Renormalization Group approaches to quantum gravity

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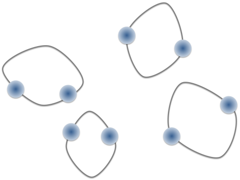
DESY, Theory Seminar, 31.3.2014

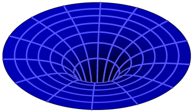
PERIMETER
INSTITUTE
FOR THEORETICAL PHYSICS

Quantum fields and gravitational fields

quantum fields:



gravity:



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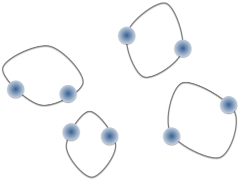

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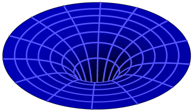
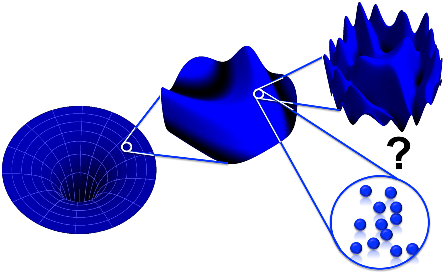


→ quantum gravity:

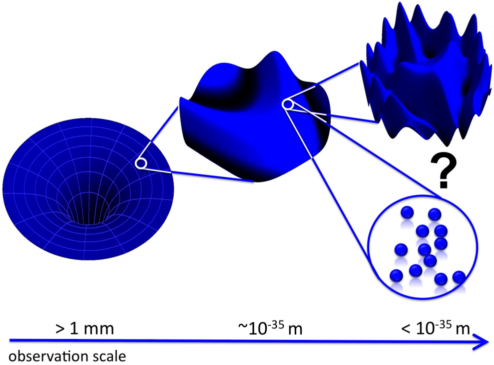


spacetime fluctuations
at the Planck scale

quantum theory of gravity in the path-integral framework:

Goal: $\int_{\text{spacetimes}} e^{iS}$

The Renormalization Group in quantum gravity



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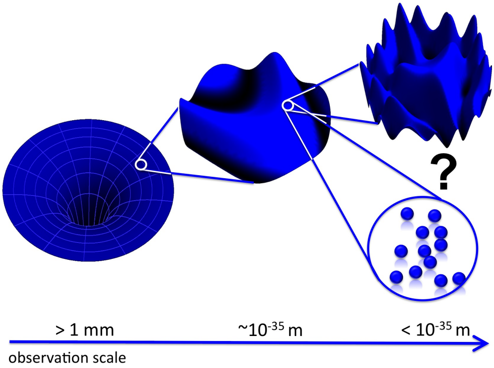

effective action for effective degrees of freedom at k:

$$e^{-\Gamma_k[\phi]} = \int_{p>k} \mathcal{D}\varphi \, e^{-S[\varphi]}$$

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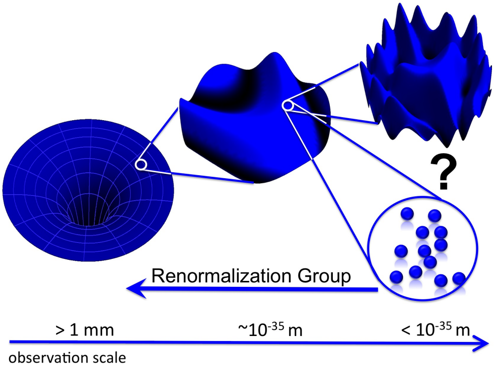

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Γ_k

microscopic action ($k \rightarrow \infty$)

family of effective actions

low-energy effective action ($k \rightarrow 0$)

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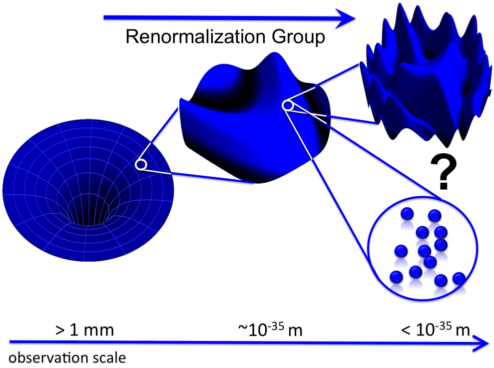

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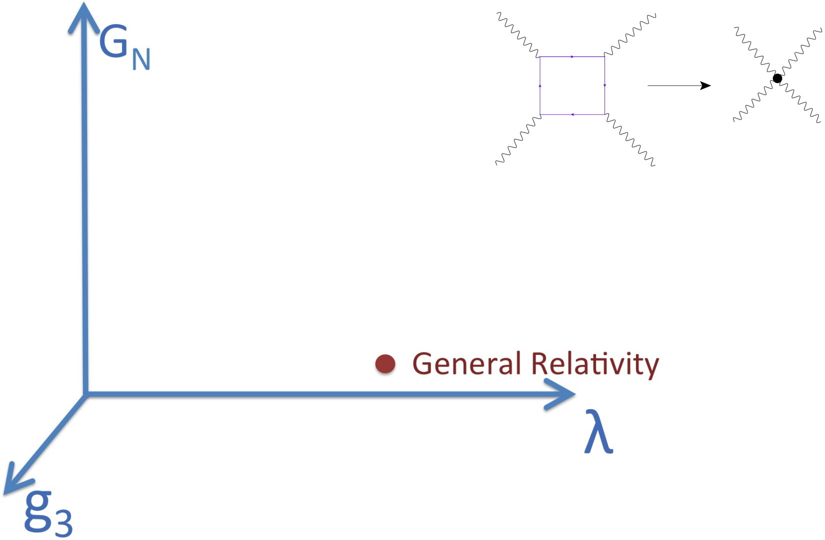

