Discrete Abelian Gauge Symmetries and Axions

Gabriele Honecker

Cluster of Excellence PRISMA & Institut für Physik, JG|U Mainz

based on JHEP 1310(2013)146, PoS Corfu2012(2013)107, Fortsch.Phys. 62(2014)115-151 with **Wieland Staessens**

BCTP Bonn, 3 June 2014

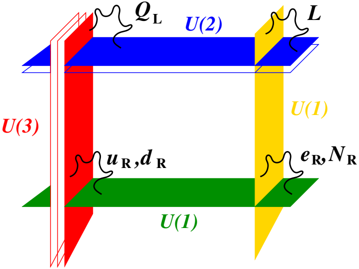
Motivation: Gauge Symmetries in String Theory

- Type II string theories: gauge theories localized on **D-branes**

- $U(1) \subset U(N)$ generically massive $\propto M_{\text{string}}$

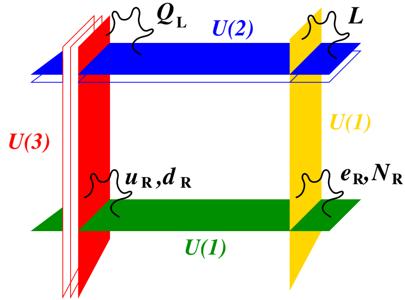
Spanish Quiver:

$$SU(3) \times SU(2) \times Y \times \begin{cases} U(1)^3_{\text{massive}} \\ (B-L) \times U(1)^2_{\text{massive}} \end{cases}$$



- $U(1)^k_{\text{massive}}$ remain as **perturbative global** symmetries

Motivation: Discrete Abelian Gauge Symmetries



$U(1)^k_{\text{massive}}$

- broken by **non-perturvative** effects, e.g. D-brane instantons
- $\mathbb{Z}_n \subset U(1)^k_{\text{massive}}$ remain as global **discrete** symmetries

$\rightsquigarrow$ constraints on **effective field theory** @ low energies

This talk:

- Conditions on the existence of $\mathbb{Z}_n$ symmetries
- Which $\mathbb{Z}_n$ occur in global (=consistent) D-brane models?

...**gauge quivers:** Richter's talk

- Relation to axions (strong CP problem, dark sector ...)

...**inflation:** Marchesano's talk

Related Works on Abelian Discrete Symmetries

SUSY field theory:

- *Discrete gauge symmetries and the origin of baryon and lepton number conservation in supersymmetric versions of the standard model* L.E.Ibáñez, G.G.Ross: Nucl.Phys.B368(1992)3-37
- *What is the discrete gauge symmetry of the MSSM?* H.K.Dreiner, C.Luhn, M.Thormeier: Phys.Rev.D73(2006)075007 Luhn's talk

$\rightsquigarrow$ R-parity ($\mathbb{Z}_2$), baryon triality ($\mathbb{Z}_3$), proton hexality ($\mathbb{Z}_6$) for e.g. **proton stability**

D-brane models:

- *Discrete gauge symmetries in D-brane models* M.Berasaluce-Gonzalez, L.E.Ibáñez, P.Soler, A.M.Uranga: JHEP1112(2011)113
- *Discrete Gauge Symmetries in Discrete MSSM-like Orientifolds* L.E.Ibáñez, A.N.Schellekens, A.M.Uranga: Nucl.Phys.B865(2012)509-540
- *String Constraints on Discrete Symmetries in MSSM Type II Quivers* P.Anastasopoulos, M.Cvetič, R.Richter, P.K.S.Vaudrevange: JHEP1303(2013)011
- *Zp charged branes in flux compactifications* M.Berasaluce-Gonzalez, P.G.Camara, F.Marchesano, A.M.Uranga: JHEP1304(2013)138
- **GH**, W. Staessens '13
  ...

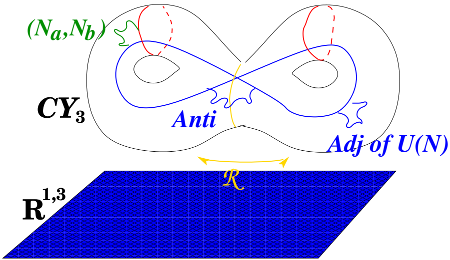
Content

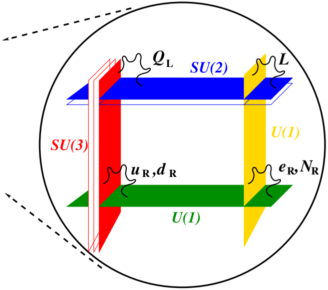
- Discrete gauge symmetries: $\mathbb{Z}_n$
  - Massive U(1)s & closed string axions
  - Conditions on $\mathbb{Z}_n$ symmetries
  - Cross-check of normalisation for n
  - Examples: D6-branes on $T^6/\mathbb{Z}_{2N}$ or $\mathbb{Z}_2 \times \mathbb{Z}_{2M}$
- Axions, strong CP problem & the dark sector
  - Open & closed string axions
  - $\not{U(1)}_{PQ}$ & Higgs-axion potential in the DFSZ model
  - soft $\not{\text{SUSY}}$
  - Bounds on M_{string}
- Conclusions

$\mathbb{Z}_n$ Symmetries

Massive U(1)s in String Theory I

- **Here:** geometric language of **Type IIA**/$\Omega\mathcal{R}$
- **same physics** for (by dualities for smooth CYs)
  - Type IIB/Ω (mirror symmetry) ... F-theory
  - hetero. w/ $U(1)$ bundles (S-dual/$SO(32)$, M-theory dual/$E_8 \times E_8$)





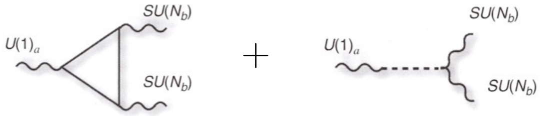
- **Global model** $\rightsquigarrow$ Non-Abelian $SU(N_b)$ gauge anomalies=0:

$$\left[\sum_a N_a \left(\Pi_a + \Pi_a'\right) - 4\,\Pi_{O6}\right] \circ \Pi_b = 0$$

upon RR tadpole cancellation

Massive U(1)s in String Theory II

- **Mixed anomalies** cancel by the **Green-Schwarz** mechanism:



- **Axions** ξ_i ($\star_4 d\xi_i \sim d\mathcal{B}_2^{(i)}$): longitudinal modes of $U(1)^k_{\text{massive}}$

$$\mathcal{S}_{CS} \supset \int_{\mathbb{R}^{1,3}} \sum_{i=0}^{h_{21}} \left(B_a^i \, \mathcal{B}_2^{(i)} \wedge \mathrm{tr} F_a + A_b^i \, \xi_i \, \mathrm{tr} F_b \wedge F_b \right)$$

with $\boxed{\mathcal{B}_2^{(i)} \propto \int_{\Pi_i^{\text{odd}}} C_5^{RR}}$; $\boxed{\xi_i \propto \int_{\Pi_i^{\text{even}}} C_3^{RR}}$

- $U(1)_X = \sum_a q_a U(1)_a$ **massless** if $\sum_a N_a q_a B_a^i = 0 \; \forall i$

- $\mathbb{Z}_n \subset U(1)^k_{\text{massive}}$ for suitable B_a^i ('mod n') due to **shift symmetry** of ξ_i

Axionic Shift Symmetry

- **Closed string axions** within $\mathcal{N}=1$ chiral multiplets:
  - axion-dilaton: $S = \phi + \mathrm{i}\,\xi_0$
  - complex structure: $U_i = c_i + \mathrm{i}\,\xi_i$ $\qquad \xi_i \subset C_3^{RR}$
  - Kähler: $T_k = v_k + \mathrm{i}\,b_k$ $\qquad b_k \subset B_2^{NSNS}$
- $\mathcal{N}=1$ **SUGRA action** independent of $\xi_i \to \xi_i + 1$

$$\mathcal{K}_{\text{closed}} = -\ln \Re(S) - \sum_i \ln \Re(U_i) - \sum_k \ln \Re(T_k)$$

  - **perturbatively:** only couplings to $(\partial_\mu \xi_i)$
  - **non-perturbative** couplings via D-brane instantons: $e^{-\mathcal{S}_{\text{inst}}}$ with $\mathcal{S}_{\text{inst}} \supset 2\pi\mathrm{i}\,\xi_i$

in IIB: $U_i \leftrightarrow T_k$

- **Discrete** $\mathbb{Z}_n$ **symmetry** preserved if

$$A^\mu \to A^\mu + \partial^\mu \lambda \qquad \xi_i \to \xi_i + \underbrace{\overline{c}_i(B_a^i)}_{0 \bmod n \quad \forall i} \lambda$$

$\rightsquigarrow$ need to determine $\overline{c}_i(B_a^i)$!

