Higher Spin Gravity and black holes

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Workshop Beyond the Standard model,
Bad Honnef

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Outline

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What is higher spin gravity

Higher Spin fields (non-interacting)

[Fronsdal]

- Spin s field described by symmetric tensor: $\phi_{\mu_1\mu_2\ldots\mu_s}(x)$
- EOM (non-interacting, massless)

$$\nabla^2\phi_{\mu_1\ldots\mu_s} - \partial_{(\mu_1}\partial^\rho\phi_{\rho\mu_2\ldots\mu_s)} + \partial_{(\mu_1}\partial_{\mu_2}\phi_{\mu_3\ldots\mu_s)\lambda}{}^\lambda = 0$$

- Higher spin gauge symmetry $\delta\phi_{\mu_1\mu_2\ldots\mu_s}(x) = \partial_{(\mu_1}\epsilon_{\mu_2\ldots\mu_s)}(x) + \ldots$, where ϵ is the infinitesimal gauge parameter (traceless)
- To guarantee Lagrangian description, ϕ has to be a double-traceless tensor field

No go theorem in flat Minkowski spacetime

- no non-trivial S-matrix of massless fields of spin $s > 2$

  [Weinberg, Coleman-Mandula, Aragone, Deser]

- note: string theory in flat spacetime has a tower of massive fields with mixed symmetry with respect to Lorentz group

  but: in high E limit massless HS fields? [Sagnotti et al]

  [see talk by Massimo Taronna]

Higher spin gravity - a special limit in string theory

Two different limits of string theory

- $\alpha' = l_s^2 \to 0$: supergravity description
  invariant under diffeomorphisms
- $\alpha' \to \infty$: tensionless limit
  Massive string states get massless in this limit:

  $$m^2 L^2 \sim \frac{L^2}{\alpha'} \to 0$$

  String theory (is expected to) reduce to higher spin gravity

  new gauge symmetry present involving higher spin fields!

  expected to be non-local

How does spacetime look like at short distances?

For example in string theory or any other sensible quantum theory of gravity?

Higher spin gravity in curved spacetime

Interacting Higher spin theories in (Anti-)de Sitter spacetime possible

- S-matrix does not exit
- but non-trivial correlation functions of massless interacting higher spin fields possible

Here I will focus on higher spin gravity in the framework of AdS/CFT!

Outline

What is AdS/CFT?

In general

Quantum gravity theory in asymptotically $d+1$ dim. AdS $\Longleftrightarrow$ Conformal field theory in d spacetime dimensions

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A specific example for AdS/CFT

$\mathcal{N} = 4$ Super Yang-Mills (SYM) theory
with gauge group $SU(N)$ and Yang-Mills coupling constant g_{YM}

is *dynamically equivalent* to

type IIB superstring theory with string length $\sqrt{\alpha'}$ and coupling constant g_s on $AdS_5 \times S^5$ with radius of curvature L and N units of $F_{(5)}$ flux on S^5.

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Mapping of parameters

g_{YM} and N are mapped to g_s and $L/\sqrt{\alpha'}$ by

$$g_{\mathrm{YM}}^2 = 4\pi g_s \qquad \text{and} \qquad g_{YM}^2 N = L/\sqrt{\alpha'}.$$

What is AdS/CFT?

$g_{\rm YM}$ and N are mapped to g_s and $L/\sqrt{\alpha'}$ by

$$g^2_{\rm YM} = 4\pi g_s \qquad \text{and} \qquad g^2_{YM} N = L/\sqrt{\alpha'}.$$

Interesting limits

- *Large N limit:* Take $N \to \infty$ but keep $\lambda = N g^2_{YM}$ fixed

  $\Rightarrow g_s \sim g_{YM} \to 0$, i.e. classical string theory on $AdS_5 \times S^5$
- *strong coupling limit:* $\lambda \to \infty$

  $L^4/\alpha'^2 \to \infty$, i.e. supergravity on $AdS_5 \times S^5$
- *weak coupling limit:* $\lambda \to 0$

  $L^4/\alpha'^2 \to 0$, i.e. higher spin gravity on $AdS_5 \times S^5$?

Examples of AdS/CFT involving higher spin gravity

AdS_4 / CFT_3 duality

[Klebanov, Polyakov]

Vasiliev higher spin gravity in AdS_4

is dual to

(2+1)-dimensional O(N) vector models

two different boundary conditions for scalar field:

- $\Delta = 1$ dual to free theory of N massless scalars
- $\Delta = 2$ dual to critical O(N) vector model

Vasiliev higher spin gravity in AdS_4

Non-linear system of eom:

- infinite set of gauge fields of spin $s = 2, 4, 6, \ldots$
- scalar field with mass $m^2 = -2/R^2_{AdS}$.

