Heterotic string models on asymmetric orbifolds

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1. Introduction
2. Model construction
3. Z3 asymmetric orbifold models
4. Enhancement point and flavor symmetries
5. Summary

Beye, T.K., Kuwakino,

1304.5621, 1311.4687, 1406.XXXX

1. Introduction

Heterotic string theory on symmetric orbifolds
is one of very useful string models on 6D compact spaces
to realize a realistic low-energy effective field theory.

compact space = T6/Zn

Its geometrical aspects are rather simple.
String theory on orbifolds is solvable analytically.
Then, in principle any (stringy) perturbative
calculations can be carried out.
………… lots of good aspects

Heterotic symmetric orbifold models

Indeed many people have been working on hetetoric (symmetric) orbifold models.
Then, we have obtained (semi-)realistic models and interesting phenomenological aspects.

Dixon, Harvey, Vafa, Witten, ‘85, ‘86
Ibanez, Nilles, Quevedo, ’87, Ibanez, Kim, Nilles, Quevedo, ‘87
Casas, Munoz, Bailin, Love, Katsuki, Kawamura, T.K.,
Ohtsubo, Ono, Tanioka, Raby, Zhang, Buchmuler, Hamaguchi,
Lebedev, Ratz, Choi, Kyae, Ramos-Sanchez, Vaudrevange,
Wingerter, Ploger, Forste, Ohki, Takahashi, Blaszczk,
Groot Nibbelink, Ruehle, Trapletti, Loukas, Fischer, Kappl,
Cabo Bizet, Mayorga Pena, Parameswaran, Schmitz, Zavala,
………… many people
sorry if I miss your names

Asymmetric orbifolds

Left-mover and right-mover are independent of each other in heterotic string theory.

symmetric orbifold

$$\text{left-mover orbifold } T^6/Z_N = \text{right-mover orbifold } T^6/Z_N$$

asymmetric orbifold

$$\text{left-mover orbifold } T^6/Z_N \neq \text{right-mover orbifold } T^6/Z_M$$

Narain, Sarmadi, Vafa, '87

Asymmetric orbifolds

Narain, Sarmadi, Vafa, '87

$$\text{left-mover orbifold } T^6/Z_N \neq \text{right-mover orbifold } T^6/Z_M$$

the left-movers of a string theory live on one orbifold,
and the right-movers live on another orbifold.
We shall call such spaces asymmetric orbifolds.

In view of the fact that the separation of left- and right-movers is already done in heterotic strings and that orbifolds are consistent, it seems somewhat surprising that these two ideas had not been combined before.

Heterotic asymmetric orbifold models

Symmetric orbifolds	Asymmetric orbifolds
	(Explicit model building except equivalent models)
Dixon, Harvey, Vafa, Witten, '85, '86	
	Narain, Sarmadi, Vafa, '87
Ibanez, Nilles, Quevedo, '87,	
Ibanez, Kim, Nilles, Quevedo, '87	
…………….	Ibanez, Mas, Nilles, Quevedo, '88
	Kakushadze, Tye, '96
uncountable papers	Ito, et. al. , '11
	Bye, T.K., Kuwakino '13
	small number of papers
	a few of papers every ten years

Asymmetric orbifolds

$$\text{left-mover orbifold } T^6/Z_N \neq \text{right-mover orbifold } T^6/Z_M$$

Why a few studies ?

Maybe one of reasons is that geometrical pictures and intuitions are poor compared with the symmetric orbifolds.

However, some aspects are quite similar to the symmetric one.

Thus heterotic asymmetric orbifolds would be as interesting as the symmetric ones.

Let’s study.

Comparison between symmetric and asymmetric orbifolds before going into detail

Moduli

symmetric orbifold

$$\text{left-mover orbifold } T^6/Z_N = \text{right-mover orbifold } T^6/Z_N$$

$$g_{i\bar{j}} \sim \psi^i{}_{-1/2}\,|0>_R \otimes \alpha^{\bar{j}}_{-1}\,|0>_L \quad \text{Zn invariant}$$

geometrical moduli appear

asymmetric orbifold

$$\text{left-mover orbifold } T^6/Z_N \neq \text{right-mover orbifold } T^6/Z_M$$

$g_{i\bar{j}}$ is projected out, but it may appear as charged matter.

The dilaton appears in both.