$\mathbb{Z}_n$ Symmetries & Green-Schwarz Couplings I

$$\mathcal{S}_{CS} \supset \int_{\mathbb{R}^{1,3}} \sum_{i=0}^{h_{21}} \left(B_a^i \, \mathcal{B}_2^{(i)} \wedge \mathrm{tr} F_a + A_b^i \, \xi_i \, \mathrm{tr} F_b \wedge F_b \right)$$

$$\text{with } \boxed{\mathcal{B}_2^{(i)} \propto \int_{\Pi_i^{\text{odd}}} C_5^{RR}} \; ; \; \boxed{\xi_i \propto \int_{\Pi_i^{\text{even}}} C_3^{RR}}$$

- Expand 3-cycles and $\Omega\mathcal{R}$-images as:

$$\Pi_a = \sum_{i=0}^{h_{21}} \left(A_a^i \, \Pi_i^{\text{even}} + B_a^i \, \Pi_i^{\text{odd}} \right), \quad \Pi_a' = \sum_{i=0}^{h_{21}} \left(A_a^i \, \Pi_i^{\text{even}} - B_a^i \, \Pi_i^{\text{odd}} \right)$$

- If $\boxed{\Pi_i^{\text{even}} \circ \Pi_j^{\text{odd}} = m_i \, \delta_{ij}}$ with $m_i \in \mathbb{Z}$
  - $\{\Pi_i^{\text{even}}, \Pi_j^{\text{odd}}\}$ span only **sublattice** of finite index:

$$\Lambda_3^{\text{even}} \oplus \Lambda_3^{\text{odd}} \subsetneq \Lambda_3$$

  - **all** known global D-brane models of **this type**
  - $A_a^i, B_a^i \in \frac{1}{m_i} \mathbb{Z}$ - how exactly?
    Need correct **normalisation** for $\mathbb{Z}_n$!

$\mathbb{Z}_n$ Symmetries & Green-Schwarz Couplings II

- $\Pi_i^{\text{even}} \circ \Pi_j^{\text{odd}} = m_i \delta_{ij} \quad \rightsquigarrow \quad d\mathcal{B}_2^{(i)} = m_i \star_4 d\xi_i$
- gauge trafos of $\boxed{U(1)_{\text{massive}} = \sum_a k_a U(1)_a}$

$$A^\mu \to A^\mu + \partial^\mu \lambda \qquad \xi_i \to \xi_i + \underbrace{\left(m_i \sum_a N_a k_a B_a^i\right)}_{\overline{c}_i(B_a^i)} \lambda$$

- $\xi_i \simeq \xi_i + 1 \rightsquigarrow \mathbb{Z}_n$ symmetry if $\boxed{m_i \sum_a N_a k_a B_a^i = 0 \text{ mod } n \ \forall i}$
- $\sum_{i=0}^{h_{21}} N_a B_a^i \int_{\mathbb{R}^{1,3}} \mathcal{B}_2^i \wedge F_a \qquad \rightsquigarrow \qquad \text{mass}_{U(1)} \propto \sum_a N_a k_a B_a^i$

Cross-checks on correct normalisation:

- K-theory constraint $\simeq \mathbb{Z}_2$ symmetry
- $\Pi_i^{\text{even}} \in \Lambda_3$ coincide with $U(N) \hookrightarrow \left\{ \begin{array}{c} USp(2N) \\ SO(2N) \end{array} \right.$

Cross-Check: K-Theory Constraint as $\mathbb{Z}_2$ Symmetry

- **K-theory** constraint $\Leftarrow$ absence of $SU(2)$ field anomalies
  - related to $\mathbb{Z}_2$ grading of $H_3(CY_3)$ in Type IIA/$\Omega\mathcal{R}$
- **probe brane** argument: Uranga '02

$$0 \bmod 2 = \Pi_{USp(2)_k} \circ \sum_a N_a \Pi_a = \sum_{i=0}^{h_{21}} A^i_{USp(2)_k} \, m_i \sum_a N_a B^i_a$$

- $(k_a, k_b, \ldots) = (1, 1, \ldots)$ if *all* $\Pi_i^{\text{even}} = \Pi_{USp(2)_i}$ ✓
- <u>*generally:*</u> N_a D-branes on Π_i^{even}: $U(N_a) \hookrightarrow \left\{ \begin{array}{c} USp(2N_a) \\ SO(2N_a) \end{array} \right.$
  - *K-theory constraint* naively *less than* $\mathbb{Z}_2$ *gauge symmetry*
- $\Pi_{USp(2)_k} \circ \Pi_a \in \mathbb{Z}$ independent of:
  - basis $\{\Pi_i^{\text{even}}, \Pi_i^{\text{odd}}\}$
  - normalisation of wrappings $\{A^i_a, B^i_a\}$

  $\rightsquigarrow$ express also $\mathbb{Z}_n$ condition via **intersection numbers**

$\mathbb{Z}_n$ Symmetries in Terms of Intersection Numbers

- ambiguities of normalisation factors m_i in B_a^i and Π_i^{odd} cancel

$U(1)_{\text{massless}} = \sum_a q_a U(1)_a$	$\mathbb{Z}_n \subset U(1)_{\text{massive}} = \sum_a k_a U(1)_a$
$\Pi_i^{\text{even}} \circ \sum_a N_a q_a \Pi_a = 0 \ \forall i$ $\Leftrightarrow \quad \sum_a N_a q_a B_a^i = 0 \ \forall i$	$\Pi_i^{\text{even}} \circ \sum_a N_a k_a \Pi_a = 0 \bmod n \ \forall i$ $\Leftrightarrow \quad m_i \sum_a N_a k_a B_a^i = 0 \bmod n \ \forall i$
$q_a \in \mathbb{Q}$	$k_a \in \mathbb{Z}$, $0 \leqslant k_a < n$, $\gcd(k_a, n) = 1$

- derivation of m_i, B_a^i for all **orbifolds** possible

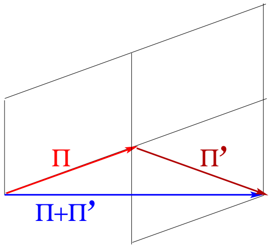
$\rightsquigarrow$ $\mathbb{Z}_n$ symmetries in any <u>*global*</u> model ✓

GH, Staessens '13

Comparison with *local* Bottom-up Models

Richter's talk

- Π_i^{even} unknown $\rightsquigarrow$ use $\left(\Pi_x + \Pi_x'\right) \in \Lambda_3^{\text{even}}$
- **caution**: $\frac{\Pi_x + \Pi_x'}{2} \notin \Lambda_3^{\text{even}}$
- $\left(\Pi_x + \Pi_x'\right) \circ \Pi_a = \Pi_x \circ \left(\Pi_a - \Pi_a'\right)$
  without factor 1/2
- gives (at most) 4 of $(h_{21} + 1)$ conditions for 4 stacks of D-branes



  - only **necessary**, **not sufficient** conditions on existence of $\mathbb{Z}_n$
  - cross-check on correct normalisation from $0 \leqslant k_a < n$

Intermezzo: $\mathbb{Z}_n$ Symmetries & D2-Brane Instantons

- $\boxed{O(1)}$ D2-instantons respect $\mathbb{Z}_n$ symmetry: $e^{-\mathcal{S}_{\mathrm{D2}}}$ contains

$$\mathcal{S}_{\mathrm{D2}} = -\frac{\mathrm{Vol(D2)}}{g_s} + 2\pi i \xi \quad \text{with} \quad \xi = \int_{\Pi_{\mathrm{D2}}} C_3^{RR} = \sum_{i=0}^{h_{21}} A^i_{\mathrm{D2}}\, \xi_i$$

$$\mathcal{S}_{\mathrm{D2}} \stackrel{\text{gauge trafo}}{\longrightarrow} \mathcal{S}_{\mathrm{D2}} + 2\pi i \lambda \underbrace{\sum_{i=0}^{h_{21}} A^i_{\mathrm{D2}} m_i \Big(\sum_a N_a k_a B^i_a\Big)}_{= \Pi_{\mathrm{D2}} \circ \Pi_{U(1)_{\text{massive}}} = 0 \bmod n}$$

