77** (2008), 043504

Model for Gauss–Bonnet Boson Stars

- **Einstein Equations are derived from the variation of the action with respect to the metric fields**

$$G_{MN} + \Lambda g_{MN} + \frac{\alpha}{2} H_{MN} = 8\pi G_5 T_{MN} \qquad (4)$$

where H_{MN} is given by

$$H_{MN} = 2\left(R_{MABC}R_N^{ABC} - 2R_{MANB}R^{AB} - 2R_{MA}R_N^A + RR_{MN}\right) - \frac{1}{2}g_{MN}\left(R^2 - 4R_{AB}R^{AB} + R_{ABCD}R^{ABCD}\right) \qquad (5)$$

- **Energy-momentum tensor**

$$T_{MN} = -g_{MN}\left[\frac{1}{2}g^{KL}\left(\partial_K\psi^*\partial_L\psi + \partial_L\psi^*\partial_K\psi\right) + U(\psi)\right] + \partial_M\psi^*\partial_N\psi + \partial_N\psi^*\partial_M\psi \qquad (6)$$

Ansatz

- The **Klein-Gordon equation** is given by:

$$\left(\Box - \frac{\partial U}{\partial |\psi|^2}\right)\psi = 0 \tag{7}$$

- $\mathcal{L}_{matter}$ is invariant under the **global U(1) transformation**

$$\psi \rightarrow \psi e^{i\chi} \quad . \tag{8}$$

- Locally conserved **Noether current** j^M

$$j^M = -\frac{i}{2}\left(\psi^* \partial^M \psi - \psi \partial^M \psi^*\right) \; ; \; j^M_{;M} = 0 \tag{9}$$

- The globally conserved **Noether charge** Q reads

$$Q = -\int d^4x \sqrt{-g} j^0 \; . \tag{10}$$

Ansatz

- **Metric in spherical Schwarzschild-like coordinates**

$$ds^2 = -N(r)A^2(r)dt^2 + \frac{1}{N(r)}dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\varphi^2 + \sin^2\theta\sin^2\varphi d\chi^2\right) \quad (11)$$

where

$$N(r) = 1 - \frac{2n(r)}{r^2} \quad (12)$$

- **Stationary Ansatz for complex scalar field**

$$\psi(r,t) = f(r)e^{i\omega t} \quad (13)$$

- **Rescaling using dimensionless quantities**

$$r \to \frac{r}{m} \ , \ \omega \to m\omega \ , \ \psi \to \eta_{\mathrm{susy}}\psi \ , \ n \to n/m^2 \ , \ \alpha \to \alpha/\sqrt{m} \quad (14)$$

Einstein equations

- **Equations for the metric functions:**

$$N'(r) = 2r\left(\frac{1-N-\frac{r^2}{3}\Lambda}{r^2+2\alpha(1-N)}\right) - \frac{2}{3}\frac{\kappa r^3}{NA^2}\left(\frac{UNA^2+\omega^2 f^2+N^2A^2f'^2}{r^2+2\alpha(1-N)}\right) \quad (15)$$

$$A' = \frac{2\kappa r^3\left(A^2N^2f'^2+\omega^2 f^2\right)}{3AN^2\left(r^2+2\alpha(1-N)\right)} \quad (16)$$

$$\kappa = 8\pi G_5 \eta_{\rm susy}^2 \quad (17)$$

- **Matter field equation:**

$$\left(r^3ANf'\right)' = r^3A\left(\frac{1}{2}\frac{\partial U}{\partial f} - \frac{\omega^2 f}{NA^2}\right) \quad (18)$$

Boundary Conditions flat space-time

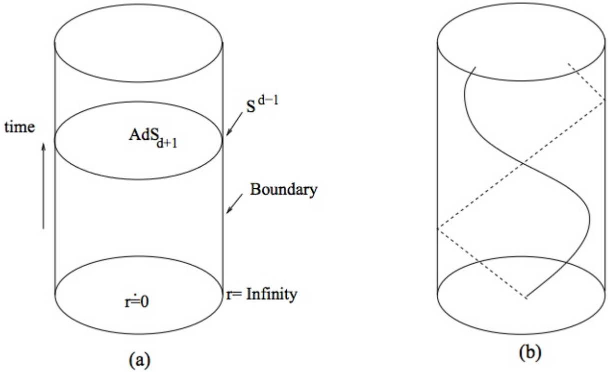
- **Appropriate boundary conditions:**

$$f'(0) = 0 \quad , \quad n(0) = 0 \; . \tag{19}$$

- We need the scalar filed to **vanish at infinity** and therefore **require** $f(\infty) = 0$, while we choose $A(\infty) = 1$ (rescaling of the time coordinate)
- If $\Lambda = 0$ the scalar field function **falls of exponentially** with

$$f(r >> 1) \sim \frac{1}{r^{\frac{3}{2}}} \exp\left(-\sqrt{1-\omega^2} r\right) + \ldots \tag{20}$$

AdS space-time



[3]

Figure : (a) Penrose diagram of AdS space-time, (b) massive (solid) and massless (dotted) geodesic.

