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Second Astro-Physics and -Particle Meeting

Time Evolution of Primordial Magnetic Fields and Present Day Intergalactic Large Scale Magnetism

Andrey Saveliev [1]

Universität Hamburg

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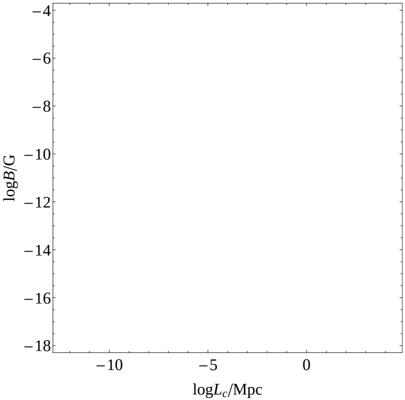
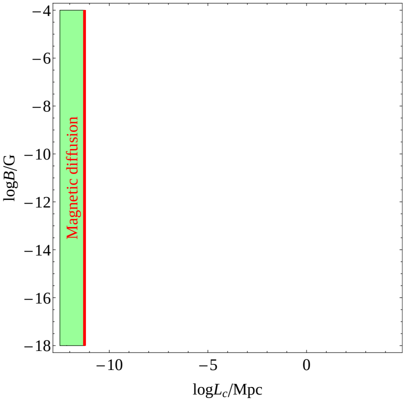
[1]with Karsten Jedamzik (Université Montpellier II), Carmelo Evoli and Günter Sigl (Universität Hamburg)

Contents

EGMF – Standard Constraints [Neronov and Semikoz, 2009]





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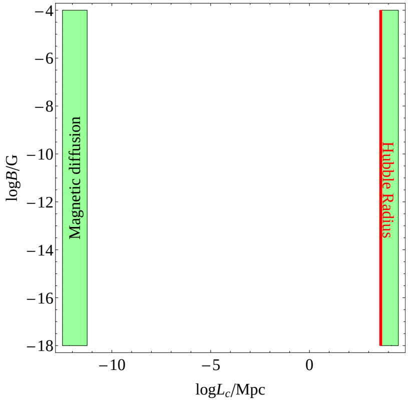

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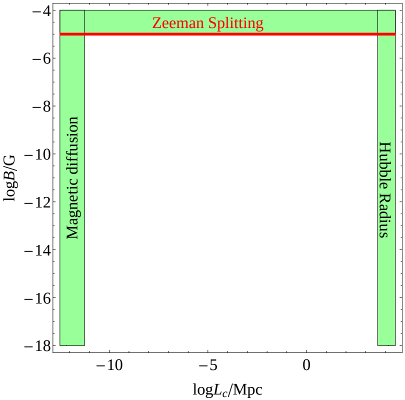

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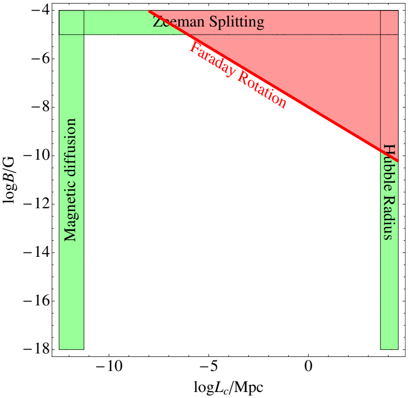

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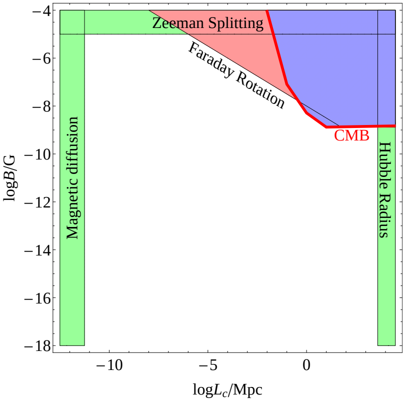