- $\boxed{U(1)}$ D2-instantons: $\mathcal{S}_{\mathrm{D2}} \supset 2\pi i \xi \quad$ with $\quad \xi = \int_{\Pi_{\mathrm{D2}} + \Pi'_{\mathrm{D2}}} C_3^{RR}$
  - $(\Pi_{\mathrm{D2}} + \Pi'_{\mathrm{D2}}) \circ \Pi_{U(1)_{\text{massive}}} = 0 \bmod n$ ✓
  - non-minimal # zero-modes ⇝ contributions to eff. action=0
- $\boxed{USp(2)}$ D2-instantons analogous ✓

Modding out Redundant $\mathbb{Z}_N$ Symmetries

- $SU(N)$ has center $\mathbb{Z}_N$
- $\mathbb{Z}_N \subset U(1)_{\text{massive}} \subset U(N)$ equivalent to $\mathbb{Z}_N^{\text{center}}$?
- for $\boxed{N \geq 3}$: representations of $SU(N)_{U(1)} \simeq U(N)$
  - $(\mathbf{N})_1$
  - $(\mathbf{N})_1 \times (\overline{\mathbf{N}})_{-1} \simeq (\mathbf{Adj})_0 + (\mathbf{1})_0$
  - $(\mathbf{N})_1 \times (\mathbf{N})_1 \simeq (\mathbf{Sym} + \mathbf{Anti})_2$

  $\rightsquigarrow$ $\mathbb{Z}_N \subset U(1)_{\text{massive}}$ provides *same* selection rules on couplings as $SU(N)$ rep.
- for $\boxed{N = 2}$:
  - $(\mathbf{Anti})_2 \simeq (\mathbf{1})_2 \quad \longleftrightarrow \quad (\mathbf{1})_0$
  - $(\mathbf{Sym})_2 \simeq (\mathbf{3})_2 \quad \longleftrightarrow \quad (\mathbf{Adj})_0 \simeq (\mathbf{3})_0$

  $\rightsquigarrow$ charges identical 'mod 2'
- **But:** non-trivial sums of $\mathbb{Z}_{N_a} \subset U(N_a)$ charges can arise

$\rightsquigarrow$ **generation dependent** $\mathbb{Z}_n$ symmetries

example of generation dependent $\mathbb{Z}_2$ later

$\mathbb{Z}_n$ Symmetries in the SUSY Standard Model

Field theory / SUSY SM: Luhn's talk

- 3 generators for $\mathbb{Z}_n$ in MSSM: $$g_n = e^{i2\pi\mathcal{R}\frac{m}{n}} \cdot e^{i2\pi\mathcal{A}\frac{k}{n}} \cdot e^{i2\pi\mathfrak{L}\frac{p}{n}}$$
  - R-parity: $\mathcal{R}_2$
  - baryon triality: $\mathcal{L}_3\mathcal{R}_3$
  - proton hexality: $\mathcal{L}_6^2\mathcal{R}_6^5$
- Q_L charge can be rotated away by $U(1)_Y$

Charges of generation-independent $\mathbb{Z}_n$ symmetries in the MSSM

Generator	Q_L	$\overline{U}_R$	$\overline{D}_R$	L	$\overline{E}_R$	$\overline{N}_R$	H_u	H_d
$\mathcal{R}$	0	$n-1$	1	0	1	$n-1$	1	$n-1$
$\mathfrak{L}$	0	0	0	$n-1$	1	1	0	0
$\mathcal{A}$	0	0	$n-1$	$n-1$	0	1	0	1

- Presence of $U(1)_{B-L}$ makes R-parity ($\mathcal{R}_2$) trivial
- Pati-Salam models: no $U(1)_{\text{massless}}$

Which $\mathbb{Z}_n$ occur in global D-brane models?

$\mathbb{Z}_n$ Symmetries on Orbifolds

$\mathbb{Z}_n$ Symmetries on Orbifolds of IIA/$\Omega\mathcal{R}$

- $\dim(\Lambda_3^{\text{even}}) = h_{21} + 1$ conditions
- phenomenologically interesting:
  - $\boxed{T^6/\mathbb{Z}_6}$: $h_{21} = 5$
  - $\boxed{T^6/\mathbb{Z}_6'}$: $h_{21} = 5\ (+6)^*$
  - $\boxed{T^6/\mathbb{Z}_2 \times \mathbb{Z}_6}$: $h_{21} = 15\ (+4)^*$
  - $\boxed{T^6/\mathbb{Z}_2 \times \mathbb{Z}_6}$: $h_{21} = 15$

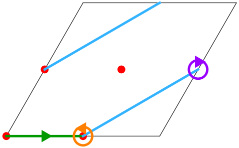
* D-branes wrap only untwisted & $\mathbb{Z}_2$ twisted cycles

- shape of Λ_3^{even} depends on lattice orientations under $\mathcal{R}$
- L-R symmetric & Pati-Salam models *'natural'* on D-branes $\rightsquigarrow$ $U(1)_Y$ & $U(1)_{B-L}$ to rotate charges to 0

Orbifolds I: Basis of Λ_3^{even} for $T^6/\mathbb{Z}_6^{(\prime)}$

- $\boxed{T^6/\mathbb{Z}_6^{(\prime)}}$:

$$\Pi_a^{\text{frac}} = \frac{1}{2}\left(\Pi_a^{\text{bulk}} + \Pi_a^{\mathbb{Z}_2}\right)$$



- 2 displacements $\sigma \in \{0, 1\}$
- 1 $\mathbb{Z}_2$ eigenvalue ± 1
- 2 Wilson lines $\tau \in \{0, 1\}$

- $2^5 = 32$ fractional 3-cycles per given bulk cycle
- only $(h_{21} + 1) = 6$ independent conditions on $\mathbb{Z}_n$

Example I: L-R Symmetric Model on $T^6/\mathbb{Z}_6$

GH, Ott '04; see also Gmeiner, **GH** '09

- $\boxed{U(3)_a \times U(2)_b \times USp(2)_c \times U(1)_d \times USp(2)_e}$
- $U(1)_{B-L} = (\frac{Q_a}{3} + Q_d)_{\text{massless}}$ & $U(1)^2_{\text{massive}}$
- $USp(2)_{x\in\{c,e\}} \to U(1)_{x,\text{massless}}$ by brane displacement
- only $x \in \{a, b, d\}$ contribute to $\mathbb{Z}_n$ conditions
- after $B - L$ rotation:

GH, Staessens '13

Discrete sym.		**Charge assignment for the MSSM states**									
$\mathbb{Z}_n$	$\subset\sum_x k_x U(1)_x$	Q_L	$\overline{U}_R$	$\overline{D}_R$	L	$\overline{E}_R$	$\overline{N}_R$	$H_u^{(1)}$	$H_u^{(2)}$	$H_d^{(1)}$	$H_d^{(2)}$
$\mathbb{Z}_2$	$Q_a + Q_d$	0	0	0	0	0	0	0	0	0	0
$\mathbb{Z}_2$	Q_b	0	1	1	0	1	1	1	1	1	1
$\mathbb{Z}_3$	Q_a	0	0	0	0	0	0	0	0	0	0

not listed: mild amount of vector-like exotics

- $(k_a, k_b, k_d) = (1, 1, 1) \simeq \mathbb{Z}_2$ of K-theory constraint
- $\mathbb{Z}_2^{(b)}$ gives **no extra constraints** beyond $SU(2)_b$ charges
  $\rightsquigarrow$ all $\mathbb{Z}_n$ appear trivial

Example II: L-R Symmetric Model on $T^6/\mathbb{Z}_6'$

Gmeiner, **GH** '07-'08

- $\boxed{U(3)_a \times U(2)_b \times USp(2)_c \times U(1)_d (\times USp(6)_{\text{hidden}})}$
- $U(1)_{B-L} = (\frac{Q_a}{3} + Q_d)_{\text{massless}}$ & $U(1)^2_{\text{massive}}$
- $USp(2)_c \to U(1)_{c,\text{massless}}$ by brane displacement σ
- $USp(6)_{\text{hidden}}$ **cannot** be broken by σ or τ (~~SUSY~~)
- after $B-L$ rotation:

GH, Staessens '13

Discrete sym.		Charge assignment for the chiral states									
$\mathbb{Z}_n$	$\subset \sum_x k_x U(1)_x$	Q_L	$\overline{U}_R$	$\overline{D}_R$	L	$\overline{L}$	$\overline{E}_R$	$\overline{N}_R$	H_u	H_d	Σ_b
$\mathbb{Z}_2$	$Q_a + Q_d$	0	0	0	0	0	0	0	0	0	0
$\mathbb{Z}_3$	Q_a	0	0	0	0	0	0	0	0	0	0
$\mathbb{Z}_6$	Q_b	0	1	1	4	4	3	3	5	5	4
	$\xrightarrow{U(1)_c}$	0	0	2	4	4	4	2	0	4	4

open string axion: $\Sigma_b \simeq (\mathbf{1}_{\overline{\mathbf{Anti}}_b})_{-2_b}$

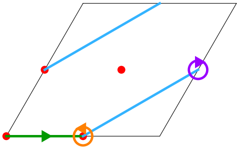
not listed: mild amount of vector-like exotics

- **non-trivial**: $\boxed{\mathbb{Z}_3 \subset U(1)_b}$

Orbifolds II: Basis of Λ_3^{even} for $T^6/\mathbb{Z}_2 \times \mathbb{Z}_6^{(\prime)}$

- $\boxed{T^6/\mathbb{Z}_2 \times \mathbb{Z}_6^{(\prime)}}$ with discrete torsion:

$$\Pi_a^{\text{frac}} = \frac{1}{4}\left(\Pi_a^{\text{bulk}} + \sum_{i=1}^{3} \Pi_a^{\mathbb{Z}_2^{(i)}}\right)$$



- 3 displacements $\sigma \in \{0,1\}$
- 2 $\mathbb{Z}_2$ eigenvalues ± 1
- 3 Wilson lines $\tau \in \{0,1\}$

- very large number of 3-cycles *per given bulk cycle*
- but: only $(h_{21}+1)$ independent Π_i^{even} $\rightsquigarrow$ classify! Förste, **GH** '10

c || to	$\Omega\mathcal{R}$ invariant for $(\eta_{(1)}, \eta_{(2)}, \eta_{(3)}) \stackrel{!}{=}$
$\Omega\mathcal{R}$	$\left(-(-1)^{\delta_2+\delta_3}, -(-1)^{\delta_1+\delta_3}, -(-1)^{\delta_1+\delta_2}\right)$
$\Omega\mathcal{R}\mathbb{Z}_2^{(1)}$	$\left(-(-1)^{\delta_2+\delta_3}, (-1)^{\delta_1+\delta_3}, (-1)^{\delta_1+\delta_2}\right)$
$\Omega\mathcal{R}\mathbb{Z}_2^{(2)}$	$\left((-1)^{\delta_2+\delta_3}, -(-1)^{\delta_1+\delta_3}, (-1)^{\delta_1+\delta_2}\right)$
$\Omega\mathcal{R}\mathbb{Z}_2^{(3)}$	$\left((-1)^{\delta_2+\delta_3}, (-1)^{\delta_1+\delta_3}, -(-1)^{\delta_1+\delta_2}\right)$

256 $\Omega\mathcal{R}$ inv. D6$_c$-branes:

- $\delta_i \equiv 2b_i\tau^i\sigma^i \in \{0,1\}$ non-tivial for **tilted** tori
- indep. of $(-1)^{\tau^{\mathbb{Z}_2^{(i)}}}$
- both $SO(2)$ & $USp(2)$

Classification of Gauge Enhancements: $T^6/\mathbb{Z}_2 \times \mathbb{Z}_6^{(\prime)}$

$\boxed{T^6/\mathbb{Z}_2 \times \mathbb{Z}_6^{(\prime)}}$ with discrete torsion:

- **256** $\Omega\mathcal{R}$-invariant 3-cycles in total
- **untilted tori** $(b_i \equiv 0 \forall i)$: $\Omega\mathcal{R}$ inv. only for
  - $c \,||$ exotic O6 & any $(\vec{\sigma}, \vec{\tau}) \rightsquigarrow USp(2N)$

$T^6/\mathbb{Z}_2 \times \mathbb{Z}_2$: Blumenhagen, Cvetič, Marchesano, Shiu '05

- **tilted tori** $(b_i \equiv \frac{1}{2} \forall i)$: $\Omega\mathcal{R}$ invariance for

GH, Ripka, Staessens '12

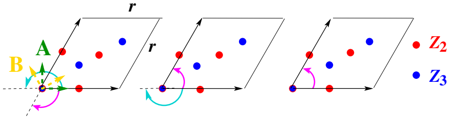
  - $c \,||$ exotic O6 & $\tau^i \sigma^i \equiv 0 \; \forall i \rightsquigarrow USp(2N)$
  - $c \,||$ exotic O6 & $\tau^i \sigma^i \equiv 1 \; \forall i \rightsquigarrow SO(2N)$
  - $c \perp$ exotic O6 & $\tau^i \sigma^i \neq \tau^j \sigma^j = \tau^k \sigma^k = 1 \rightsquigarrow SO(2N)$
  - $c \perp$ exotic O6 & $\tau^i \sigma^i \neq \tau^j \sigma^j = \tau^k \sigma^k = 0 \rightsquigarrow USp(2N)$
- **mixed set-up:** $(b_1, b_2, b_3) = (0, \frac{1}{2}, \frac{1}{2})$ on $T^6/\mathbb{Z}_2 \times \mathbb{Z}_6$

work in progress Ecker, **GH**, Staessens

- full **classification** of $USp(2)$ **needed** for
  - K-theory constraint
  - $O(1)$ D2-brane instantons
- only $(h_{21} + 1) = 16$ indep. conditions on $\mathbb{Z}_n$ symmetries

Example: A Pati-Salam Model on $T^6/\mathbb{Z}_2 \times \mathbb{Z}_6'$

- $\mathbb{Z}_2 \times \mathbb{Z}_6'$ shifts: $\vec{v} = (\frac{1}{2}, \frac{-1}{2}, 0)$, $\vec{w}' = (\frac{-1}{3}, \frac{1}{6}, \frac{1}{6})$ on $SU(3)^3$



- $\Pi_a^{\text{frac}} = \frac{1}{4}\left(X_a\,\rho_1 + Y_a\,\rho_2 + \sum_{k=1}^{3}\sum_{\alpha=1}^{5}\left[x_{a,\alpha}^{(k)}\,\varepsilon_\alpha^{(k)} + y_{a,\alpha}^{(k)}\,\tilde{\varepsilon}_\alpha^{(k)}\right]\right)$

  with $\rho_1 \circ \rho_2 = -\varepsilon_\alpha^{(k)} \circ \tilde{\varepsilon}_\alpha^{(k)} = 4$

- $\Omega\mathcal{R}$-even & odd 3-cycles:

  GH, Staessens '13

$\Pi_0^{\text{even},\mathbf{1}} = \rho_1,$ $\quad$ $\Pi_0^{\text{odd},\mathbf{1}} = -\rho_1 + 2\,\rho_2,$

$\Pi_{\alpha\in\{1,2,3\}}^{\text{even},\mathbb{Z}_2^{(k)}} = \varepsilon_\alpha^{(k)},$ $\quad$ $\Pi_{\alpha\in\{1,2,3\}}^{\text{odd},\mathbb{Z}_2^{(k)}} = -\varepsilon_\alpha^{(k)} + 2\,\tilde{\varepsilon}_\alpha^{(k)},$

$\Pi_4^{\text{even},\mathbb{Z}_2^{(k)}} = \varepsilon_4^{(k)} + \varepsilon_5^{(k)},$ $\quad$ $\Pi_4^{\text{odd},\mathbb{Z}_2^{(k)}} = 2\,(\tilde{\varepsilon}_4^{(k)} + \tilde{\varepsilon}_5^{(k)}) - (\varepsilon_4^{(k)} + \varepsilon_5^{(k)}),$

$\Pi_5^{\text{even},\mathbb{Z}_2^{(k)}} = 2\,(\tilde{\varepsilon}_4^{(k)} - \tilde{\varepsilon}_5^{(k)}) - (\varepsilon_4^{(k)} - \varepsilon_5^{(k)}),$ $\quad$ $\Pi_5^{\text{odd},\mathbb{Z}_2^{(k)}} = \varepsilon_4^{(k)} - \varepsilon_5^{(k)},$