Comparison between symmetric and asymmetric orbifolds before going into detail

Degeneracy factors

symmetric orbifold with the twist g

$$n_g = \det{}'(1-g)$$

the number of fixed points e.g. n= 27 for Z3

asymmetric orbifold

$$n_g = \sqrt{(\det(1-\theta_L))\det(1-\theta_R)/v}$$

Geometrical meaning is not so clear.

$$\text{For } \theta_L = \theta_R, \ (n_g)^{symm} = (n_g)^{asymm}.$$

Comparison between symmetric and asymmetric orbifolds before going into detail

Left-mover maassless spectra

$$\sum(\text{conformal dimensions}) = 1$$

The twisted sector in the Z3 symmetric orbifold has the conformal dimension, 1/3, because of twisted B.C..

Conformal dimension = 1/3 (1/4) for SU(3) 3 (SU(2) 2).

For example, the quark doublets have the conformal dimensions, 7/12, and they cann't have other non-Abelian charges in the twisted sector.

Asymmetric side

$$\text{For example, when } \theta_L = 1,\ h = 0.$$

Quark doublets in the twisted sector can have another non-Abelian charge, SU(3) or SU(2) flavor symmetries.

Asymmetric orbifolds

Some aspects are different between symmetric and asymmetric orbifold models, while other aspects are similar.

Anyway, heterotic asymmetric orbifolds would be as intereting as the symmetric ones.

Let's study.

2. Model construction

total extra dimensions 6+16 = 22 for left-movers
6 = 6 for right-movers

Our starting point is 22D torus compactification
for left-movers
and 6D torus compactification
for right-movers.

We need $\Gamma^{22,6}$ Narain lattice.

even and self-dual Lorentzian lattice

Narain lattice

The classification of (22,6) Narain lattices is not clear (at least to me).

We have classification for even and self-dual Euclidian lattices.

We use the technical tool:

lattice engineering technique,

Euclidian lattice → Lorentzian lattice

Lerche, Schellekens, Warner, '89

Lattice engineering technique

Lerche, Schellekens, Warner, '89

E6 x SU(3) is a maximal subgroup of E8.
E8 lattice, which is even self-dual, is decomposed as

$$(0_{E6}, 0_{SU(3)}) \cup (1_{E6}, 1_{SU(3)}) \cup (2_{E6}, 2_{SU(3)})$$

1 and 2 are fundamental and anti-fundamental weights of E6 and SU(3).

We call E6 and SU(3) dual to each other.

Lattice engineering technique

Lerche, Schellekens, Warner, '89

lattice engineering

$$\text{relacing SU(3) (left-mover)} \rightarrow \overline{\text{E}}_6 \text{(right-mover)},$$
$$0_{SU(3)} \rightarrow 0_{\overline{E}6},\quad 1_{SU(3)} \rightarrow 1_{\overline{E}6},\quad 2_{SU(3)} \rightarrow 2_{\overline{E}6},$$

They have the same modular transformation properties.

E8 Euclidian lattice →

$$(0_{E6}, 0_{\overline{E}6}) \cup (1_{E6}, 1_{\overline{E}6}) \cup (2_{E6}, 2_{\overline{E}6})$$

(6,6) Narain lattice

Euclidian lattice → Lorentzian lattice

Lattice engineering technique

Lerche, Schellekens, Warner, '89

$G \times G_{dual}$ = maximal subgroup of G_{Esd}

G_{Esd} is the group factor of an even self-dual lattice, $\Gamma_{G_{Esd}}$.

G and $\overline{G}_{dual}$ have the same modular transformation propetires.

lattice engineering

We replace $G \to \overline{G}_{dual}$.

Even self-dual lattices

8D even self-dual Euclidean lattice E8

16D E8xE8, SO(32)

24D 23 Niemeier lattices

$$D24, D16\times E8, (E8)^3, A24, (D12)^2, A17\times E7,$$
$$D10\times(E7)^2, A15\times D9, (D8)^3, (A12)^2,$$
$$A11\times D7\times E6, (E6)^4, (A9)^2\times D6, (D6)^4,$$
$$(A8)^3, (A7)^2\times(D5)^2, (A6)^4, (A5)^4\times D4,$$
$$(D4)^6, (A4)^6, (A3)^8, (A2)^{12}, (A1)^{24}$$

lattice engineering

$$\text{We replace } G \to \overline{G}_{dual}.$$

We can construct (22,6) Narain lattices.