[3] J. Maldacena, The gauge/gravity duality, arXiv:1106.6073v1

Boundary Conditions for asymptotic AdS space time

- If $\Lambda < 0$ the scalar field function **falls of** with

$$\phi(r >> 1) = \frac{\phi_\Delta}{r^\Delta} \ , \ \ \Delta = 2 + \sqrt{4 + L_{eff}^2} \ . \tag{21}$$

- Where L_{eff} is the effective AdS-radius:

$$L_{eff}^2 = \frac{2\alpha}{1 - \sqrt{1 - \frac{4\alpha}{L^2}}} \tag{22}$$

- With L being the AdS-radius which is related to Λ as:

$$\Lambda = \frac{-6}{L^2} \tag{23}$$

- Chern-Simons limit:

$$\alpha = \frac{L^2}{4} \tag{24}$$

Expressions for Charge Q and Radius R

- **The explicit expression for the Noether charge**

$$Q = 2\pi^2 \int_0^\infty \mathrm{d}r \; r^3 \frac{\omega f^2}{AN} \tag{25}$$

- Define the **radius of the boson star** as an averaged radial coordinate[4]

$$R = \frac{2\pi^2}{Q} \int_0^\infty \mathrm{d}r \; r^4 \frac{\omega f^2}{AN} \tag{26}$$

[4] D. Astefanesei and E. Radu, Nucl. Phys. B **665** (2003) 594 [gr-qc/0309131].

Expressions for Mass M

- **Mass for** $\kappa = 0$

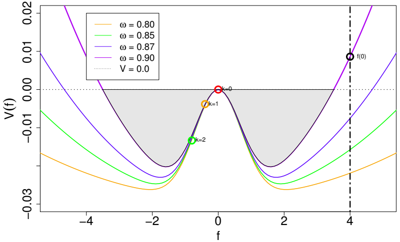
$$M = 2\pi^2 \int_0^\infty \mathrm{d}r\, r^3 \left(N f'^2 + \frac{\omega^2 f^2}{N} + U(f) \right) \tag{27}$$

- **Mass for** $\kappa > 0$ we define the **gravitational mass**[5] by the asymptotic behaviour:

$$M_G \sim n(r \to \infty)/\kappa \tag{28}$$

[5] Y. Brihaye and B. Hartmann, Nonlinearity **21** (2008), 1937, D. Astefanesei and E. Radu, Phys. Lett. B **587** (2004) 7

Finding solutions: fixing ω



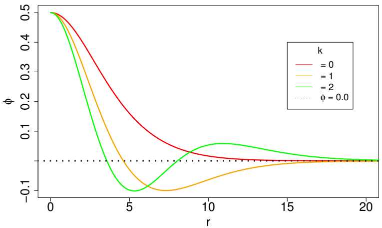


Figure : Effective potential $V(f) = \omega^2 f^2 - U(f)$.

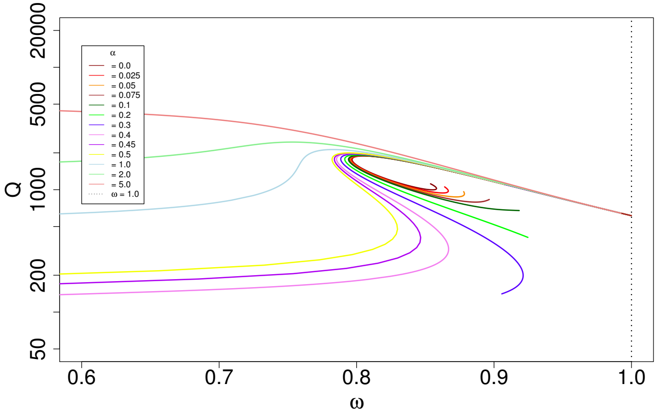


Figure : Charge Q in dependence on the frequency ω for $\kappa = 0.05$ and different values of α

