# Branes @ singularities
# Discrete Symmetries

Amihay Hanany

# Branes @ singular CY cone

- When a D3 brane is placed at the tip of a singular CY cone
- Splits into a collection of fractional branes
- Open strings stretched between fractional branes
- Open strings combine to form a closed string

# Perturbative Open Strings

- Supersymmetry is N=1 in 3+1d
- A fractional brane has a U(n) gauge theory - vector multipet
- An open string has a collection of massless modes - chiral multiplets
- Open strings forming a closed string give interactions

# Low energy effective theory on the brane

- Quiver Gauge Theories (Open strings)
- Superpotentials given as sums over monomials of chiral multiplets
- Each such term represents a process where a collection of open strings combine to form a closed string

# Quiver Gauge Theories and CY singularities

- The details of the quiver and the superpotential depend on the CY singularity
- Develop a method to compute this efficiently for a given singularity

# CY singularities

- No known classification, but classes
- Orbifolds of $C^3$
- Abelian
- Non Abelian
- Toric singularities
- del Pezzo singularities

# Example: Quiver, W

## 18 Model 16: $\mathbb{C}^3/\mathbb{Z}_3$ $(1,1,1)$, $\mathbf{dP}_0$

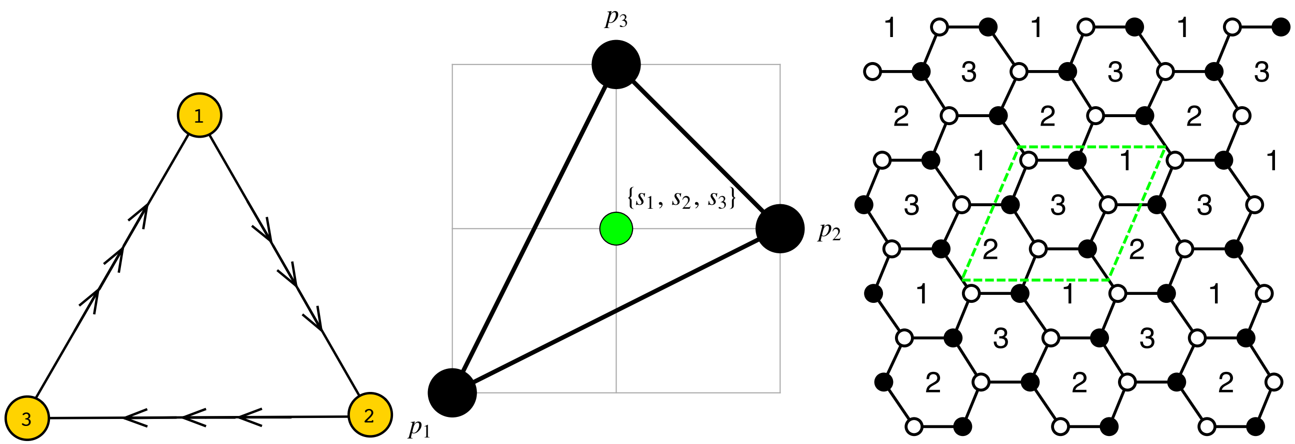



**Figure 36**. The quiver, toric diagram, and brane tiling of Model 16.

The superpotential is

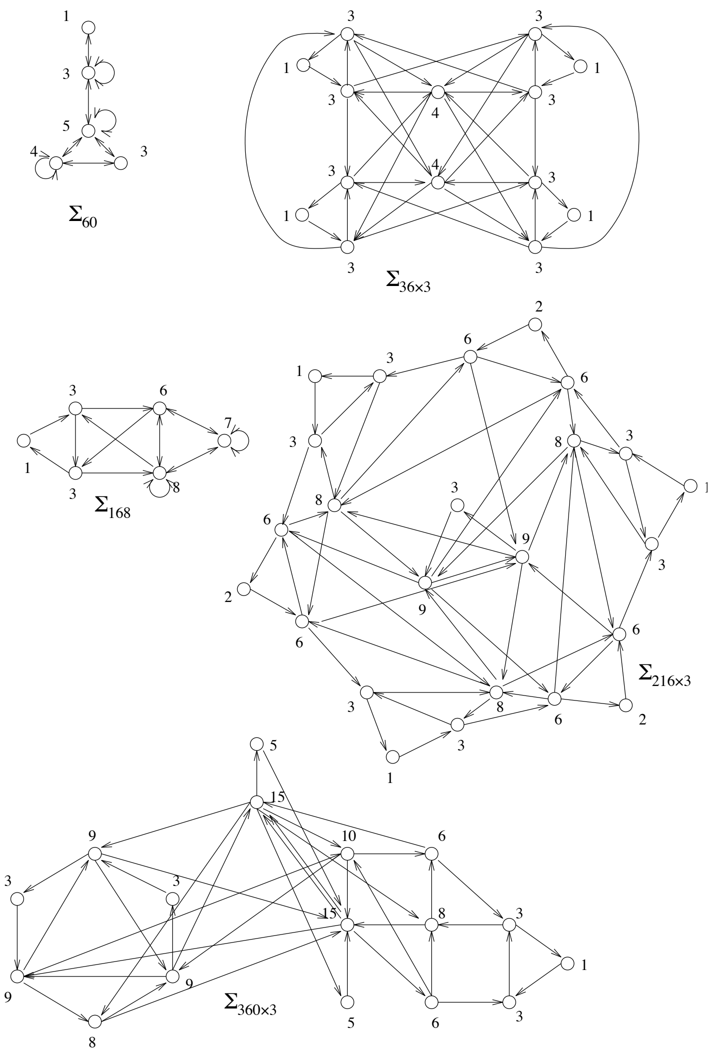
$$
\begin{aligned}
W = & +X^1_{12}X^3_{23}X^2_{31} + X^2_{12}X^1_{23}X^3_{31} + X^3_{12}X^2_{23}X^1_{31} \\
& -X^1_{12}X^1_{23}X^1_{31} - X^3_{12}X^3_{23}X^3_{31} - X^2_{12}X^2_{23}X^2_{31}
\end{aligned}
\tag{18.1}
$$

# An orbifold

- one fractional brane per irreducible representation of the discrete group
- rank of gauge group - dimension of irrep
- Quiver encodes the tensor product of irreducible representations with respect to a defining 3 dimensional representation

# Classification of orbifolds of $C^3$

- 4 infinite series: Z(n)
- Z(n) x Z(m)
- $\Delta(3n^2)$
- $\Delta(6n^2)$
- 5 exceptional discrete groups of orders 60, 108, 168, 648, 1080




e 5: $\Sigma \subset$ full $SU(3)$. Only $\Sigma_{36\times3,216\times3,360\times3}$ belong only to the full $SU(3$
ve the one loop $\beta$-function vanishing condition manifesting as the label of
ng to $\frac{1}{3}$ of that of the incoming and outgoing neighbours respectively.
entation for these graphs are given in Appendix IV.

# Counting Abelian Orbifolds

| | $\mathbb{C}^D/\Gamma_N$ | | | | | |
|---|---|---|---|---|---|---|
| | $D$ | | | | | |
| | 2 | 3 | 4 | 5 | 6 | 7 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 2 | 1 | 1 | 2 | 2 | 3 | 3 |
| 3 | 1 | 2 | 3 | 4 | 6 | 7 |
| 4 | 1 | 3 | 7 | 10 | 17 | 23 |
| 5 | 1 | 2 | 5 | 8 | 13 | 19 |
| 6 | 1 | 3 | 10 | 19 | 40 | 65 |
| 7 | 1 | 3 | 7 | 13 | 27 | 46 |
| 8 | 1 | 5 | 20 | 45 | 106 | |
| 9 | 1 | 4 | 14 | 33 | 72 | |
| 10 | 1 | 4 | 18 | 47 | 127 | |
| 11 | 1 | 3 | 11 | 30 | 79 | |
| 12 | 1 | 8 | 41 | 129 | 391 | |
| 13 | 1 | 4 | 15 | 43 | 129 | |
| 14 | 1 | 5 | 28 | 96 | 321 | |
| 15 | 1 | 6 | 31 | 108 | | |
| 16 | 1 | 9 | 58 | 224 | | |
| 17 | 1 | 4 | 21 | 78 | | |
| 18 | 1 | 8 | 60 | 264 | | |
| 19 | 1 | 5 | 25 | 102 | | |
| 20 | 1 | 10 | 77 | 357 | | |
| 21 | 1 | 8 | 49 | 226 | | |
| 22 | 1 | 7 | 54 | 277 | | |
| 23 | 1 | 5 | 33 | 163 | | |
| 24 | 1 | 15 | 144 | 813 | | |
| $N$ 25 | 1 | 7 | 50 | 260 | | |
| 26 | 1 | 8 | 72 | 425 | | |
| 27 | 1 | 9 | 75 | 436 | | |
| 28 | 1 | 13 | 123 | 780 | | |
| 29 | 1 | 6 | 49 | 297 | | |
| 30 | 1 | 14 | 158 | 1092 | | |
| 31 | 1 | 7 | 55 | | | |