Towards Z3 orbifold models

In order to construct asymmetric Z3 orbifold models, we need right-mover lattices including

$$\bar{E}6, \qquad (\bar{A}2)^3, \qquad \bar{D}4\times\bar{A}2$$

We construct (22,6) Narain lattice including these right-mover lattices by starting with Euclidean lattice and using lattice engineering technique.

Towards Z3 orbifold models

$$A11 \times D7 \times E6 \rightarrow A8 \times U(1) \times A2 \times D7 \times E6$$
$$\rightarrow A8 \times U(1) \times \overline{E}6 \times D7 \times E6$$

lots of lattices including $\overline{E}6, \;\; (\overline{A}2)^3, \;\; \overline{D}4 \times \overline{A}2$

see Beye, T.K., Kuwakino, 1304.5621 classification

Various left-mover lattices
→ various gauge symmetries before orfolding

We have understood the starting points
towards Z3 asymmetric orbifold construction.

3. Z3 asymmetric orbifold models

Beye, T.K., Kuwakino, 1311.4687

Example of asymmetric orbifold

$$\theta_R = (1/3, 1/3, -2/3)$$
$$\theta_L = 1$$

22D Left-mover lattice is devided by shift V.

Various possibilities for shift V with 3V on lattice

Massless spectrum

unbroken gauge group

$$P_L V = \text{integer}$$

untwisted matter fields

$$P_L V = 1/3, \quad (\text{mod } 1)$$

twisted massless condition

$$(P_L + V)^2/2 - 1 = 0$$

There is no oscillator modes, because N=integer.

In some models, we can introduce Wilson lines.

Model 1

Beye, T.K., Kuwakino, 1311.4687

Starting lattice, $(A_3)^7 \times U(1) \times \bar{E}_6$ (22,6) Narain lattice

$$(A_3)^8 \to (A_3)^7 \times U(1) \times A_2 \to (A_3)^7 \times U(1) \times \bar{E}_6$$

We divide right-movers by Z3 twist and
left-movers by Z3 shift V.

Unbroken gauge group

$$SU(3)_C \times SU(2)_L \times U(1)_Y \times SU(3)^2 \times SU(4) \times U(1)^9$$

U(1)Y is anomaly-free.

degeneracy factor in the twisted sector = 3
We did not introduce WL.

Model 1

Beye, T.K., Kuwakino, 1311.4687

full massless spectrum

3 x 12 multiplets in untwisted sector
3 x 37 multiplets in twisted sector
maybe smaller number than symmetric models

3 generations of (MSSM) quarks and leptons
+ vector-like matter fields

top quark has O(1) of Yukawa coupling,
but the charm Yukawa coupling is also strong.

U(1)Y is anomaly-free.

Model 2

Beye, T.K., Kuwakino, 1311.4687

Starting lattice,

$$(A_1)^2\times(A_4)^4\times U(1)^2\times A_2\times(\bar{A}_2)^3 \quad \text{(22,6) Narain lattice}$$

$$(A_4)^6\to(A_2\times A_1\times U(1))^2\times(A_4)^4$$
$$\to(A_1)^2\times(A_4)^4\times U(1)^2\times(\bar{A}_2)^2$$

$$\times(A_2\times\bar{A}_2)$$

We divide right-movers by Z3 twist and left-mover by Z3 shift V.

Unbroken gauge group

$$SU(3)_C\times SU(2)_L\times SU(2)_R\times U(1)_{B-L}$$
$$\times SU(2)_F\times SU(3)^2\times SU(4)^2\times U(1)^6$$

U(1)(B-L) is anomaly-free.

SU(2)F flavor symmetry

degeneracy factor in the twisted sector = 1

Model 2

Beye, T.K., Kuwakino, 1311.4687

full massless spectrum

3 x 5 multiplets in untwisted sector
1x 65 multiplets in twisted sector
maybe smaller number than symmetric models

3 generations of (would-be) quarks and leptons
+ vector-like matter fields

SU(2) flavor symmetry
Left-handed quarks (1+2) under SU(2) flavor symmetry,
in the twisted sector
top Yukawa coupling is of O(1).

Other models

We have studied asymmetric Z3 orbifold models
with only right-movers twisted and vanishing Wilson lines.

Similarly, we can construct asymmetric Z3 orbifold models
with left-movers also twisted partly and/or
non-vanishing Wilson lines.

Also we can study other ZN asymmetric orbifold models
in a similar way.