- Intersection numbers

$$\Pi_{\tilde{\alpha}}^{\text{even},\mathbb{Z}_2^{(k)}} \circ \Pi_{\tilde{\beta}}^{\text{odd},\mathbb{Z}_2^{(l)}} = \delta^{kl}\delta_{\tilde{\alpha}\tilde{\beta}} \times \begin{cases} 8 & \tilde{\alpha} = 0 \\ -8 & 1\ldots3 \\ -16 & 4 \\ 16 & 5 \end{cases} \quad \text{with} \quad \mathbb{Z}_2^{(0)} \equiv \mathbb{1}$$

- wrapping numbers *a priori* A_a^i, $B_a^i \in \frac{1}{8}\,\mathbb{Z}$

A Pati-Salam model on $T^6/\mathbb{Z}_2 \times \mathbb{Z}_6'$: $\mathbb{Z}_n$ conditions

$$
\sum_a k_a N_a \left(\begin{array}{c}
Y_a \\ \hline
-y^{(1)}_{a,1} \\
-y^{(1)}_{a,2} \\
-y^{(1)}_{a,3} \\
-(y^{(1)}_{a,4} + y^{(1)}_{a,5}) \\
2\,(x^{(1)}_{a,4} - x^{(1)}_{a,5}) + (y^{(1)}_{a,4} - y^{(1)}_{a,5}) \\ \hline
-y^{(2)}_{a,1} \\
-y^{(2)}_{a,2} \\
-y^{(2)}_{a,3} \\
-(y^{(2)}_{a,4} + y^{(2)}_{a,5}) \\
2\,(x^{(2)}_{a,4} - x^{(2)}_{a,5}) + (y^{(2)}_{a,4} - y^{(2)}_{a,5}) \\ \hline
-y^{(3)}_{a,1} \\
-y^{(3)}_{a,2} \\
-y^{(3)}_{a,3} \\
-(y^{(3)}_{a,4} + y^{(3)}_{a,5}) \\
2\,(x^{(3)}_{a,4} - x^{(3)}_{a,5}) + (y^{(3)}_{a,4} - y^{(3)}_{a,5})
\end{array}\right)
\stackrel{!}{=} 0 \bmod n \stackrel{!}{=} \sum_a k_a N_a
\left(\begin{array}{c}
\frac{Y_a - \sum_{i=1}^3 [y^{(i)}_{a,1} + y^{(i)}_{a,2} + y^{(i)}_{a,3}]}{4} \\
\frac{Y_a - [y^{(1)}_{a,1} + y^{(1)}_{a,2} + y^{(1)}_{a,3}]}{2} \\
\frac{Y_a - [y^{(2)}_{a,1} + y^{(2)}_{a,2} + y^{(2)}_{a,3}]}{2} \\
-\frac{y^{(2)}_{a,1} + y^{(2)}_{a,3} + y^{(3)}_{a,1} + y^{(3)}_{a,3}}{2} \\
-\frac{y^{(1)}_{a,1} + y^{(1)}_{a,3} + y^{(3)}_{a,2} + y^{(3)}_{a,3}}{2} \\
-\frac{y^{(1)}_{a,2} + y^{(1)}_{a,3} + y^{(2)}_{a,2} + y^{(2)}_{a,3}}{2} \\ \hline
\frac{Y_a + [y^{(1)}_{a,3} + x^{(1)}_{a,4} + y^{(1)}_{a,4} - x^{(1)}_{a,5}] + \sum_{j=2}^3 [y^{(j)}_{a,2} - (y^{(j)}_{a,4} + y^{(j)}_{a,5})]}{4} \\
\frac{Y_a + \sum_{j=1,2} [y^{(j)}_{a,1} - x^{(j)}_{a,4} + x^{(j)}_{a,5} + y^{(j)}_{a,5}] + [y^{(3)}_{a,3} + x^{(3)}_{a,4} + y^{(3)}_{a,4} - x^{(3)}_{a,5}]}{4} \\
\frac{Y_a + [y^{(2)}_{a,2} - (y^{(2)}_{a,4} + y^{(2)}_{a,5})]}{2} \\
\frac{Y_a + [y^{(1)}_{a,1} - x^{(1)}_{a,4} + x^{(1)}_{a,5} + y^{(1)}_{a,5}]}{2} \\
-\frac{y^{(2)}_{a,4} + y^{(2)}_{a,5} + y^{(3)}_{a,4} + y^{(3)}_{a,5}}{2} \\
-\frac{x^{(1)}_{a,4} - x^{(1)}_{a,5} + y^{(1)}_{a,5} + x^{(2)}_{a,4} - x^{(2)}_{a,5} + y^{(2)}_{a,5}}{2} \\ \hline
\frac{Y_a + \sum_{i=1}^3 [y^{(i)}_{a,3} + x^{(i)}_{a,4} + y^{(i)}_{a,4} - x^{(i)}_{a,5}]}{4} \\
\frac{Y_a + y^{(1)}_{a,3} + x^{(1)}_{a,4} + y^{(1)}_{a,4} - x^{(1)}_{a,5}}{2} \\
\frac{Y_a + y^{(2)}_{a,3} + x^{(2)}_{a,4} + y^{(2)}_{a,4} - x^{(2)}_{a,5}}{2} \\
\frac{Y_a + \sum_{i=1}^3 y^{(i)}_{a,3}}{2}
\end{array}\right)
$$

A Pati-Salam model on $T^6/\mathbb{Z}_2 \times \mathbb{Z}_6'$: spectrum

GH, Ripka, Staessens '12

$$SU(4)_a \times SU(2)_b \times SU(2)_c \times SU(2)_d \times SU(2)_e \times U(1)^5_{\text{massive}}$$

- Standard Model particles plus **one Higgs**

$$(\mathbf{4},\overline{\mathbf{2}},\mathbf{1};\mathbf{1},\mathbf{1})+2\,(\mathbf{4},\mathbf{2},\mathbf{1};\mathbf{1},\mathbf{1})+(\overline{\mathbf{4}},\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1})+2\,(\overline{\mathbf{4}},\mathbf{1},\overline{\mathbf{2}};\mathbf{1},\mathbf{1})+(\mathbf{1},\mathbf{2},\overline{\mathbf{2}};\mathbf{1},\mathbf{1})$$

$\rightsquigarrow$ **one massive generation** at leading order by charge selection rules

- chiral w.r.t. anomalous $U(1)^5_{\text{massive}}$

$$(\mathbf{1},\mathbf{2},\mathbf{1};\overline{\mathbf{2}},\mathbf{1})+3(\mathbf{1},\overline{\mathbf{2}},\mathbf{1};\overline{\mathbf{2}},\mathbf{1})+(\mathbf{1},\overline{\mathbf{2}},\mathbf{1};\mathbf{1},\overline{\mathbf{2}})+(\mathbf{1},\mathbf{1},\overline{\mathbf{2}};\mathbf{2},\mathbf{1})+3(\mathbf{1},\mathbf{1},\mathbf{2};\mathbf{2},\mathbf{1})+(\mathbf{1},\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{2})$$

but non-chiral w.r.t. $SU(4)_a \times SU(2)_b \times SU(2)_c$

- non-chiral w.r.t. to full $U(4)_a \times U(2)^4$ with **GUT Higgses**

$$2\,\left[(\mathbf{4},\mathbf{1},\mathbf{1};\overline{\mathbf{2}},\mathbf{1})+h.c.\right]+\left[(\mathbf{1},\mathbf{1},\mathbf{1};\mathbf{2},\mathbf{2})+h.c.\right]+(\mathbf{1},\mathbf{1},\mathbf{1};\mathbf{4}_{\mathbf{Adj}},\mathbf{1})$$
$$+\,2\,\left[(\mathbf{1},\mathbf{1},\mathbf{1};\mathbf{3}_{\mathbf{S}},\mathbf{1})+(\mathbf{1},\mathbf{1},\mathbf{1};\mathbf{1}_{\mathbf{A}},\mathbf{1})+h.c.\right]+\left[(\mathbf{1},\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{3}_{\mathbf{S}})+(\mathbf{1},\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1}_{\mathbf{A}})+h.c.\right]$$

