Bethe Forum – Bonn

Discrete symmetries and their stringy origin

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Phenomenology of discrete symmetries

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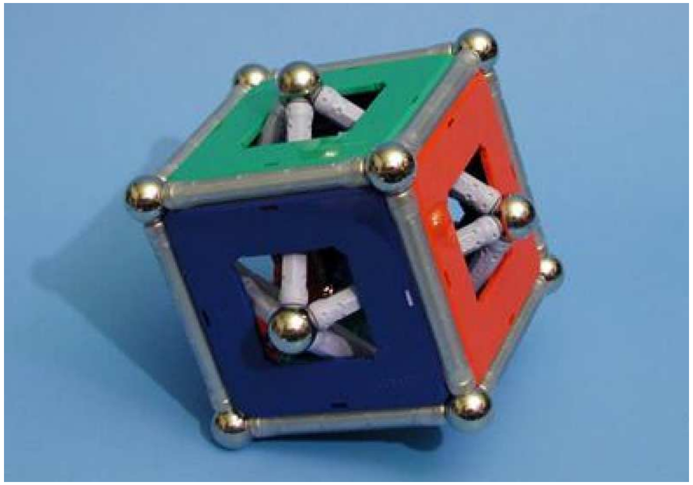
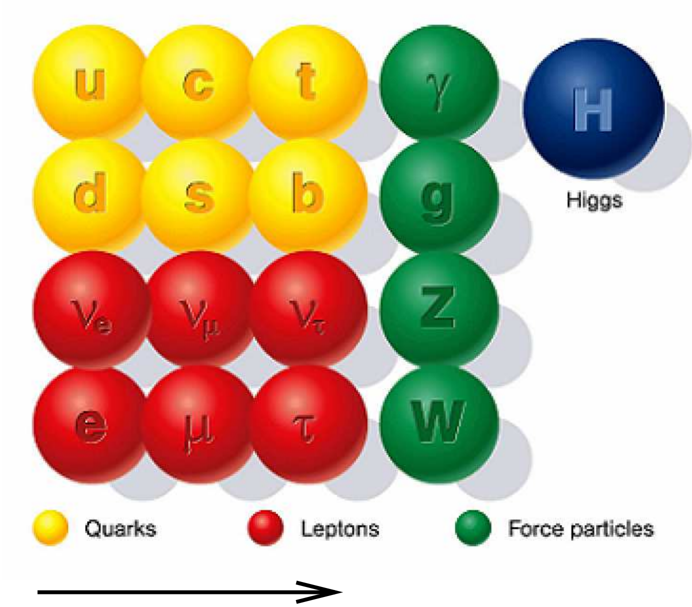


Outline

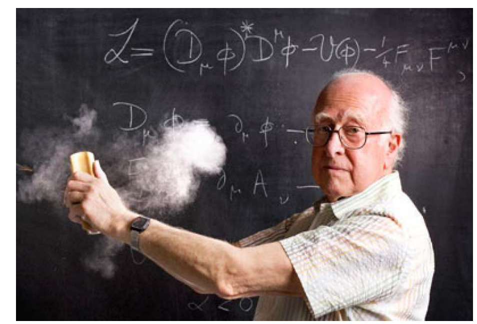
- supersymmetric Standard Model
  - R-parity and matter parity
  - possible $U(1)_{B-L}$ origin
- Abelian family symmetries
  - hierarchies from $U(1)_{\mathrm{FN}}$
  - residual discrete symmetries
- non-Abelian family symmetries
  - simple mixing patterns from non-Abelian symmetries
  - direct and indirect implementation
  - situation after 2012 (Daya Bay & RENO)
  - sum rules for mixing angles
  - benchmark example based on S_4 and CP symmetry

Standard Model (of particle physics)



families or generations

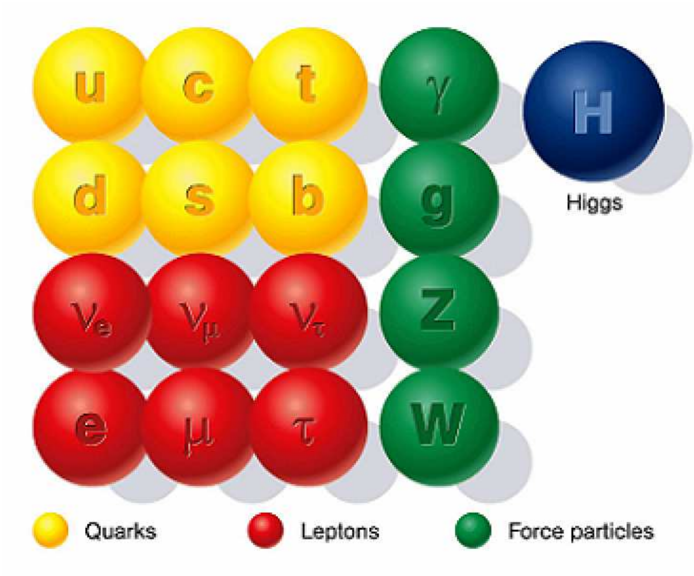
- highly successful theory
- based on gauge symmetry $SU(3)_C \times SU(2)_W \times U(1)_Y$
- broken by Higgs vacuum



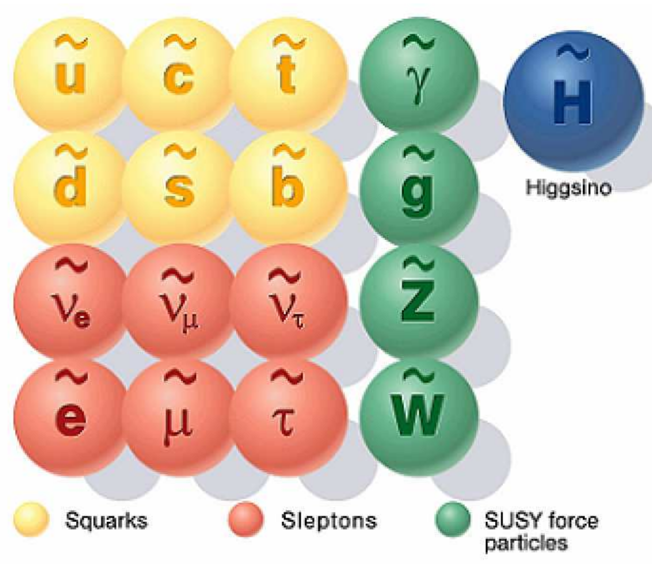
Why look beyond?

- hierarchy problem (protecting the Higgs mass)
- origin of three families of quarks & leptons
- neutrino masses and mixing
- baryogenesis, dark matter, dark energy, etc.

Supersymmetric Standard Model



$\times 2$



$\times 2$

- addresses hierarchy problem
- most general space-time symmetry (extension of Poincaré symmetry)

- supersymmetry must be broken – at TeV scale?
- in total > 300 parameters
- impose extra symmetries, e.g. R-parity $\sim$ matter parity
  $\rightarrow$ MSSM with 124 parameters

R-parity and matter parity

superfield formalism, e.g.

$$\boxed{Q = \widetilde{q} + q\,\theta + F_q\,\theta^2}$$

$q =$ quark

$\widetilde{q} =$ squark

$\theta =$ superspace variable

- R-parity (R_p) defined on component fields, e.g. q and $\widetilde{q}$

R_p	quarks	leptons	Higgs	squarks	sleptons	Higgsino
additive Z_2	0	0	0	1	1	1

- matter parity (M_p) defined on superfields, e.g. Q

M_p	(s)quarks	(s)leptons	Higgs(inos)
additive Z_2	1	1	0

R_p and M_P allow and forbid exactly the same terms in Lagrangian

Matter parity from $U(1)_{B-L}$

matter parity
- forbids (renormalizable) B and L violation
- introduced to stabilize proton
- violated by quantum gravity effects Krauss, Wilczek (1989)
- unless gauge origin, e.g. from breaking $U(1)_{B-L}$

	Q	U^c	D^c	L	E^c	H_u	H_d	ϕ
$U(1)_{B-L}$	$\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	-1	1	0	0	$\pm\frac{2}{3}$
$3 \times U(1)_{B-L}$	1	-1	-1	-3	3	0	0	± 2
M_p	1	1	1	1	1	0	0	0

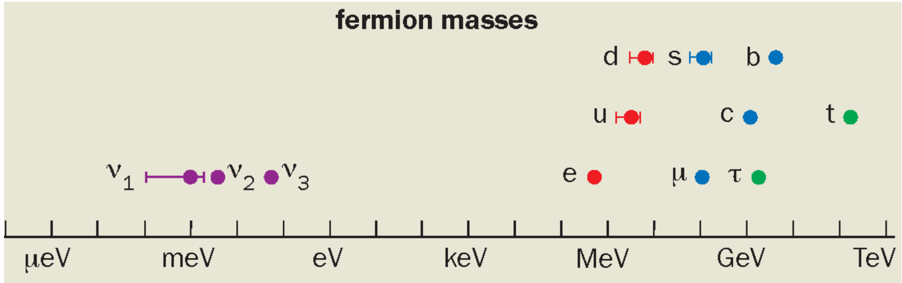
$\rightarrow$ vev of field ϕ breaks $U(1)_{B-L}$ spontaneously down to M_p

"Higgs"-type field
neutral under
SM gauge group

$U(1)_{\mathrm{FN}}$ family symmetry

Froggatt, Nielsen (1979)

