

Three-loop Heavy-Flavour Corrections to Deep Inelastic Scattering

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Unpolarized Deep-Inelastic Scattering (DIS)

$$Q^2 := -q^2, \quad x := \frac{Q^2}{2P \cdot q} \quad \text{Bjorken-}x$$

$$\frac{d\sigma}{dQ^2 dx} \sim W_{\mu\nu} L^{\mu\nu}$$

$$W_{\mu\nu}(q, P, s) = \frac{1}{4\pi} \int d^4\xi \exp(iq\xi) \langle P, s | [J_\mu^{em}(\xi), J_\nu^{em}(0)] | P, s \rangle =$$

$$\frac{1}{2x} \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) F_L(x, Q^2) + \frac{2x}{Q^2} \left(P_\mu P_\nu + \frac{q_\mu P_\nu + q_\nu P_\mu}{2x} - \frac{Q^2}{4x^2} g_{\mu\nu} \right) F_2(x, Q^2)$$

Structure functions $F_{2,L}$ contain light and heavy quark contributions

Fit theory prediction to data and obtain α_s

$\alpha_s(M_Z^2)$ from NNLO DIS(+) analyses [from ABM13]

	$\alpha_s(M_Z^2)$	
BBG	$0.1134^{+0.0019}_{-0.0021}$	valence analysis, NNLO
GRS	0.112	valence analysis, NNLO
ABKM	0.1135 ± 0.0014	HQ: FFNS $N_f = 3$
JR	0.1128 ± 0.0010	dynamical approach
JR	0.1140 ± 0.0006	including NLO-jets
MSTW	0.1171 ± 0.0014	(2013)
MSTW	$0.1155 - 0.1175$	Tevatron jets (NLO) incl.
ABM11 _J	$0.1134 - 0.1149 \pm 0.0012$	(without jets)
ABM13	0.1133 ± 0.0011	(without jets)
ABM13	0.1132 ± 0.0011	(without jets)
CTEQ	$0.1159 - 0.1162$	(without jets)
CTEQ	0.1140	(without jets)
NN21	$0.1174 \pm 0.0006 \pm 0.0001$	
Gehrman et al.	$0.1131^{+0.0028}_{-0.0022}$	e^+e^- thrust
Abbate et al.	0.1140 ± 0.0015	e^+e^- thrust
BBG	$0.1141^{+0.0020}_{-0.0022}$	valence analysis, N ³ LO

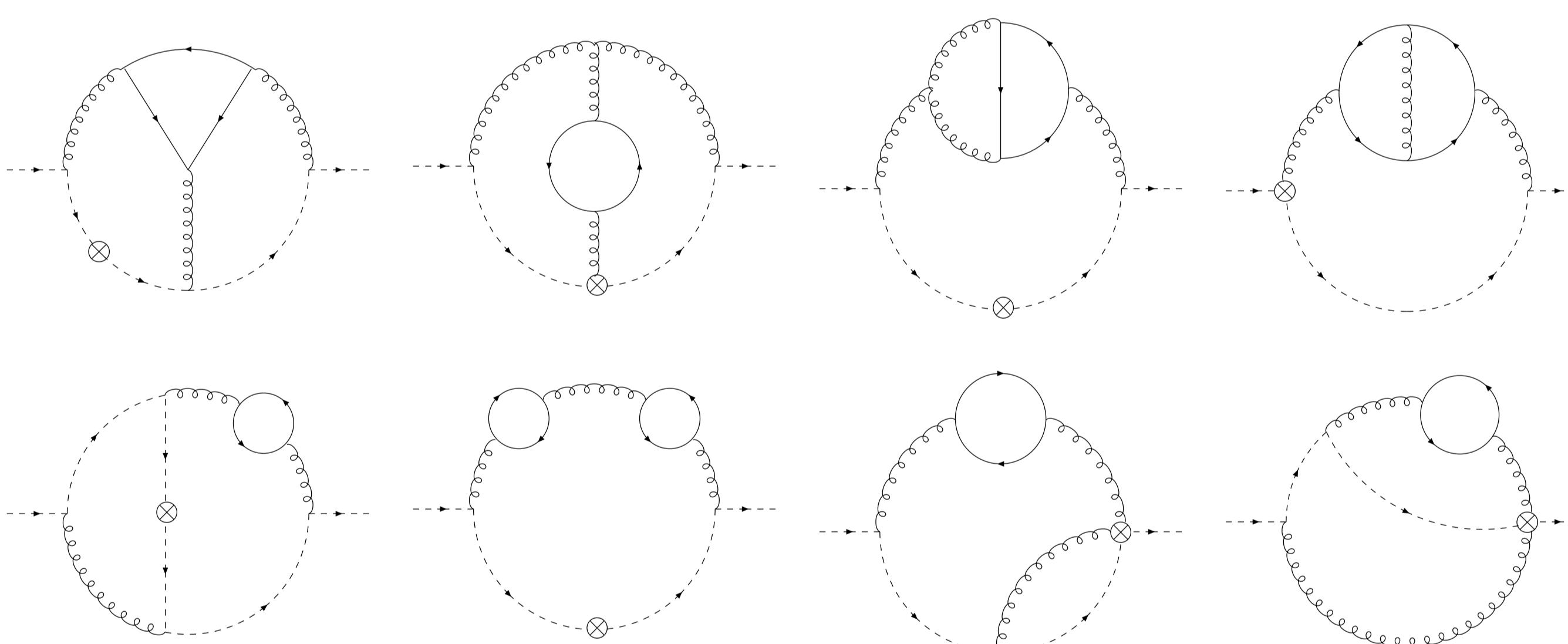
$$\Delta_{\text{TH}} \alpha_s = \alpha_s(\text{N}^3\text{LO}) - \alpha_s(\text{NNLO}) + \Delta_{\text{HQ}} = +0.0009 \pm 0.0006_{\text{HQ}}$$

