Conditions on holographic entangling surfaces for black hole geometries in higher derivative gravity

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based on J. Erdmenger, M.F., C. Sleight: arXiv:1401.5075

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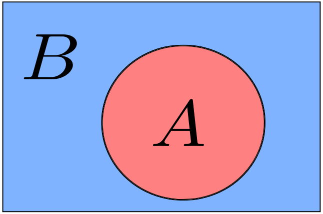
Overview

- Recap:
  - (Holographic) entanglement entropy
  - Higher curvature theories
- Closed curves extremising the entropy functional
  - Gauss-Bonnet gravity
- Possible solutions
  - Causal influence argument
  - Lewkowycz-Maldacena approach

Entanglement entropy

Entanglement entropy $\mathcal{S}_A$ defines the entropy of a subsystem A with respect to the total system $A \cup B$.



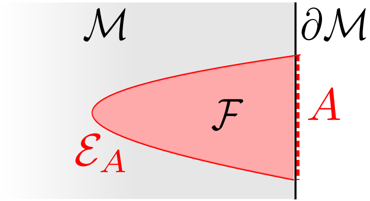
$$\mathcal{S}_A = -\mathrm{Tr}_A[\rho_A \log(\rho_A)]$$

with reduced density matrix $\rho_A \equiv \mathrm{Tr}_B[\rho_{A\cup B}]$.

[see e.g. Nielsen, Chuang]

Holographic entanglement entropy

In AdS/CFT correspondence, a gravity theory in the bulk spacetime $\mathcal{M}$ is related to a CFT on the asymptotic boundary $\partial\mathcal{M}$.



$$\mathcal{S}_A = \frac{\mathrm{Area}\left(\mathcal{E}_A\right)}{4G_N}$$

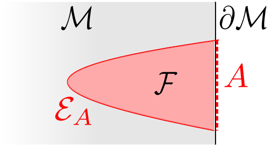
where $\mathcal{E}_A$ is a hypersurface in the bulk.

[Ryu, Takayanagi]

Holographic entanglement entropy

More concretely:

$$\mathcal{S}_A = \min \frac{\text{Area}\,(\mathcal{E}_A)}{4G_N}$$



- $\mathcal{E}_A$ has to be anchored at the boundary of A, ∂A: $\mathcal{E}_A \cap \partial\mathcal{M} = \partial A$.
- $\mathcal{E}_A$ has to be *homologous* to A, i.e. $\exists\mathcal{F}$ s.t. $\partial\mathcal{F} = \mathcal{E}_A \cup A$.
- $\mathcal{F}$ should be spacelike.
- If several such curves exist, $\mathcal{E}_A$ should be the one with minimal entropy.

[Ryu, Takayanagi; Fursaev; Hubeny, Rangamani, Takayanagi]

Higher curvature theories

When the bulk gravity action has higher curvature terms, it it proposed that the entanglement entropy gets similar corrections:

$$S = \frac{1}{16\pi G_N} \int d^{d+1}x \sqrt{-g} \left[R + 2\Lambda + aR^2 + bR_{\mu\nu}R^{\mu\nu} + cR_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta} \right],$$

$$\mathcal{S}_A = \frac{1}{4G_N} \int_{\mathcal{E}_A} d^{d-1}y \sqrt{\gamma} \left[1 + 2aR + b\left(R_{\parallel} - \frac{1}{2}k^2 \right) + 2c\left(R_{\parallel\parallel} - \mathrm{Tr}(k)^2 \right) \right].$$

γ: induced metric, $R_{\parallel}, R_{\parallel\parallel}$: projected curvature tensors, $k^2, \mathrm{Tr}(k)^2$: extrinsic curvature terms.

[Fursaev; Dong; Camps]

Higher curvature theories

$$\mathcal{S}_A = \frac{1}{4G_N}\int_{\mathcal{E}_A} d^{d-1}y\sqrt{\gamma}\left[1 + 2aR + b\left(R_{\|} - \frac{1}{2}k^2\right) + 2c\left(R_{\|\|} - \mathrm{Tr}(k)^2\right)\right]$$

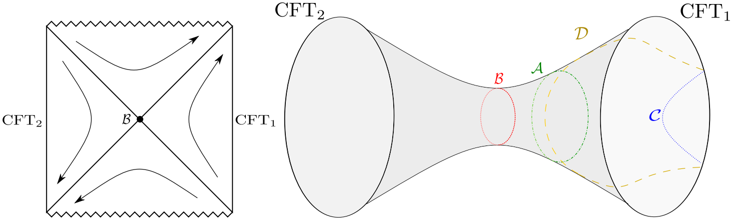
γ: induced metric, $R_{\|}, R_{\|\|}$: projected curvature tensors, $k^2, \mathrm{Tr}(k)^2$: extrinsic curvature terms.

Question:

If at all, under which conditions does searching for curves $\mathcal{E}_A$ minimising these functionals yield the correct results?

Closed extremal curves

While curves anchored at the boundary ($\mathcal{C}, \mathcal{D}$) may describe the entanglement entropy of a subregion of that boundary, closed extremal curves in the bulk ($\mathcal{B}, \mathcal{A}$) define the entropy of the whole left or right boundary.



- In Einstein-Hilbert gravity: Only bifurcation surface $\mathcal{B}$. [Headrick]
- Higher curvature functionals allow for *additional* extremal curves such as $\mathcal{A}$!