Fermion masses



- fermion mass terms: $M_{ij}\,\overline{\psi}_i\,\psi_j$
- masses = eigenvalues of M

$$m_u : m_c : m_t \sim \lambda^8 : \lambda^4 : 1$$

$$m_d : m_s : m_b \sim \lambda^4 : \lambda^2 : 1$$

$$m_e : m_\mu : m_\tau \sim \lambda^{4\,\mathrm{or}\,5} : \lambda^2 : 1 \qquad (\lambda \sim 0.22)$$

- neutrinos massless in Standard Model
- observation of neutrino oscillation $\rightarrow\ m_\nu \neq 0$
- three scenarios:

$$m_{\nu_1} : m_{\nu_2} : m_{\nu_3} \sim \begin{cases} \lambda^x : \lambda : 1 & \text{(normal hierarchy)} \\ 1 : 1 : \lambda^x & \text{(inverted hierarchy)} \\ 1 : 1 : 1 & \text{(quasi degenerate)} \end{cases}$$

Froggatt-Nielsen mechanism

- Yukawa terms $\boxed{Y_{ij}\,\overline{\psi}_i\,\psi_j H}$ forbidden by $U(1)_{\mathrm{FN}}$
- family dependent charges
- introduce flavon field ϕ which allows effective terms $\boxed{c_{ij}\,\overline{\psi}_i\,\psi_j H\left(\frac{\phi}{\Lambda}\right)^{x_{ij}}}$
- flavon ϕ acquires a VEV $\rightarrow\ Y_{ij} = c_{ij}\left(\frac{\langle\phi\rangle}{\Lambda}\right)^{x_{ij}}$

fields	Q_1	Q_2	Q_3	D_1^c	D_2^c	D_3^c	H_d	ϕ
$U(1)_{\mathrm{FN}}$	6	4	0	5	3	3	-3	-2

$$\frac{\langle\phi\rangle}{\Lambda} \sim \lambda$$

$$c_{ij}\,Q_i\,D_j^c\,H_d\left(\frac{\phi}{\Lambda}\right)^{x_{ij}} \rightarrow \quad c_{ij}\,Q_i\,D_j^c\,H_d\,\lambda^{x_{ij}} \rightarrow \quad Y_d \sim \begin{pmatrix} \lambda^4 & \lambda^3 & \lambda^3 \\ \lambda^3 & \lambda^2 & \lambda^2 \\ \lambda & 1 & 1 \end{pmatrix}$$

- hierarchies arise from spontaneous breakdown of $U(1)_{\mathrm{FN}}$
- often residual discrete Z_N symmetry, e.g. matter parity
- $\mathcal{O}(1)$ coefficients c_{ij} not fixed

Scale of family symmetry breaking

- models of flavor are typically formulated at high energies
- separate family symmetry from EW symmetry breaking

M_{Pl}

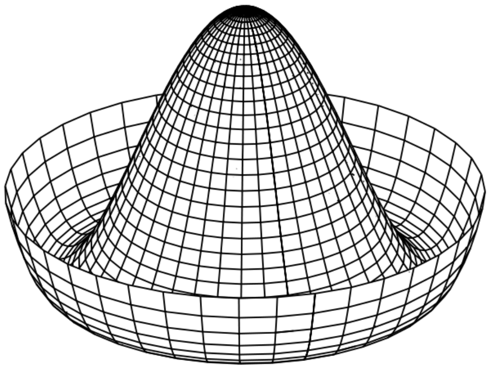
flavon VEV $\langle \phi \rangle$ breaks family symmetry

M_{GUT} $\rightarrow$ Yukawas Y_{ij}

M_{seesaw}

Higgs VEV $\langle H \rangle$ breaks electroweak symmetry

M_{EW} $\rightarrow$ fermion masses $M_{ij} = Y_{ij} \langle H \rangle$



Discrete symmetries from $U(1)_{\mathrm{FN}}$

- simple conditions to obtain a particular Z_N symmetry from $U(1)_{\mathrm{FN}}$
- in $U(1)_{\mathrm{FN}}$ charge normalization with $X_\phi = -1$

	$X_{H_d} - X_{L_1}$	$3X_{Q_1} + X_{L_1}$
M_p	integer $- \frac{1}{2}$	integer
P_6	integer $- \frac{1}{2}$	integer $\pm \frac{1}{3}$
B_3	integer	integer $\pm \frac{1}{3}$

Dreiner, Luhn, Murayama, Thormeier (2006)

M_p lightest SUSY particle (LSP) is stable $\rightarrow$ dark matter candidate
allows for non-renormalizable operator $QQQL$ $\rightarrow$ proton decay

P_6 LSP is stable $\rightarrow$ dark matter candidate
forbids $QQQL$ $\rightarrow$ proton stabilized

B_3 LSP not stable $\rightarrow$ less missing E_T
forbids $QQQL$ $\rightarrow$ proton stabilized
renormalizable L violation $\rightarrow$ neutrino masses without seesaw

Discrete Z_4^R symmetry from $U(1)^R_{\text{FN}}$

- superspace variable θ charged under R-symmetries

	Q	U^c	D^c	L	E^c	H_u	H_d	θ
Z_4^R	1	1	1	1	1	0	0	1

Lee, Raby, Ratz, Ross et al. (2010)

- solves the μ-problem by forbidding $\mu H_u H_d$
- forbids $QQQL$

- Z_4^R can be obtained from $U(1)^R_{\text{FN}}$ if

$X_{H_d} - X_{L_1}$	$3X_{Q_1} + X_{L_1}$	X_θ	X_ϕ
integer $\pm \frac{1}{4}$	integer	integer $\pm \frac{1}{4}$	-1

Dreiner, Luhn, Opferkuch (2013)

Non-Abelian family symmetries

Fermion mixings

- mismatch of flavor (weak) and mass eigenstates

$$\Psi_{\text{flavor}} \;=\; V^\dagger\,\Psi_{\text{mass}}$$

- quark sector: V_L^u and V_L^d

$$U_{\text{CKM}} \;=\; V_L^u\,V_L^{d\,\dagger} \;\approx\; \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix} \qquad \lambda \sim 0.22$$

- lepton sector: V_L^e and V_L^ν

$$U_{\text{PMNS}} \;=\; V_L^e\,V_L^{\nu\,\dagger} \;\approx\; \begin{pmatrix} 0.82 & 0.55 & 0.15 \\ 0.37 & 0.57 & 0.70 \\ 0.39 & 0.59 & 0.68 \end{pmatrix}$$

www.nu-fit.org (2013)

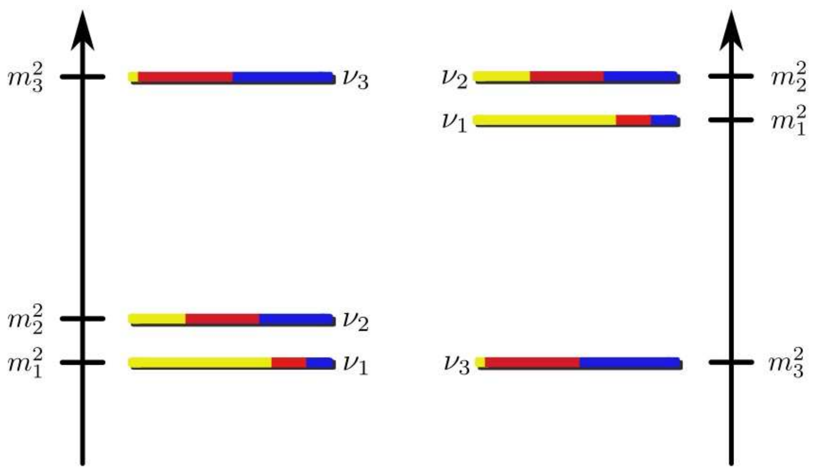
mixing $\Longleftrightarrow$ each family knows of the existence of the others!

Three neutrino flavor mixing

(in diagonal charged lepton basis)

flavor — PMNS mixing — mass

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$



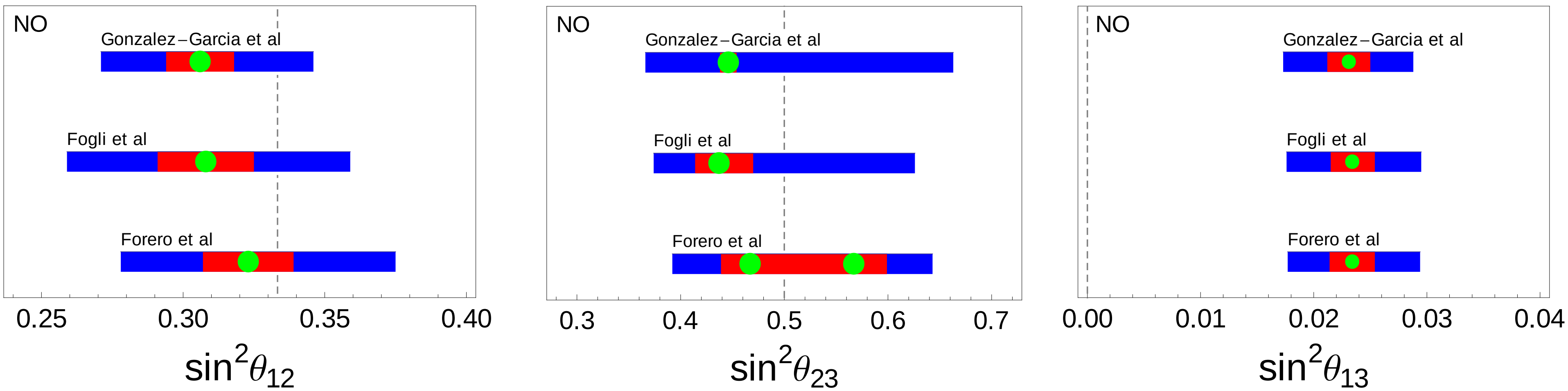
atmospheric — reactor + Dirac — solar — Majorana

$$U_{\text{PMNS}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\frac{\alpha_2}{2}} & 0 \\ 0 & 0 & e^{i\frac{\alpha_3}{2}} \end{pmatrix}$$