NNLO accuracy is needed to analyze the world data. \Rightarrow NNLO HQ corrections needed.

Precision of experimental data requires the NNLO calculation of massive structure functions

Goals

- NNLO VFNS will be provided
- Improved values of α_s and m_c
- Better constraint on sea quarks and gluon pdfs

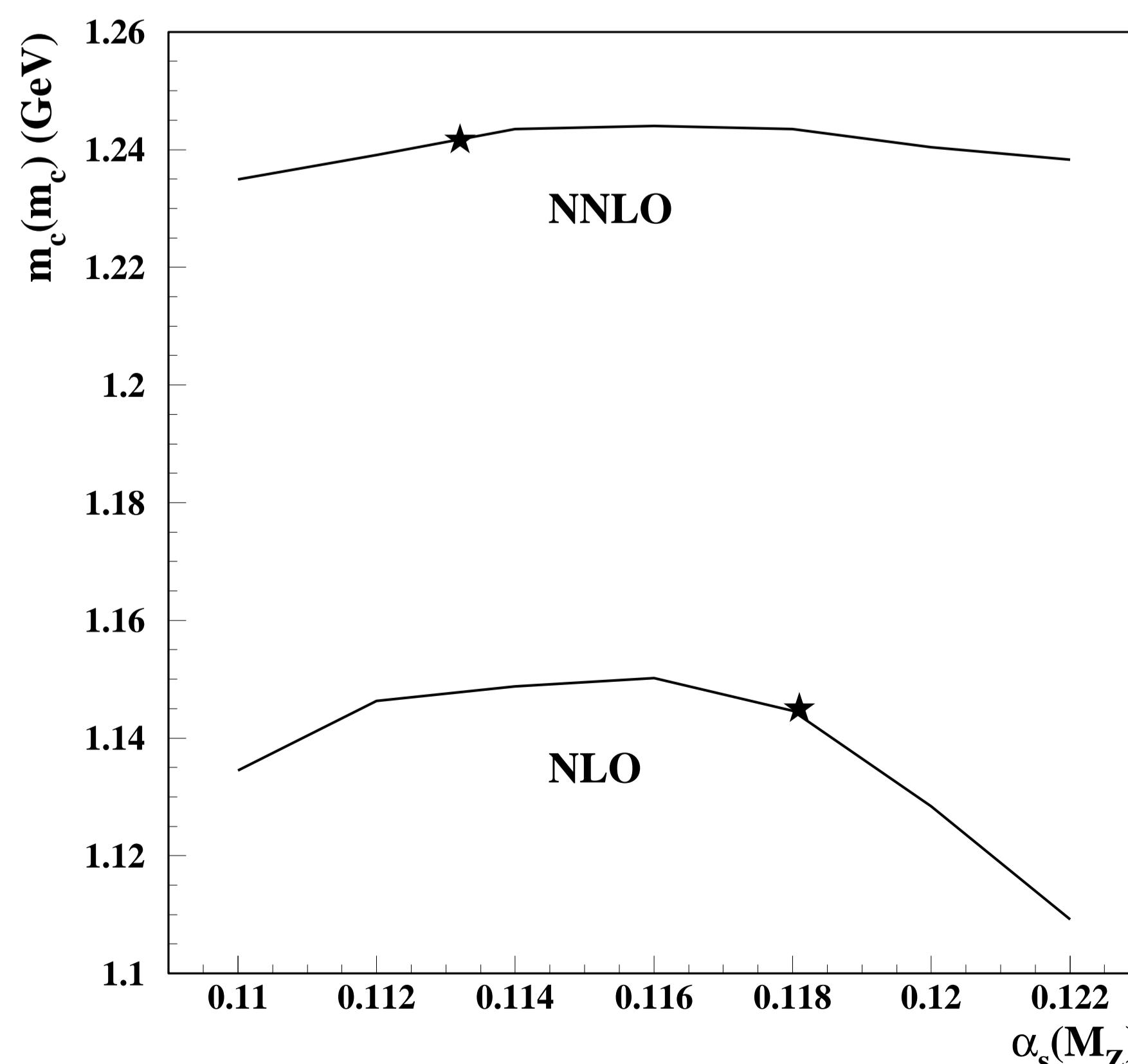


Results for a typical diagram

$$I(x) = \frac{1}{(1+N)(2+N)x} \left\{ \zeta_2 [2L(\{-1\}, x) - 2(-1+2x)L(\{1\}, x) - 4L(\{1, 1\}, x)] - 3L(\{-1, 0, 0, 1\}, x) + 2L(\{-1, 0, 1, 1\}, x) - 2xL(\{0, 0, 1, 1\}, x) + 3xL(\{0, 1, 0, 1\}, x) - xL(\{0, 1, 1, 1\}, x) + (-3+2x)L(\{1, 0, 0, 1\}, x) + 2xL(\{1, 0, 1, 1\}, x) - (-1+5x)L(\{1, 1, 0, 1\}, x) + xL(\{1, 1, 1, 1\}, x) - 2L(\{1, 0, 0, 1, 1\}, x) + 3L(\{1, 0, 1, 0, 1\}, x) - L(\{1, 0, 