Examples of AdS/CFT involving higher spin gravity

AdS_3 / CFT_2 duality

[Gaberdiel, Gopakumar, '10]

Vasiliev higher spin gravity in AdS_3

is dual to

W_N minimal model in the 't Hooft limit

- coset representation for W_N models

$$\frac{su(N)_k \oplus su(N)_1}{su(N)_{k+1}}$$

- 't Hooft limit

$$N, k \to \infty \,, \quad \lambda \equiv \frac{N}{k+N} \quad \text{fixed}$$

Why is higher spin gravity for AdS/CFT interesting?

- From conceptional point of view:

  What is the gravity dual of non-interaction field theoies? Of minimal CFTs?
- For condensed matter applications:

  Higher Spin Gavity in 4D dual to O(N) models in the large N-limit.

  How do we compute entanglement entropy in higher spin gravity?
- For (quantum) gravity applications:

  Higher Spin Gravity as toy-model to study properties of black holes in asymptotically AdS.

  Can we study black hole creation and evaporation explicitly since we have both sides under full control?

  What is geometry in higher spin gravity?

Why Higher Spin gravity in context of AdS/CFT?

In this talk:

I focus on higher spin gravity in three spacetime dimensions.

advantage:

We do not have to take into account the infinite tower of higher spins since we can truncate to a finite order.

Here:

I consider only 'minimal' extensions of Einstein Gravity by adding a spin-3 degree of freedom.

Outline

Review: 3D Gravity as Chern-Simons theory

Action

$$S = \frac{1}{16\pi G}\int_{\mathcal{M}} d^3x\sqrt{-g}(R + \frac{2}{L^2}) - \int_{\partial\mathcal{M}} \omega^a \wedge e_a$$

or equivalently

$$S = S_{CS}[A] - S_{CS}[\overline{A}] \qquad A = \omega + e\,,\ \overline{A} = \omega - e$$

$$S_{CS}[A] = \frac{k}{4\pi}\int \mathrm{Tr}\left[A \wedge dA + \frac{2}{3}A \wedge A \wedge A\right]$$

- gauge fields $A, \overline{A} \in sl(2, \mathbb{R})$
- k is the Chern-Simons level, $k = \frac{L}{4G}$.

Equations of motion

$$F = dA + A \wedge A = 0\,, \qquad \overline{F} = d\overline{A} + \overline{A} \wedge \overline{A} = 0$$

Metric can be computed by

$$g_{\mu\nu} = \frac{1}{2}\mathrm{Tr}(e_\mu e_\nu)\,, \qquad e = e_\mu dx^\mu\,.$$

3D Higher Spin Gravity as Chern-Simons theory

3D Gravity coupled to spin-3 field given by

$$S = S_{CS}[A] - S_{CS}[\overline{A}] \qquad A = \omega + e\,,\ \overline{A} = \omega - e$$

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Equations of motion

$$F = dA + A \wedge A = 0\,, \qquad \overline{F} = d\overline{A} + \overline{A} \wedge \overline{A} = 0$$

Metric and Spin-3 field can be computed by

$$g_{\mu\nu} = \frac{1}{2}\mathrm{Tr}(e_\mu e_\nu)\,, \qquad \phi_{\mu\nu\rho} = \frac{1}{6}\mathrm{Tr}(e_{(\mu} e_\nu e_{\rho)}) \qquad e = e_\mu dx^\mu .$$

3D Higher Spin Gravity as Chern-Simons theory II

Gauge connection for AdS in Poincare patch

$$A = A_+ \, dx^+ + A_- \, dx^- + L_0 \, d\rho, \qquad \overline{A} = \overline{A}_+ \, dx^+ + \overline{A}_- \, dx^- - \overline{L}_0 \, d\rho$$

$$A_+ = e^{\rho} L_1, \qquad \overline{A}_- = -e^{\rho} L_{-1}, \qquad A_- = \overline{A}_+ = 0$$

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$$A_+ = e^\rho L_1, \qquad \overline{A}_- = -e^\rho L_{-1}, \qquad A_- = \overline{A}_+ = 0$$

Gauge Transformation

$$\begin{aligned} A &\to g^{-1} A\, g + g^{-1} dg \\ \overline{A} &\to \tilde{g}\, \overline{A}\, \tilde{g}^{-1} - d\tilde{g}\, \tilde{g}^{-1} \end{aligned}$$

where g and $\tilde{g}$ are functions of spacetime coordinates and are valued in $SL(3,\mathbb{R})$.

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Remarks

- Some of the gauge transformations (namely $g, \tilde{g} \in SL(2, \mathbb{R}) \subset SL(3, \mathbb{R})$) correspond to diffeomorphisms and frame rotations.
- Higher spin gauge transformations may change the causal structure of the spacetime. What is the notion of geometry in higher spin gravity?