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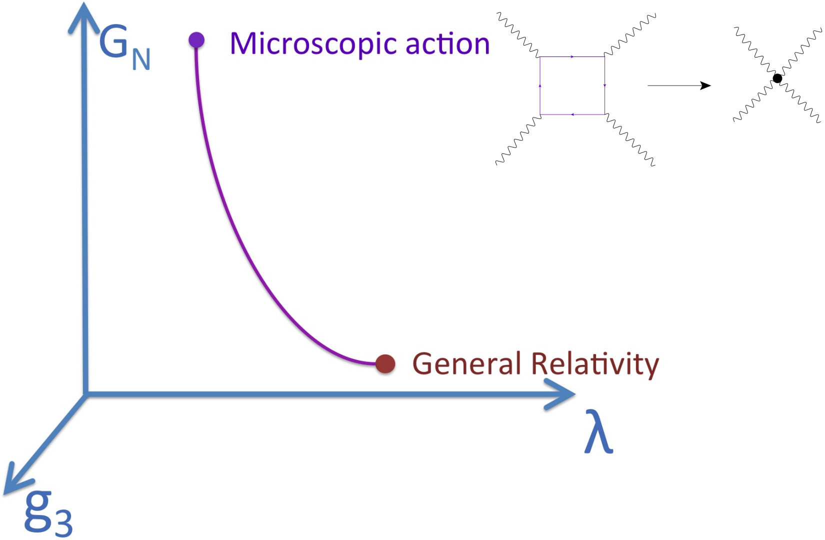
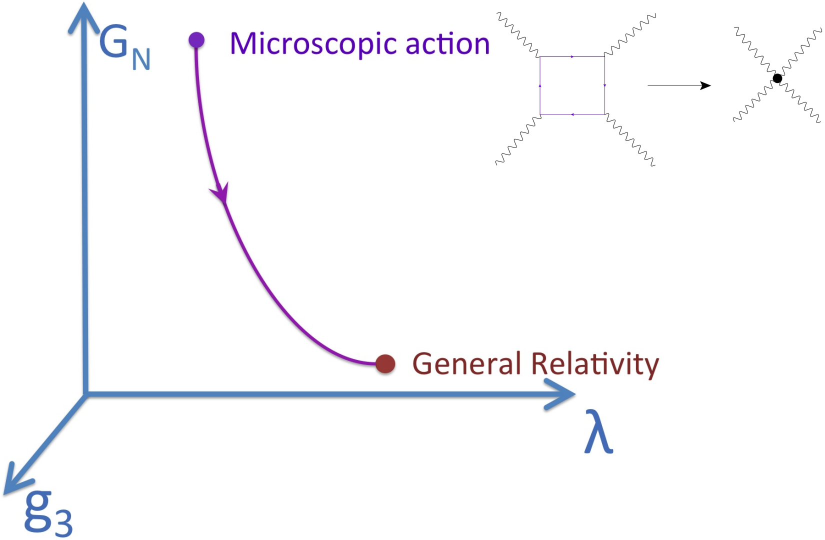
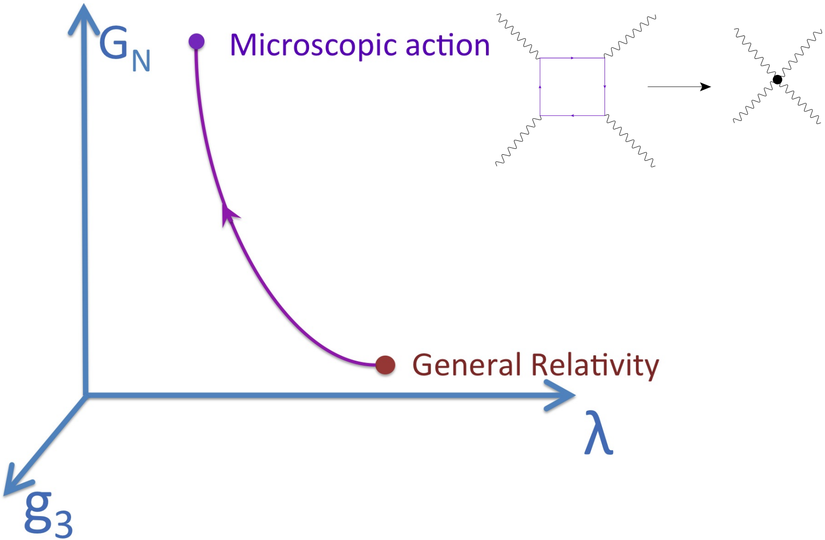