1, 1, 1\}, x) + 2L(\{1, 1, 0, 0, 1\}, x) + 2L(\{1, 1, 0, 1, 1\}, x) - 5L(\{1, 1, 1, 0, 1\}, x) + L(\{1, 1, 1, 1, 1\}, x) \right\}$$

$$I(N) = \frac{1}{(N+1)(N+2)(N+3)} \left\{ \frac{648 + 1512N + 1458N^2 + 744N^3 + 212N^4 + 32N^5 + 2N^6}{(1+N)^3(2+N)^3(3+N)^3} - 2 \frac{(-1)^N + N + (-1)^N N}{(1+N)} \zeta_3 - (-1)^N S_{-3} - \frac{N}{6(1+N)} S_1^3 + \frac{1}{24} S_4^4 - \frac{(7+22N+10N^2)}{2(1+N)^2(2+N)} S_2 - \frac{19}{8} S_2^2 - \frac{1+4N+2N^2}{2(1+N)^2(2+N)} S_1^2 + \frac{9}{4} S_2 - \frac{(-9+4N)}{3(1+N)} S_1 - \frac{1}{4} S_4 - 2(-1)^N S_{-2,1} + \frac{(-1+6N)}{(1+N)} S_{2,1} + \frac{54+207N+246N^2+130N^3+32N^4+3N^5}{(1+N)^3(2+N)^2(3+N)^2} S_1 + 4\zeta_3 S_1 - \frac{(-2+7N)}{2(1+N)} S_2 S_1 + \frac{13}{3} S_3 S_1 - 7S_{2,1} S_1 - 7S_{3,1} + 10S_{2,1,1} \right\}$$