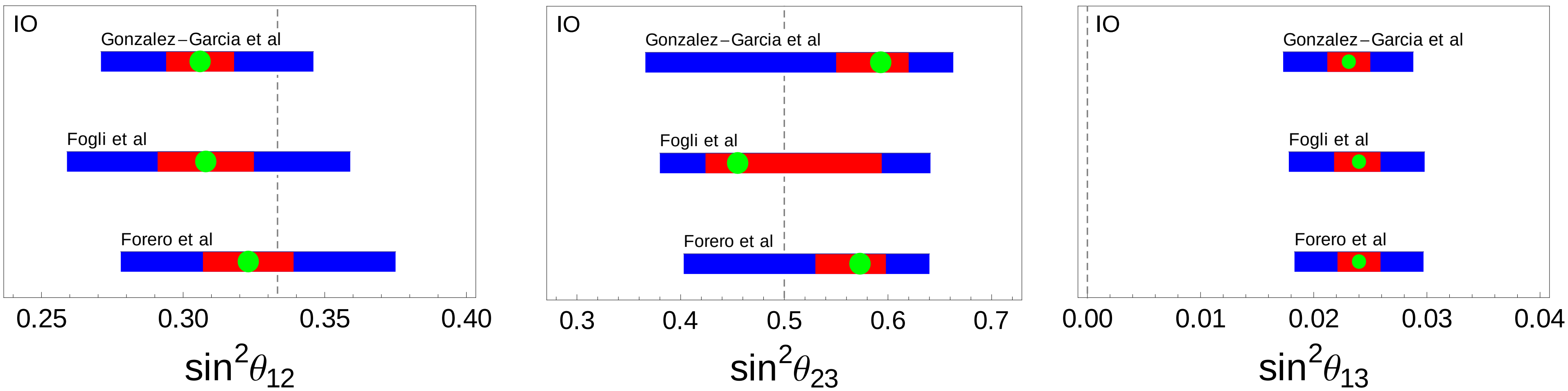
$\theta_{23} \approx 45°$ — $\theta_{13} \approx 9°$ — $\theta_{12} \approx 33°$

Global neutrino fits

normal mass ordering



inverted mass ordering



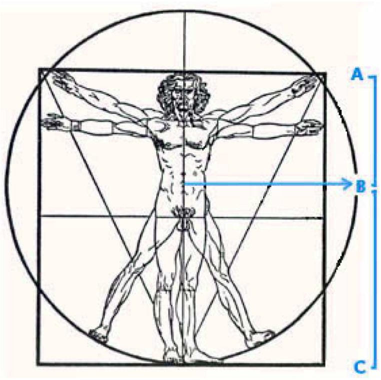
Simple mixing patterns – tri-bimaximal

Harrison Perkins Scott

$$U_{\rm PMNS} \approx U_{\rm TB} \equiv \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$$

$$\theta_{12} \approx 35.3^\circ \qquad \theta_{23} = 45^\circ \qquad \theta_{13} = 0^\circ$$

Simple mixing patterns – golden ratio



$$\varphi = \frac{1+\sqrt{5}}{2} \approx 1.618$$

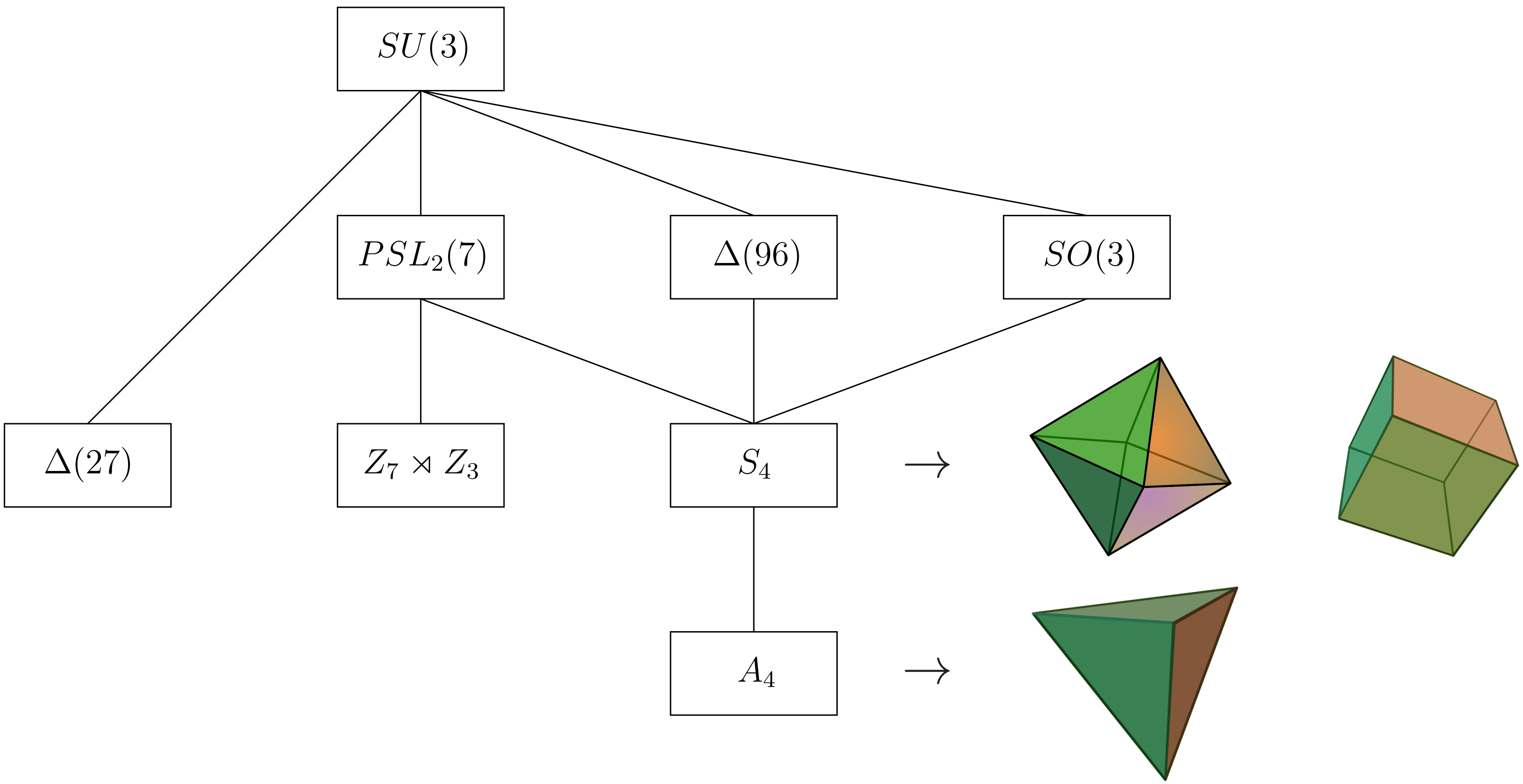
$$\tan\theta_{12} = \frac{1}{\varphi}$$

$$U_{\rm PMNS} \quad \approx \quad U_{\rm GR} \quad \equiv \quad \begin{pmatrix} \cos\theta_{12} & \sin\theta_{12} & 0 \\ -\frac{\sin\theta_{12}}{\sqrt{2}} & \frac{\cos\theta_{12}}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{\sin\theta_{12}}{\sqrt{2}} & \frac{\cos\theta_{12}}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$$

$$\theta_{12} \approx 31.7^\circ \qquad \theta_{23} = 45^\circ \qquad \theta_{13} = 0^\circ$$

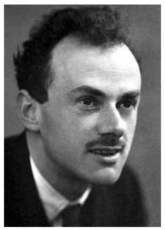
Candidates

- unify three families in multiplets of family symmetry
- underlying group should have two- or three-dimensional representations



Symmetries of the mass matrices (in flavor basis)

charged leptons



Dirac

$$M_\ell = \mathrm{diag}\,(m_e, m_\mu, m_\tau)$$

symmetric under diagonal phase transformation h

$$\boxed{M_\ell = h^T M_\ell\, h^*}$$

e.g. $h = \mathrm{diag}\,(1, e^{\frac{4\pi i}{3}}, e^{\frac{2\pi i}{3}})$

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neutrinos

Majorana

$$M_\nu = U_{\mathrm{PMNS}}\, \mathrm{diag}\,(m_{\nu_1}, m_{\nu_2}, m_{\nu_3})\, U^T_{\mathrm{PMNS}}$$

symmetry of M_ν depends on U_{PMNS}

$$\boxed{M_\nu = k^T M_\nu\, k}$$

$k = U^*_{\mathrm{PMNS}}\, \mathrm{diag}\,(+1, -1, -1)\, U^T_{\mathrm{PMNS}}$

four different k $\rightarrow$ generate $Z_2 \times Z_2$ symmetry group

Klein symmetry: $\mathcal{K} = \{1, k_1, k_2, k_3\}$

for $U_{\mathrm{PMNS}} = U_{\mathrm{TB}}$:

$$k_1 = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}, \quad k_2 = -\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad k_3 = k_1 k_2$$