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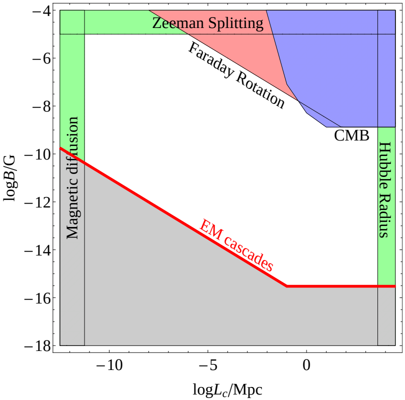
EGMF – Lower Bound on B? [Neronov and Semikoz, 2009]



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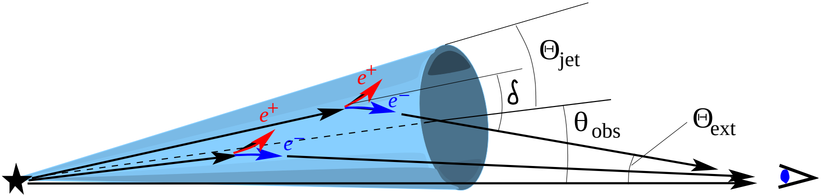
- Lower bound on B from Fermi LAT data?

[Neronov et al., 2010]



Gamma rays emmitted from a blazar develop an electromagnetic cascade due to interactions with the Extragalactic Backgriound Light (EBL) via pair production and Inverse Compton (IC) scattering. The interaction of this cascade with the EGMF results in several observational features.

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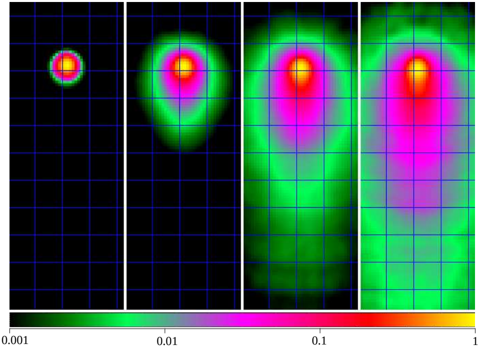

Appearance of a point-like source at $\theta_{obs} = 3$ for magnetic fields $B = 10^{-17}$ G, 10^{-16} G, 10^{-15} G and 10^{-14} G [Neronov et al., 2010]

- Point-like sources appear extensive [Dolag et al., 2009], [Neronov et al., 2010]

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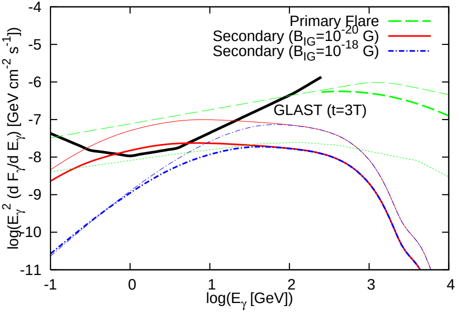

Spectrum of the time-delayed spectrum of the 2005 flare of Mrk 501 for different values of the EGMF after 0.5 days (thin) and 1.5 days (thick) [Murase et al., 2008]

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- Time-delayed echos of primary gamma rays [Plaga, 1994], [Murase et al., 2008]

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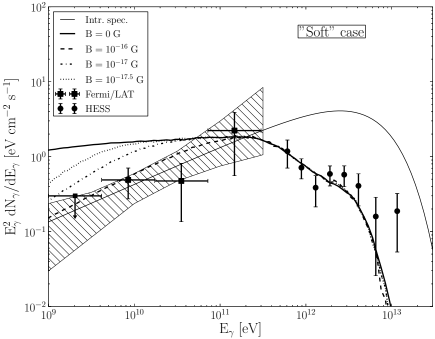

Predicted gamma ray flux of 1ES0229+200 for different magnetic fields with data points of Fermi LAT and HESS [Vovk et al., 2012]

- Point-like sources appear extensive [Dolag et al., 2009], [Neronov et al., 2010]
- Time-delayed echos of primary gamma rays [Plaga, 1994], [Murase et al., 2008]
- Suppression of observed photon flux in the GeV region [d'Avezac et al., 2007], [Neronov and Vovk, 2010], [Vovk et al., 2012]