3D Higher Spin Gravity as Chern-Simons theory IV

$$A = A_+\, dx^+ + A_-\, dx^- + L_0\, d\rho, \qquad \overline{A} = \overline{A}_+\, dx^+ + \overline{A}_-\, dx^- - \overline{L}_0\, d\rho$$

Asymptotic symmetry group

$$A_+ = e^{\rho} L_1 - \frac{2\pi}{k}\, \mathcal{L}(x^+)\, e^{-\rho} L_{-1} - \frac{\pi}{2k}\, \mathcal{W}(x^+)\, e^{-2\rho} W_{-2}, \qquad A_- = 0$$

$$\overline{A}_- = -\left(e^{\rho} L_{-1} - \frac{2\pi}{k}\, \overline{\mathcal{L}}(x^-)\, e^{-\rho} L_1 - \frac{\pi}{2k}\, \overline{\mathcal{W}}(x^-)\, e^{2\rho} W_2 \right), \qquad \overline{A}_+ = 0$$

where $\mathcal{L}(x^+) \sim \hat{T}$ and $\overline{\mathcal{L}}(x^+) \sim \hat{\overline{T}}$

asymptotic symmetry group: $\mathrm{Vir} \oplus \mathrm{Vir}$

[see talk by Stefan Fredenhagen]

3D Higher Spin Gravity as Chern-Simons theory IV

$$A = A_+ \, dx^+ + A_- \, dx^- + L_0 \, d\rho, \qquad \overline{A} = \overline{A}_+ \, dx^+ + \overline{A}_- \, dx^- - \overline{L}_0 \, d\rho$$

Asymptotic symmetry group

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where $\mathcal{L}(x^+) \sim \hat{T}$ and $\overline{\mathcal{L}}(x^+) \sim \hat{\overline{T}}$

$\mathcal{W}(x^+) \sim \text{spin} - 3 \text{ operator}$

asymptotic symmetry group: $W_3 \oplus W_3$

[see talk by Stefan Fredenhagen]

Outline

Black holes in 3D Higher Spin Gravity I

Can we find black holes in 3D Higher spin gravity? *Yes, ...*

- BTZ black hole is also a solution of 3D higher spin gravity.

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- There exist also black holes with higher spin charge [Gutperle, Kraus, '11, MA, Gutperle, Kraus, Perlmutter, '11]

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$$A = A_+ \, dx^+ + A_- \, dx^- + A_\rho \, d\rho, \qquad \overline{A} = \overline{A}_+ \, dx^+ + \overline{A}_- \, dx^- + \overline{A}_\rho \, d\rho$$

with

$$\begin{aligned} A_\rho &= L_0 \\ A_+ &= e^\rho L_1 - \frac{2\pi}{k} \mathcal{L} \, e^{-\rho} L_{-1} - \frac{\pi}{2k} \mathcal{W} \, e^{-2\rho} W_{-2} \\ A_- &= \mu \left(e^{2\rho} W_2 - \frac{4\pi}{k} \mathcal{L} \, W_0 + \frac{4\pi^2}{k^2} \mathcal{L}^2 \, e^{-2\rho} W_{-2} + \frac{4\pi}{k} \mathcal{W} \, e^{-\rho} L_{-1} \right) \sim \mu A_+^2 \end{aligned}$$

Black holes in 3D Higher Spin Gravity II

Thermodynamics of charged higher spin black holes are only consistent if Holonomy condition is satisfied.

The Holonomy condition

The holonomies associated with the Euclidean time circle

$$\omega = 2\pi(\tau A_+ - \overline{\tau} A_-) \qquad \overline{\omega} = 2\pi(\tau \overline{A}_+ - \overline{\tau} \overline{A}_-)$$

have eigenvalues $(0, 2\pi i, -2\pi i)$ as in the case of the BTZ black hole.

Gauge invariant characterization of higher spin black holes!

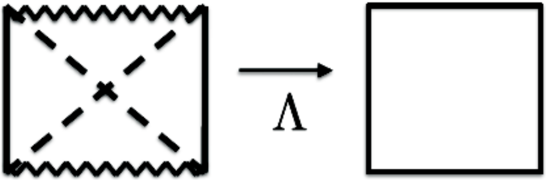
Entropy:

$$S \neq \frac{A}{4G}$$

can be reproduced from CFT side (and also from entanglement entropy, Wald formula)

Black holes in 3D Higher Spin Gravity III

The causal structure is not invariant under higher spin transformations. [MA, Gutperle, Kraus, Perlmutter, '11]



For example, a higher spin black hole in one gauge can look like a traversable wormhole in another gauge, even though they describe the same physics.

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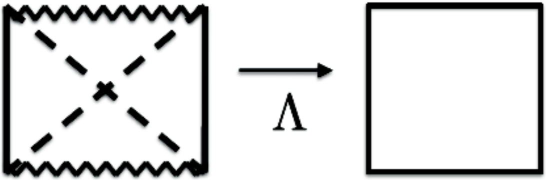

For example, a higher spin black hole in one gauge can look like a traversable wormhole in another gauge, even though they describe the same physics.