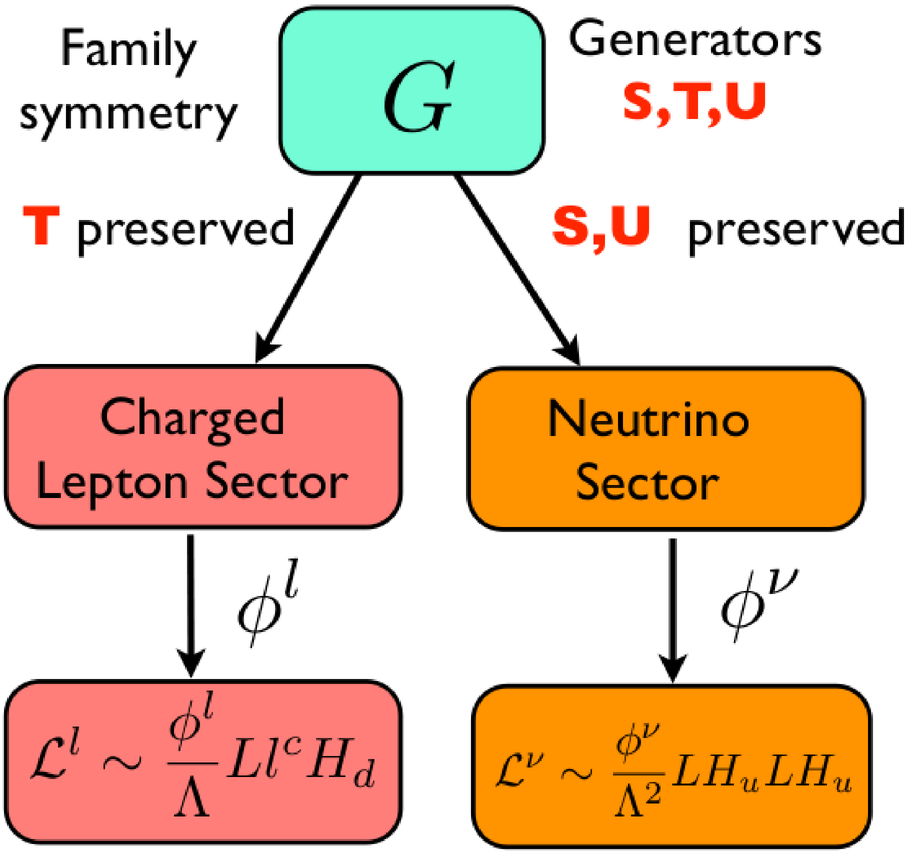
Origin of the Klein symmetry $\mathcal{K}$

- direct models
  - $\boxed{\text{Klein symmetry } \mathcal{K} \;\subset\; \text{family symmetry } \mathcal{G}}$
  - flavons ϕ are multiplets of $\mathcal{G}$
  - their VEVs $\langle\phi\rangle$ break $\mathcal{G}$ down to $\mathcal{K}$ in neutrino sector
  - for TB mixing (k_1, k_2, h) generate permutation group S_4
- indirect models
  - $\boxed{\text{Klein symmetry } \mathcal{K} \;\not\subset\; \text{family symmetry } \mathcal{G}}$
  - $\mathcal{G}$ responsible for generating particular flavon VEV configurations $\langle\phi\rangle$
  - for TB mixing – from e.g. $\Delta(27)$, $Z_7 \rtimes Z_3$

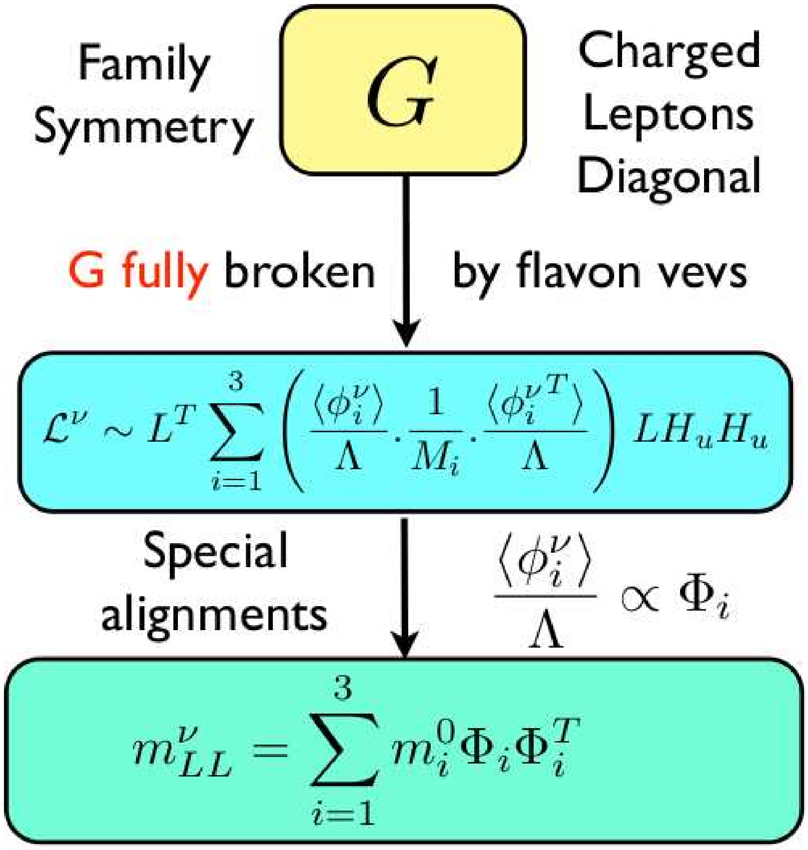
$$\langle\phi_1\rangle \propto \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix} \qquad \langle\phi_2\rangle \propto \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \qquad \langle\phi_3\rangle \propto \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$$

$$\Rightarrow \quad \mathcal{L}_\nu \quad \sim \quad \nu\,(\phi_1\phi_1^T + \phi_2\phi_2^T + \phi_3\phi_3^T)\,\nu\; H\,H$$

Two options



(direct models)



(indirect models)

Building a direct model with tri-bimaximal mixing

- choose family symmetry group – S_4
- identify suitable flavon VEV configurations

$$S\langle\phi^\nu\rangle = U\langle\phi^\nu\rangle = \langle\phi^\nu\rangle \qquad T\langle\phi^\ell\rangle = \langle\phi^\ell\rangle$$

S_4	S	U	T	$\langle\phi^\nu\rangle$	$\langle\phi^\ell\rangle$
$\mathbf{1}, \mathbf{1}'$	1	± 1	1	$\mathbf{1}$	$\mathbf{1}, \mathbf{1}'$
$\mathbf{2}$	$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	$\begin{pmatrix} \omega & 0 \\ 0 & \omega^2 \end{pmatrix}$	$\mathbf{2} \propto \begin{pmatrix} 1 \\ 1 \end{pmatrix}$	$-$
$\mathbf{3}, \mathbf{3}'$	$\frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}$	$\mp \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix}$	$\mathbf{3}' \propto \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$	$\mathbf{3}, \mathbf{3}' \propto \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

- control coupling of flavons to fermions by extra Z_N or $U(1)$ symmetry

$$\frac{\phi^\nu}{\Lambda^2} L H_u L H_u \qquad \frac{\phi^\ell}{\Lambda} L\, \ell^c H_d$$

- with type I seesaw

$$\mathcal{L}_\nu \;\sim\; L\, H_u\, \nu^c \;+\; \phi^\nu \nu^c \nu^c$$

Building an indirect model with tri-bimaximal mixing

- family symmetry $\mathcal{G} \subset SU(3)$
- diagonal charged leptons
- type I seesaw with 2 or 3 ν^c_a in singlet representation of $\mathcal{G}$
- diagonal right-handed neutrino mass matrix (e.g. due to Z_2 symmetry)

$$\mathcal{L}_\nu \sim \sum_a \frac{\phi^\nu_a}{\Lambda} L\, H_u\, \nu^c_a + M_a \nu^c_a \nu^c_a$$

- $\phi^\nu_a \sim \overline{\mathbf{3}}$ and $L \sim \mathbf{3}$ of $\mathcal{G}$
- $\mathcal{G}$ or $SU(3)$ invariant $\rightarrow \phi^\nu_{a\,1} L_1 + \phi^\nu_{a\,2} L_2 + \phi^\nu_{a\,3} L_3 = {\phi^\nu_a}^T L$
- integrate out ν^c_a (seesaw formula)

$$\mathcal{L}_\nu \sim L^T \sum_{a=1} \left(\frac{\langle \phi^\nu_a \rangle}{\Lambda} \cdot \frac{1}{M_a} \cdot \frac{\langle \phi^\nu_a \rangle^T}{\Lambda} \right) L H_u H_u$$

- tri-bimaximal if $\left[\langle \phi^\nu_1 \rangle \propto \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix} \right] \quad \langle \phi^\nu_2 \rangle \propto \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \quad \langle \phi^\nu_3 \rangle \propto \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$

