Excecrices for lecture 2

Excercise 1 : The correlator from an Ansatz
In the lecture we have seen that the correlator must satisfy

$$\Pi_n = \Pi_{-n} \quad , \quad \Pi_n = \frac{\hbar}{\mu}\delta_{n,0} + \frac{\gamma}{\mu}\left(\Pi_{n+1} + \Pi_{n-1}\right)$$

Now suppose that Π_n has the form

$$\Pi_n = A\ B^{|n|}$$

By inspecting Π_0 and Π_1, find A and B. Then, take the continuum limit

$$n \to \frac{x}{\Delta} \quad , \quad \gamma \to \frac{1}{\Delta} \quad , \quad \mu \to \frac{2}{\Delta} + m^2\Delta \quad , \quad \Delta \to 0$$

to find the result in the continuum limit :

$$\Pi(x) = \frac{\hbar}{2m}\exp(-m|x|) \quad .$$

Solution 1 :
Using the Ansatz for Π_1 we find from the SDe

$$\Pi_1 = \frac{\gamma}{\mu}(\Pi_0 + \Pi_2) \quad \Rightarrow \quad AB = \frac{\gamma}{\mu}(A + AB^2)$$

so that A drops out, and we have for B

$$B^2 + \frac{\mu}{\gamma}B + 1 = 0 \quad \Rightarrow \quad B = \frac{\mu}{2\gamma} - \sqrt{\left(\frac{\mu}{2\gamma}\right)^2 - 1}$$

In the continuum limit, we find $B = 1 - m\Delta + \mathcal{O}\left(\Delta^2\right)$. The SDe for Π_0 reads

$$\Pi_0 = \frac{\hbar}{\mu} + \frac{2\gamma}{\mu}\Pi_1 \quad \Rightarrow \quad A = \frac{\hbar}{\mu} + \frac{2\gamma}{\mu}AB$$

so that

$$A = \frac{\hbar}{\mu - 2\gamma B}$$

In the continuum limit $A = \hbar/(2m) + \mathcal{O}\left(\Delta\right)$. The correlator becomes

$$\Pi(x) = \Pi_{|x|/\Delta} = \frac{\hbar}{2m}\left(1 - m\Delta\right)^{|x|/\Delta} = \frac{\hbar}{2m}e^{-|x|m}$$

where we have used the fact that

$$\lim_{K\to\infty}\left(1 - \frac{z}{K}\right)^K = e^{-z}$$

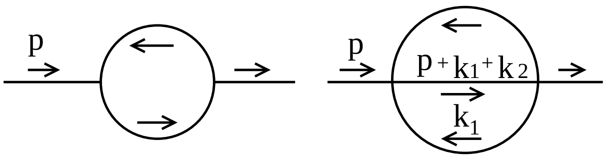
Excercise 2 : To mass or not to mass
Show that in the Feynman rules the variable m must have the dimension of inverse length, and therefore *cannot* be simply the mass M of the particle, which is given in kilograms. Try to find the relation between m and M (Hint: Arthur Holly C.).

Solution 2 :
In order to convert the dimension of quantities we can employ the natural constants c and $\hbar$. The combination of M, c and $\hbar$ that has the dimension of inverse length is

$$m = \frac{Mc}{\hbar}$$

which is the inverse of the Compton wavelength of the particle.

Excercise 3 : Some actual (gasp!) loop calculations
Here are two diagrams occurring in one-dimensional $\varphi^{3/4}$ theory :



1. The momentum flow is indicated by arrows. Find the missing momenta.
2. Write the diagrams completely, including coupling constants, symmetry factors, etcetera.
3. Use the standard integral result

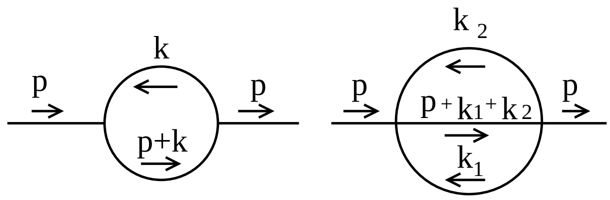
$$\int_{-\infty}^{\infty} dz \, \frac{1}{(z^2+a^2)((z+q)^2+b^2)} = \frac{\pi}{ab}\frac{a+b}{q^2+(a+b)^2}$$

to work out the values of the diagrams.

Now you have done an actual two-loop calculation !

Solution 3 :

1. The completed diagrams:



2. The first diagram reads

$$\frac{\hbar^2 {\lambda_3}^2}{2(p^2+m^2)^2}\frac{1}{2\pi}\int dk \frac{1}{(k^2+m^2)((p+k)^2+m^2)}$$

and the second one

$$\frac{\hbar^3 {\lambda_4}^2}{6(p^2+m^2)}\frac{1}{(2\pi)^2}\int dk_1 \int dk_2 \frac{1}{({k_1}^2+m^2)({k_2}^2+m^2)((p+k_1+k_2)^2+m^2)}$$

3. Using the standard integrals gives

$$\frac{\hbar^2 {\lambda_3}^2}{2m}\frac{1}{(p^2+m^2)^2}\frac{1}{p^2+(2m)^2}$$

for the first diagram, and

$$\frac{\hbar^3 {\lambda_4}^2}{8m^2}\frac{1}{(p^2+m^2)^2}\frac{1}{p^2+(3m)^2}$$