The Renormalization Group in quantum gravity



test continuum limit of discrete models (matrix/ tensor models)

The Renormalization Group in quantum gravity

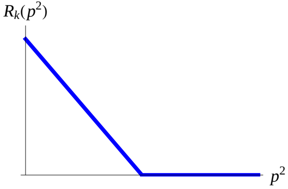


find out whether quantum gravity is asymptotically safe

Functional Renormalization Group

include high- energy quantum fluctuations first:

$$e^{-\Gamma_k[\phi]} = \int \mathcal{D}\varphi \, e^{-S[\varphi] - \frac{1}{2}\int_p \varphi(p)\, R_k(p)\, \varphi(-p)}$$



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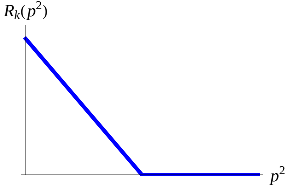

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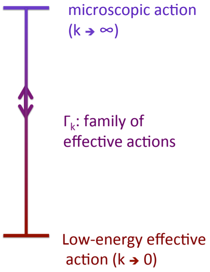
scale-dependent action:

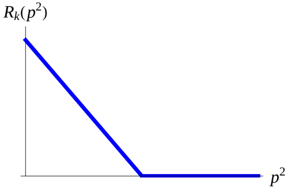


$\boldsymbol{\Gamma_k}$ contains effect of quantum fluctuations above **momentum scale** k

$\Gamma_{k\to 0} = \Gamma$ $\qquad$ $\Gamma_{k\to\Lambda\to\infty} \to S$

$\Gamma = \Gamma_{\mathrm{EH}}$ $\qquad$ S: prediction!

Functional Renormalization Group



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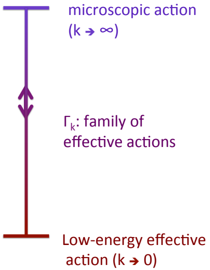
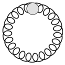


Wetterich equation [1993]:

$$\partial_k \Gamma_k = \frac{1}{2}\mathrm{STr}\left(\Gamma_k^{(2)} + R_k\right)^{-1} \partial_k R_k =$$



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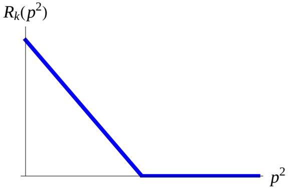

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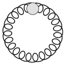
$$\partial_k \Gamma_k = \frac{1}{2}\mathrm{STr}\left(\Gamma_k^{(2)} + R_k\right)^{-1} \partial_k R_k =$$  $\quad\quad$ $\Gamma_k = \sum_i g_i(k)\mathcal{O}_i$

$\to$ non-perturbative $\beta_g = k\partial_k g(k) \to$ Quantum Gravity [Reuter, 1996]

Functional Renormalization Group

$\partial_k \Gamma_k = \frac{1}{2}\mathrm{STr}\left(\Gamma_k^{(2)} + R_k\right)^{-1} \partial_k R_k$: exact one-loop equation

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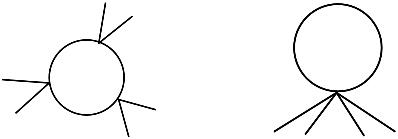
$\Gamma_k = \sum_i \bar{g}_i(k)\mathcal{O}_i[g_{\mu\nu}] \rightarrow$ infinitely many coupled equations

example: $\lambda\phi^4$ theory:



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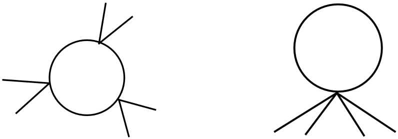

in practice:

1. truncated ansatz for $\Gamma_k = \sum_i^I \bar{g}_i(k)\mathcal{O}_i \to$ evaluate β functions for $g_i(k)$
2. enlarge truncation for Γ_k
3. Convergence of results?

FRG in different physical systems: Does it work?

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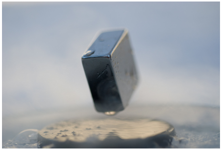
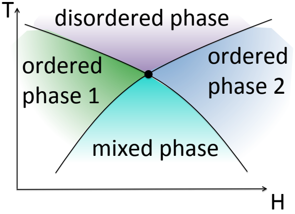
multicritical points in condensed matter systems with competing orders





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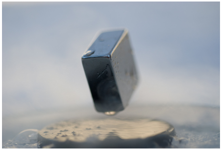
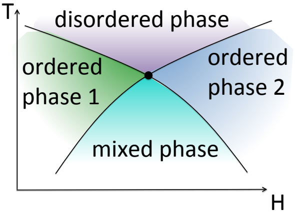