Aligning triplet flavons in $\Delta(27)\,,\ Z_7 \rtimes Z_3\,,\ A_4$

$$\boxed{V(\phi) \;=\; -\,m^2 \textstyle\sum_i {\phi_i}^\dagger \phi_i \;+\; \lambda \left(\sum_i {\phi_i}^\dagger \phi_i\right)^2 \;+\; \Delta V}$$

central terms in ΔV

$(i)\quad \kappa \sum_i \phi_i^\dagger \phi_i \phi_i^\dagger \phi_i \qquad \kappa > 0 \;\rightarrow\; \langle \phi \rangle \;\propto\; \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

$\qquad \kappa < 0 \;\rightarrow\; \langle \phi \rangle \;\propto\; \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

$(ii)\quad \tilde{\kappa} \sum_{i,j} (\phi_i^\dagger \tilde{\phi}_i)(\tilde{\phi}_j^\dagger \phi_j) \qquad \tilde{\kappa} > 0 \;\rightarrow\;$ orthogonality condition $\langle \phi \rangle \perp \langle \tilde{\phi} \rangle$

e.g. $\langle \tilde{\phi} \rangle \;\propto\; \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$

... $\langle \widetilde{\phi} \rangle \;\propto\; \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix}$

Flavon alignment in supersymmetry

- SUSY unbroken at scale of family symmetry breaking
- introduce so-called driving fields X which couple to flavons
- flavon superpotential W^{flavon} linear in X due to $U(1)_R$ symmetry
- F-terms of driving fields need to vanish

$$\boxed{F^*_{X_i} = -\frac{\partial W^{\text{flavon}}}{\partial X_i} = 0}$$

- two examples in S_4

$$W^{\text{flavon}} \sim X_{\mathbf{1}}\phi_{\mathbf{2}}\phi_{\mathbf{2}} = X_{\mathbf{1}}(\phi_{\mathbf{2},1}\phi_{\mathbf{2},2} + \phi_{\mathbf{2},2}\phi_{\mathbf{2},1}) = 2X_{\mathbf{1}}\phi_{\mathbf{2},1}\phi_{\mathbf{2},2}$$

$$\longrightarrow \langle\phi_{\mathbf{2}}\rangle \propto \begin{pmatrix}1\\0\end{pmatrix} \quad \text{or} \quad \langle\phi_{\mathbf{2}}\rangle \propto \begin{pmatrix}0\\1\end{pmatrix}$$

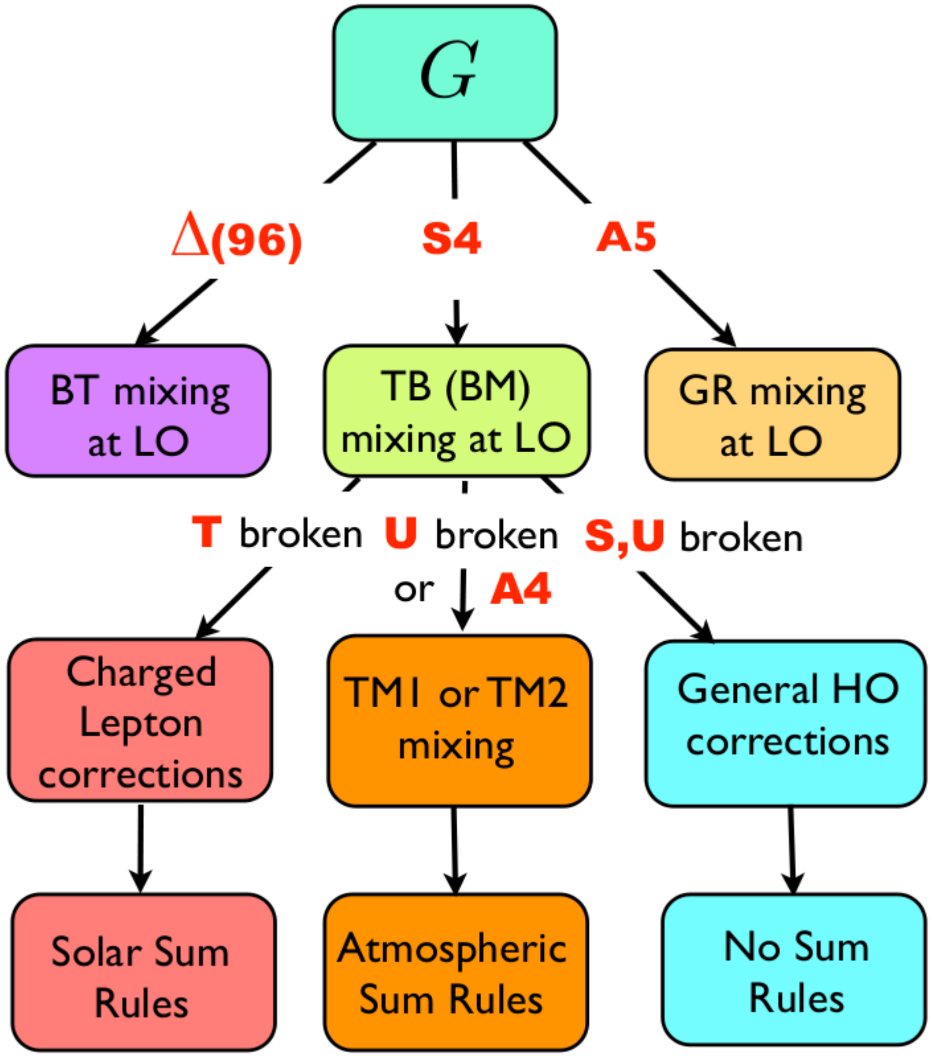
$$W^{\text{flavon}} = g_0 X_{\mathbf{3}}\phi_{\mathbf{3'}}\phi_{\mathbf{2}} + X_{\mathbf{3'}}\left(g_1\phi_{\mathbf{3'}}\phi_{\mathbf{3'}} + g_2\phi_{\mathbf{3'}}\phi_{\mathbf{2}} + g_3\phi_{\mathbf{3'}}\phi_{\mathbf{1}}\right)$$

$$\longrightarrow \langle\phi_{\mathbf{3'}}\rangle = \varphi_{\mathbf{3'}}\begin{pmatrix}1\\1\\1\end{pmatrix} \qquad \langle\phi_{\mathbf{2}}\rangle = \varphi_{\mathbf{2}}\begin{pmatrix}1\\1\end{pmatrix} \qquad \varphi_{\mathbf{2}} = -\tfrac{g_3}{2g_2}\varphi_{\mathbf{1}}$$

- flavon alignments independent of g_i

How to accommodate $\theta_{13} \sim 9^\circ$

Direct models after 2012



mixing patterns:

	θ_{13}	θ_{23}	θ_{12}
TB	0°	45°	35.3°
BM	0°	45°	45°
GR	0°	45°	31.7°
BT	12.2°	36.2°	36.2°
TM	$\neq 0^\circ$	$\neq 45^\circ$	35.3°

TB = tri-bimaximal
BM = bimaximal
GR = golden ratio
BT = bi-trimaximal
TM = trimaximal

Solar mixing sum rule

- T symmetry of charged lepton sector "slightly" broken (e.g. GUTs)
- $U_{\text{PMNS}} = V_{\ell_L} V_{\nu_L}^\dagger \quad \text{and} \quad V_{\nu_L}^\dagger = U_{\text{TB}}$

$$U_{\text{PMNS}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & \hat{s}_{23} \\ 0 & -\hat{s}_{23}^* & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & \hat{s}_{13} \\ 0 & 1 & 0 \\ -\hat{s}_{13}^* & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & \hat{s}_{12} & 0 \\ -\hat{s}_{12}^* & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$s_{12}\, e^{i\delta_{12}} \approx \frac{1}{\sqrt{3}} \left(e^{i\delta_{12}^\nu} - \theta_{12}^\ell\, e^{i\delta_{12}^\ell} + \theta_{13}^\ell\, e^{i(\delta_{13}^\ell - \delta_{23}^\nu)} \right)$$

$$s_{23}\, e^{i\delta_{23}} \approx \frac{1}{\sqrt{2}} \left(e^{i\delta_{23}^\nu} - \theta_{23}^\ell\, e^{i\delta_{23}^\ell} \right)$$