# Large order behavior

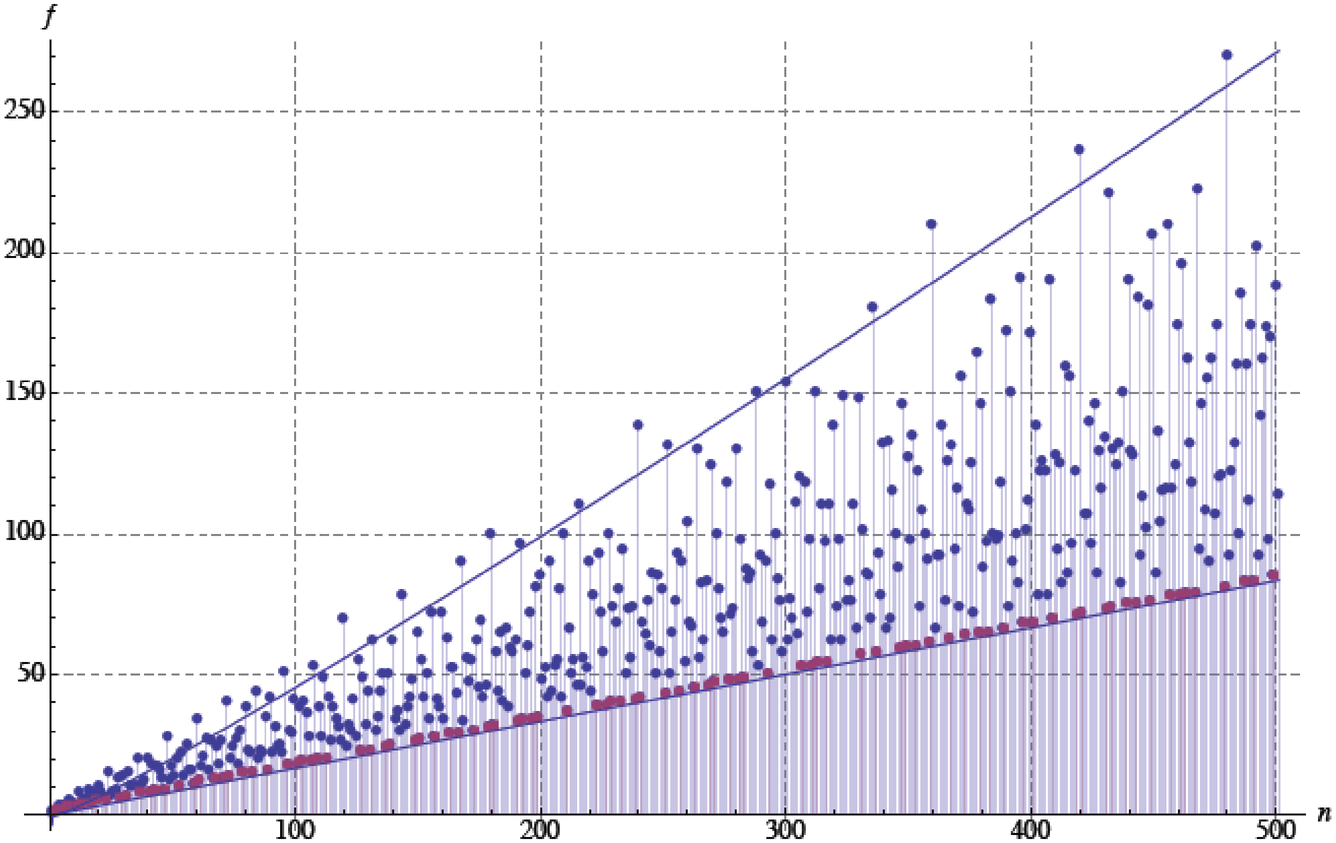



Figure 1: Scatter plot of the sequence $f^{\triangle}$ for a hexagonal lattice. Prime numbers are emphasized in red. The two lines correspond to $n/6$ and $e^{\gamma} n \log \log n/6$.

# Toric Singularities

- a toric singularity is a fibration of a 3 torus over a 3 dimensional real base.
- The data of the singularity is encoded in terms of a polygon with vertices on a 2 dimensional lattice
- Abelian orbifolds of C3 have toric diagrams which are triangles

# Example: Quiver, W

## 18 Model 16: $\mathbb{C}^3/\mathbb{Z}_3$ $(1,1,1)$, $\mathbf{dP}_0$

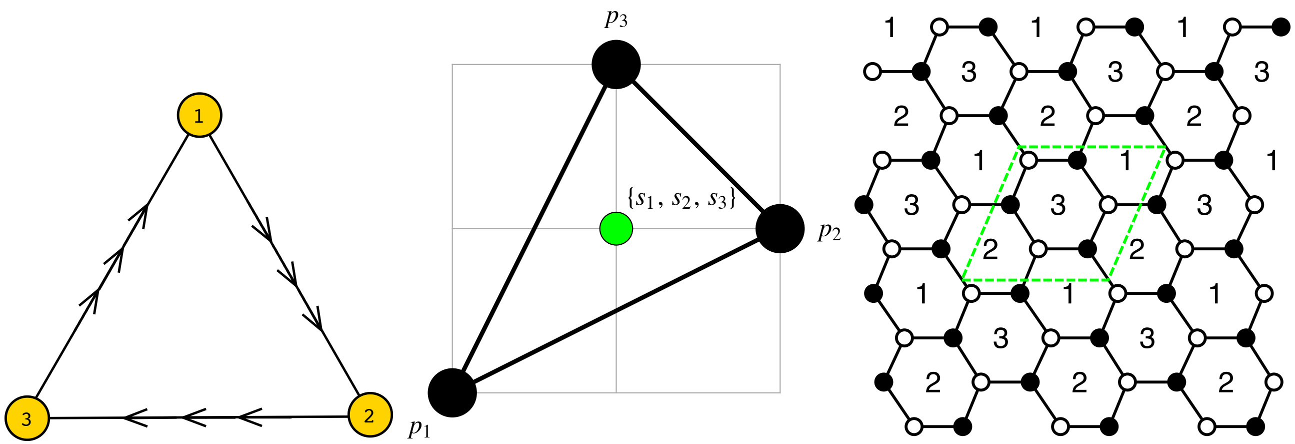
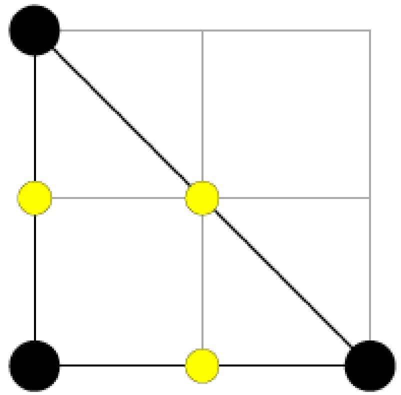
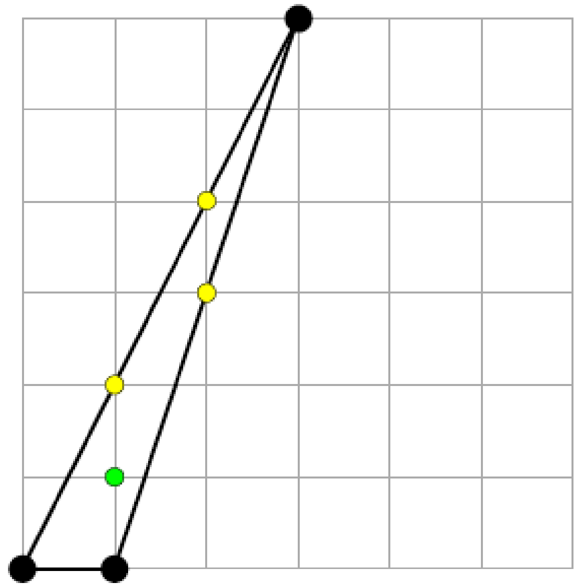



**Figure 36**. The quiver, toric diagram, and brane tiling of Model 16.

The superpotential is

$$
\begin{aligned}
W = & +X_{12}^1 X_{23}^3 X_{31}^2 + X_{12}^2 X_{23}^1 X_{31}^3 + X_{12}^3 X_{23}^2 X_{31}^1 \\
& -X_{12}^1 X_{23}^1 X_{31}^1 - X_{12}^3 X_{23}^3 X_{31}^3 - X_{12}^2 X_{23}^2 X_{31}^2
\end{aligned} \qquad (18.1)
$$

| | | | | |
|---|---|---|---|---|
| (4.3) | 4 | $\mathbb{C}^3/\mathbb{Z}_2 \times \mathbb{Z}_2$ | $\begin{pmatrix} (1,0,1) \\ (0,1,1) \end{pmatrix}$ |  |

| | | | | | |
|---|---|---|---|---|---|
| (6.3) | 6 | $\mathbb{C}^3/\mathbb{Z}_6$ | $\begin{pmatrix} (1,2,3) \\ (0,0,0) \end{pmatrix}$ |  | 6 |

# Look for a graphical representation

- 3 objects in N=1 supersymmetry in 3+1d:
- gauge fields - vector multiplets
- matter fields - chiral multiplets
- interactions - superpotential

# Brane Tilings

- bi-partite tilings of the torus
- or periodic bi-partite tilings of the plane
- Write a Lagrangian according to the rules:

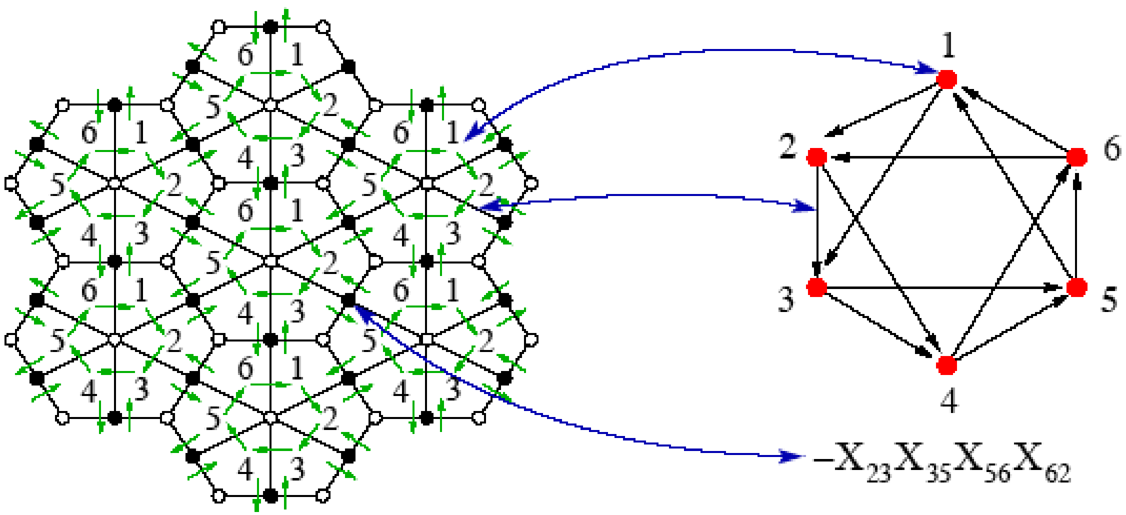
# Brane Tilings Dictionary

- Face (tile) - U(N) Gauge group; U(N) V-plet
- Edge - A bi-fundamental chiral multiplet
- Node - Interaction term in W
- +(-) sign for a white (black) node

# Conditions for conformal invariance

- Locally flat tiles (NSVZ)
- Locally flat nodes (W has R charge 2)
- Periodic, bi-partite, 2d tilings
- R charges are angles in the tiling

# Example: dP3 tiling and quiver



# Example
# Quiver, Tiling, W

## 3 Model 1: $\mathbb{C}^3/\mathbb{Z}_3 \times \mathbb{Z}_3$ $(1,0,2)(0,1,2)$

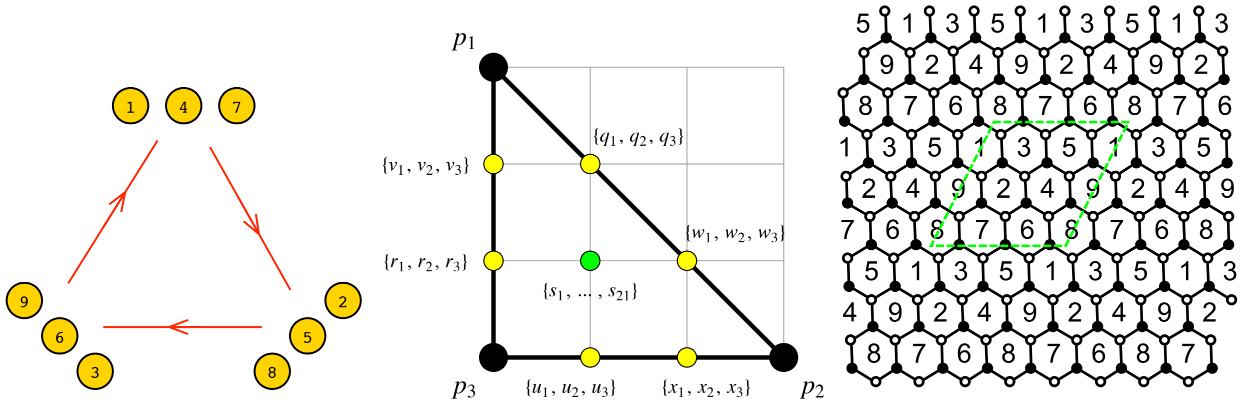



**Figure 4**. The quiver, toric diagram, and brane tiling of Model 1. The red arrows in the quiver indicate all possible connections between blocks of nodes.