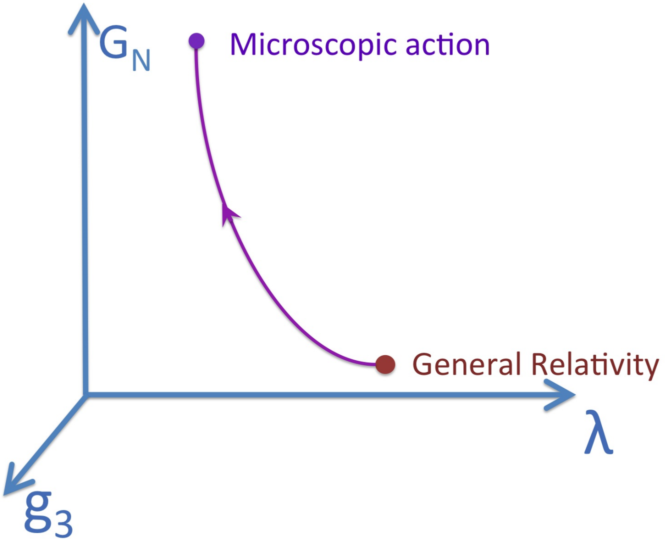


scaling in the vicinity of critical point: can be determined from Renormalization Group fixed-point scaling

results: scaling exponent for $O(2) \oplus Z_2$:

$y_{2,2} = 1.790$ [AE, Mesterházy, Scherer, Phys. Rev. E 88 (2013) 042141];
$y_{2,2} = 1.7906(3)$ (Monte Carlo) [Hasenbusch, Vicari (2007)]

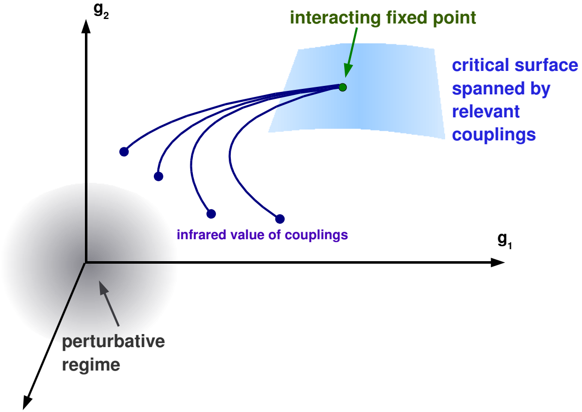
The Renormalization Group in quantum gravity



find out whether quantum gravity is asymptotically safe

Asymptotic safety

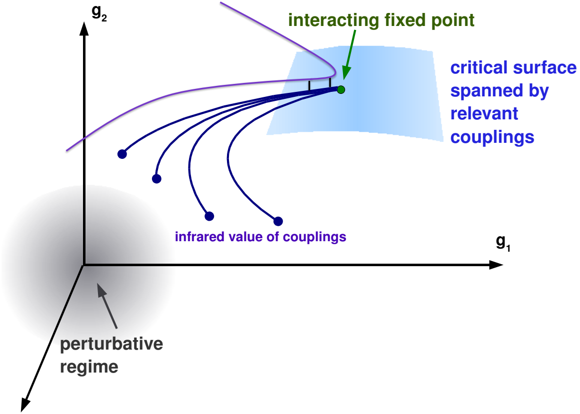
Fundamental QFT:
$\int_{p>k} \mathcal{D}\varphi \, e^{-S}$
couplings stay finite
for $k \to \infty$
scale-free!



Fundamental theory:

Running dimensionless couplings approach fixed point towards UV

Asymptotic safety



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Running dimensionless couplings approach fixed point towards UV

Predictive theory:

Finite number of relevant (UV-attractive) couplings:

$$g_i(k) = g_* + c \left(\frac{k}{k_0}\right)^{-\theta_i} \qquad \theta_i = d_i + \eta_i$$

Asymptotically Safe Quantum Gravity: Evidence

$\Gamma_{k\,\mathrm{EH}} = \frac{-1}{16\pi G_N(k)} \int \sqrt{g}(R - 2\bar{\lambda}(k))$

$G = G_N k^2$ and $\lambda = \bar{\lambda}/k^2$

fixed point in dimensionless couplings $\rightarrow$ scale-free regime

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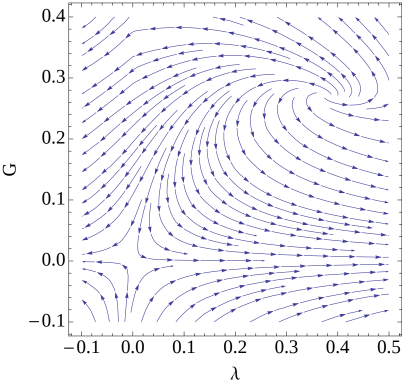

Compatibility with observations:
G and λ relevant directions

Semiclassical gravity?