Pati-Salam model cont'd: $\mathbb{Z}_n$ Symmetries in $U(1)^5_{\text{massive}}$

- 5 independent $\mathbb{Z}_n$ symmetries ($h_{21} = 15$) G.H., Staessens '13
- family-independent & trivial:
  - $\mathbb{Z}_4 \subset U(1)_a \subset U(4)_a$
  - $\mathbb{Z}_2 \subset U(1)_x \subset U(2)_{x, x \in \{b,c,d,e\}}$
    - $\mathbb{Z}_2 \subset U(1)_c$: **R-parity** $\mathcal{R}_2$
    - $\mathbb{Z}_2 \subset U(1)_{d,e}$: only non-trivial on exotic matter
- **family-dependent**:
  - $\mathbb{Z}_4 \subset \frac{1}{2} \sum_{x \in \{b,c,d,e\}} U(1)_x \rightsquigarrow$ **selection** rule on **Yukawas**

Discrete charges for the five-stack Pati-Salam model on $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_6' \times \Omega\mathcal{R})$

Discrete symmetries		**Charge assignment for the 'chiral' states**											
$\mathbb{Z}_n$	$U(1) = \sum_x k_x U(1)_x$	(Q_L, L) ab	ab'	(Q_R, R) ac	ac'	(H_d, H_u)	X_{bd}	$X_{bd'}$	$X_{be'}$	X_{cd}	$X_{cd'}$	$X_{ce'}$
$\mathbb{Z}_2$	$U(1)_e$	0	0	0	0	0	0	0	1	0	0	1
	$U(1)_d$	0	0	0	0	0	1	1	0	1	1	0
	$U(1)_c$	0	0	1	1	1	0	0	0	1	1	1
	$U(1)_b$	1	1	0	0	1	1	1	1	0	0	0
$\mathbb{Z}_4$	$U(1)_a$	1	1	3	3	0	0	0	0	0	0	0
	$U(1)_b + U(1)_c + U(1)_d + U(1)_e$	3	1	1	3	0	0	2	2	0	2	2

Reduction of the *Family Dependent* Symmetry: $\mathbb{Z}_4 \to \mathbb{Z}_2$

- unwritten lore: **mod out centers** of $SU(N)$:
  $((\mathbb{Z}_4)^2 \times (\mathbb{Z}_2)^3)/(\mathbb{Z}_4 \times (\mathbb{Z}_2)^4) \simeq \boxed{\mathbb{Z}_2}$
- search consistent charge assignment by hand:
  - $(\mathbf{4}, \overline{\mathbf{2}}, \mathbf{1}, \mathbf{1}, \mathbf{1}).(\overline{\mathbf{4}}, \mathbf{1}, \mathbf{2}, \mathbf{1}, \mathbf{1}).(\mathbf{1}, \mathbf{2}, \overline{\mathbf{2}}, \mathbf{1}, \mathbf{1})$ *perturbatively* allowed
  - $(\mathbf{4}, \overline{\mathbf{2}}, \mathbf{1}, \mathbf{1}, \mathbf{1}).(\overline{\mathbf{4}}, \mathbf{1}, \mathbf{1}, \mathbf{2}, \mathbf{1}).(\mathbf{1}, \overline{\mathbf{2}}, \mathbf{1}, \overline{\mathbf{2}}, \mathbf{1})$ *pert.* forbidden by $U(1)_b$
    - $\mathbb{Z}_4$ charge: 2 mod 4
  - $(\mathbf{4}, \overline{\mathbf{2}}, \mathbf{1}, \mathbf{1}, \mathbf{1}).(\overline{\mathbf{4}}, \mathbf{1}, \overline{\mathbf{2}}, \mathbf{1}, \mathbf{1}).(\mathbf{1}, \mathbf{2}, \overline{\mathbf{2}}, \mathbf{1}, \mathbf{1})$ *pert.* forbidden by $U(1)_c$
  - ...

	(Q_L, L) ab	ab'	(Q_R, R) ac	ac'	(H_d, H_u)	X_{bd}	$X_{bd'}$	$X_{be'}$	X_{cd}	$X_{cd'}$	$X_{ce'}$
$\mathbb{Z}_2$	0	1	0	1	0	0	1	1	0	1	1

- $\mathbb{Z}_2$ remains **family-dependent**
- **cannot** be obtained from 'mod 2' on $\mathbb{Z}_4$ charges
  $\rightsquigarrow$ unwritten lore doesn't really help

General Characteristics of Global D6-Brane Models

Global D-brane models: GH, Staessens '13

- $U(1)_{B-L}$ in L-R sym. models makes $\mathbb{Z}_2$'s (R-parity) trivial
- L-R models: $\mathbb{Z}_3 \subset U(1)_a \subset U(3)_a$ trivial
- Pati-Salam models: no $\mathbb{Z}_3 \not\subset U(1)_a \subset U(4)_a$

- most $\mathbb{Z}_n$ give no new coupling selection rules beyond $SU(N)$
- **family-dependent** $\mathbb{Z}_n$ for *very special* D-brane set-up
  - *naive way* to mod out centers of $SU(N)$ *wrong!*

Axions, CP, Dark Sector

Axions and the Strong CP Problem

- **Axions** originally invoked to solve **strong CP-problem**

  $\mathcal{L}_\alpha \supset \frac{1}{2}\left(\partial_\mu \alpha\right)\left(\partial^\mu \alpha\right) - \frac{1}{32\pi^2}\frac{\alpha(x)}{f_\alpha}\,\mathrm{Tr}\left(G_{\mu\nu}\tilde{G}^{\mu\nu}\right)$

  - *global* Pecci-Quinn symmetry $U(1)_{PQ}$ — Pecci, Quinn '77
  - **axion** α arises from rewriting two Higgs doublets
  - ~~electro-weak~~ & ~~PQ~~ scales identical
  - axions $\leftrightarrow$ photon conversion assumed *(Primakoff effect)*
    $\rightsquigarrow$ astrophysical & lab searches (e.g. ALPs@DESY)

    experimentally excluded

- modified models contain **SM singlet field** σ
  - σ couples to Higgs doublets $\rightsquigarrow$ new terms in V_{Higgs}
  - ~~PQ~~ by $\langle\sigma\rangle$ at higher energy than $SU(2)_L \times U(1)_Y$

  e.g. Zhitnitsky '80; Dine, Fischler, Srednicki '81; . . . ; Dreiner, Staub, Ubaldi '14

- realisation in **D-brane models** — cf. Berenstein, Perkins '12
  - $U(1)_{PQ} \rightarrow U(1)_{\text{massive}}$
  - 'exotic' scalars abundant - adjustments to **SUSY** required
  - suitable SUSY breaking minimum of V_{Higgs}?

  GH, Staessens '13

Open String Axions & DFSZ Model

- $U(1)_{PQ}$ must allow:

$$\mathcal{L}_{\text{Yukawa}} = f_u\, Q_L \cdot H_u\, u_R + f_d\, Q_L \cdot H_d\, d_R + f_e\, L \cdot H_d\, e_R + f_\nu\, L \cdot H_u\, \nu_R$$

- introduce **SM singlet** σ with $U(1)_{PQ} \simeq U(1)_{\text{massive}}$ charge
- (H_u, H_d) charged under $U(1)_{PQ}$

  $\rightsquigarrow Q_L$ **or** (u_R, d_R) have $U(1)_{PQ}$ charge

- **Higgs potential** of the DFSZ model

$$\begin{aligned} V_{\text{DFSZ}}(H_u, H_d, \sigma) \quad = \quad & \lambda_u (H_u^\dagger H_u - v_u^2)^2 + \lambda_d (H_d^\dagger H_d - v_d^2)^2 + \lambda_\sigma (\sigma^* \sigma - v_\sigma^2)^2 \\ & + (a\, H_u^\dagger H_u + b\, H_d^\dagger H_d)\, \sigma^* \sigma + c\, (H_u \cdot H_d\, \sigma^2 + h.c.) \\ & + d\, |H_u \cdot H_d|^2 + e\, |H_u^\dagger H_d|^2 \end{aligned}$$

- **SUSY** version: $V = V_F + V_D + V_{\text{soft}}$
- modify $c\,(H_u \cdot H_d\, \sigma^2 + h.c.) \longrightarrow c\,(H_u \cdot H_d\, \sigma + h.c.)$; $\sigma \sim e^{ia}$

Matter	Q_L	$\bar{u}_R$	$\bar{d}_R$	H_u	H_d	L	$\bar{e}_R$	$\bar{\nu}_R$	Σ
$U(1)_{PQ}$	∓ 1	0	0	± 1	± 1	∓ 1	0	0	∓ 2