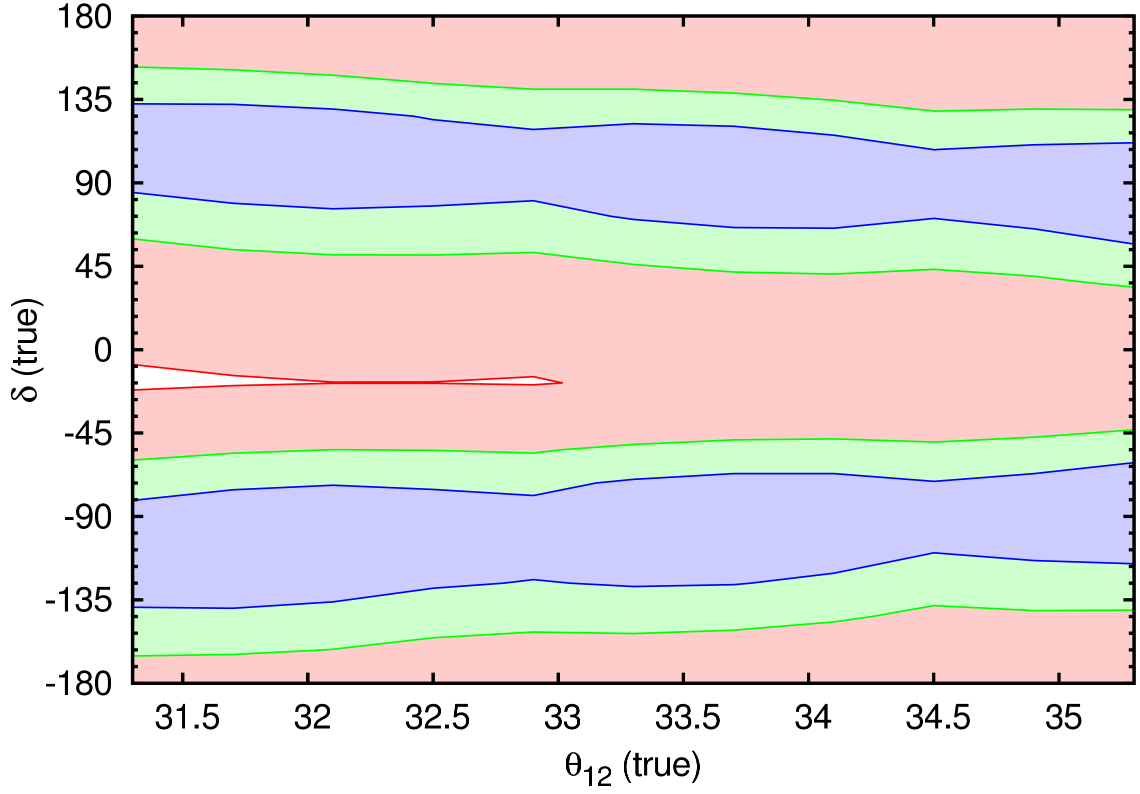
$$s_{13}\, e^{i\delta_{13}} \approx \frac{1}{\sqrt{2}} \left(-\theta_{12}^\ell\, e^{i(\delta_{12}^\ell + \delta_{23}^\nu)} - \theta_{13}^\ell\, e^{i\delta_{13}^\ell} \right)$$

$$c_{ij} = \cos\theta_{ij}$$
$$\hat{s}_{ij} = \sin\theta_{ij}\, e^{-i\delta_{ij}}$$

- $\theta_{12}^\ell \sim \theta_C \sim 0.22 \quad \rightarrow \quad \theta_{13} \sim 9^\circ$
- first order relation $\quad \boxed{\theta_{12} \approx 35.3^\circ + \theta_{13} \cos\delta}$

Testing the solar sum rule

- JUNO will measure θ_{12} with high precision
- wide-band superbeam (LBNO/LBNE/LBNF) could access Dirac phase δ
- expected sensitivity for ruling out solar sum rule



Ballett et al.
(in preparation)

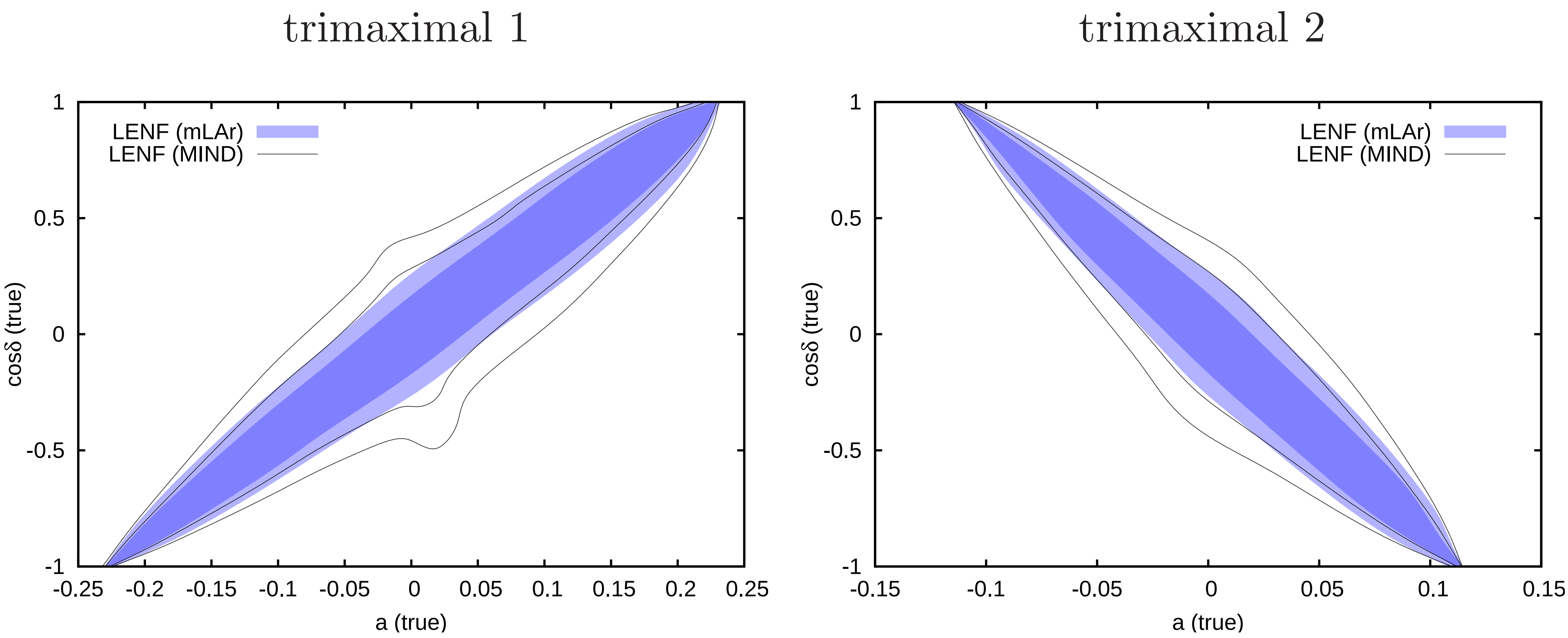
Atmospheric mixing sum rule

- U symmetry of neutrino sector "slightly" broken $\rightarrow$ $U_{\text{PMNS}}^{13} \neq 0$
- conserve one Z_2 symmetry of Klein symmetry $Z_2^S \times Z_2^U$

	trimaximal 1 (TM_1)	trimaximal 2 (TM_2)
unbroken Z_2	$SU = -\frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & 2 & -1 \\ 2 & -1 & 2 \end{pmatrix}$	$S = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}$
PMNS mixing	$\frac{1}{\sqrt{6}}\begin{pmatrix} 2 & \cdot & \cdot \\ -1 & \cdot & \cdot \\ -1 & \cdot & \cdot \end{pmatrix}$	$\frac{1}{\sqrt{3}}\begin{pmatrix} \cdot & 1 & \cdot \\ \cdot & 1 & \cdot \\ \cdot & 1 & \cdot \end{pmatrix}$
solar angle	$\theta_{12} \approx 34.2^\circ$	$\theta_{12} \approx 35.8^\circ$
first order relation	$\boxed{\theta_{23} \approx 45^\circ + \sqrt{2}\,\theta_{13}\cos\delta}$	$\boxed{\theta_{23} \approx 45^\circ - \frac{1}{\sqrt{2}}\,\theta_{13}\cos\delta}$

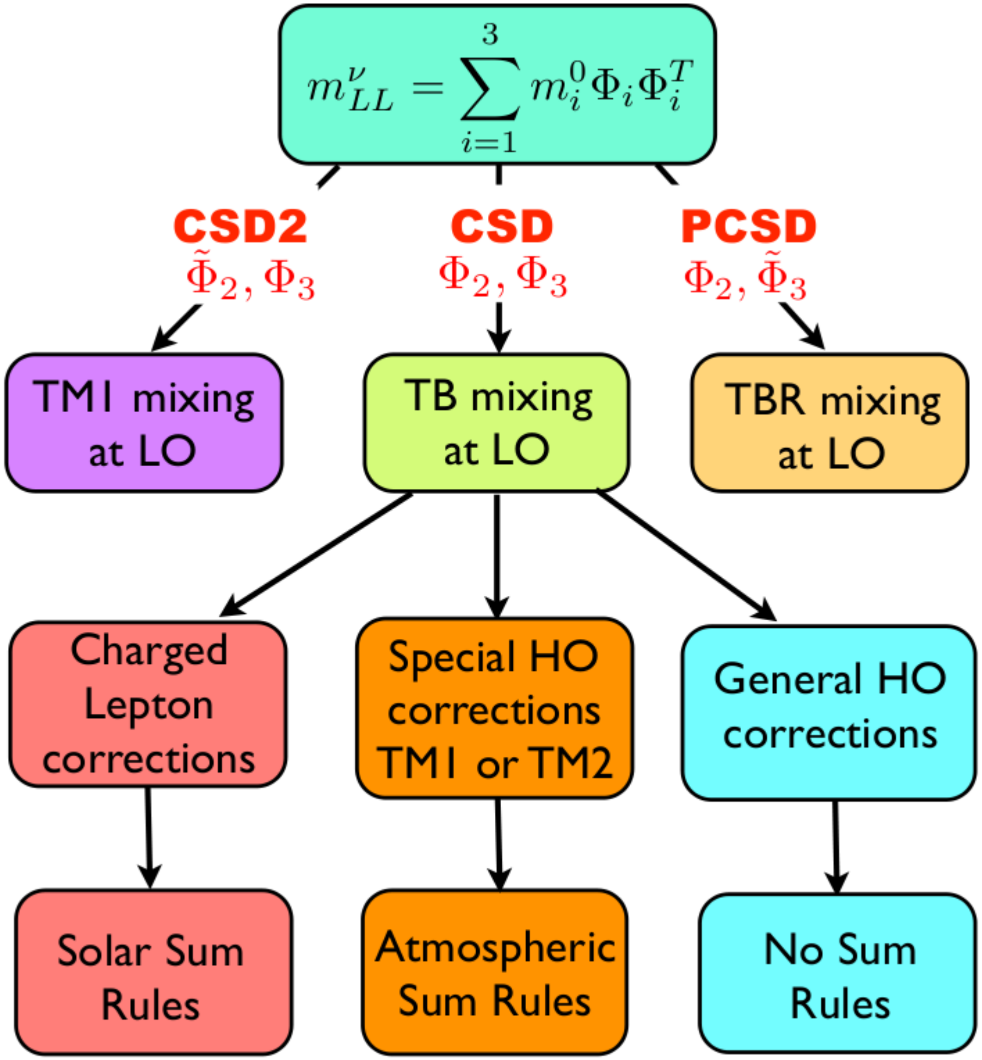
Testing the atmospheric sum rule

- low energy neutrino factory could measure θ_{23} and δ to high precision
- expected sensitivity for ruling out atmospheric sum rule