4. Enhancement point and flavor symmetry

Beye, T.K., Kuwakino, 1406.XXXX

Starting point of asymmetric orbifold models is Narain models.

Gauge enhancement point in the moduli space

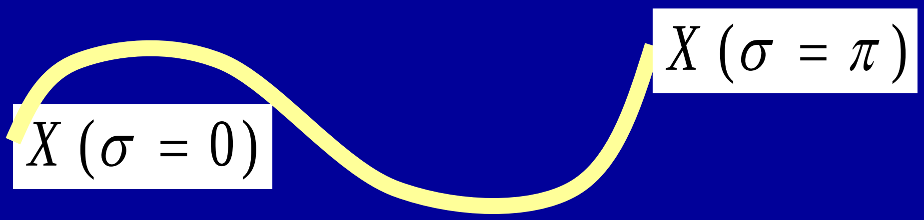
Flavor symmetries: the topic in the last week

Heterotic sting theory on symmetric orbifolds leads to certain non-Abelian discrete flavor symmetries.

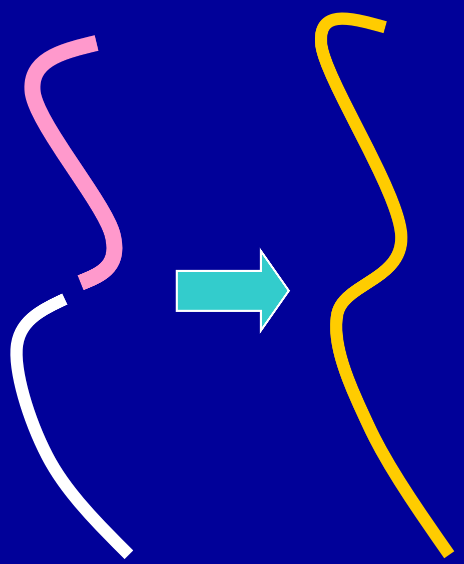
T.K., Nilles, Ploger, Raby, Ratz, '07

We revisit this from the viewpoint of the enhancement of point in the moduli space.

Coupling selection rule and symmetries



A string can be specified by its boundary condition.



Two strings can be connected to become a string if their boundary conditions fit each other.

→ coupling selection rule
symmetry

Heterotic orbifold models

S1/Z2 Orbifold

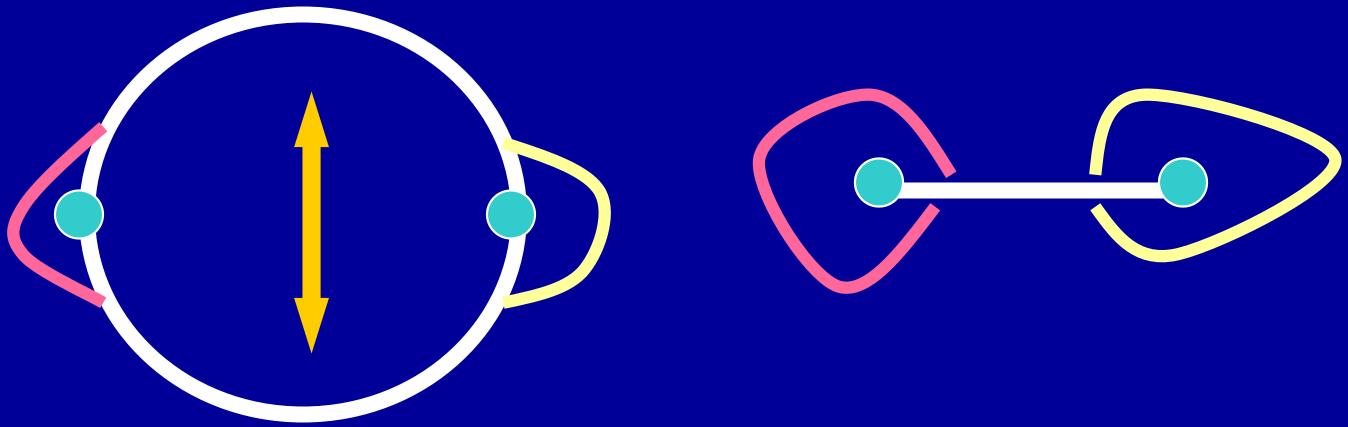
There are two singular points,
which are called fixed points.