Gauss–Bonnet boson stars with $\Lambda = 0$

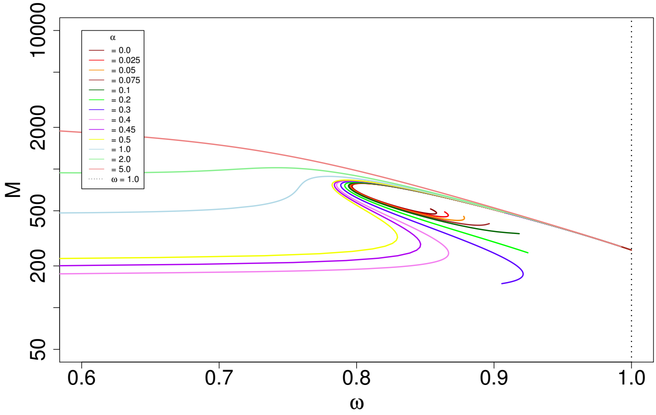


Figure : Mass M in dependence on the frequency ω for $\kappa = 0.05$ (left) and $\kappa = 0.02$ (right) and different values of α

Gauss–Bonnet boson stars with $\Lambda = 0$

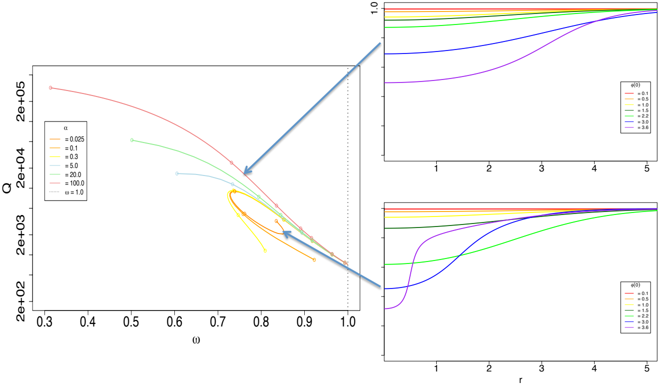


Figure : Metric function $A(r)$ for different values of α and $f(0)$, $\kappa = 0.05$.

Gauss–Bonnet boson stars with $\Lambda = 0$

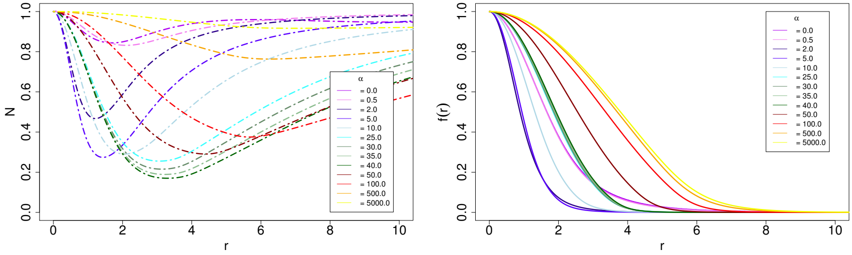


Figure : $f(r)$ and $N(r)$ for fixed $f(0)$, different values of α (large alpha regime) and $\kappa = 0.05$.

Gauss–Bonnet boson stars with $\Lambda = 0$

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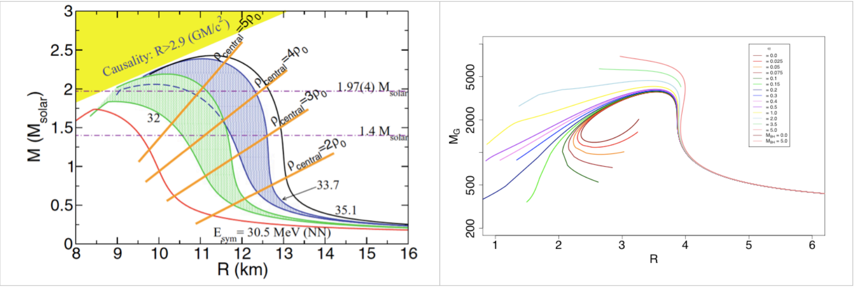

Figure : Mass-radius relation for the equation of state of neutron stars (left) in comparison to the mass-radius relation of Gauss-Bonnet boson stars (right).