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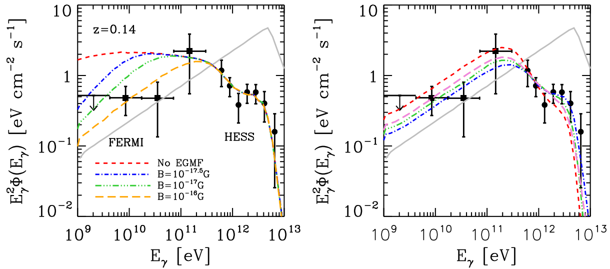
- After refining the statistical analysis method the zero EGMF hypothesis has been claimed to be true [Arlen et al., 2012]
- The electromagnetic cascade might heat up the intergalactic medium (IGM) and therefore rapidly lose energy [Broderick et al., 2012],[Schlickeiser et al., 2012]; this again results in a suppresion of the spectrum at GeV energies [Saveliev et al., 2013a]



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- Cosmological scenario: Strong seed magnetic fields are generated in the Early Universe, e.g. at a phase transition (QCD, electroweak) [Sigl et al., 1997] or during inflation [Turner and Widrow, 1988], and some of the initial energy content is transfered to larger scales.

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- Basics for the time evolution: Homogeneous and isotropic magnetohydrodynamics in an expanding Universe.

Magnetohydrodynamics (MHD)

Primordial Magnetic fields - Basic MHD

Magnetohydrodynamics (MHD)

- Maxwell's equations:
  $\nabla \cdot \mathbf{B} = 0, \ \nabla \times \mathbf{E} = -\partial_t \mathbf{B}, \ \nabla \times \mathbf{B} = 4\pi \mathbf{j}$

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- Navier-Stokes equations:
  $\rho \left(\partial_t \mathbf{v} + (\mathbf{v} \cdot \nabla)\, \mathbf{v}\right) = -\nabla p + \mu \Delta \mathbf{v} + (\lambda + \mu) \nabla \left(\nabla \cdot \mathbf{v}\right) + \mathbf{f}$

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For the magnetic field and the turbulent fluid it follows therefore

$$\partial_t \mathbf{B} = \frac{1}{4\pi\sigma} \Delta \mathbf{B} + \nabla \times (\mathbf{v} \times \mathbf{B})$$
$$\partial_t \mathbf{v} = -\left(\mathbf{v} \cdot \nabla\right) \mathbf{v} + \frac{(\nabla \times \mathbf{B}) \times \mathbf{B}}{4\pi\rho} + \mathbf{f}_v .$$

Primordial Magnetic fields - Basic MHD

The aspect of interest is the distribution of energies on different scales k, i.e. the magnetic spectral energy density M of the magnetic fields and the kinetic magnetic spectral energy density U

$$\epsilon_B = \frac{1}{8\pi V}\int \mathrm{d}^3x\, \mathbf{B}^2(\mathbf{x}) = \int \frac{\mathrm{d}^3k}{8\pi}\,|\hat{\mathbf{B}}(\mathbf{k})|^2 \equiv \rho \int \mathrm{d}k\, M_k$$

$$\epsilon_K = \frac{\rho}{2V}\int \mathrm{d}^3x\, \mathbf{v}^2(\mathbf{x}) = \frac{\rho}{2}\int \mathrm{d}^3k\,|\hat{\mathbf{v}}(\mathbf{k})|^2 \equiv \rho \int \mathrm{d}k\, U_k$$

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In addition, for magnetic helicity one can define the spectral helicity density $\mathcal{H}$ by

$$h_B = \frac{1}{V} \int \mathrm{d}^3x\, \mathbf{A}(\mathbf{x}) \cdot \mathbf{B}(\mathbf{x}) = i \int \mathrm{d}^3\mathrm{k} \left(\frac{\mathbf{k}}{\mathrm{k}^2} \times \widehat{\mathbf{B}}(\mathbf{k}) \right) \cdot \widehat{\mathbf{B}}(\mathbf{k})^* \equiv \rho \int \mathrm{d}k\, \mathcal{H}_k$$