Need matter in order to decide what is the correct causal structure:

- we have to consider Vasiliev theory
- calculate correlation functions of scalar fields on both asymptotic AdS sides
- correlation functions behave as in a black hole background (also in wormhole gauge)

Outline

Review: Entanglement entropy in CFT & AdS/CFT

Entanglement entropy in CFT

Consider quantum system described by a density matrix ϱ, and divide it into two subsystems A and $B = A^c$. Reduced density matrix ϱ_A of subsystem A:

$$\varrho_A = \mathrm{Tr}_{A^c}\,\varrho$$

Entanglement entropy S_{EE} = von Neumann entropy associated with ϱ_A:

$$S_{EE} = -\mathrm{Tr}_A \varrho_A \log \varrho_A\,.$$

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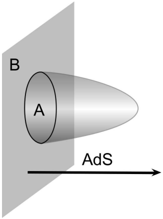
Gravity dual of entanglement entropy (supergravity limit)



Construct minimal spacelike surface $m(A)$ which is anchored at the boundary ∂A of the region A and extends into the bulk spacetime.

$$S_{EE} = \frac{m(A)}{4G_N} \,.$$

Entanglement entropy in higher spin gravity I

Geodesics will not work: What is spacetime geometry in higher spin gravity? Can we find a bulk object that correctly calculates the entanglement entropy?

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Proposal for Entanglement Entropy in Higher Spin Gravity [MA, Castro, Iqbal, '13; see also de Boer, Jottar,'13 for a similar proposal]

Entanglement Entropy may be calculated from a Wilson line in infinite dim. rep.

$$W_{\mathcal{R}}(C) = \mathrm{tr}_{\mathcal{R}}(\mathcal{P} \exp \int_{\mathcal{C}} \mathcal{A}) = \int \mathcal{D}U \exp(-S(U, P; \mathcal{A})_C)$$

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- $\mathcal{R}$ contains information about quantum numbers of probe
- $U(s) \in SL(3, \mathbb{R})$: field capturing the dynamics of the probe
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$$S(U,P;\mathcal{A})_C = \int ds\left(\mathrm{Tr}\,(PU^{-1}D_sU) + \lambda_2(\mathrm{Tr}\,(P^2) - c_2) + \lambda_3(\mathrm{Tr}\,(P^3) - c_3)\right)$$

where $D_sU = \frac{d}{ds}U + A_sU - U\overline{A}_s\ , \quad A_s \equiv A_\mu\frac{dx^\mu}{ds}\ ,$

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where $D_sU = \frac{d}{ds}U + A_sU - U\bar{A}_s$, $\quad A_s \equiv A_\mu \frac{dx^\mu}{ds}$,

Entanglement entropy: Take $c_3 = 0$ and $\sqrt{2c_2} \to \frac{c}{6}$ and compute $S_{EE} = -\log(W_{\mathcal{R}}(\mathcal{C}))$

Entanglement entropy in higher spin gravity II

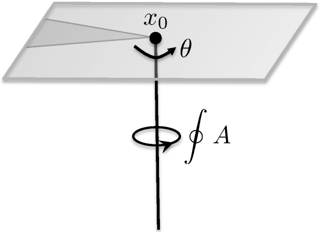
Why do we think our proposal is correct?

- Perfect agreement with CFT results (where available)

Entanglement entropy in higher spin gravity II

Why do we think our proposal is correct?

- Perfect agreement with CFT results (where available)
- Wilson line induces conical defect if backreaction is included

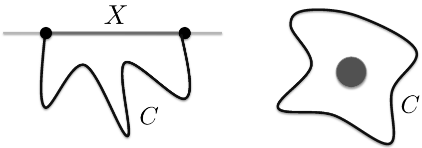


$$\sqrt{2c_2} \to \frac{c}{6}$$

[see also Lewkowycz, Maldacena,'13]

Entanglement entropy in higher spin gravity III

Two possible choices for C:



Entanglement entropy in higher spin gravity III

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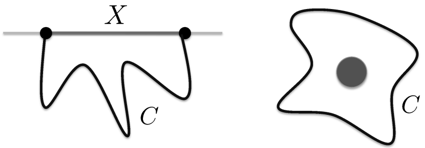
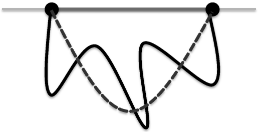


Wilson Line does not depend on path. Geodesic equation is irrelevant to entanglement entropy.



Can we conclude that large regions of spacetime are highly entangled with each other?

Outline

Summary I

In this talk

We focused on gravity + spin-3 field in AdS_3

- Black hole solution in higher spin gravity with non-trivial higher spin charge
  - Causal structure/curvature singularities not invariant under higher spin gauge transformations
- Entanglement Entropy dual to a Wilson line in an infinite dimensional rep.

Generalizations & Outlook

Generalization to Vasiliev theory (gravity + infinite tower of higher spin fields + matter)

dual to minimal two-dimensional CFTs (which can be solved) [Gaberdiel, Gopakumar]

- Higher spin black hole solutions known
- Have to construct Wilson Line

Next: Study Black hole creation & evaporation in this theory!

Summary II

Possible Caveats

- Is higher spin gravity in 3D non-trivial enough to create black holes from scalar fields?
- Can we “higgs” higher spin gravity to say something useful for ordinary gravity in AdS?
- Can we learn something for four-dimensional asymptotically flat black holes from AdS black holes?