$$X \sim -X$$

$$X - e/2 \sim -(X - e/2)$$

Heterotic orbifold models

S1/Z2 Orbifold



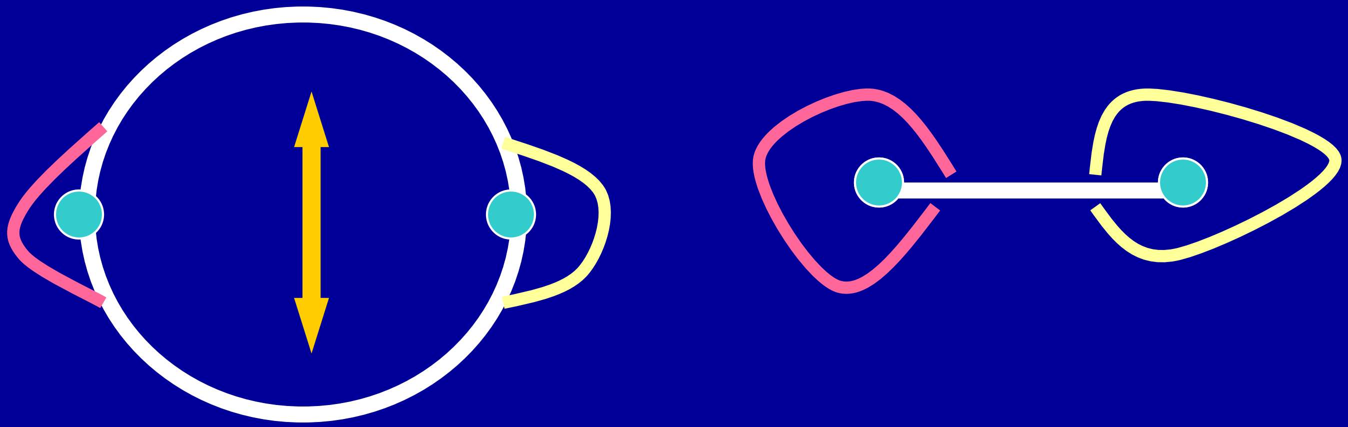
$$X(\sigma=\pi) = -X(\sigma=0)$$

$$X(\sigma=\pi) - e/2 = -(X(\sigma=0) - e/2)$$

$$X(\sigma=\pi) = -X(\sigma=0) + ne,\quad n = 0,1 \ (\mathrm{mod}\ 2)$$

Heterotic orbifold models

S1/Z2 Orbifold



twisted string

$$X(\sigma = \pi) = -X(\sigma = 0) + n\,e, \quad n = 0, 1 \pmod 2$$

untwisted string $\quad X(\sigma = \pi) = X(\sigma = 0)$

$$X(\sigma = \pi) = (-1)^m X(\sigma = 0) + n\,e,$$

$$m, n = 0, 1 \pmod 2$$

Z2 x Z2 in Heterotic orbifold models

S1/Z2 Orbifold

$$X(\sigma = \pi) = (-1)^m X(\sigma = 0) + n\,e,$$

$$m, n = 0, 1 \pmod{2}$$

two Z2's

twisted string

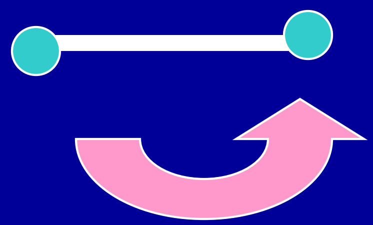
$$\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

untwisted string

Z2 even for both Z2

Geometrical symmetry

S1/Z2 Orbifold



String theory has two Z2's.

In addition, the Z2 orbifold has the geometrical symmetry, i.e. Z2 permutation.