[M. Reuter, 1996; M. Reuter, F.Saueressig, 2001; D. Litim, 2004]

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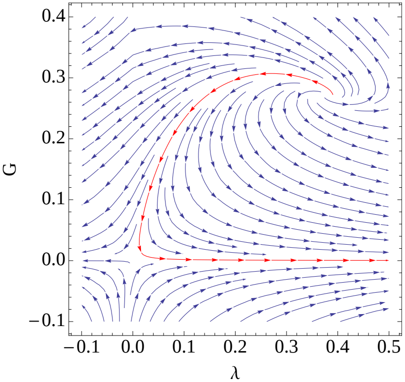

Compatibility with observations:
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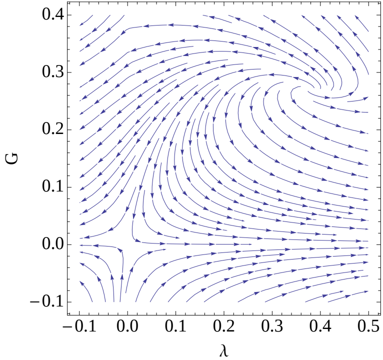
Semiclassical gravity?

trajectory with $G_N \rightarrow \mathrm{const}$ and $\bar{\lambda} \rightarrow \mathrm{const}$ and measured values in infrared

[M. Reuter, 1996; M. Reuter, F.Saueressig, 2001; D. Litim, 2004]

Asymptotically Safe Quantum Gravity: Evidence



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fixed-point action: *prediction*

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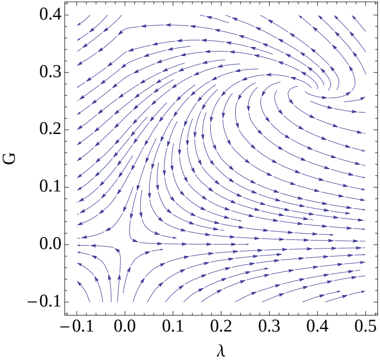

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fixed-point action: *prediction*

$$\Gamma_k = \Gamma_{k\,\mathrm{EH}} + \Gamma_{\mathrm{gauge-fixing}} + \Gamma_{\mathrm{ghost}} + \int \sqrt{g}\,(f(R) + R_{\mu\nu}R^{\mu\nu} + ...)$$

E. Manrique, M. Reuter, F. Saueressig (2009, 2010);

I. Donkin, J. Pawlowski (2012);

A. Codello, G. D'Odorico, C. Pagani (2013)

A.E., H.Gies, M.Scherer (2009), A.E., H. Gies (2010),

A.E. (2013)

A. Codello, R. Percacci, C. Rahmede (2008);

D.Benedetti, F. Caravelli (2012);

K. Falls, D. Litim, K. Nikolakopoulos (2013);

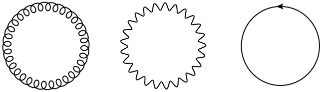
J. Dietz, T. Morris (2013);

M. Demmel, F. Saueressig, O. Zanusso (2014)

D. Benedetti, P. Machado, F. Saueressig (2009)

Does matter matter?

quantum fluctuations of all fields drive Renormalization Group flow:



Quantum gravity and matter

Quantum gravity and matter

Quantum gravity will have an effect on matter:

$$\beta_{g_{\mathrm{matter}}} = f(g_{\mathrm{matter}}) + c_1\, G\, g_{\mathrm{matter}} + c_2\, G^2 + \ldots$$

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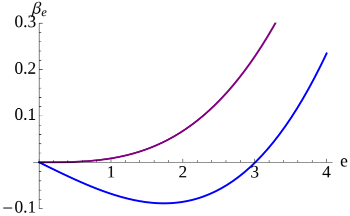
- possible UV fixed point for Higgs sector [Zanusso, Zambelli, Vacca, Percacci (2009); A.E. (2012)]
- possible UV fixed point for QED [Harst, Reuter (2011); Doná, A.E., Percacci (2013)] (cf. [Robinson, Wilczek (2005)])

$$\beta_e = \frac{e^3}{12\pi^2} N_D - \frac{3}{8\pi} e \left(G - 4G\lambda^2\right) + \ldots$$



Can predict e at low energies?

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Matter will have an effect on quantum gravity

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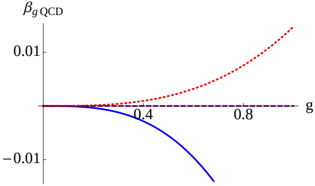
analogy: Quantum Chromodynamics:



Asymptotic freedom only for $N_f < 16.5$

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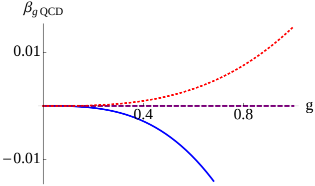

Asymptotic freedom only for $N_f < 16.5$

Possibility to test quantum gravity models!

Is quantum gravity compatible with Standard Model matter?

Matter matters

with P. Doná, R. Percacci (2013): Truncation of the effective action:

$\Gamma_k = \Gamma_{k\,\mathrm{Einstein-Hilbert}} + \Gamma_{k\,\mathrm{matter}}$ with minimally coupled matter:

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N_S scalars: $S_S = \frac{Z_S}{2}\int d^dx\sqrt{g}\,g^{\mu\nu}\sum_{i=1}^{N_s}\partial_\mu\phi^i\partial_\nu\phi^i$

N_D Dirac fermions $S_D = iZ_D\int d^dx\sqrt{g}\,\sum_{i=1}^{N_D}\bar{\psi}^i\not{\nabla}\psi^i$