The superpotential is

$$\begin{aligned} W = & +X_{15}X_{56}X_{61} + X_{29}X_{91}X_{12} + X_{31}X_{18}X_{83} + X_{42}X_{23}X_{34} + X_{53}X_{37}X_{75} + X_{67}X_{72}X_{26} \\ & +X_{78}X_{89}X_{97} + X_{86}X_{64}X_{48} + X_{94}X_{45}X_{59} - X_{15}X_{59}X_{91} - X_{29}X_{97}X_{72} - X_{31}X_{12}X_{23} \\ & -X_{42}X_{26}X_{64} - X_{53}X_{34}X_{45} - X_{67}X_{75}X_{56} - X_{78}X_{83}X_{37} - X_{86}X_{61}X_{18} - X_{94}X_{48}X_{89} \end{aligned}$$

# Quiver, Toric diagram, Tiling, W

## 18 Model 16: $\mathbb{C}^3/\mathbb{Z}_3$ $(1,1,1)$, $\mathbf{dP}_0$

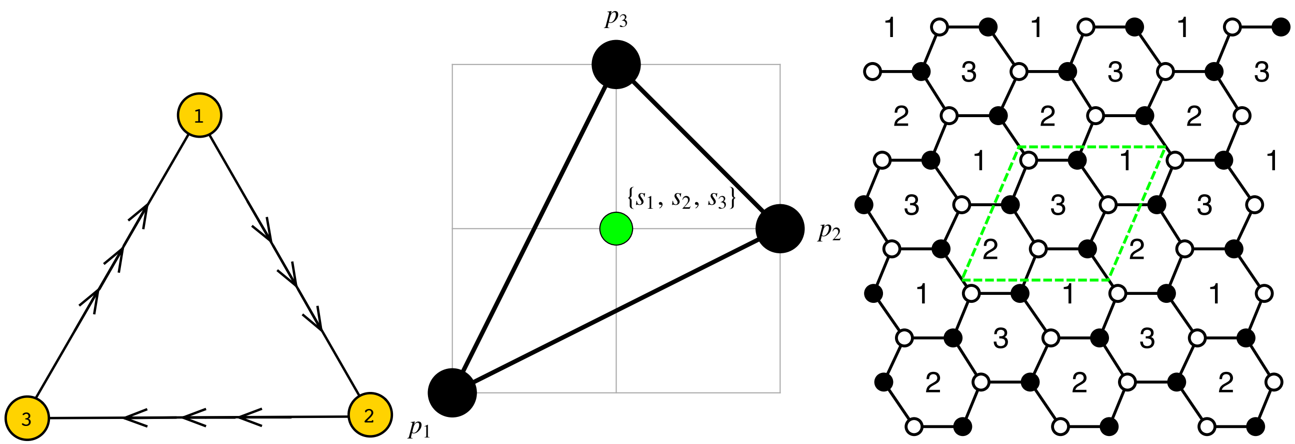
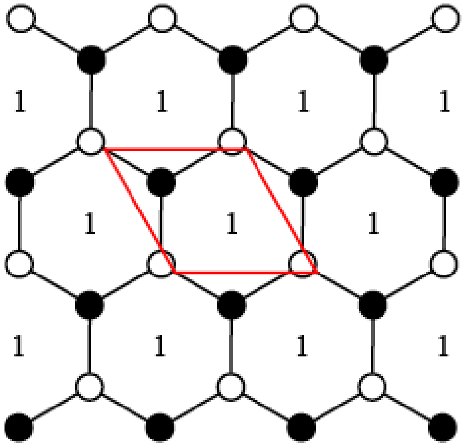



**Figure 36**. The quiver, toric diagram, and brane tiling of Model 16.

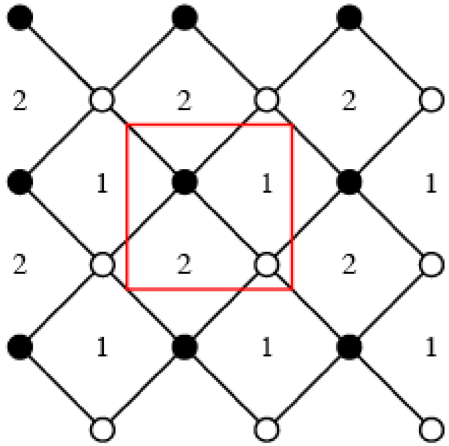
The superpotential is

$$\begin{aligned} W = &+X_{12}^1 X_{23}^3 X_{31}^2 + X_{12}^2 X_{23}^1 X_{31}^3 + X_{12}^3 X_{23}^2 X_{31}^1 \\ &-X_{12}^1 X_{23}^1 X_{31}^1 - X_{12}^3 X_{23}^3 X_{31}^3 - X_{12}^2 X_{23}^2 X_{31}^2 \end{aligned} \quad (18.1)$$

# Brane Tilings
# $N_T$=2, G=1,2

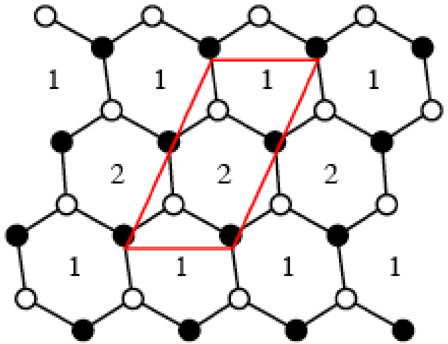


(1.1) $\mathbb{C}^3$

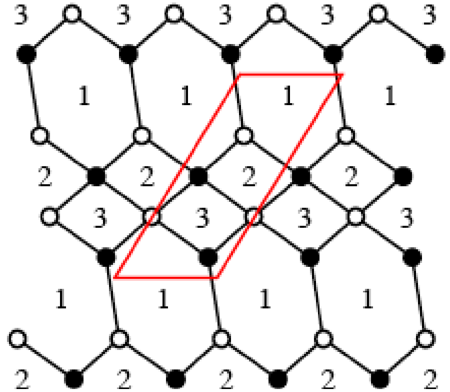


(1.2) $\mathcal{C}$

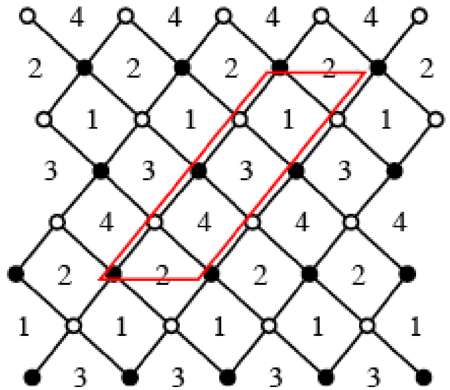
# Brane Tilings
# $N_T$=4, G=2,3,4



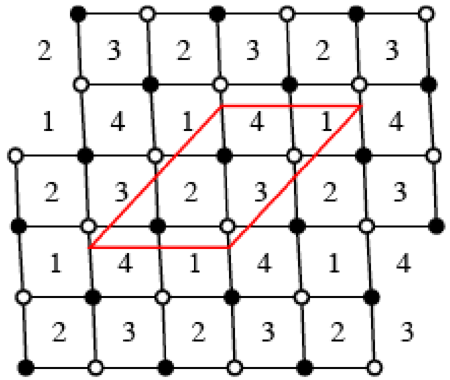
(2.1) $\mathbb{C}^2/\mathbb{Z}_2 \times \mathbb{C}$



(2.2) $SPP$

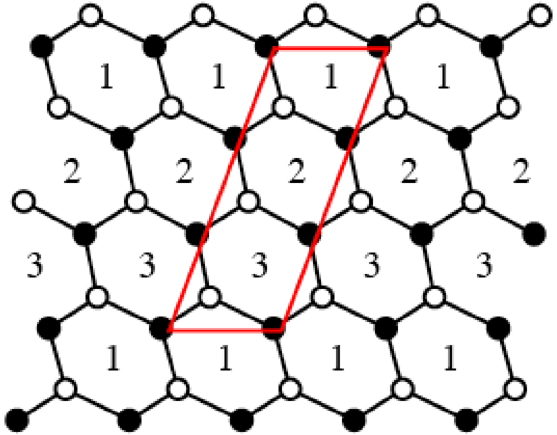


(2.4) $L^{222}$ (I)

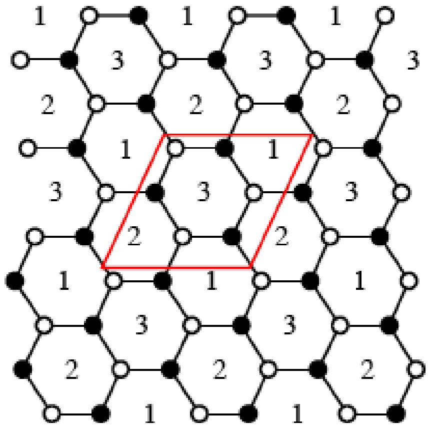


(2.5) $\mathbb{F}_0$ (I)