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For more details

Wilson Lines & Entanglement Entropy in higher spin gravity
MA, Castro, Iqbal, arXiv: 1306.4338

Review on Higher Spin black holes
MA, Gutperle, Kraus, Perlmutter, arXiv: 1208.5182

or just ask me!

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Thank you!

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Review: Spin–3 gravity

How to embed $sl(2,\mathbb{R})$ into $sl(3,\mathbb{R})$?

There are two different embeddings possible (leading to different CFTs). Here we use:

$sl(3,\mathbb{R})$ has eight generators which we split into:

- L_{-1}, L_0, L_1 generators of $sl(2,\mathbb{R})$, with commutation relations $[L_i, L_j] = (i-j)L_{i+j}$
- W_j, $(j = -2, -1, ..., 2)$ satisfying $[L_j, W_m] = (2j - m)W_{j+m}$

Spin-3 black hole in "wormhole gauge"

gauge connection

$$A = A_+\,dx^+ + A_-\,dx^- + A_\rho\,d\rho, \qquad \overline{A} = \overline{A}_+\,dx^+ + \overline{A}_-\,dx^- + \overline{A}_\rho\,d\rho$$

with

$$\begin{aligned}
A_\rho &= L_0 \qquad \overline{A}_\rho = -L_0 \\
A_+ &= e^\rho L_1 - \frac{2\pi}{k}\mathcal{L}\,e^{-\rho}L_{-1} - \frac{\pi}{2k}\mathcal{W}\,e^{-2\rho}W_{-2} \\
A_- &= \mu\left(e^{2\rho}W_2 - \frac{4\pi}{k}\mathcal{L}\,W_0 + \frac{4\pi^2}{k^2}\mathcal{L}^2\,e^{-2\rho}W_{-2} + \frac{4\pi}{k}\mathcal{W}\,e^{-\rho}L_{-1}\right) \sim \mu A_+^2 \\
\overline{A}_- &= -\left(e^\rho L_{-1} - \frac{2\pi}{k}\overline{\mathcal{L}}\,e^{-\rho}L_1 - \frac{\pi}{2k}\overline{\mathcal{W}}\,e^{-2\rho}W_2\right) \\
\overline{A}_+ &= -\overline{\mu}\left(e^{2\rho}W_{-2} - \frac{4\pi}{k}\overline{\mathcal{L}}\,W_0 + \frac{4\pi^2}{k^2}\overline{\mathcal{L}}^2\,e^{-2\rho}W_2 + \frac{4\pi}{k}\overline{\mathcal{W}}\,e^{-\rho}L_1\right) \sim \overline{\mu}\overline{A}_-^2
\end{aligned}$$

From now on: Non rotating case

$$\overline{\mathcal{L}} = \mathcal{L}, \qquad \overline{\mathcal{W}} = -\mathcal{W}, \qquad \overline{\mu} = -\mu$$

Spin-3 black hole in "wormhole gauge"

Parameters in non rotating case

- Four free parameters: $\mathcal{L}, \mathcal{W}, \mu$ and $\beta = T^{-1}$, given by $t \simeq t + i\beta$
- but only a two–parameter family of physically admissable solutions:

  If T and μ (or equivalently $\tau = i\beta/(2\pi)$ and $\alpha = \overline{\tau}\mu$) are specified

  $\Rightarrow \mathcal{L}(\tau, \alpha)$ and $\mathcal{W}(\tau, \alpha)$ should be determined thermodynamically

$$Z(\tau, \alpha) = \operatorname{Tr} e^{8\pi^2 i[\tau \hat{\mathcal{L}} + \alpha \hat{\mathcal{W}}]}$$

$$\mathcal{L} = \langle \hat{\mathcal{L}} \rangle = -\frac{i}{8\pi^2} \frac{\partial \ln Z}{\partial \tau} \qquad \mathcal{W} = \langle \hat{\mathcal{W}} \rangle = -\frac{i}{8\pi^2} \frac{\partial \ln Z}{\partial \alpha}$$

  and therefore have to satisfy the *integrability condition*

$$\frac{\partial \mathcal{L}}{\partial \alpha} = \frac{\partial \mathcal{W}}{\partial \tau}$$

How do we determine $\mathcal{L}(\tau, \alpha)$ and $\mathcal{W}(\tau, \alpha)$ from the holographic perspective?