N_V Abelian vector bosons:
$S_V = \frac{Z_V}{4}\int d^dx\sqrt{g}\sum_{i=1}^{N_F}g^{\mu\nu}g^{\kappa\lambda}F^i_{\mu\kappa}F^i_{\nu\lambda} + \frac{Z_V}{2\xi}\int d^dx\sqrt{\bar{g}}\sum_{i=1}^{N_F}\left(\bar{g}^{\mu\nu}\bar{D}_\mu A^i_\nu\right)^2$

Matter matters

with P. Doná, R. Percacci (2013): Truncation of the effective action:

$\Gamma_k = \Gamma_{k\,\mathrm{Einstein-Hilbert}} + \Gamma_{k\,\mathrm{matter}}$ with minimally coupled matter:

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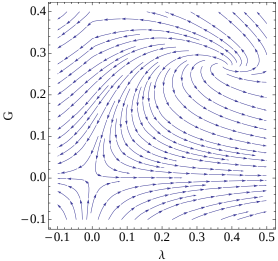
$\rightarrow \beta_G, \beta_\lambda, \eta_h, \eta_c,$
η_F, η_D, η_S

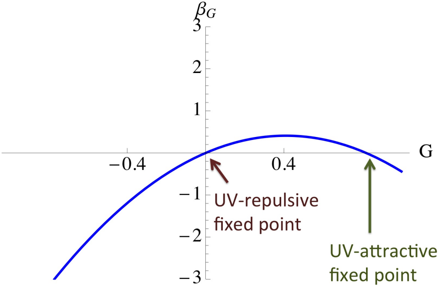


???

Perturbative analysis

(neglect graviton and matter wave function renormalizations)

$$\beta_G = 2G + \frac{G^2}{6\pi}\left(\qquad\qquad\qquad\qquad - 46\right),$$



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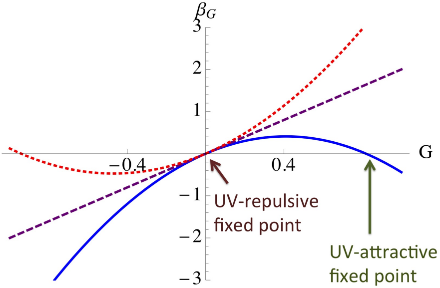
(neglect graviton and matter wave function renormalizations)

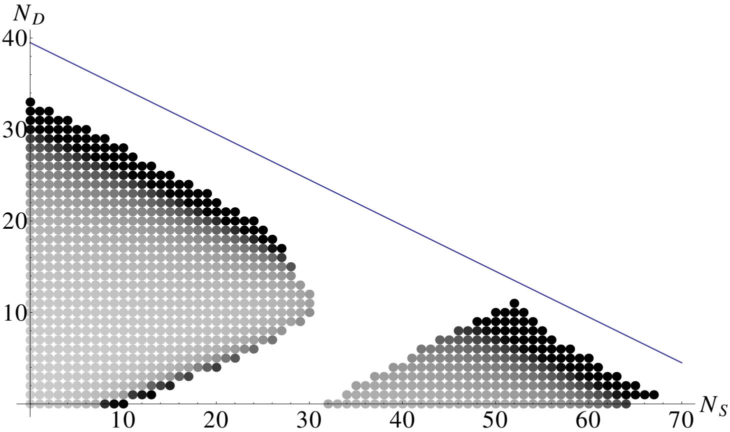
$$\beta_G = 2G + \frac{G^2}{6\pi}\left(N_S + 2N_D - 4N_V - 46\right),$$



→ for a given number of vectors N_V, there is an upper limit on the number of scalars N_S and Dirac fermions N_D!

Matter matters in asymptotically safe quantum gravity!

Full analysis for $N_V = 12$



upper limit on N_D and N_S

Standard Model: $N_V = 12$, $N_D = 45/2$, $N_S = 4$: compatible with gravitational fixed point

Specific matter models

Standard Model: $(N_S = 4, N_D = 45/2, N_V = 12)$ ✓

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supersymmetric extension (MSSM: $N_S = 49, N_D = 61/2, N_V = 12$) ✗

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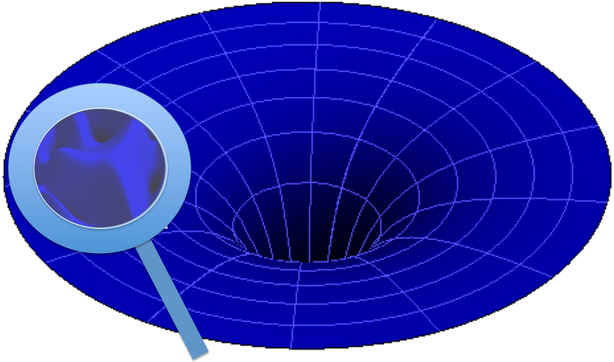
Prediction:
Only specific models with restricted matter content are compatible with Asymptotically Safe Quantum Gravity within our truncation!

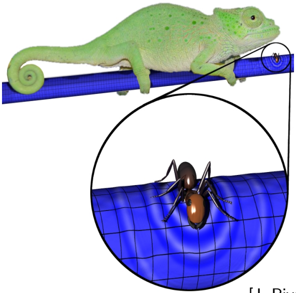
Tests of quantum gravity



Does testing quantum gravity require galaxy-size accelerators?