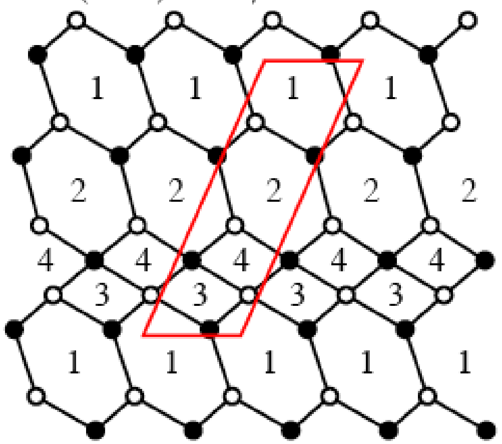
# Brane Tilings
# $N_T$=6, G=3,4



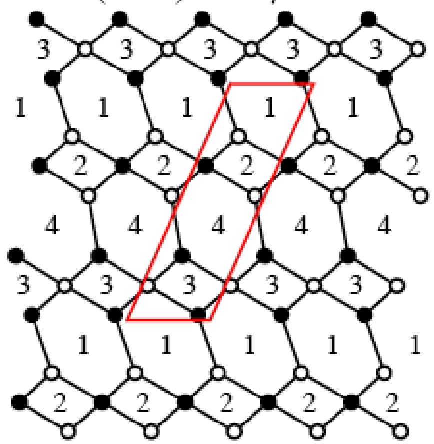
(3.1) $\mathbb{C}^2/\mathbb{Z}_3 \times \mathbb{C}$



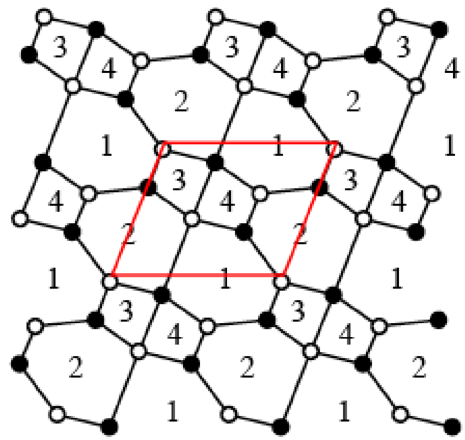
(3.2) $\mathbb{C}^3/\mathbb{Z}_3$



(3.4) $L^{131}$

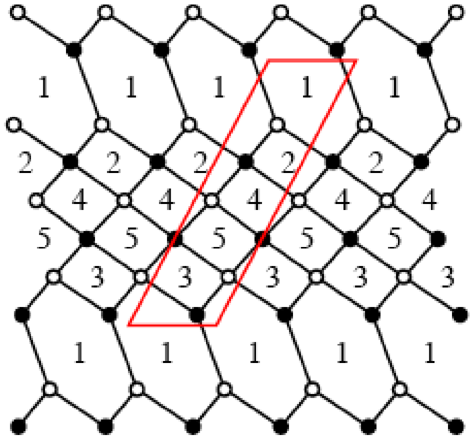


(3.5) $L^{222}$ (II)

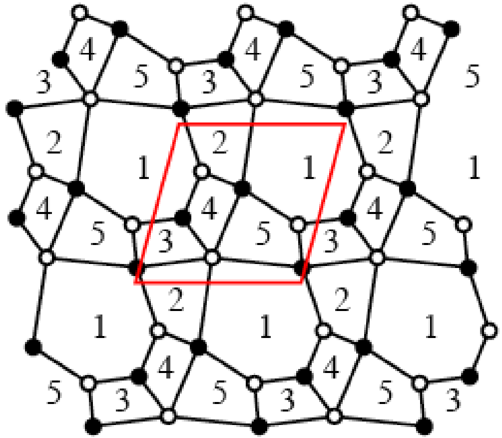


(3.6) $dP_1$

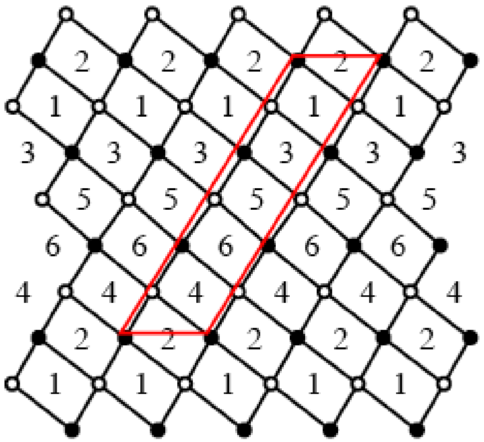
# Brane Tilings
# $N_T$=6, G=5,6



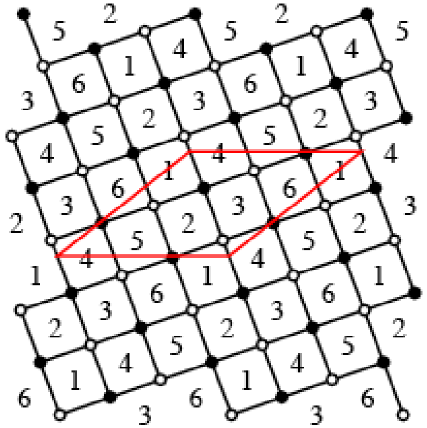
(3.13) $L^{232}$ (I)



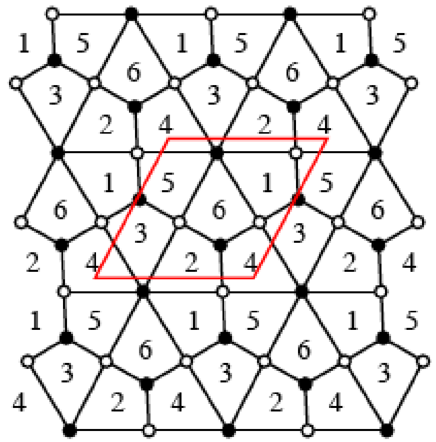
(3.14) $dP_2$ (I)



(3.26) $L^{333}$ (I)



(3.27) $Y^{3,0}$ (I)



(3.28) $dP_3$ (I)

# N=1 supersymmetric gauge theories

- Consider an SU(n) gauge theory
- Two types of gauge invariant operators
- Mesons - delta contraction
- Baryons - epsilon contraction
- Goes through for a product gauge group

# Moduli Space of Vacua

- Given a supersymmetric gauge theory
- Study its moduli space
- VEV to mesons - Mesonic moduli space
- VEV to baryons - Baryonic moduli space
- Combined mesonic baryonic moduli space

# Brane Tilings Mesonic Moduli space

- Mesonic moduli space is singular toric CY3
- Type IIB D3 brane at the tip of this cone
- $AdS_5 \times SE^5$
- N branes mesonic moduli space is $S^N(CY3)$

# Moduli Space

- An N=1 supersymmetric theory
- Set of all fields subject to F terms - F flat
- D terms divided into Abelian & non Abelian
- mesonic space: gauge invariants under both
- combined space: under non Abelian only

# Dimensions

- Mesonic moduli space: 3N
- Set G the number of gauge groups
- Number of Abelian D terms is G-1
- Combined space has dimension 3N+G-1
- singular CY cone

# Master Space

- For one brane, N=1,
- master space of dimension G+2
- singular toric CY cone
- set of fields subject to F terms (F flat)

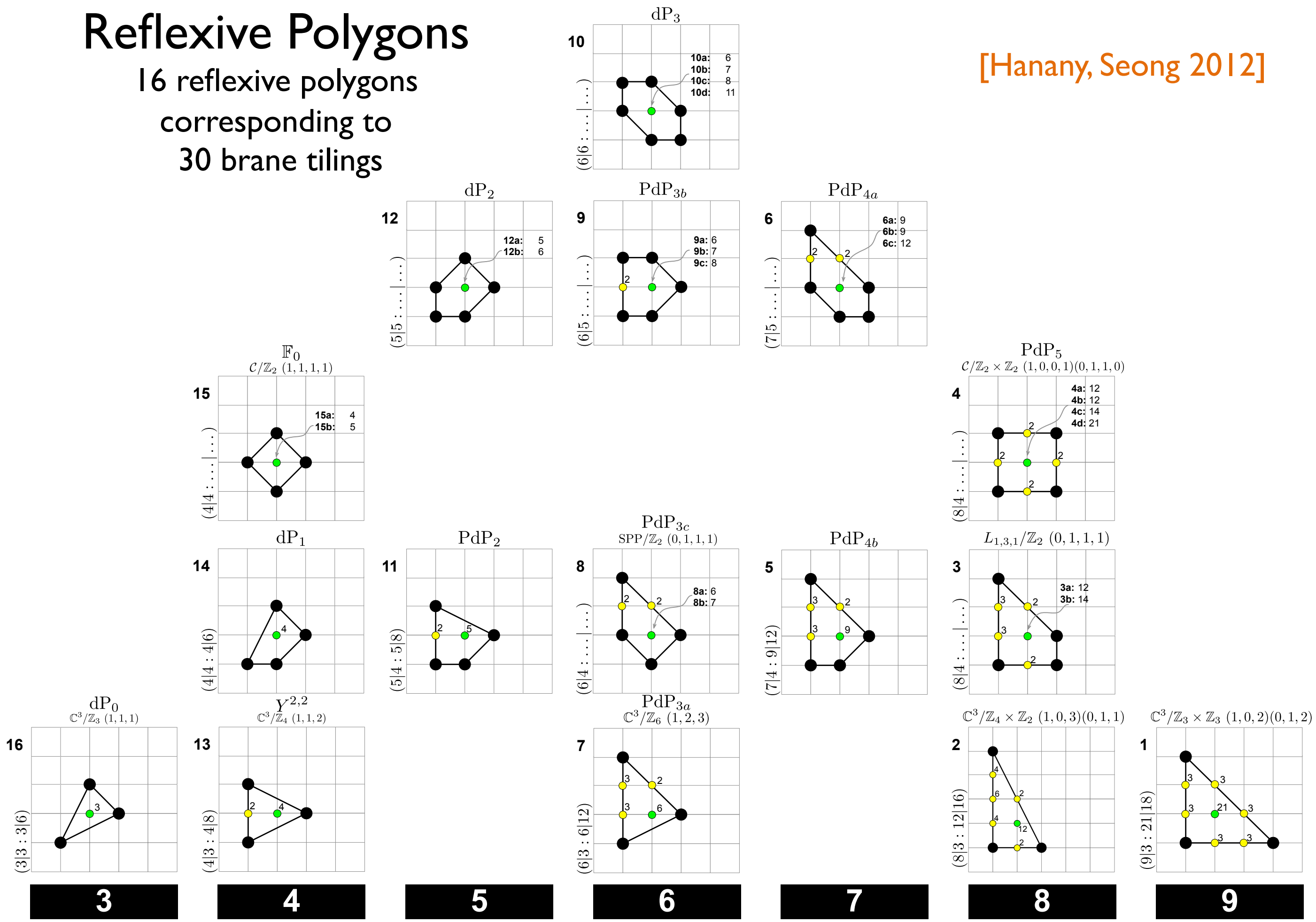
# Master Space Examples

- For $C^3$ no baryonic moduli
- G=1, master space is $C^3$
- For conifold, G=2, master space is $C^4$
- For $C^3/Z_2$, G=2: conifold x C
- For $C^3/Z_3$, G=3, master space is
- complex cone over $P^2$ x $P^2$