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

Non-Abelian discrete flavor symmetry

The full symmetry includes

$$\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

D4 symmetry

the twisted strings on the two fixed points

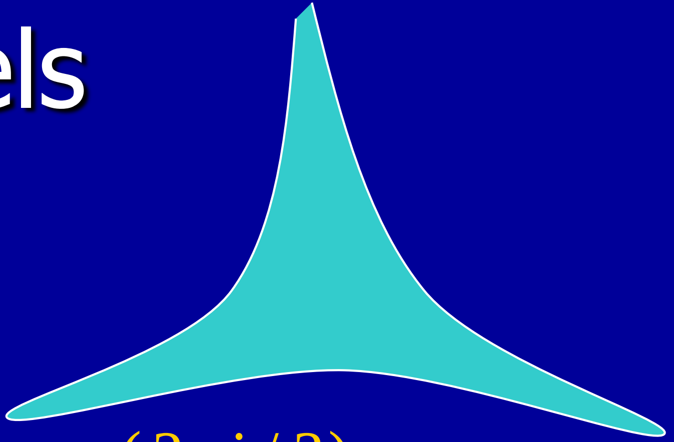
D4 doublet

untwisted strings D4 singlets

T.K., Raby, Zhang, '05

T.K, Nilles, Ploger, Raby,Ratz, '07

Heterotic orbifold models



T2/Z3 Orbifold

two Z3's

$$\begin{pmatrix} \omega & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega \end{pmatrix}, \quad \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{pmatrix}, \quad \omega = \exp(2\pi i/3)$$

Z3 orbifold has the S3 geometrical symmetry,

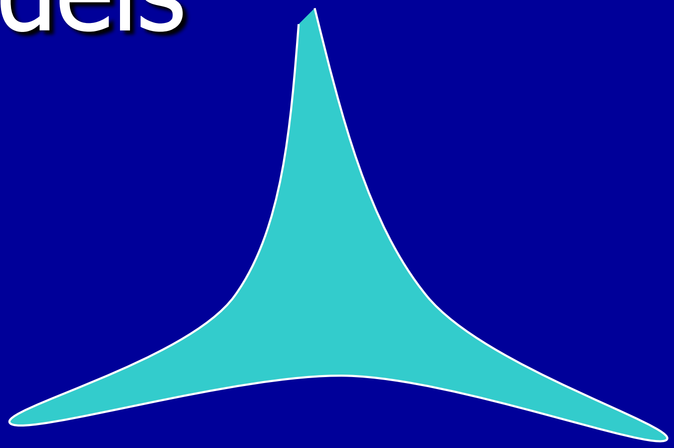
$$\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, \quad \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

Their closed algebra is Delta(54).

T.K., Nilles, Ploger, Raby, Ratz, '07

Heterotic orbifold models

T2/Z3 Orbifold



has Delta(54) symmetry.

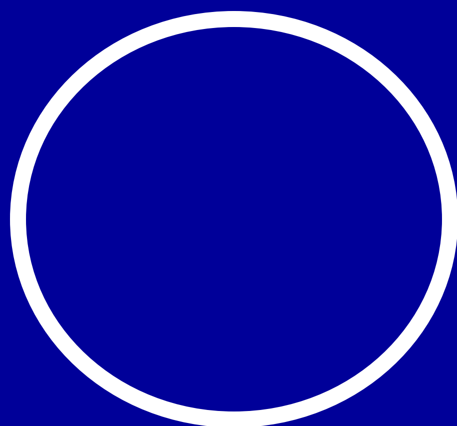
localized modes on three fixed points → Delta(54) triplet

bulk modes → Delta(54) singlet

T.K., Nilles, Ploger, Raby, Ratz, ‘07

S1 at an enhancement point

S1 before orbifolding



U(1) $i\partial X$

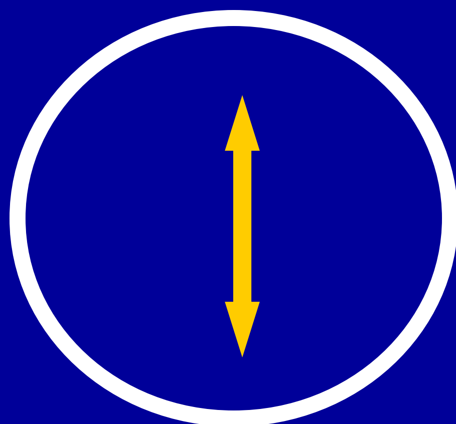
other massless modes at a special point

SU(2) non-zero roots $E_{\pm} = \exp[\pm i\sqrt{2}x]$

Gauge symmetry is enhanced to SU(2).

S1/Z2 at an enhancement point

S1/Z2



U(1) $\quad i\partial X \quad$ not Z2 invarinat

$$i\partial X' = (E_{+} + E_{-})/\sqrt{2}$$ is invariant.

SU(2) is broken to U(1) by orbifolding.