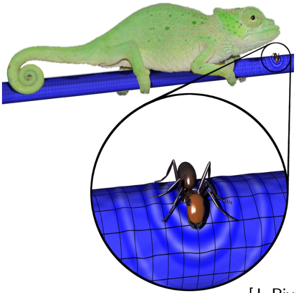
Possibly could test Asymptotically Safe Quantum Gravity at LHC, 14 TeV:
Look for Beyond-Standard-Model particle physics

Extra dimensions



[J. Pivarski]

Extra dimensions



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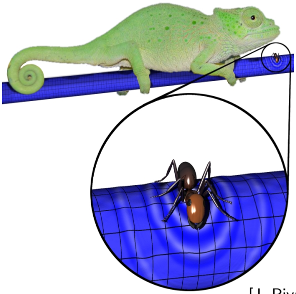
Extra dimensions in asymptotic safety?

pure-gravity fixed point exists in $d \geq 4$

(Einstein-Hilbert [Fischer, Litim, 2006] and higher derivatives [Ohta, Percacci, 2013])

Extra dimensions



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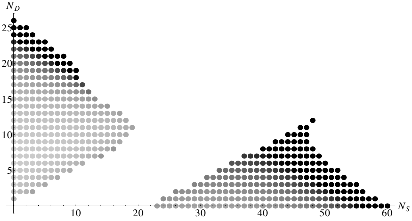
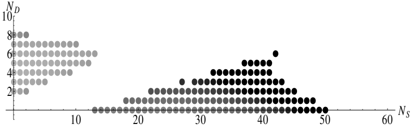


$5d$



$6d$

Asymptotic safety: Summary

Continuum quantum field theory for gravity $\rightarrow$ interacting UV fixed point

$\Rightarrow$ nonperturbative tool: functional Renormalization Group

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- Pure gravity fixed point persists in extended truncations
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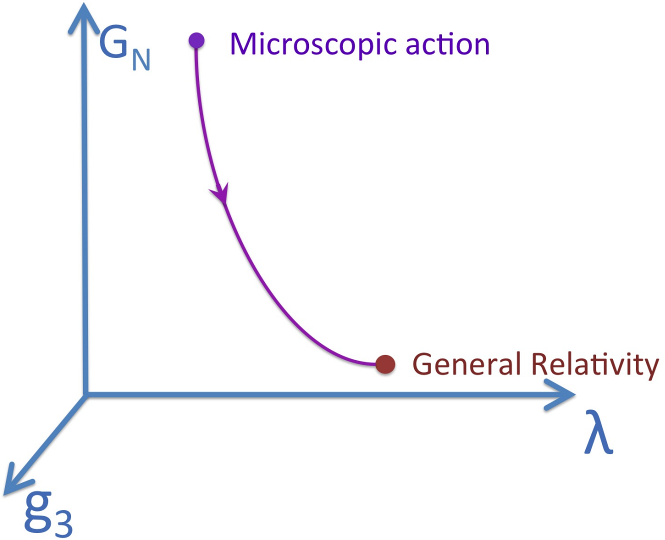
- Pure gravity fixed point persists in extended truncations
- Coupling to Standard Model matter consistent (not all BSM-scenarios compatible!)

New test of quantum gravity at LHC and future accelerators could be possible!

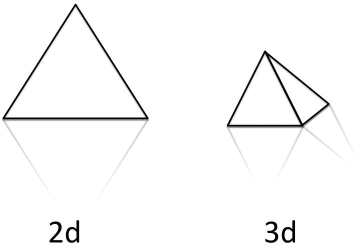
The Renormalization Group in quantum gravity

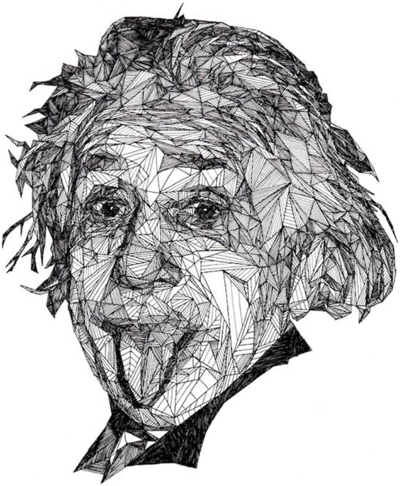


test continuum limit of discrete models (matrix/ tensor models)

Building a spacetime

building blocks of spacetime:



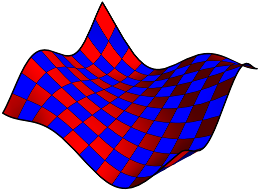


[by Josh Bryan]

Building a spacetime

$$\int \mathcal{D}g_{\mu\nu} e^{-S[g_{\mu\nu}]} \longrightarrow \sum_{\mathcal{T}} e^{-S[\mathcal{T}]}$$

superposition of quantum spacetimes $\rightarrow$ sum over triangulations



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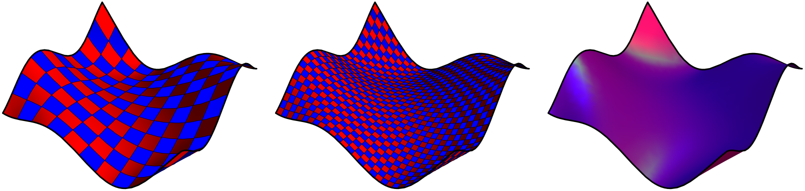