# Reflexive Polygons

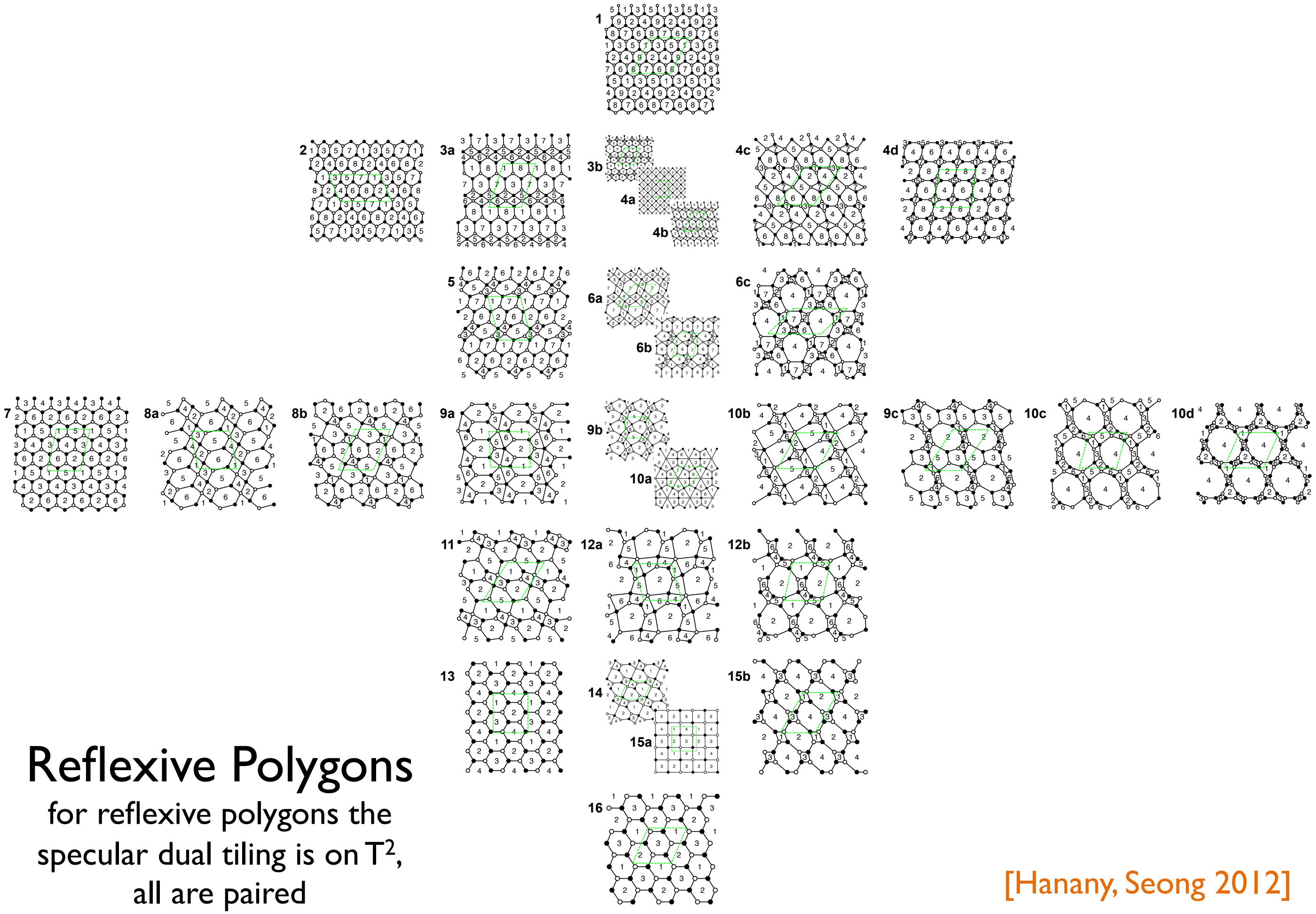
16 reflexive polygons
corresponding to
30 brane tilings

[Hanany, Seong 2012]



# Reflexive Duality

- Chiral ring of the mesonic moduli space
- generated by chiral gauge inv operators
- charged under 3 U(1) symmetries
- Form a lattice
- Reflexive dual

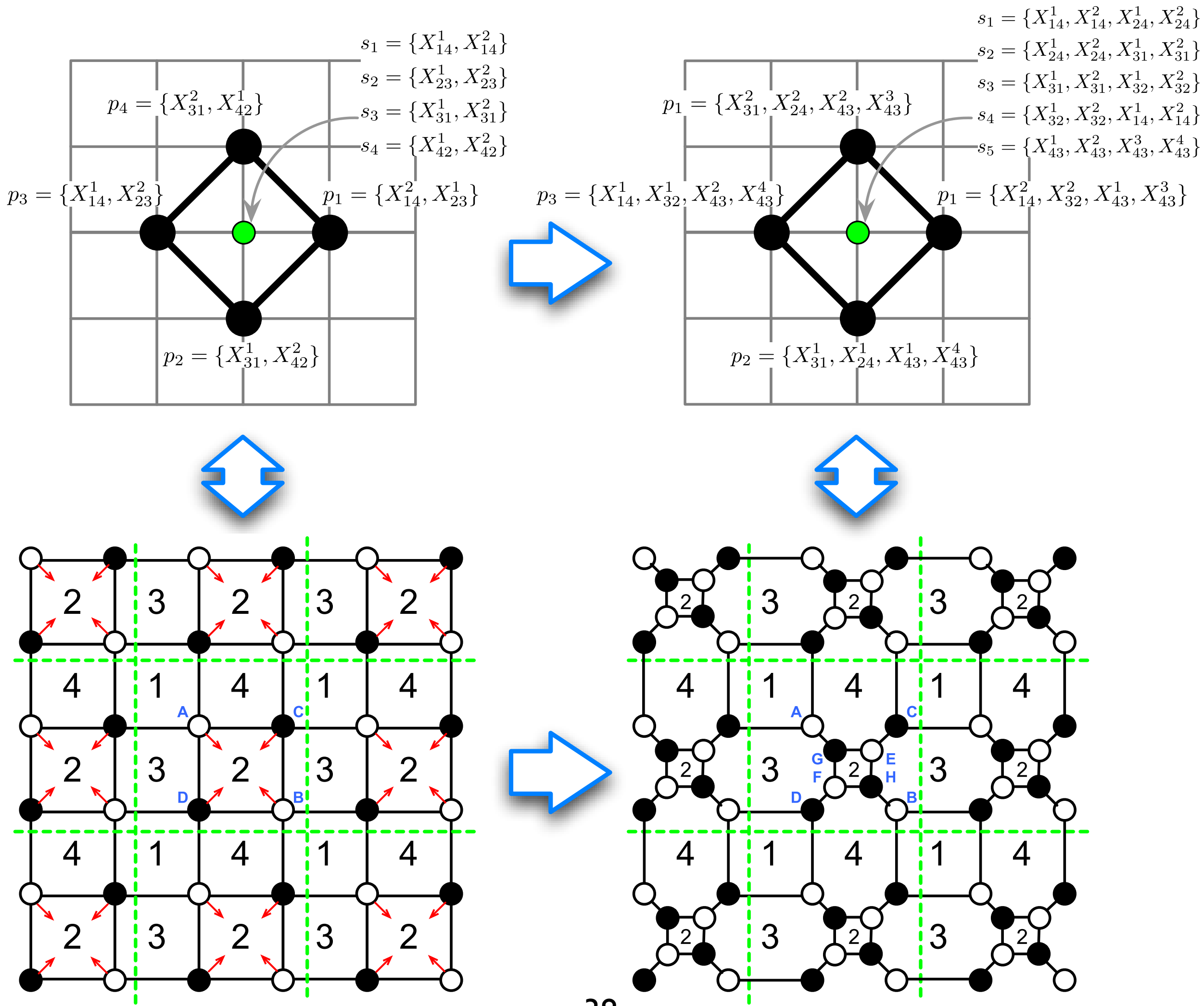


# Reflexive Polygons

for reflexive polygons the
specular dual tiling is on $T^2$,
all are paired

[Hanany, Seong 2012]

# Seiberg Duality F0



# F0 I

## 17 Model 15: $\mathcal{C}/\mathbb{Z}_2\ (1,1,1,1),\ \mathbb{F}_0$

### 17.1 Model 15 Phase a

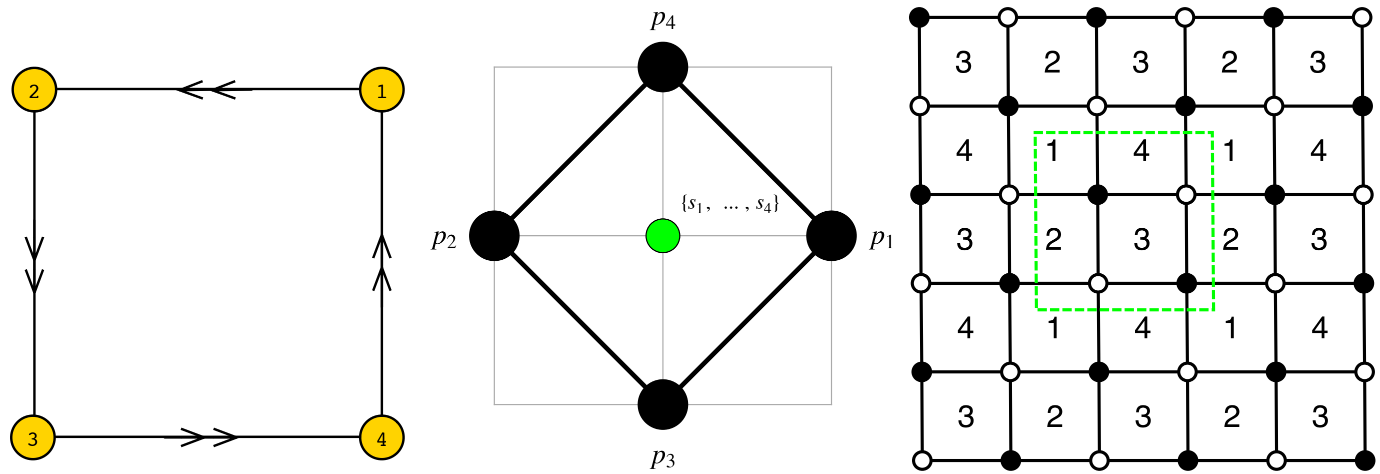



**Figure 34**. The quiver, toric diagram, and brane tiling of Model 15a.

The superpotential is

$$W = +X^1_{12}X^1_{23}X^2_{34}X^2_{41} + X^2_{12}X^2_{23}X^1_{34}X^1_{41} - X^1_{12}X^2_{23}X^2_{34}X^1_{41} - X^2_{12}X^1_{23}X^1_{34}X^2_{41} \quad . \tag{17.1}$$