Therefore: Switch to Fourier (k-)space:
$B(\mathbf{x}) \longrightarrow \hat{B}(\mathbf{q})$, $v(\mathbf{x}) \longrightarrow \hat{v}(\mathbf{q})$

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$$\partial_t \hat{\mathbf{B}}(\mathbf{q}) = -\frac{1}{4\pi\sigma} q^2 \hat{\mathbf{B}}(\mathbf{q}) + \frac{iV^{\frac{1}{2}}}{(2\pi)^{\frac{3}{2}}} \mathbf{q} \times \left[\int \mathrm{d}^3 k \left(\hat{\mathbf{v}}(\mathbf{q}-\mathbf{k}) \times \hat{\mathbf{B}}(\mathbf{k}) \right) \right]$$

$$\partial_t \hat{\mathbf{v}}(\mathbf{q}) = -\frac{iV^{\frac{1}{2}}}{(2\pi)^{\frac{3}{2}}} \int \mathrm{d}^3 k \left[\left(\hat{\mathbf{v}}(\mathbf{q}-\mathbf{k}) \cdot \mathbf{k} \right) \hat{\mathbf{v}}(\mathbf{k}) \right]$$
$$+ \frac{iV^{\frac{1}{2}}}{(2\pi)^{\frac{3}{2}}} \frac{1}{4\pi\rho} \int d^3 k \left[\left(\mathbf{k} \times \hat{\mathbf{B}}(\mathbf{k}) \right) \times \hat{\mathbf{B}}(\mathbf{q}-\mathbf{k}) \right].$$

Primordial Magnetic Fields - Correlation Function

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In Fourier space this means that the most general Ansatz is [von Kármán and Howarth, 1938, Junklewitz and Enßlin, 2011]

$$\langle \hat{B}_l(\mathbf{k})\hat{B}_m(\mathbf{k}')\rangle \sim \delta(\mathbf{k}-\mathbf{k}')[(\delta_{lm}-\frac{k_l k_m}{k^2})M(k)-\frac{i}{8\pi}\epsilon_{lmj}k_j\mathcal{H}(k)]$$

$$\langle \hat{v}_l(\mathbf{k})\hat{v}_m(\mathbf{k}')\rangle \sim \delta(\mathbf{k}-\mathbf{k}')[(\delta_{lm}-\frac{k_l k_m}{k^2})U(k)-\frac{i\rho}{2k^2}\epsilon_{lmj}k_j\mathcal{H}^{\mathrm{K}}(k)]$$

Master Equations for the Time Evolution of M, U and $\mathcal{H}$

$$\left\langle \partial_t M_q \right\rangle = \int_0^\infty \mathrm{d}k \Bigg(\Delta t \Bigg\{ -\frac{2}{3} q^2 \left\langle M_q \right\rangle \langle U_k \rangle - \frac{4}{3} q^2 \left\langle M_q \right\rangle \langle M_k \rangle$$

$$+ \frac{1}{3} \frac{1}{(4\pi)^2} q^2 k^2 \left\langle \mathcal{H}_q \right\rangle \langle \mathcal{H}_k \rangle + \int_0^\pi \mathrm{d}\theta \left[\frac{1}{2} \frac{q^4}{k_1^4} \left(q^2 + k^2 - qk \cos\theta \right) \sin^3\theta \, \langle M_k \rangle \left\langle U_{k_1} \right\rangle \right] \Bigg\} \Bigg)$$

$$\left\langle \partial_t U_q \right\rangle = \int_0^\infty \mathrm{d}k \Bigg(\Delta t \Bigg\{ -\frac{2}{3} q^2 \langle M_k \rangle \left\langle U_q \right\rangle - \frac{2}{3} q^2 \left\langle U_q \right\rangle \langle U_k \rangle$$

$$+ \int_0^\pi \mathrm{d}\theta \Bigg[\frac{1}{4} \frac{q^3 k}{k_1^4} \left(qk \sin^2\theta + 2k_1^2 \cos\theta \right) \sin\theta \, \langle M_k \rangle \left\langle M_{k_1} \right\rangle + \frac{1}{4} \frac{q^4 k}{k_1^4} (3k - q\cos\theta) \sin^3\theta \, \langle U_k \rangle \left\langle U_{k_1} \right\rangle$$