- identify $\Sigma = (\textbf{Anti})_{U(2)_b}$ in global D-brane model

e.g. SM on $T^6/\mathbb{Z}_6$: **GH**, Ott '04 & $T^6/\mathbb{Z}_6'$: Gmeiner, **GH** '08

Mixing of Open and Closed String Axions

GH, Staessens '13

- open string axion a from $\sigma = \frac{v+s(x)}{\sqrt{2}} e^{i\frac{a(x)}{v}}$
- **open** axion a mixes with **closed** axion ξ ($\leftarrow U(1)_{\text{massive}}$)

$$\zeta = \frac{M_{\text{string}}\,\xi + qv\,a}{\sqrt{M_{\text{string}}^2 + q^2 v^2}}, \qquad \alpha = \frac{M_{\text{string}}\,a - qv\,\xi}{\sqrt{M_{\text{string}}^2 + q^2 v^2}}$$

$$\Rightarrow \qquad \mathcal{L}_{\text{CP-odd}} = \frac{1}{2}\left(\partial_\mu \zeta + m_B B_\mu\right)^2 + \frac{1}{2}(\partial_\mu \alpha)^2$$

- axion **decay constant** f_α from dim. reduction:
$\mathcal{L}_{\rm anom} = \frac{1}{16\pi^2}\frac{\zeta(x)}{f_\zeta}\mathrm{Tr}(G_{\mu\nu}\tilde{G}^{\mu\nu}) + \frac{1}{32\pi^2}\frac{\alpha(x)}{f_\alpha}\mathrm{Tr}(G_{\mu\nu}\tilde{G}^{\mu\nu})$

$$\text{with} \qquad f_\zeta = \frac{\sqrt{M_{\text{string}}^2 + (qv)^2}}{2}, \qquad f_\alpha = \frac{M_{\text{string}}\, qv \sqrt{M_{\text{string}}^2 + (qv)^2}}{(M_{\text{string}}^2 - (qv)^2)}$$

- For $\boxed{M_{\text{string}} \gg v}$: $\zeta \simeq \xi_{\text{closed}}$, $\alpha \simeq a_{\text{open}}$

Soft ~~SUSY~~ Terms

Origin of $\boxed{V = V_F + V_D + V_{\rm soft}}$

$$V_{\rm DFSZ}(H_u, H_d, \sigma) = \lambda_u(H_u^\dagger H_u - v_u^2)^2 + \lambda_d(H_d^\dagger H_d - v_d^2)^2 + \lambda_\sigma(\sigma^*\sigma - v_\sigma^2)^2$$
$$+(a\, H_u^\dagger H_u + b\, H_d^\dagger H_d)\,\sigma^*\sigma + c\,(H_u \cdot H_d\,\sigma + h.c.)$$
$$+d\,|H_u \cdot H_d|^2 + e\,|H_u^\dagger H_d|^2$$

in **SUSY field theory**

- $\mathcal{W} = \mu \Sigma H_d \cdot H_u$
- $K^{\rm SUSY}(\Phi^\dagger e^{2gV}\Phi) = \Phi^\dagger e^{2gV}\Phi$
- $\mathcal{W}_{\rm soft} = \eta\, c H_u \cdot H_d\, \Sigma \rightsquigarrow \mathcal{A}$-terms
- $K_{\rm soft} = \eta\bar{\eta}\, m_\Phi^2\, \Phi^\dagger e^{2gV}\Phi \rightsquigarrow m_{\rm soft}$

in **Type II string models**

- strongly coupled hidden group e.g. $USp(6)$ in $T^6/\mathbb{Z}_6'$ model
- gaugino condensate: $\langle \lambda\lambda \rangle = \Lambda_c^3 \rightsquigarrow M^2_{\not{SUSY}} = \langle F^{\mathcal{H}} \rangle \sim \frac{\Lambda_c^3}{M_{\rm Planck}}$
- gravity mediation to SM sector

Lower Bounds on M_{string}

- typical phenomenological constraints from $f_\zeta \sim M_{\text{string}}$, $f_\alpha \sim qv$: $M_{\text{string}} \geq 10^9$ GeV
- supplemented by constraints on gauge couplings
  - $\frac{M^2_{\text{Planck}}}{M^2_{\text{string}}} \stackrel{\text{examples}}{=} \frac{4\pi \; v_1 v_2 v_3}{g^2_{\text{string}}}$
  - @ tree-level: $\frac{4\pi}{g^2_{SU(N_a)}} = \frac{\sqrt{v_1 v_2 v_3}}{8\pi^3 3^{1/4} \, g_{\text{string}}} \times \mathcal{O}(1)_{\text{model}}$
  - @ 1-loop: linear dep. on v_i, $\ln \frac{v_1 v_3}{v_2^2}$ $\Leftarrow$ cancellations possible

$\rightsquigarrow$ M_{string} can be lowered to intermediary scale by exponentially large volumes:

GH, Staessens '13

M_{string} as a function of v_i and g_{string}								
$g_{\text{string}} = 0.1$			$g_{\text{string}} = 0.01$			$g_{\text{string}} = 0.001$		
$v_1 v_3$	$v^2_{2,\max}$	M_{string}	$v_1 v_3$	$v^2_{2,\max}$	M_{string}	$v_1 v_3$	$v^2_{2,\max}$	M_{string}
10^8	9.7×10^9	1.6×10^{10} GeV	10^6	1.5×10^{10}	1.6×10^{10} GeV	10^2	1.5×10^6	1.6×10^{12} GeV
10^{10}	1.5×10^{14}	2.8×10^9 GeV	10^8	1.6×10^{14}	1.5×10^8 GeV	10^4	1.6×10^{10}	1.5×10^{10} GeV
10^{12}	1.5×10^{18}	2.8×10^8 GeV	10^{10}	1.6×10^{18}	1.5×10^6 GeV	10^6	1.6×10^{14}	1.5×10^8 GeV

Conclusions:

- Conditions on $\boxed{\mathbb{Z}_n}$ expressed via **intersection numbers**:
  - independent of choice of basis & parameterisation: **correct normalisations** (cross-check: K-theory $\leftrightarrow \mathbb{Z}_2$)
  - many 'probe branes', but only $(h_{21}+1)$ conditions per orbifold
- $\mathbb{Z}_N \subset U(N)$ automatic
- $U(1)_{\text{massless}}$, e.g. $Y, (B-L)$: $\rightsquigarrow$ many $\mathbb{Z}_n$ **trivial**
- **Pati-Salam** example:
  - R-parity $\subset U(2)_R$
  - **family-dependent** $\mathbb{Z}_4$ $(\mathbb{Z}_2)$ constrains Yukawas

. . . more details in **GH**, Staessens '13

- $U(1)_{PQ} \simeq U(1)_{\text{massive}}$
- Mixing of **axions** from open/closed string sector
- intermediary M_{string} and exponentially large volumes?

. . . more details in **GH**, Staessens '13

The String Theory Universe

& 20th European Workshop on String Theory
2nd COST MP1210 Meeting

22–26 September 2014
Philosophicum, JGU Mainz

www.strings2014.uni-mainz.de

The conference is dedicated to all aspects of superstring, supergravity and supersymmetric theories and is embedded in the MITP programme **String Theory and its Applications**.

Organizers
Johanna Erdmenger | Munich
Mirjam Cvetič | Philadelphia
Fernando Marchesano | Madrid
Carlos Núñez | Swansea
Timo Weigand | Heidelberg

Local Organizer
Gabriele Honecker | Mainz

International Advisory Committee
Ana Achúcarro | Leiden
Matthias Blau | Bern
Jan de Boer | Amsterdam
Anna Ceresole | Torino
Roberto Emparan | Barcelona
Jerome Gauntlett | London
Elias Kiritsis | Heraklion
Charlotte Kristjansen | Copenhagen
María A. Lledó | Valencia
Yolanda Lozano | Oviedo
Dieter Lüst | Munich
Silvia Penati | Milano
Antoine Van Proeyen | Leuven

Overview Talks
Paul Chesler | Harvard
Fernando Marchesano | Madrid
Dario Martelli | London
Tadashi Takayanagi | Kyoto
Ivonne Zavala | Groningen

Special Interest Talks
Lutz Köpke | Mainz
IceCube Neutrino Observatory

Ana Achúcarro | Leiden
Strings and the Cosmic Microwave Background

MITP Public Lecture
Dieter Lüst | Munich
Strings im Multiversum
Mainzer Wissenschaftsmarkt
Saturday, 13 September 2014 at 6pm.