# F0 II

### 17.2 Model 15 Phase b

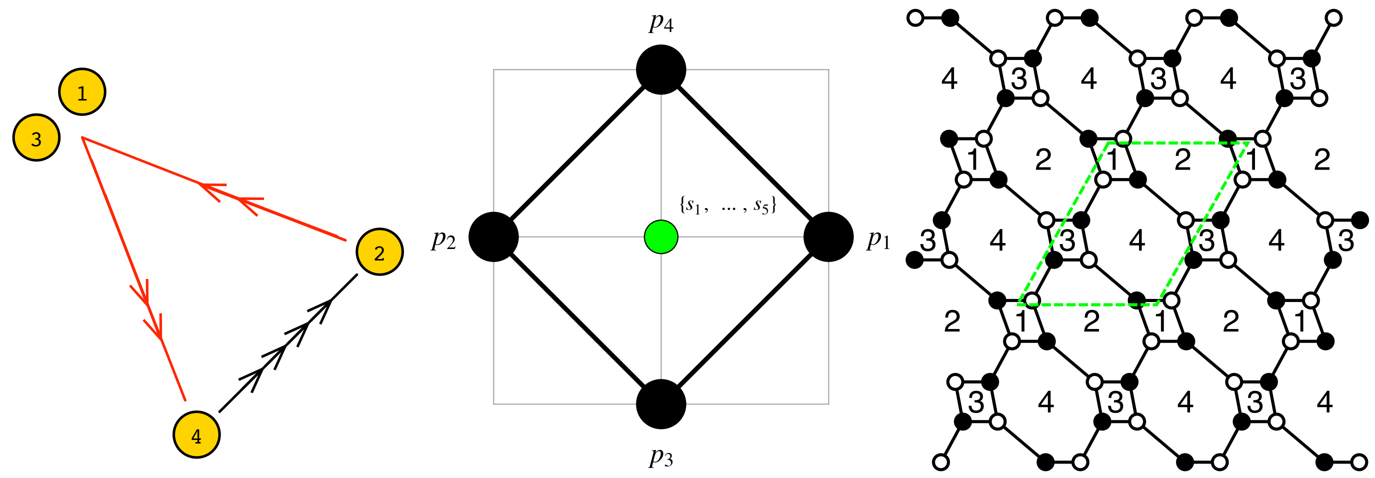
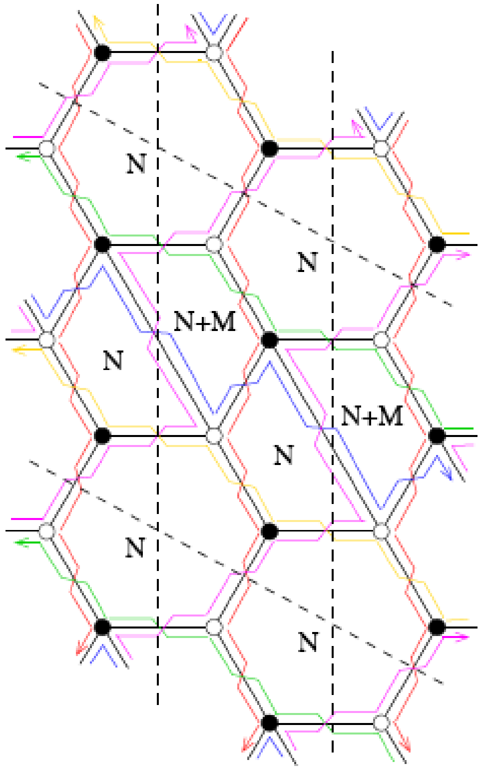


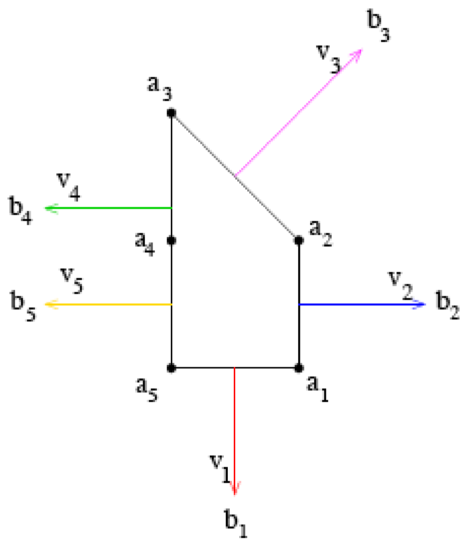

**Figure 35**. The quiver, toric diagram, and brane tiling of Model 15b. The red arrows in the quiver indicate all possible connections between blocks of nodes.

The superpotential is

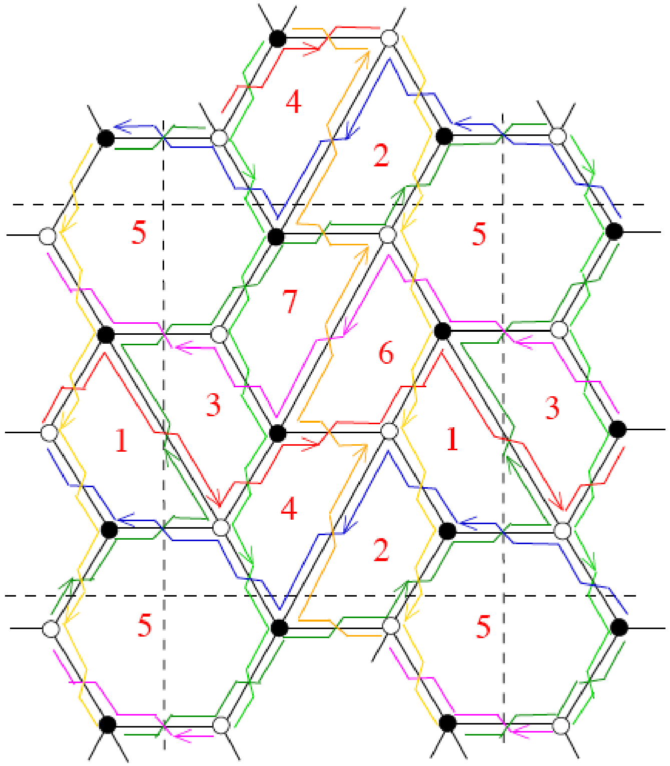
$$\begin{aligned} W = &+X^1_{21}X^1_{14}X^1_{42} + X^2_{21}X^2_{14}X^2_{42} + X^1_{23}X^2_{34}X^3_{42} + X^2_{23}X^1_{34}X^4_{42} \\ &-X^1_{21}X^2_{14}X^3_{42} - X^2_{21}X^1_{14}X^4_{42} - X^1_{23}X^1_{34}X^2_{42} - X^2_{23}X^2_{34}X^1_{42} \end{aligned} \tag{17.13}$$