$$+ \frac{1}{(16\pi)^2} \frac{q^3 k^2}{k_1^2} \left(-2q - q\sin^2\theta + 2k\cos\theta \right) \sin\theta \, \langle \mathcal{H}_k \rangle \left\langle \mathcal{H}_{k_1} \right\rangle \Bigg] \Bigg\} \Bigg)$$

$$\langle \partial_t \mathcal{H}_q \rangle = \int_0^\infty \mathrm{d}k \Bigg(\Delta t \Bigg\{ \frac{4}{3} k^2 \langle M_q \rangle \langle \mathcal{H}_k \rangle - \frac{4}{3} q^2 \langle M_k \rangle \langle \mathcal{H}_q \rangle - \frac{2}{3} q^2 \langle U_k \rangle \langle \mathcal{H}_q \rangle$$

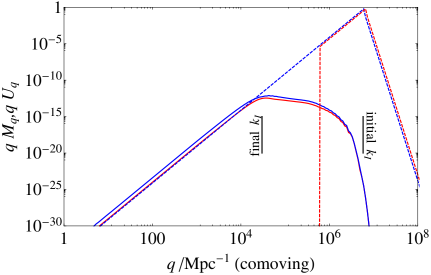
$$+ \int_0^\pi \mathrm{d}\theta \left[\frac{1}{2} \frac{q^4 k^2}{k_1^4} \sin^3\theta \left\langle U_{k_1} \right\rangle \langle \mathcal{H}_k \rangle \right] \Bigg\} \Bigg)$$

Energy/helicity conservation: $\partial_t \epsilon_\mathrm{B} = \rho \int \mathrm{d}q \left(\partial_t M_q + \partial_t U_q \right) = 0$ and $\partial_t h_\mathrm{B} = \rho \int \mathrm{d}q \partial_t \mathcal{H}_q = 0$

Results on the Time Evolution of Primordial Magnetic Fields without Helicity

[Saveliev et al., 2012]

- Starting either with an initial power-law ...



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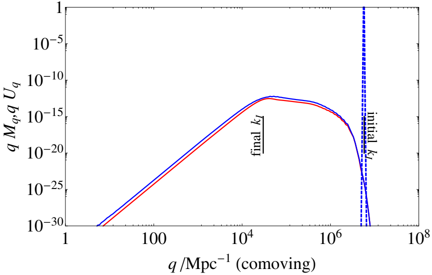
- Starting either with an initial power-law ...
- ... or a concentration of the spectral energies on a single scale the qualitative result is similar: a tendence to equipartition and both $M_q \propto q^4$ (i.e. $B \propto q^{\frac{5}{2}}$) and $U_q \sim q^4$ at large scales.



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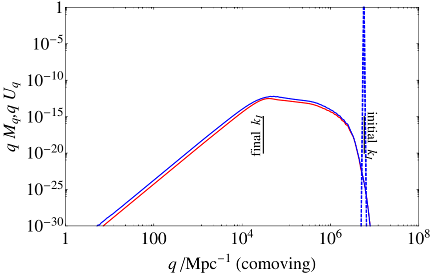
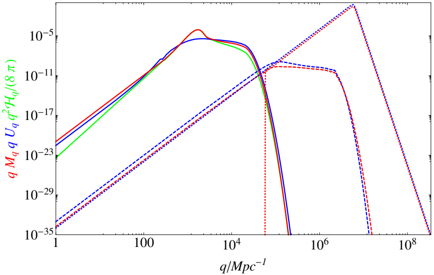


- A rough estimate for B (for the QCD phase transition) is given by $B(200\,\mathrm{pc}) \lesssim 5 \times 10^{-12}\,\mathrm{G}$

Results on the Time Evolution of Primordial Magnetic Fields with Helicity

- Including magnetic helicity for the same initial conditions results in an inverse cascade, a fast transport of big amounts of magnetic energy to large scales. This is due to helicity conservation.