For BTZ, relation between energy and temperature is fixed by demanding absence of conical singularity at horizon in Euclidean signature

Spin-3 black hole in "wormhole gauge"

How are the charges $\mathcal{L}$ and $\mathcal{W}$ related to τ and α – a naive first attempt

Metric of spin–3 black hole

$$ds^2 = d\rho^2 - \mathcal{F}(\rho)dt^2 + \mathcal{G}(\rho)d\phi^2$$

with

$$\mathcal{F}(\rho) = \left(2\mu e^{2\rho} + \frac{\pi}{k}\mathcal{W}e^{-2\rho} - \frac{8\pi^2}{k^2}\mu\mathcal{L}^2 e^{-2\rho}\right)^2 + \left(e^{\rho} - \frac{2\pi}{k}\mathcal{L}e^{-\rho} + \frac{4\pi}{k}\mu\mathcal{W}e^{-\rho}\right)^2$$

Since we demand $g_{tt}(\rho = \rho_+) = 0$

$$e^{2\rho_+} = \frac{2\pi}{k}\left(\mathcal{L} - 2\mu\mathcal{W}\right), \qquad k + 32\mu^2\pi\left(\mu\mathcal{W} - \mathcal{L}\right) = 0$$

The temperature is fixed by avoiding the conical singularity. Then we can determine

$$\mathcal{W}(\tau,\alpha) \quad \text{and} \quad \mathcal{L}(\tau,\alpha)$$

However, these charge assignments do not satisfy the integrability condition!

Attempt failed!

Spin-3 black hole in "wormhole gauge"

puzzle

But if we use another charge assignment then the metric of the spin–3 black hole

$$ds^2 = d\rho^2 - \mathcal{F}(\rho)dt^2 + \mathcal{G}(\rho)d\phi^2$$

$\mathcal{F}(\rho)$ and $\mathcal{G}(\rho)$ are always positive!

- no event horizon since $\mathcal{F}$ never vanishes
- geometry possesses a globally defined timelike Killing vector

For $\rho \to \pm\infty$: $\mathcal{F}(\rho), \mathcal{G}(\rho) \sim e^{4|\rho|} \Rightarrow$ asymptotic AdS_3 region with radius 1/2.

Geometry is a "wormhole" connecting two asymptotic AdS_3 regions

Why do we call this gauge connection a higher spin black hole?

Spin-3 black hole in "wormhole gauge"

Wish list

We want to find a charge assignment $\mathcal{L}(\tau, \alpha)$ and $\mathcal{W}(\tau, \alpha)$ such that

- the integrability condition
$$\frac{\partial \mathcal{L}}{\partial \alpha} = \frac{\partial \mathcal{W}}{\partial \tau}$$
is satisfied,

  i.e. in other words the thermodynamic quantities associated with the black hole will obey the first law of thermodynamics
- in the limit $\mu \to 0$ the solution goes smoothly over to BTZ, in particular $\mathcal{W} \to 0$
- it is possible to find *another* gauge in which there exists a horizon and the geometry is smooth near the horizon.

  The charge assignments are compatible with the smoothness in that geometry

Outline

The Holonomy condition

The Holonomy condition

The holonomies associated with the Euclidean time circle

$$\omega = 2\pi(\tau \mathcal{A}_+ - \overline{\tau} \mathcal{A}_-) \qquad \overline{\omega} = 2\pi(\tau \overline{\mathcal{A}}_+ - \overline{\tau} \overline{\mathcal{A}}_-)$$

have eigenvalues $(0, 2\pi i, -2\pi i)$ as in the case of the BTZ black hole.

Alternative formulation (presented only for ω)

$$\det(\omega) = 0\,, \qquad \mathrm{Tr}(\omega^2) + 8\pi^2 = 0$$

Remarks

The holonomy condition

- is gauge invariant,
- reproduces known BTZ results in the uncharged case $\mu \to 0, \mathcal{W} \to 0$.

The Holonomy condition

Let us apply the holonomy condition to the spin–3 black hole in "wormhole" gauge.

$$
\begin{aligned}
0 &= 2048\pi^2\alpha^3\mathcal{L}^3 - 576\pi k\tau^2\alpha\mathcal{L}^2 - 864\pi k\alpha^2\tau\mathcal{W}\mathcal{L} - 864\pi k\alpha^3\mathcal{W}^2 - 27k^2\tau^3\mathcal{W} \\
0 &= 256\pi^2\alpha^2\mathcal{L}^2 + 24\pi k\tau^2\mathcal{L} + 72\pi k\tau\alpha\mathcal{W} + 3k^2
\end{aligned}
$$

To check integrability

- Solve the second equation for $\mathcal{W}$ and calculate $\frac{\partial\mathcal{W}}{\partial\tau}$
- Plug $\mathcal{W}$ into the first equation. We can now determine $\frac{\partial\mathcal{L}}{\partial\alpha}$ and $\frac{\partial\mathcal{L}}{\partial\tau}$
- Show that

$$\frac{\partial\mathcal{L}}{\partial\alpha} = \frac{\partial\mathcal{W}}{\partial\tau}$$

Integrability is indeed satisfied!

Last task on our list:

Find a gauge transformation to turn the "wormhole" into a smooth black hole!

Outline

Transformation of the "wormhole" into a black hole

Consider new connections $(\mathcal{A}, \overline{\mathcal{A}})$ related to the wormhole connection $(A, \overline{A})$

$$\begin{aligned} A &= g^{-1}(\rho)\mathcal{A}(\rho)g(\rho) + g^{-1}(\rho)dg(\rho) \\ \overline{A} &= g(\rho)\overline{\mathcal{A}}(\rho)g^{-1}(\rho) - dg(\rho)g^{-1}(\rho) \end{aligned}$$

The new connection should be smooth at the horizon.