Result for m_c



Operator Matrix Element

$$a_{Qq}^{(3),\text{PS}}(N) = \text{C}_F^2 \text{T}_F \left\{ \frac{64(N^2+N+2)^2}{(N-1)N^2(N+1)^2(N+2)} S_{2,2}(2, \frac{1}{2}) - \frac{64(N^2+N+2)^2}{(N-1)N^2(N+1)^2(N+2)} S_{3,1}(2, \frac{1}{2}) \right. \\ \left. + 2^N \left[\frac{32P_3 S_{2,1}(1, \frac{1}{2}, N)}{(N-1)N^3(N+1)^2(N+2)} - \frac{32P_3 S_{1,1,1}(\frac{1}{2}, 1, 1, N)}{(N-1)^2N^3(N+1)^2(N+2)} + \frac{32P_3 S_{1,1,1}(1, \frac{1}{2}, N)}{(N-1)^3N^4(N+1)^2(N+2)} \right] \right. \\ \left. + 2^{-N} \left[\frac{64(N^2+N+2)^2 S_{1,2,1}(2, \frac{1}{2}, 1, N)}{(N-1)N^2(N+1)^2(N+2)} + \frac{64(N^2+N+2)^2 S_{1,2,1}(2, 1, \frac{1}{2}, N)}{(N-1)N^2(N+1)^2(N+2)} + \dots \right] \right. \\ \left. + \text{C}_F^2 \text{T}_F^2 \text{N}_F \left\{ \frac{16(N^2+N+2)^2 S_1(N)^3}{27(N-1)N^2(N+1)^2(N+2)} + \frac{16P_3 S_1(N)^2}{27(N-1)N^3(N+1)^3(N+2)^2} \right. \right. \\ \left. \left. + \left[\frac{208(N^2+N+2)^2 S_2}{9(N-1)N^2(N+1)^2(N+2)} - \frac{32P_{23}}{81(N-1)N^4(N+1)^2(N+2)^3} \right] S_1 \right. \right. \\ \left. \left. + \frac{32P_{31}}{243(N-1)N^5(N+1)^5(N+2)} + \frac{224(N^2+N+2)^2}{9(N-1)N^2(N+1)^2(N+2)} \zeta_3 + \dots \right\} \right. \\ \left. + \text{C}_F \text{C}_A \text{T}_F \left\{ \frac{2(N^2+N+2)^2 S_1(N)^4}{9(N-1)N^2(N+1)^2(N+2)} + \frac{4(N^2+N+2) P_6 S_1(N)^3}{27(N-1)^2 N^3(N+1)^3(N+2)^2} \right. \right. \\ \left. \left. + \frac{16P_2 S_3(2, N)}{(N-1)N^3(N+1)^2} - \frac{16P_2 S_{1,2}(2, 1, N)}{(N-1)N^3(N+1)^2} + \frac{16P_2 S_{1,1,1}(2, 1, \frac{1}{2}, N)}{(N-1)N^3(N+1)^2} \right\} \right. \\ \left. + 2^{-N} \left[\frac{32(N^2+N+2)^2 S_{1,2,1}(2, \frac{1}{2}, 1, N)}{(N-1)N^2(N+1)^2(N+2)} + \frac{32(N^2+N+2)^2 S_{1,2,1}(2, 1, \frac{1}{2}, N)}{(N-1)N^2(N+1)^2(N+2)} \right. \right. \\ \left. \left. + \frac{32(N^2+N+2)^2 S_{1,2,1}(\frac{1}{2}, 1, 1, N)}{(N-1)N^2(N+1)^2(N+2)} - \frac{32(N^2+N+2)^2 S_{1,2,1}(2, 1, \frac{1}{2}, N)}{(N-1)N^2(N+1)^2(N+2)} \right. \right. \\ \left. \left. + \frac{32(N^2+N+2)^2 S_{1,1,1}(2, \frac{1}{2}, 1, N)}{(N-1)N^2(N+1)^2(N+2)} - \frac{32(N^2+N+2)^2 S_{1,1,1}(2, 1, \frac{1}{2}, N)}{(N-1)N^2(N+1)^2(N+2)} \right. \right. \\ \left. \left. - \frac{32(N^2+N+2)^2 S_{1,1,1,1}(2, \frac{1}{2}, 1, 1, N)}{(N-1)N^2(N+1)^2(N+2)} - \frac{32(N^2+N+2)^2 S_{1,1,1,1}(2, 1, \frac{1}{2}, 1, N)}{(N-1)N^2(N+1)^2(N+2)} + \dots \right\} + \dots \right]$$