[6] S. GANDOLFI et al, PHYSICAL REVIEW C 85, 032801(R) (2012)

Gauss–Bonnet boson stars with $\Lambda < 0$

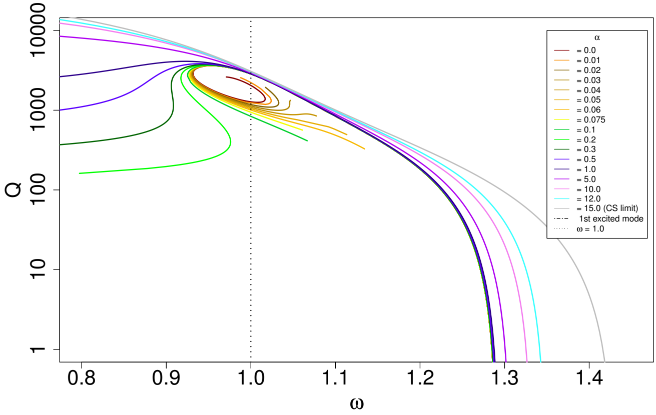

Figure : Charge Q in dependence on the frequency ω for $\Lambda = -0.01$, $\kappa = 0.02$ and different values of α. ω_{max} **shift**: $\omega_{max} = \frac{\Delta}{L_{eff}}$

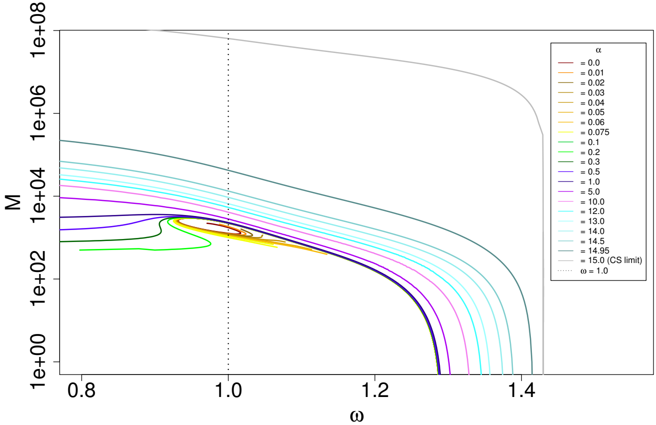

Figure : Mass M in dependence on the frequency ω for $\Lambda = -0.01$, $\kappa = 0.02$ and different values of α

Excited Gauss–Bonnet boson stars with $\Lambda = 0$

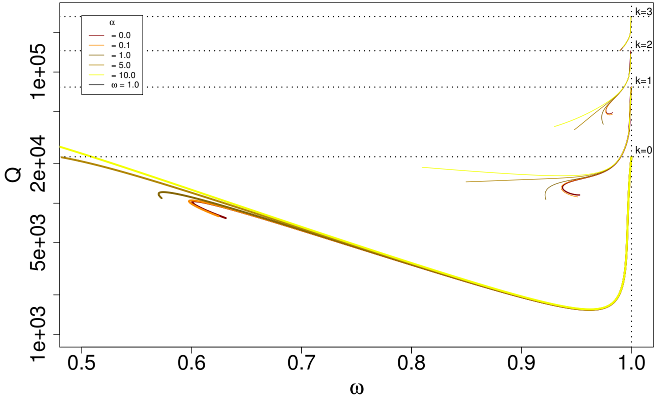

Figure : Charge Q in dependence on the frequency ω for $\Lambda = 0$, $\kappa = 0.0$, and different values of α

Excited Gauss–Bonnet boson stars with $\Lambda < 0$

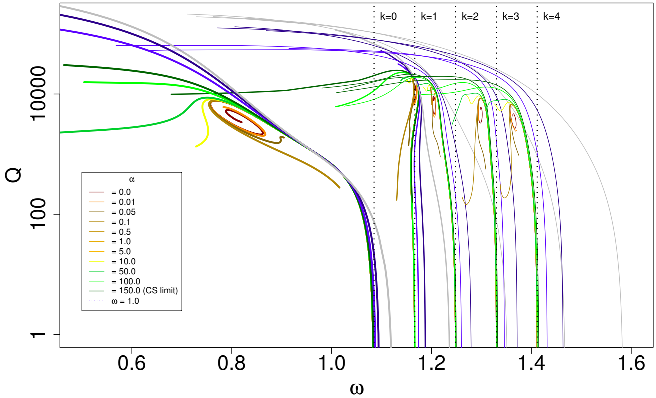

Figure : Charge Q in dependence on the frequency ω for $\Lambda = -0.01$, $\kappa = 0.02$, and different values of α. ω_{max} **shift**: $\omega_{max} = \frac{\Delta+2k}{L_{eff}}$

Outlook

- **Rotating Gauss-Bonnet Boson Stars** paper with Vyes Brihaye online: arXiv:1310.7223
- Gauss-Bonnet Boson Stars and **AdS/CFT correspondance**
- **Stability analysis** of Gauss-Bonnet Boson Stars
- Boson Stars in **general Lovelock theory**

Thank You!