Which conditions do we have to satisfy? How does $g(\rho)$ look like?

smoothness conditons, part I

The metric and spin-3 field corresponding to $(A, \overline{A})$ look like

$$\begin{aligned} ds^2 &= g_{\rho\rho}(\rho)d\rho^2 + g_{tt}(\rho)dt^2 + g_{\phi\phi}(\rho)d\phi^2 \\ \varphi_{\alpha\beta\gamma}dx^\alpha dx^\beta dx^\gamma &= \varphi_{\phi\rho\rho}(\rho)d\phi d\rho^2 + \varphi_{\phi tt}(\rho)d\phi dt^2 + \varphi_{\phi\phi\phi}(\rho)d\phi^3 \end{aligned}$$

Solution should describe smooth black hole with event horizon at $\rho = \rho_+$

Transformation of the "wormhole" into a black hole

smoothness conditons, part II

Let us introduce $r = \rho - \rho_+$. We will see $g_{rr}(0) > 0$ and therefore

$$g_{tt}(0) = g'_{tt}(0) = 0 \qquad \text{and} \quad g_{\phi\phi}(0) > 0$$

The metric around $r = 0$ looks like

$$ds^2 \approx g_{rr}(0)dr^2 - \frac{1}{2}g''_{tt}(0)r^2 dt_E^2 + g_{\phi\phi}(0)d\phi^2$$

and therefore the temperature has to be

$$\beta = 2\pi\sqrt{\frac{2g_{rr}(0)}{-g''_{tt}(0)}}$$

A similar argument for the spin–3 field φ gives $\varphi_{\phi tt}(0) = \varphi'_{\phi tt}(0) = 0$ and

$$\beta = 2\pi\sqrt{\frac{2\varphi_{\phi rr}(0)}{-\varphi''_{\phi tt}(0)}}$$

Transformation of the "wormhole" into a black hole

smoothness conditons, part III

one more condition to ensure that some curvature invariants involving covariant derivatives do not diverge:

All functions should be smooth at the horizon and even under reflection about $r = 0$

$$\begin{aligned} g_{rr}(-r) = g_{rr}(r)\,, \quad g_{tt}(-r) &= g_{tt}(r)\,, \quad g_{\phi\phi}(-r) = g_{\phi\phi}(r) \\ \varphi_{\phi rr}(-r) = \varphi_{\phi rr}(r)\,, \quad \varphi_{\phi tt}(-r) &= \varphi_{\phi tt}(r)\,, \quad \varphi_{\phi\phi\phi}(-r) = \varphi_{\phi\phi\phi}(r) \end{aligned}$$

This is enforced by demanding a twisted vielbein reflection symmetry

$$\begin{aligned} e_t(-r) &= -h(r)^{-1} e_t(r) h(r) \\ e_\phi(-r) &= h(r)^{-1} e_\phi(r) h(r) \\ e_r(-r) &= h(r)^{-1} e_r(r) h(r) \end{aligned}$$

with $h(r) \in SL(3, \mathbb{R})$.

How do we determine $g(r)$ and $h(r)$?

Transformation of the "wormhole" into a black hole

Ansatz for $g(r)$ and $h(r)$

$$\begin{aligned} g(r) &= e^{F(r)(W_1 - W_{-1}) + G(r)L_0} \\ h(r) &= e^{H(r)(W_1 + W_{-1})} \end{aligned}$$

This is verified by perturbation theory around BTZ perturbatively in the charge.

$F(r), G(r)$ and $H(r)$ are very difficult to determine but are known explicitly in terms of

$$\zeta = \sqrt{\frac{k}{32\pi\mathcal{L}^3}}\mathcal{W}, \quad \gamma = \sqrt{\frac{2\pi\mathcal{L}}{k}}\mu, \quad \zeta = \frac{C-1}{C^{3/2}}$$

$$\begin{aligned} \tan H(r) &= -\frac{\sinh(r)}{\sqrt{C-2-\cosh(r)}} & G(r) &= -\frac{\log(X)}{Y} \\ F(r) &= \frac{\sqrt{C}}{2}\cosh(r)G(r) & X(r) &= \sqrt{\frac{C+Y-1}{C-Y-1}} \\ Y^2 &= 1 + C\cosh^2(r) \end{aligned}$$