Ballett et al. (2013)

Indirect models after 2012



flavon alignments:

	$\langle\Phi_2\rangle$	$\langle\Phi_3\rangle$
CSD	$\frac{1}{\sqrt{3}}\begin{pmatrix}1\\1\\1\end{pmatrix}$	$\frac{1}{\sqrt{2}}\begin{pmatrix}0\\1\\-1\end{pmatrix}$
CSD2	$\frac{1}{\sqrt{5}}\begin{pmatrix}1\\2\\0\end{pmatrix}$	$\frac{1}{\sqrt{2}}\begin{pmatrix}0\\1\\-1\end{pmatrix}$
PCSD	$\frac{1}{\sqrt{3}}\begin{pmatrix}1\\1\\1\end{pmatrix}$	$\frac{1}{\sqrt{2}}\begin{pmatrix}\epsilon\\1\\-1\end{pmatrix}$

Variations of constrained sequential dominance (CSD)

$$\boxed{m_\nu = m_2^0\, \Phi_2 \Phi_2^T + m_3^0\, \Phi_3 \Phi_3^T} \qquad m_2^0 \ll m_3^0$$

- CSD

tri-bimaximal

$$\frac{m_2^0}{3}\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \frac{m_3^0}{2}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{pmatrix} \Longrightarrow \boxed{\begin{array}{ll} \theta_{13} = 0 & m_1^\nu = 0 \\ \theta_{23} = 45^\circ & m_2^\nu = m_2^0 \\ \theta_{12} = 35.3^\circ & m_3^\nu = m_3^0 \end{array}}$$

- CSD2

trimaximal 1 (TM_1)

$$\frac{m_2^0}{5}\begin{pmatrix} 1 & 2 & 0 \\ 2 & 4 & 0 \\ 0 & 0 & 0 \end{pmatrix} + \frac{m_3^0}{2}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{pmatrix} \qquad \boxed{\begin{array}{ll} \theta_{13} \approx \frac{\sqrt{2}}{3}\frac{m_2^\nu}{m_3^\nu} & m_1^\nu = 0 \\ \theta_{23} \approx 45^\circ + \sqrt{2}\theta_{13}\cos\delta & m_2^\nu \approx \frac{3}{5} m_2^0 \\ \theta_{12} \approx 35.3^\circ & m_3^\nu \approx m_3^0 \end{array}}$$

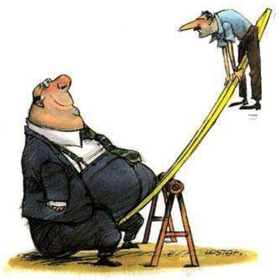
- PCSD (partially CSD)

tri-bimaximal-reactor

$$\frac{m_2^0}{3}\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \frac{m_3^0}{2}\begin{pmatrix} \epsilon^2 & \epsilon & -\epsilon \\ \epsilon & 1 & -1 \\ -\epsilon & -1 & 1 \end{pmatrix} \Longrightarrow \boxed{\begin{array}{ll} \theta_{13} \approx \frac{\epsilon}{\sqrt{2}} & m_1^\nu = 0 \\ \theta_{23} \approx 45^\circ & m_2^\nu \approx m_2^0 \\ \theta_{12} \approx 35.3^\circ & m_3^\nu \approx m_3^0 \end{array}}$$

An S_4 benchmark model

Direct model of leptons based on S_4



- discrete family symmetry with 24 elements
- irreducible S_4 representations: $\mathbf{1} \quad \mathbf{1'} \quad \mathbf{2} \quad \mathbf{3} \quad \mathbf{3'}$
- diagonal charged leptons (enforced by T symmetry)
- in neutrino sector $\quad (L \sim N^c \sim \mathbf{3} \qquad H_u \sim \mathbf{1})$

$$W_\nu \;\sim\; y_D L N^c H_u + (y_{\mathbf{3'}}\phi_{\mathbf{3'}} + y_{\mathbf{2}}\phi_{\mathbf{2}} + y_{\mathbf{1}}\phi_{\mathbf{1}}) N^c N^c + \frac{y_{\mathbf{1'}}}{M}\widetilde{\phi}_{\mathbf{1'}}\phi_{\mathbf{2}} N^c N^c$$

S_4 irrep	S	U	VEV alignment
$\mathbf{3'}$	$\frac{1}{3}\begin{pmatrix}-1 & 2 & 2\\ 2 & -1 & 2\\ 2 & 2 & -1\end{pmatrix}$	$\begin{pmatrix}1 & 0 & 0\\ 0 & 0 & 1\\ 0 & 1 & 0\end{pmatrix}$	$\langle\phi_{\mathbf{3'}}\rangle \propto \begin{pmatrix}1\\1\\1\end{pmatrix}$
$\mathbf{2}$	$\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$	$\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}$	$\langle\phi_{\mathbf{2}}\rangle \propto \begin{pmatrix}1\\1\end{pmatrix}$
$\mathbf{1}$	1	1	$\langle\phi_{\mathbf{1}}\rangle \propto 1$
$\mathbf{1'}$	1	-1	$\langle\widetilde{\phi}_{\mathbf{1'}}\rangle \propto 1$

U broken & S conserved $\longrightarrow$ TM_2 mixing

Flavon alignment

$$\langle\phi_{\mathbf{3'}}\rangle = v_{\mathbf{3'}}\begin{pmatrix}1\\1\\1\end{pmatrix} \qquad \langle\phi_{\mathbf{2}}\rangle = v_{\mathbf{2}}\begin{pmatrix}1\\1\end{pmatrix} \qquad \langle\phi_{\mathbf{1}}\rangle = v_{\mathbf{1}} \qquad \langle\widetilde{\phi}_{\mathbf{1'}}\rangle = \widetilde{v}_{\mathbf{1'}}$$

- SUSY unbroken at scale of family symmetry breaking
- F-terms of driving fields $\phi^0_{\mathbf{r}}$ need to vanish

$$\begin{aligned} W_\nu^{\text{flavon}} \quad = \quad & \phi^0_{\mathbf{3'}}(g_1\,\phi_{\mathbf{3'}}\phi_{\mathbf{3'}} + g_2\,\phi_{\mathbf{3'}}\phi_{\mathbf{2}} + g_3\,\phi_{\mathbf{3'}}\phi_{\mathbf{1}}) \\ & + \phi^0_{\mathbf{3}}(g_4\,\phi_{\mathbf{3'}}\phi_{\mathbf{2}}) \\ & + \phi^0_{\mathbf{1}}(g_5\,\phi_{\mathbf{3'}}\phi_{\mathbf{3'}} + g_6\,\phi_{\mathbf{2}}\phi_{\mathbf{2}} + g_7\,\phi_{\mathbf{1}}\phi_{\mathbf{1}}) \\ & + \widetilde{\phi}^0_{\mathbf{1}}(g_8\,\widetilde{\phi}_{\mathbf{1'}}\widetilde{\phi}_{\mathbf{1'}} + M^2) \end{aligned}$$

- previously assumed flavon alignments independent of g_i with

$$v_{\mathbf{2}} = -\frac{g_3}{2g_2}v_{\mathbf{1}} \qquad v^2_{\mathbf{3'}} = -\frac{1}{3g_5}\left(g_7 + \frac{g_3^2 g_6}{2g_2^2}\right)v^2_{\mathbf{1}} \qquad \widetilde{v}^2_{\mathbf{1'}} = -\frac{1}{g_8}M^2$$

Imposing CP symmetry (straightforward in S_4)

$$M_R = y_{\mathbf{3'}} v_{\mathbf{3'}} \begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix} + y_{\mathbf{2}} v_{\mathbf{2}} \begin{pmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} + y_{\mathbf{1}} v_{\mathbf{1}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} + \frac{y_{\mathbf{1'}}}{M} \widetilde{v}_{\mathbf{1'}} v_{\mathbf{2}} \begin{pmatrix} 0 & 1 & -1 \\ 1 & -1 & 0 \\ -1 & 0 & 1 \end{pmatrix}$$

$$v_{\mathbf{2}} = -\frac{g_3}{2g_2} v_{\mathbf{1}} \qquad v_{\mathbf{3'}}^2 = -\frac{1}{3g_5}\left(g_7 + \frac{g_3^2 g_6}{2g_2^2}\right) v_{\mathbf{1}}^2 \qquad \widetilde{v}_{\mathbf{1'}}^2 = -\frac{1}{g_8} M^2$$