S1/Z2 at an enhancement point

S1/Z2

U(1) theory $i\partial X'$

Z2 odd untwisted matter

$$E_{+}' = i\partial X/\sqrt{2} - (E_{+} - E_{-})/2 \quad : \mathrm{U}_1$$

$$E_{+}' = i\partial X/\sqrt{2} + (E_{+} - E_{-})/2 \quad : \mathrm{U}_2$$

U(1) charges $\pm\sqrt{2}$

two twisted (shifted) matter

U(1) charges $\pm\sqrt{2}/4$

S1/Z2 at an enhancement point

U(1) symmetry

Discrete Z2 permutation symmetry

$$U_+ \leftrightarrow U_- \qquad T_+ \leftrightarrow T_-$$

remnant of SU(2)

S1/Z2

Deformation from the enhancement point by moduli VEV

$$E_+' = i\partial X / \sqrt{2} - (E_+ - E_-)/2 \quad : \mathrm{U}_+$$

$$E_+' = i\partial X / \sqrt{2} + (E_+ - E_-)/2 \quad : \mathrm{U}_-$$

$$< U_+ > = < U_- >$$

U(1) is broken to Z4, and permutation Z2 symmetry reamins.

The full symmetry is D4.

S1/Z2

$$< U_{+} > = < U_{-} >$$

D4

$$< U_{+} > \neq < U_{-} >$$

The other directions of VEVs lead to just Z4.

T2/Z3

Heterotic string on T2 with the enahncemnt point can have SU(3) gauge symmetry.

Orbifolding breaks SU(3) to U(1)xU(1)

and S3.

remnant of SU(3)

T2/Z3

Charged untwisted fields

$$E_1' = i\partial X / \sqrt{2} + \ldots\ldots\ldots\ldots \quad : \mathrm{U}_1 \ (\sqrt{2}, 0)$$

$$E_2' = i\partial X / \sqrt{2} + \ldots\ldots\ldots\ldots \quad : \mathrm{U}_2 \ (-\sqrt{2}/2, \sqrt{6}/2)$$

$$E_3' = i\partial X / \sqrt{2} + \ldots\ldots\ldots\ldots \quad : \mathrm{U}_3 \ (-\sqrt{2}/2, -\sqrt{6}/2)$$

There are three twsited matter fields with charges,

$$\mathrm{T}_1 \ -(\sqrt{2}, 0)/3$$

$$\mathrm{T}_2 \ -(-\sqrt{2}/2, \sqrt{6}/2)/3$$

$$\mathrm{T}_3 \ -(-\sqrt{2}/2, -\sqrt{6}/2)/3$$

T2/Z3

Charged untwisted fields

$$E_1' = i\partial X / \sqrt{2} + \ldots\ldots\ldots\ldots\ldots \quad : \mathrm{U}_1 \ (\sqrt{2}, 0)$$

$$E_2' = i\partial X / \sqrt{2} + \ldots\ldots\ldots\ldots\ldots \quad : \mathrm{U}_2 \ (-\sqrt{2}/2, \sqrt{6}/2)$$

$$E_3' = i\partial X / \sqrt{2} + \ldots\ldots\ldots\ldots\ldots \quad : \mathrm{U}_3 \ (-\sqrt{2}/2, -\sqrt{6}/2)$$

$$< U_1 > = < U_2 > = < U_3 >$$

U(1) x U(1) is broken to Z3xZ3, and permutation S3 symmetry reamins.

The full symmetry is Delta(54).

T2/Z3

$$\langle U_1 \rangle = \langle U_2 \rangle = \langle U_3 \rangle$$

Delta(54)

The other directions of VEVs,

$$\langle U_1 \rangle = 0, \langle U_2 \rangle = \langle U_3 \rangle, \ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

Z3xS3, Z6x(U(1) xS2), Z3xZ3,

-> S1/Z2

Non-Abelian discrete symmetries

We have not used explicitly the geometrical symmetries, but gauge symmetries, although of course gauge symmetries and geometrical symmetries are tightly related.

Our approach can be extended into string models, where geometrical pictures are not so clear such as asymmetric orbifolds, Gepner models, etc.

Applications

We can start e.g.
field-theoretical models with many U(1)s
and their permutation symmetries.
Then, break it by orbifolding.
We would get various
non-Abelian disctete symmetries.

Summary

- We have studied on asymmetric heterotic orbifold models, in particular Z3.
- We have constructed models with MSSM matter contents + vector-like matter as well as SU(2) flavor symmetry models.
- We also discussed the gauge origin of non-Abelian discrete symmetries.

Summary

Perturbative calculations such as higher-order couplings are not clear.

Introduction of WLs is important.

We can extend our analysis to other asymmetric ZN orbifolds.

Further studies: Dilaton stabilizatin, …