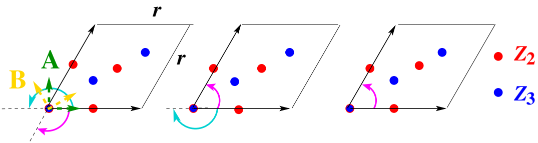
Working Groups
Gauge/Gravity Duality
String Phenomenology
Cosmology and Quantum Gravity

mitp Mainz Institute for Theoretical Physics

Technical Details

IIA/$\Omega\mathcal{R}$ on $T^6/\mathbb{Z}_2 \times \mathbb{Z}_6'$ with discrete torsion: geometry

$\mathbb{Z}_2 \times \mathbb{Z}_6'$ shifts: $\vec{v} = (\frac{1}{2}, \frac{-1}{2}, 0)$, $\vec{w}' = (\frac{-1}{3}, \frac{1}{6}, \frac{1}{6})$ on $SU(3)^3$



- $\boxed{\Pi_a^{\rm rigid} = \frac{1}{4}\left(\Pi_a^{\rm bulk} + \sum_{i=1}^3 \Pi_a^{\mathbb{Z}_2^{(i)}}\right)}$
- $\boxed{\Pi_a^{\rm bulk} = X_a\, \rho_1 + Y_a\, \rho_2}$ with

Förste, G.H. JHEP 1101 (2011) 091

$$X_a \equiv n_a^1 n_a^2 n_a^3 - m_a^1 m_a^2 m_a^3 - \sum_{i\neq j\neq k\neq i} n_a^i m_a^j m_a^k \in \mathbb{Z}, \qquad Y_a \equiv \sum_{i\neq j\neq k\neq i} \left(n_a^i n_a^j m_a^k + n_a^i m_a^j m_a^k\right) \in \mathbb{Z}$$

$$\rho_1 \equiv 2\sum_{k=0}^{5} \omega^k(\pi_{135}), \qquad \rho_1 \equiv 2\sum_{k=0}^{5} \omega^k(\pi_{136}) \qquad \text{with} \quad \rho_1 \circ \rho_2 = 4$$

$T^6/\mathbb{Z}_2 \times \mathbb{Z}_6'$ geometry cont'd

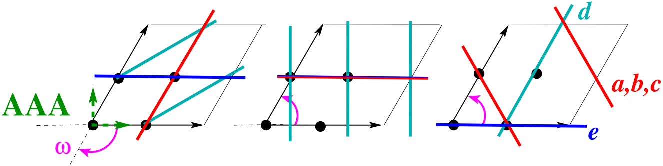
- $\boxed{\Pi_a^{\text{rigid}} = \frac{1}{4}\left(\Pi_a^{\text{bulk}} + \sum_{i=1}^3 \Pi_a^{\mathbb{Z}_2^{(i)}}\right)}$
- $\boxed{\Pi^{\mathbb{Z}_2^{(i)}} = \sum_{\alpha=1}^5 \left(x^i_{\alpha,a}\,\varepsilon_\alpha^{(i)} + y^i_{\alpha,a}\,\tilde{\varepsilon}_\alpha^{(i)}\right)}$ with
  - 3 equivalent $\mathbb{Z}_2^{(i)}$ twisted sectors:
    $\varepsilon^{(i)}_{\alpha=1} = 2\sum_{k=0}^2 \omega^k (e_{41}{}^{(i)} \otimes \pi_{2i-1})$,
    $\tilde{\varepsilon}^{(i)}_{\alpha=1} = 2\sum_{k=0}^2 \omega^k (e_{41}{}^{(i)} \otimes \pi_{2i})$
    with $\varepsilon_\alpha^{(i)} \circ \tilde{\varepsilon}^{(j)}_{\beta=1} = -4\,\delta^{ij}\,\delta_{\alpha\beta}$
  - exceptional wrappings $(x^i_{\alpha,a}, y^i_{\alpha,a}) \sim (n^i_a, m^i_a)$
  - sign factors from
    - $\mathbb{Z}_2$ eigenvalues ± 1
    - Wilson lines $\tau \in \{0,1\}$
- example for a short $\Omega\mathcal{R}$-even cycle:

$$\Pi^{\text{frac},\Omega\mathcal{R}}_{(\sigma^i)=(1,1,1)} \overset{\tau^i\equiv\tau}{=} \frac{\Pi_0^{\text{even}}}{4} + \sum_{i=1}^3 \frac{(-1)^{\tau^{\mathbb{Z}_2^{(i)}}}}{4}\left(-\Pi_3^{\text{even},\mathbb{Z}_2^{(i)}} + (-1)^\tau \frac{-\Pi_4^{\text{even},\mathbb{Z}_2^{(i)}} + \Pi_5^{\text{even},\mathbb{Z}_2^{(i)}}}{2}\right)$$

A typical global Pati-Salam model on $T^6/\mathbb{Z}_2 \times \mathbb{Z}_6'$

G.H., Ripka, Staessens '12

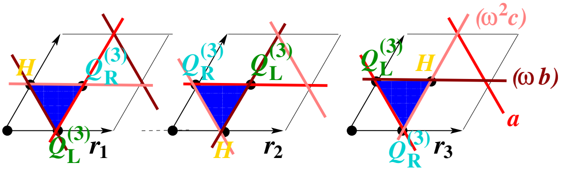
brane	$(n^i, m^i)_{i=1,2,3}$	$\mathbb{Z}_2$	$(\vec{\tau})$	$(\vec{\sigma})$	group	(X, Y)
a		$(+++)$	$(0,0,1)$		$U(4)_a$	
b	(0,1;1,0,1,-1)	$(--+)$	$(0,1,1)$	$(\vec{1})$	$U(2)_b$	(1,0)
c		$(-+-)$	$(1,0,1)$		$U(2)_c$	
d	(1,1;1,-2;0,1)	$(+++)$	$(0,0,1)$	$(\vec{1})$	$U(2)_d$	(3,0)
e	(1,0;1,0;1,0)	$(+--)$	$(1,1,1)$	$(1,1,0)$	$U(2)_e$	(1,0)



- a, b, c at $(\frac{\pi}{3}, 0, -\frac{\pi}{3})$, d at $(\frac{\pi}{6}, -\frac{\pi}{2}, \frac{\pi}{3})$ e at $(0,0,0)$
- all $U(1)^5$ anomalous & massive at $M_{\text{string}} \leftrightarrow h_{21} = 15(\mathbb{Z}_2)$
- $SU(4)_a \times SU(2)_b \times SU(2)_c \times SU(2)_d \times SU(2)_e$ with
  - 3 generations of quarks + leptons
  - **one Higgs** (H_d, H_u)
  - ~~**Adj**~~ on $a, b, c, e \longleftrightarrow 1 \times \mathbf{Adj}_d$

Yukawa interactions for the typical Pati-Salam model

- charge selection rules not sufficient on $T^6/\mathbb{Z}_{2N}$, $T^6/\mathbb{Z}_2 \times \mathbb{Z}_{2M}$ due to various sectors $a(\omega^k b)_{k\in\{0,1,2\}}$ G.H., Vanhoof '12



- Pati-Salam model: one heavy generation by G.H., Ripka, Staessens '12
  $W_{Q_L^{(3)} Q_R^{(3)} H} \sim e^{-\sum_{i=1}^3 v_i/8}$ with Kähler moduli $v_i \equiv \frac{\sqrt{3}}{2}\frac{r_i^2}{\alpha'}$
- non-chiral $\left[(\mathbf{4},\mathbf{1},\mathbf{1};\overline{\mathbf{2}},\mathbf{1}) + (\mathbf{1},\mathbf{1},\mathbf{1};\mathbf{2},\mathbf{2}) + (\mathbf{1},\mathbf{1},\mathbf{1};\mathbf{1_A},\mathbf{1}) + h.c.\right]$ massive via couplings to $(\mathbf{1},\mathbf{1},\mathbf{1};\mathbf{4_{Adj}},\mathbf{1})$
- several types of $(\mathbf{1},\mathbf{2}_x,\mathbf{2}_y,\mathbf{1},\mathbf{1})_{x,y\in\{b,c,d,e\}}$ massive through 3-point couplings among each other and with SM Higgs
- other masses through higher order or non-perturbative (instanton) couplings $\rightsquigarrow$ need to be computed!