- CP symmetry $\rightarrow$ couplings y_i and g_i real
- phases of $v_{\mathbf{1}}$, $v_{\mathbf{2}}$, $v_{\mathbf{3'}}$ identical up to π or $\pm\pi/2$
- absorb phase of $v_{\mathbf{1}}$ into redefinition of N^c

	$v_{\mathbf{1}}$	$v_{\mathbf{2}}$	$v_{\mathbf{3'}}$	$\widetilde{v}_{\mathbf{1'}}$
(A)	real	real	real	real
(B)	real	real	real	imaginary
(C)	real	real	imaginary	real
(D)	real	real	imaginary	imaginary

Predictions with CP symmetry

- seesaw mechanism

$$M_\nu = m_D \, {M_R}^{-1} \, m_D^T \qquad \text{with} \qquad m_D \propto \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\longrightarrow \quad \boxed{U_{\mathrm{PMNS}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} U_R} \qquad \text{with} \qquad U_R^T \, M_R \, U_R = M_R^{\mathrm{diag}}$$

- resulting PMNS parameters

	θ_{13}	θ_{23}	θ_{12}	δ	(α_1, α_2)
(A)	free	$45^\circ \mp \frac{1}{\sqrt{2}}\theta_{13}$	35.3°	0 or π	0 or π
(B)	free	45°	35.3°	$\pm\frac{\pi}{2}$	0 or π
(C)	unphysical: two degenerate neutrino masses				
(D)	unphysical: $\theta_{13} = 35.3^\circ$				

$\Longrightarrow$ five low-energy predictions with imposed CP symmetry (up to a finite choice)

Conclusion and outlook

- symmetries reduce number of free parameters
- discrete symmetries essential in supersymmetry (proton decay)
  - $Z_n^{(R)}$ symmetries from $U(1)_{\mathrm{FN}}^{(R)}$
  - R-parity violating symmetries might be interesting
- observed pattern in neutrino mixing
  - suggestive of non-Abelian (discrete) family symmetries
  - $\theta_{13} \approx 9^\circ$ from Daya Bay and RENO
  - deviations from simple mixing patterns
  - testable(!) solar/atmospheric sum rules
- other aspects not covered here
  - CP violation and discrete symmetries $\rightarrow$ Mu-Chun's talk
  - SUSY flavor problem
  - quarks and grand unified models

Thank you

Details of the direct S_4 model

King, Luhn (2011)

matter	L	τ^c	μ^c	e^c	N^c	H_u	H_d
S_4	$\mathbf{3}$	$\mathbf{1}'$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}$
Z_3^ν	1	2	2	2	2	0	0
Z_3^ℓ	0	2	1	0	0	0	0

$$\langle \varphi_\ell \rangle = \begin{pmatrix} 0 \\ v_\ell \\ 0 \end{pmatrix} \qquad \langle \eta_\mu \rangle = \begin{pmatrix} 0 \\ w_\mu \end{pmatrix} \qquad \langle \eta_e \rangle = \begin{pmatrix} w_e \\ 0 \end{pmatrix}$$

flavons	φ_ℓ	η_μ	η_e	$\phi_{\mathbf{3}'}$	$\phi_{\mathbf{2}}$	$\phi_{\mathbf{1}}$	$\widetilde{\phi}_{\mathbf{1}'}$
S_4	$\mathbf{3}'$	$\mathbf{2}$	$\mathbf{2}$	$\mathbf{3}'$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}'$
Z_3^ν	0	0	0	2	2	2	0
Z_3^ℓ	1	1	2	0	0	0	0

$$\langle \phi_{\mathbf{3}'} \rangle = v_{\mathbf{3}'} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \qquad \langle \phi_{\mathbf{2}} \rangle = v_{\mathbf{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \qquad \langle \phi_{\mathbf{1}} \rangle = v_{\mathbf{1}} \qquad \langle \widetilde{\phi}_{\mathbf{1}'} \rangle = \widetilde{v}_{\mathbf{1}'}$$

Charged lepton sector

$$\boxed{W_\ell \;\sim\; \left[\tfrac{1}{M}(L\varphi_\ell)_{1'}\,\tau^c + \tfrac{1}{M^2}(L\varphi_\ell)_2\,\eta_\mu\,\mu^c + \tfrac{1}{M^2}(L\varphi_\ell)_2\,\eta_e\,e^c\right]H_d}$$

- Z_3^ℓ controls pairing of flavons with right-handed charged fermions
- different S_4 contractions of $(L\varphi_\ell)$ pick out different L_i components

$$(L\varphi_\ell)_{1'} \;=\; L_1\varphi_{\ell 1} + L_2\varphi_{\ell 3} + L_3\varphi_{\ell 2} \;\rightarrow\; L_3$$

$$(L\varphi_\ell)_2 \;=\; \begin{pmatrix} L_1\varphi_{\ell 3} + L_2\varphi_{\ell 2} + L_3\varphi_{\ell 1} \\ L_1\varphi_{\ell 2} + L_2\varphi_{\ell 1} + L_3\varphi_{\ell 3} \end{pmatrix} \;\rightarrow\; \begin{pmatrix} L_2 \\ L_1 \end{pmatrix}$$

- mass matrix diagonal by construction
- m_τ heavier than m_μ and m_e
- hierarchy between m_μ and m_e due to hierarchy of VEVs w_μ and w_e
- just a toy model of charged lepton sector (with GUTs off-diagonals)

Parameter counting in neutrino sector

- without family symmetry

$$\left.\begin{array}{ll} Y_\nu & 18 \text{ d.o.f.} \\ M_R & 12 \text{ d.o.f.} \end{array}\right\} \; 18 \text{ of which are physical} \; \left\{\begin{array}{ll} \text{neutrino masses} & 3+3 \\ \text{PMNS mixing} & 3+3 \\ \text{Casas-Ibarra } R & 3+3 \end{array}\right.$$

- in S_4 model

$$\boxed{W_\nu \;\sim\; L N^c H_u + (\phi_{\mathbf{3'}} + \phi_{\mathbf{2}} + \phi_{\mathbf{1}}) N^c N^c + \frac{1}{M} \widetilde{\phi}_{\mathbf{1'}} \phi_{\mathbf{2}} N^c N^c}$$

(i) with arbitrary flavon configurations

$$2 \times (\; 1 \;\; + \;\; 3 \; + \; 2 \; + \; 1 \;\;\;\; + \;\;\;\; 1 \;) \; = \; 16$$

(ii) with flavon alignment

$$2 \times (\; 1 \;\; + \;\; 1 \; + \; 1 \; + \; 1 \;\;\;\; + \;\;\;\; 1 \;) \; = \; 10$$

$$\left.\begin{array}{ll} Y_\nu & 2 \text{ parameters} \\ M_R & 8 \text{ parameters} \end{array}\right\} \; 8 \text{ of which are physical}$$

$\longrightarrow$ correlation between low-energy observables guaranteed

Combining family and CP symmetry

- theory symmetric under $\mathcal{G}$ transformation

$$\psi \xrightarrow{\mathcal{G}} \rho(g)\psi$$

- "generalized" CP transformation

Grimus, Rebelo (1995)
Branco, González Felipe, Joaquim (2011)

$$\psi \xrightarrow{\mathrm{CP}} X\psi^*$$

- sequence of transformations $\mathrm{CP} \circ \mathcal{G} \circ (\mathrm{CP})^{-1}$ is again $\mathcal{G}$ transformation

$$\psi \xrightarrow{\mathrm{CP}} X\psi^* \xrightarrow{\mathcal{G}} X\rho^*(g)\psi^* \xrightarrow{(\mathrm{CP})^{-1}} X\rho^*(g)X^{-1}\psi$$

- find all X which satisfy consistency condition $\boxed{X\rho^*(g)X^{-1} = \rho(g')}$
- these matrices form a representation of the automorphism group of $\mathcal{G}$
- $\mathcal{G} = S_4$: $X = \rho(h)$ with $h \in S_4$

Holthausen, Lindner, Schmidt (2012)
Feruglio, Hagedorn,Ziegler (2012)

Imposing CP symmetry in S_4

- naive expectation: all coupling constants real
- closer look at term $\boxed{y\,\psi\,\chi\,\phi}$ in S_4 where
  (i) $X = \rho(h)$
  (ii) $\rho^*(h) = X^{-1}\rho(h')X = \rho(\tilde{h})$
  (iii) Clebsch coefficients are real

$$y \times c_{ijk}\,\psi_i\,\chi_j\,\phi_k \quad \xrightarrow{\text{CP}} \quad y \times c_{ijk}\,[X\psi^*]_i\,[X\chi^*]_j\,[X\phi^*]_k$$

$$= y \times c_{ijk}\,[\rho(h)\psi^*]_i\,[\rho(h)\chi^*]_j\,[\rho(h)\phi^*]_k$$

$$= \left\{y^* \times c_{ijk}\,[\rho^*(h)\psi]_i\,[\rho^*(h)\chi]_j\,[\rho^*(h)\phi]_k\right\}^*$$

$$= \left\{y^* \times c_{ijk}\,[\rho(\tilde{h})\psi]_i\,[\rho(\tilde{h})\chi]_j\,[\rho(\tilde{h})\phi]_k\right\}^*$$

$$= \left\{y^* \times c_{ijk}\,\psi_i\,\chi_j\,\phi_k\right\}^*$$

- CP maps term $y\,\psi\,\chi\,\phi$ onto its hermitian conjugate if y is real

$\longrightarrow$ naive expectation correct in S_4 (in basis with real Clebsches)