[Saveliev et al., 2013b]



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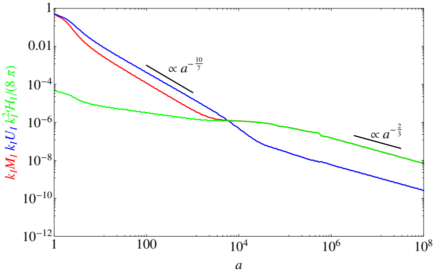
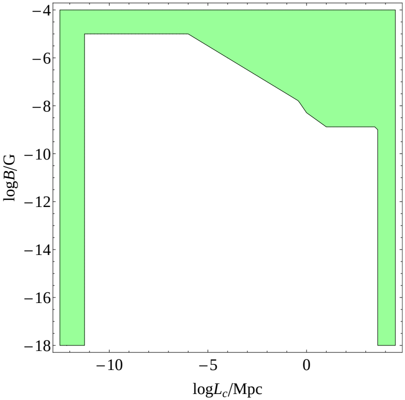


  - Two regimes are visible: When helicity is small, the considerations of the non-helical case are valid; once helicity reaches its maximal value, the behaviour changes dramatically

Additional EGMF Constraints from Primordial Fields



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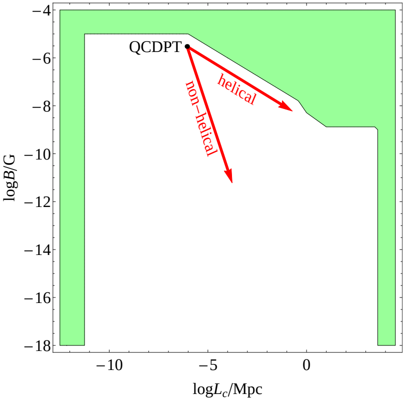

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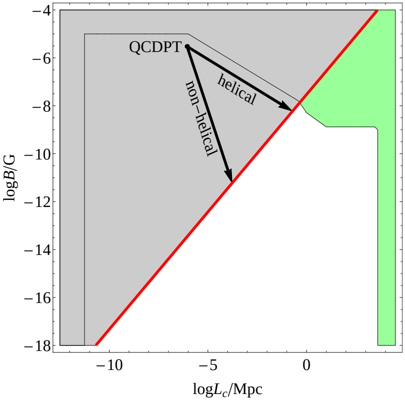

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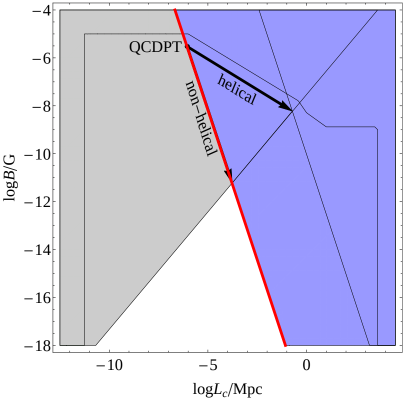

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- Considering the power-law slope for the spectral energies, causality dictates further limits.

Conclusions and Outlook

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- The expicit computation of the back-reaction of the magnetic field on the medium gives the result of a power-law behavior with $M_q \sim q^4$ (i.e. $B \sim q^{\frac{5}{2}}$) and $U_q \sim q^4$ and equipartition at large scales.

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- In the future comparison of the presented results from a semi-analytical approach with numerical simulations $\longrightarrow$ Collaboration with Robi's group

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- One possible origin is the time evolution of primordial magnetic fields during which energy, among other things, can be transported from smaller to larger scales
- Helicity enhances this effect by creating an inverse cascade which results in much higher magnetic fields today compared to the non-helical case
- The expicit computation of the back-reaction of the magnetic field on the medium gives the result of a power-law behavior with $M_q \sim q^4$ (i.e. $B \sim q^{\frac{5}{2}}$) and $U_q \sim q^4$ and equipartition at large scales.
- In the future comparison of the presented results from a semi-analytical approach with numerical simulations $\longrightarrow$ Collaboration with Robi's group

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