Outline

Properties of the black hole

Let us study the final form of the transformed metric

$$
\begin{aligned}
g_{rr} &= \frac{(C-2)(C-3)}{\left(C-2-\cosh^2(r)\right)^2} \\
g_{tt} &= -\frac{8\pi\mathcal{L}}{k}\frac{C-3}{C^2}\frac{\left(a_t+b_t\cosh^2(r)\right)\sinh^2(r)}{\left(C-2-\cosh^2(r)\right)^2} \\
g_{\phi\phi} &= \frac{8\pi\mathcal{L}}{k}\frac{C-3}{C^2}\frac{\left(a_\phi+b_\phi\cosh^2(r)\right)\sinh^2(r)}{\left(C-2-\cosh^2(r)\right)^2}+\frac{8\pi\mathcal{L}}{k}\left(1+\frac{16}{3}\gamma^2+12\gamma\zeta\right)
\end{aligned}
$$

coefficients $a_{t,\phi}$ and $b_{t,\phi}$ are functions of γ and C

similar expression also available for the spin–3 field

Smoothness conditions

The smoothness conditions $\beta = 2\pi\sqrt{\frac{2g_{rr}(0)}{-g''_{tt}(0)}} = 2\pi\sqrt{\frac{2\varphi_{\phi rr}(0)}{-\varphi''_{\phi tt}(0)}}$ are equivalent to the holonomy conditons $\det(\omega)=0, \mathrm{Tr}(\omega^2)+8\pi^2=0$

Properties of the black hole

Limiting cases

- the uncharged BTZ limit corresponds to taking $\zeta, \gamma \to 0$, and $C \to \infty$.
- C has to lie in the range $3 \leq C < \infty$
- For $C = 3$ we have $\zeta_{max} = \sqrt{\frac{4}{27}}$, $\quad \gamma_{max} = \sqrt{\frac{3}{16}}$

  this corresponds to an extremal black hole with $T = 0$.

boundary asymptotics

The metric coefficients diverge at $r = r_\star$ where

$$\cosh^2(r_\star) = C - 2$$

The leading behavior of the metric near $r_\star$ is

$$ds^2 \approx \frac{1}{4}\frac{dr^2}{(r_\star - r)^2} + \frac{2\pi\mathcal{L}}{k}\frac{C-3}{C^2(C-2)}\frac{-[a_t + b_t(C-2)]dt^2 + [a_\phi + b_\phi(C-2)]d\phi^2}{(r_\star - r)^2}$$

i.e. AdS_3 with radius 1/2.

Properties of the black hole

Black hole entropy

The black hole entrpy is given by

$$S = 4\pi\sqrt{2\pi k\mathcal{L}}f(y)$$

where $y = \frac{27}{2}\zeta^2$ and

$$f(y) = \cos\theta\,, \quad \theta = \frac{1}{6}\arctan\left(\frac{\sqrt{y(2-y)}}{1-y}\right)\,, \quad 0 \leq \theta \leq \frac{\pi}{6}$$

In terms of C we get the simple expression

$$f(y) = \sqrt{1 - \frac{3}{4C}}$$

Note that the entropy is not given by

$$S = \frac{1}{4G}\mathcal{A}$$

where $\mathcal{A}$ is the area of the horizon.

Entanglement entropy in higher spin gravity

How to get the geodesics for 3D spin-2 gravity?

Just set $U(s) = 1$ (not possible for higher spin gravity)

Geodesic equation

$$\frac{d}{ds}\left((A-\overline{A})_\mu \frac{dx^\mu}{ds}\right) + [\overline{A}_\mu, A_\nu]\frac{dx^\mu}{ds}\frac{dx^\nu}{ds} = 0$$

Proper distance appears in on-shell action

$$\begin{aligned} S_C &= \sqrt{c_2}\int_C ds \sqrt{\mathrm{Tr}\left((A-\overline{A})_\mu (A-\overline{A})_\nu \frac{dx^\mu}{ds}\frac{dx^\nu}{ds}\right)} \\ &= \sqrt{2c_2}\int_C ds \sqrt{g_{\mu\nu}(x)\frac{dx^\mu}{ds}\frac{dx^\nu}{ds}} \end{aligned}$$

and thus

$$S_{EE} = e^{-S_C}$$

Entanglement entropy in higher spin gravity

Infinite dimensional highest-weight state $|h, w\rangle$ with definite eigenvalues under the elements of the $SL(3,\mathbb{R})$ Cartan L_0, W_0:

$$L_0|h, w\rangle = h|h, w\rangle \ , \qquad W_0|h, w\rangle = w|h, w\rangle \ ,$$

and which is annihilated by the positive modes of the algebra:

$$L_1|h, w\rangle = 0 \ , \qquad W_{1,2}|h, w\rangle = 0 \ .$$

We may now generate other excited states by acting with L_{-1}, $W_{-1,-2}$ on this ground state, filling out an infinite dimensional unitary and irreducible representation.

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Relationship between Casimirs c_2, c_3 and h, w

$$C_2 = \tfrac{1}{2}L_0^2 + \tfrac{3}{8}W_0^2 + \cdots \ , \qquad C_3 = \tfrac{3}{8}W_0\left(L_0^2 - \tfrac{1}{4}W_0^2\right) + \cdots \ .$$

Acting with C_2 and C_3 on the highest weight state $|h, w\rangle$ we find

$$c_2 = \tfrac{1}{2}h^2 + \tfrac{3}{8}w^2 \ , \qquad c_3 = \tfrac{3}{8}w\left(h^2 - \tfrac{1}{4}w^2\right) \ .$$

We consider heighest weight representation with $w = 0, h = c/6$ implying $c_3 = 0$.