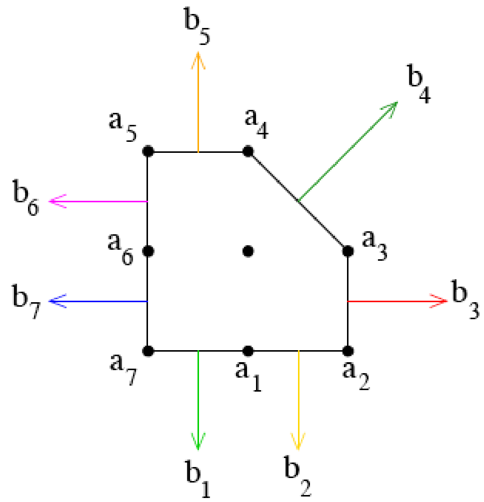
# Zig Zag Paths SPP



# Zig Zag paths PdP4

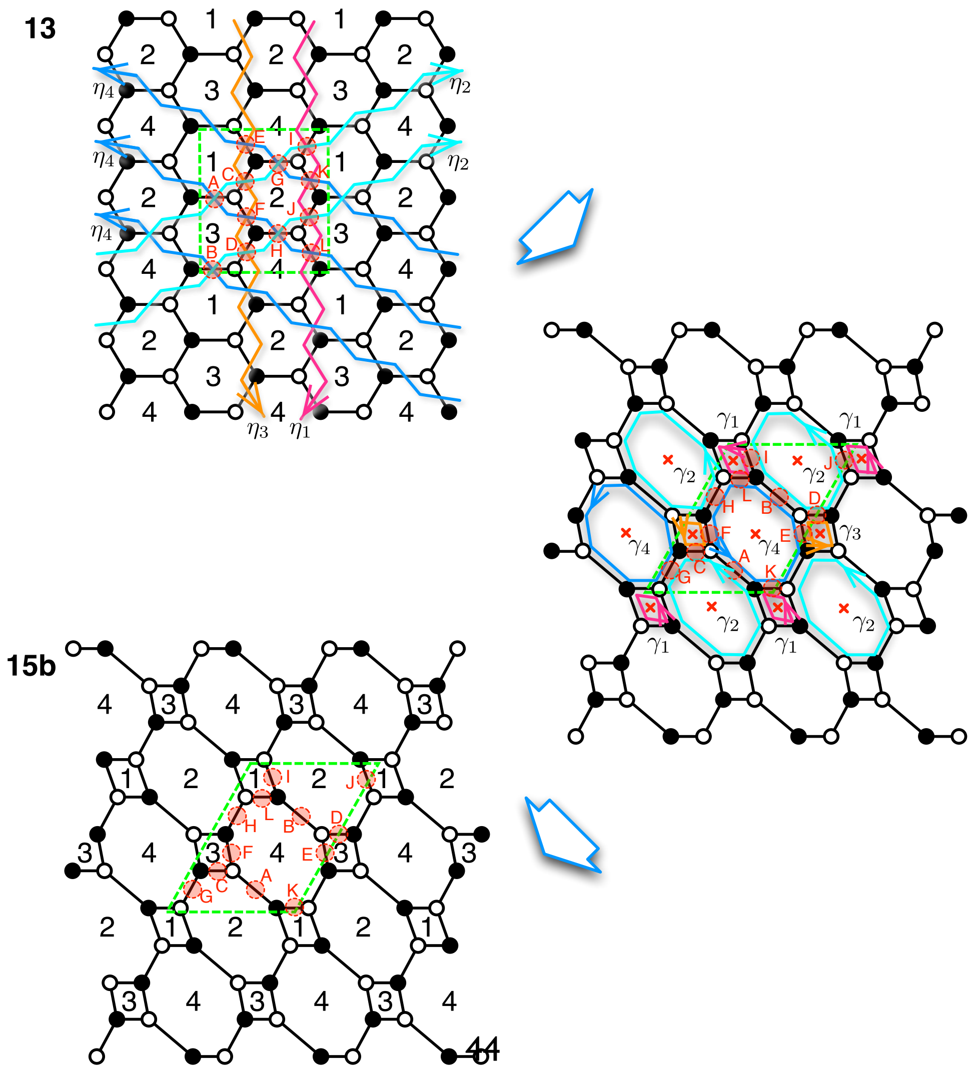


# Zig Zag path

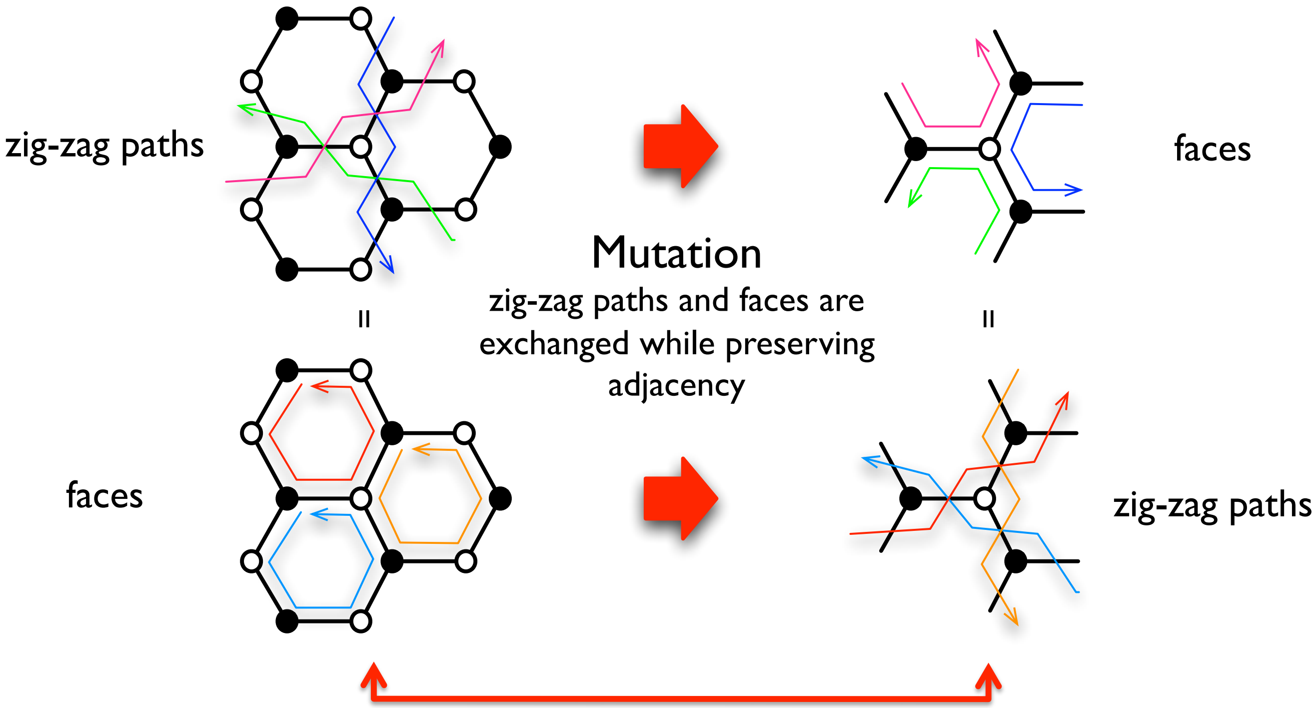
- Each edge at the intersection of precisely 2 zig zag paths
- 1-1 with external legs in the dual to the toric diagram
- closed paths

# Specular Duality



# Specular Duality

Brane Tilings can have also the *same* Master Space (N=1)




Brane Tilings (Quiver Theories) are not necessary the same

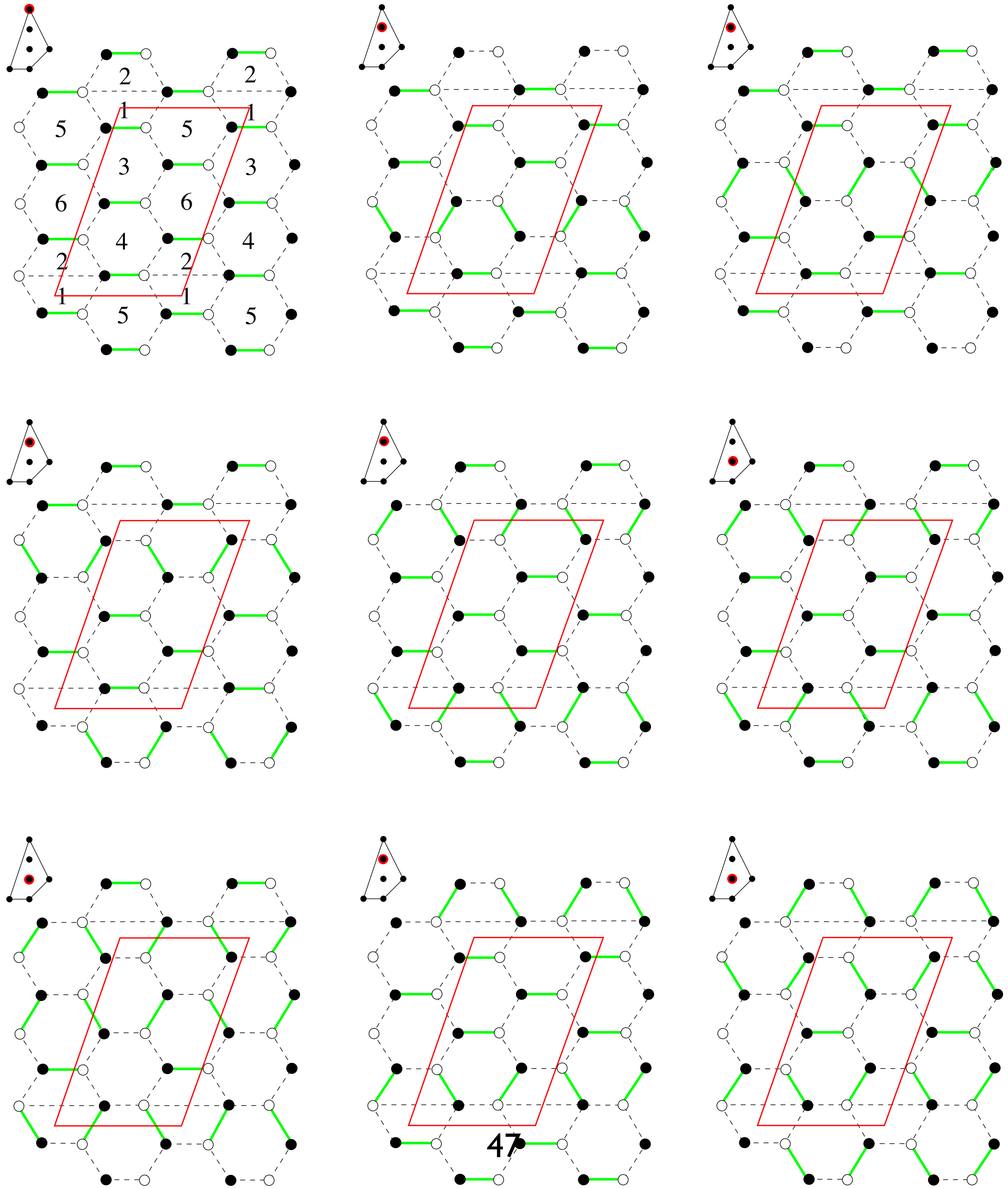
## Specular Duality

[Feng, He, Kennaway, Vafa 2005]
[Hanany, Seong 2012]

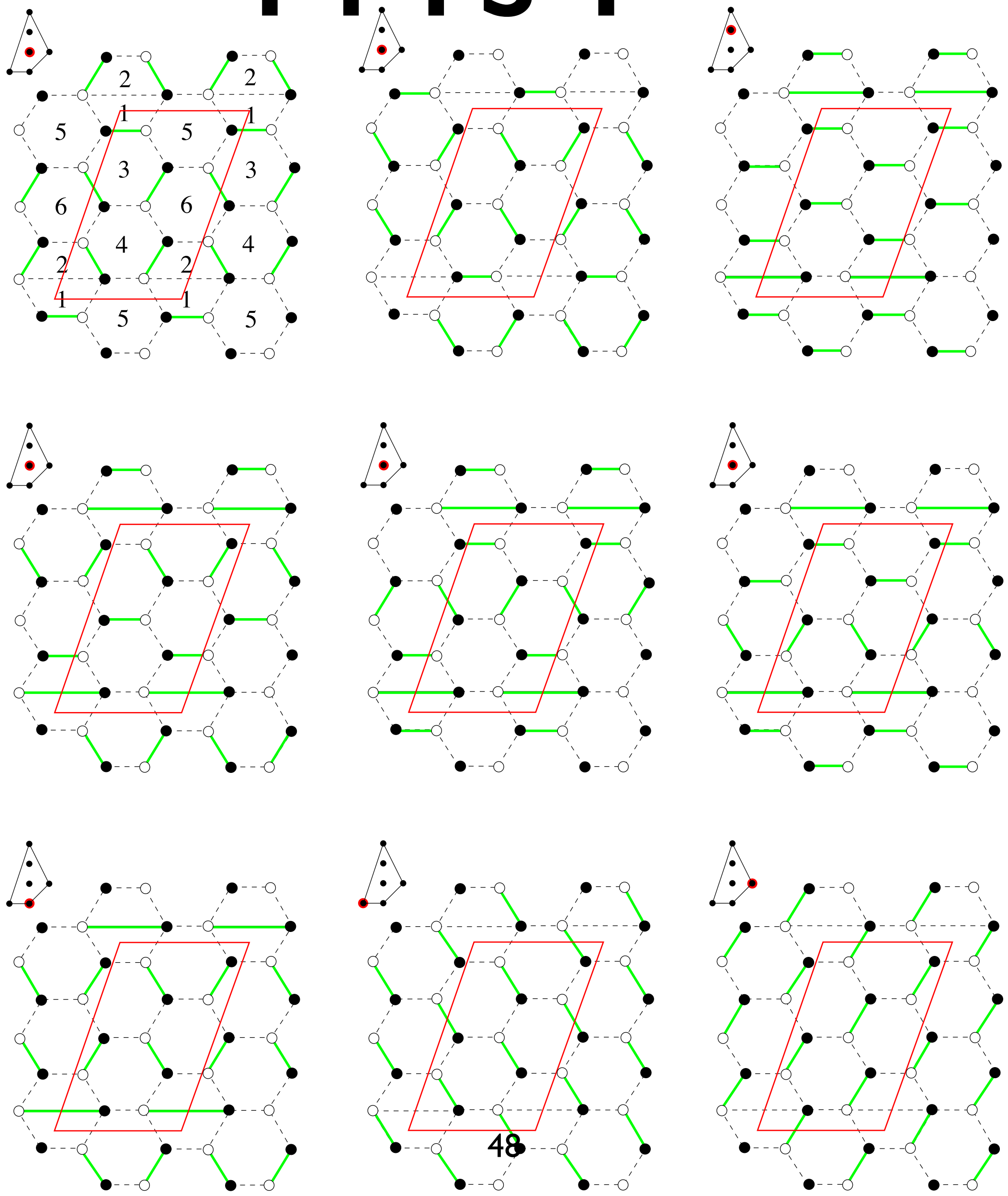
# Specular Duality

- Master space is isomorphic
- $C^3/Z_4$ specular dual to F0II
- W of Abelian theory remains the same
- internal pm's exchange with external pm's
- baryonic symmetry -- mesonic symmetry