Gauss-Bonnet Gravity

- Gauss-Bonnet Gravity is an often studied theory in 5 dimensions with action

$$S = \frac{1}{16\pi G_N} \int d^5x \sqrt{-g} \left[R + \frac{12}{L^2} + \lambda \frac{L^2}{2} \left(R_{\mu\nu\alpha\beta} R^{\mu\nu\alpha\beta} - 4 R_{\mu\nu} R^{\mu\nu} + R^2 \right) \right].$$

- It is ghost free and unitary for $-\frac{7}{36} \leq \lambda \leq \frac{9}{100}$.
- Entropy functional:

$$\mathcal{S}_{EE} = \frac{1}{4G_N} \int d^3y \sqrt{\gamma} \left(1 + \lambda L^2 \mathcal{R} \right),$$

with the *intrinsic* curvature scalar $\mathcal{R}$.

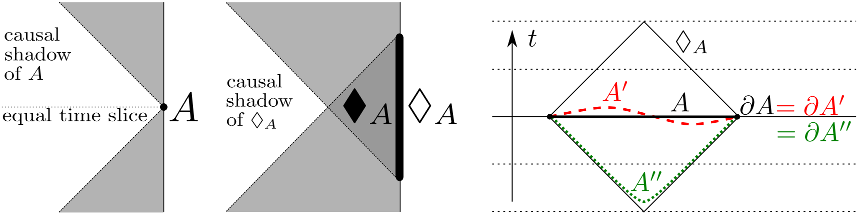
In *spherically symmetric spacetimes*, we find additional closed bulk extremal curves with **lower** entropy for $\lambda < 0$.

[see e.g. Edelstein; Camanho, Edelstein for reviews]

Causal influence argument

Proposed in the framework of ordinary Einstein-Hilbert gravity when dealing with spacetimes which are not globally static:

$\mathcal{E}_A$ should not be in causal contact with A **or** any spacelike deformation $(A', A'', ..)$ thereof.



But this only works in the Lorentzian setting. Can it be shown to be a corollary of a more general condition?

[Headrick, Hubeny, Lawrence, Rangamani, to appear]

Lewkowycz-Maldacena approach

- Replica trick on the boundary introduces conical singularity in the bulk
- For spherical surfaces in GB gravity, divergencies read:

$$qq\text{-component:} \qquad -\frac{\epsilon}{q}\left[k\left(1+\frac{2\lambda L^2}{r^2}\right)-\lambda L^2\frac{2}{9}q^{2\epsilon}k^3\right]$$

$$ij\text{-component:} \qquad 4\lambda L^2\gamma_{ij}\left[\frac{\epsilon}{q}q^{4\epsilon}k^3\frac{-2}{27}+\frac{\epsilon^2}{q^2}q^{4\epsilon}k^2\frac{2}{9}\right]$$

with ϵ, q small.

- Leading divergence gives same equation as extremisation of entropy functional.
- Can subleading terms rule out additional extremal curves?

[Lewkowycz, Maldacena; Bhattacharyya, Sharma, Sinha; Chen, Zhang]

Summary

- Entropy functionals where proposed for higher curvature theories.
- These functionals allow for pathological additional extremal surfaces.
- In Lorentzian settings, imposing causality would rule these curves out, leaving only the physical ones.
- Would be interesting to derive the invalidity of these additional curves from the replica trick.

Thank you for your attention

New Massive Gravity

- NMG is a gravity theory in 3 dimensions with action

$$S = \frac{1}{16\pi G_N} \int d^3x \sqrt{-g} \left[\sigma R - 2\lambda m^2 + \frac{1}{m^2} \left(R_{\mu\nu} R^{\mu\nu} - \frac{3}{8} R^2 \right) \right].$$

- Allows for the BTZ black holes with curvature ℓ as solutions when $\frac{1}{\ell^2} = 2m^2 \left(\sigma \pm \sqrt{1+\lambda} \right)$.

- Suffers from ghosts, unitarity violation for certain values of σ, λ, m^2.

[Bergshoeff, Hohm, Townsend]

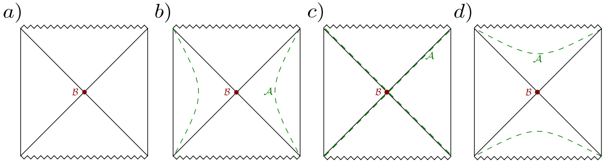
- Entropy functional:

$$\mathcal{S}_{EE} = \frac{1}{4G_N} \int d\tau \sqrt{g_{\tau\tau}} \left[\sigma + \frac{1}{m^2} \left(R_{\parallel} - \frac{1}{2} k^2 - \frac{3}{4} R \right) \right]$$

New Massive Gravity

$$\mathcal{S}_{EE} = \frac{1}{4G_N}\int d\tau\sqrt{g_{\tau\tau}}\left[\sigma + \frac{1}{m^2}\left(R_{\parallel} - \frac{1}{2}k^2 - \frac{3}{4}R\right)\right]$$

allows for additional closed bulk extremal curves with lower entropy than $\mathcal{B}$.



Bifurcation surface $\mathcal{B}$: $\mathcal{S}_{\mathcal{B}} = \frac{2\pi\ell\sqrt{M}}{4G_N}\left(\sigma + \frac{1}{2\ell^2 m^2}\right)$, $r_{\mathcal{B}} = \ell\sqrt{M}$.

Additional curves $\mathcal{A}$: $\mathcal{S}_{\mathcal{A}} = \frac{2\pi\sigma}{4G_N}\sqrt{\frac{2M}{\sigma m^2}}$, $r_{\mathcal{A}} = \sqrt{\frac{M}{2\sigma m^2}}$.