# PM's $Y^{32}$



# PM's $Y^{32}$

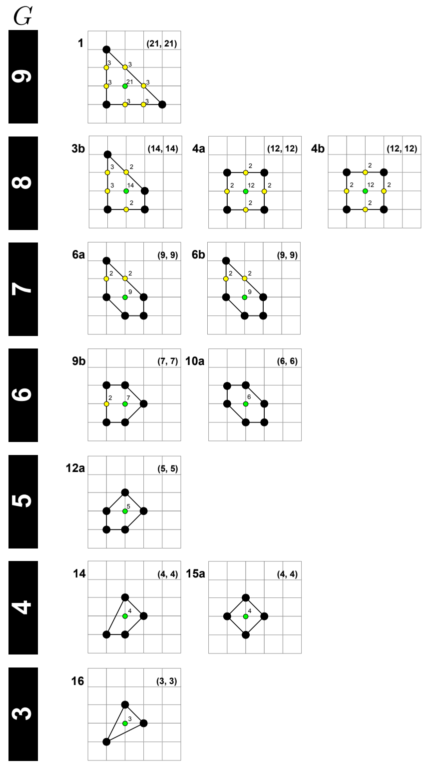


# Perfect Matchings

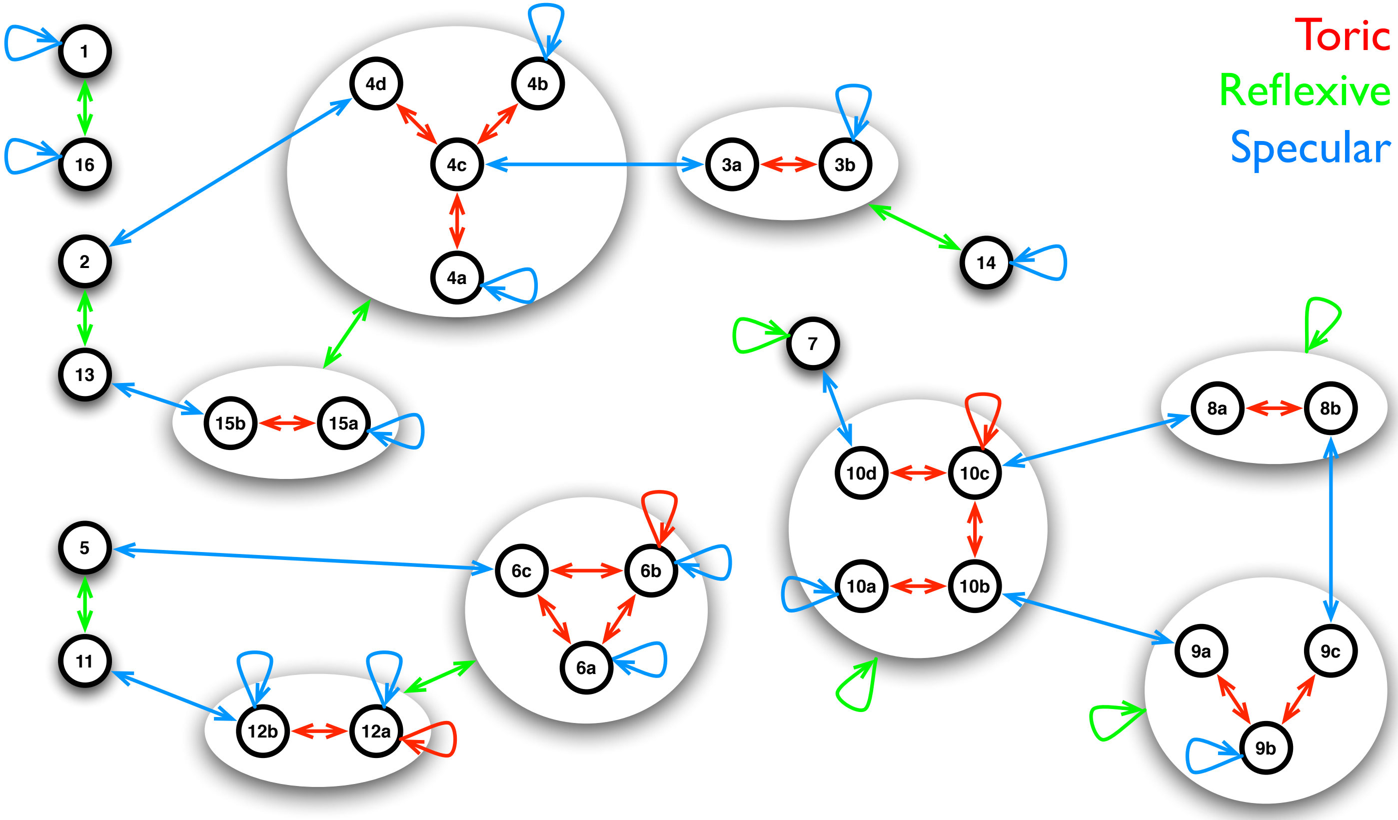
- generators on the master space
- A solution to F term equations
- points in toric diagram

# Specular Duality

# Specular Self duals



# Specular Duality



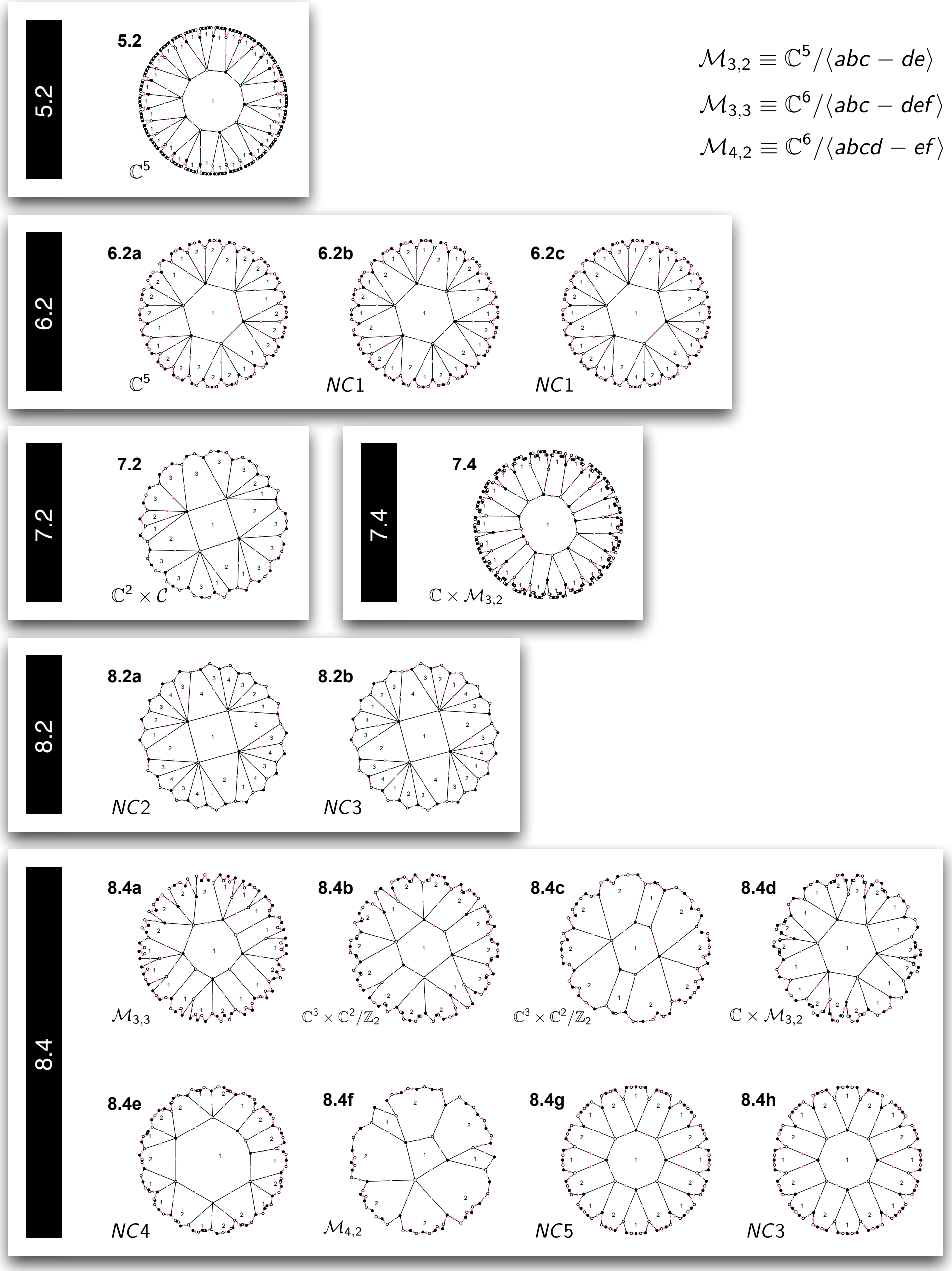
# Summary

- Brane Tilings
- Mesonic, Baryonic, combined
- Master Space, G+2, singular toric CY cone
- Zig Zag Paths
- Reflexive Polygons
- Specular Duality (baryons vs mesons)

$$\mathcal{M}_{3,2} \equiv \mathbb{C}^5 / \langle abc - de \rangle$$

$$\mathcal{M}_{3,3} \equiv \mathbb{C}^6 / \langle abc - def \rangle$$

$$\mathcal{M}_{4,2} \equiv \mathbb{C}^6 / \langle abcd - ef \rangle$$



# Thank You!