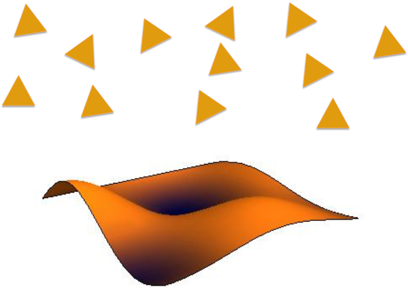
discretization: “unphysical” tool

Main challenge: What is the continuum limit?
(e.g. Causal Dynamical Triangulations program [Ambjorn, Jurkiewicz, Loll], Tensor Track[Bonzom, Ben Geloun, Freidel, Gurau, Oriti, Rivasseau])

Building a spacetime

Continuum limit from fixed point of Renormalization Group

"condensation" of building blocks



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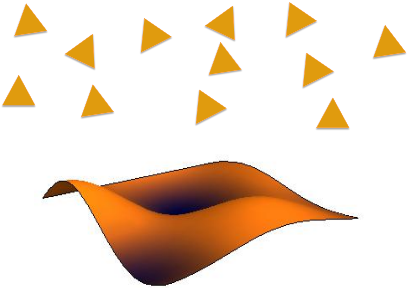
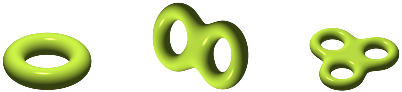


two-dimensional gravity: exact results known $\rightarrow$ testing ground for Functional RG methods!

Two-dimensional quantum gravity

gravity in two dimensions: $\frac{1}{4\pi}\int d^2x\sqrt{g}R = \chi \rightarrow$ action is topological

Euler character $\chi = 2 - 2h$ with h number of handles



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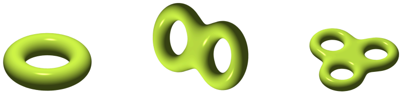

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How to evaluate partition function? $\rightarrow$ use triangulation!

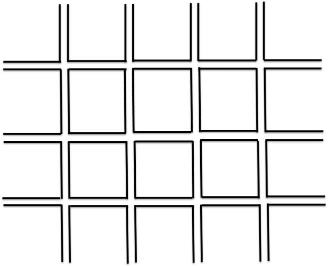
Matrix model for two-dimensional quantum gravity

$Z_{grav} \sim \sum_{\mathcal{T}} e^{-S[\mathcal{T}]}$,

reformulate as hermitian matrix model: ϕ_{ij} NxN matrix

$Z_{grav} \sim \ln\left(\int d\phi e^{-\mathrm{tr}\phi^2 + g\mathrm{tr}\phi^4}\right)$

Feynman diagrams:



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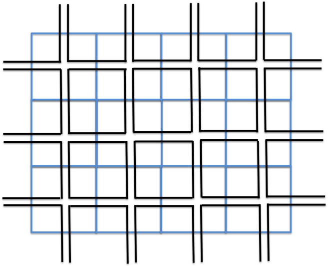

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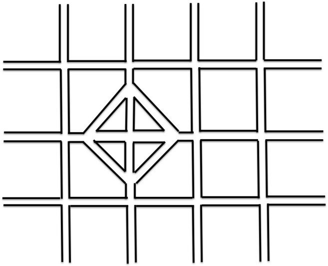

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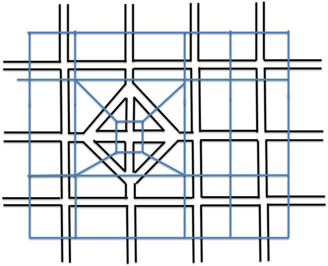
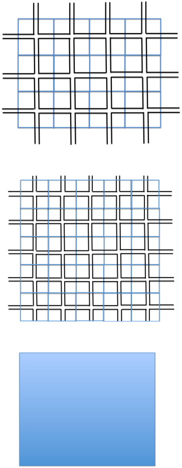
Feynman diagram expansion yields sum over all “squarulations”



Matrix model for two-dimensional quantum gravity

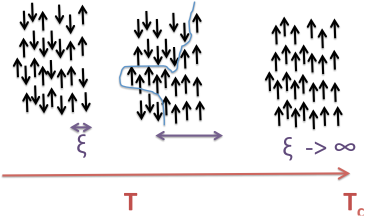
continuum limit: $$N^2 Z_{grav} = \ln\left(\int d\phi\, e^{-\mathrm{tr}\phi^2 + g\,\mathrm{tr}\phi^4}\right)$$



$g \to g_c$

$\langle A \rangle \to \infty$

(cf. phase transition)
$\to$ universal behavior!



Large N limit

$Z_{grav} = \sum_h N^{2(1-h)} Z_h \quad \Rightarrow \quad N \to \infty$: $h = 0$ dominates

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$N \to \infty$ and $g \to g_c$ such that N^{-2h} is compensated

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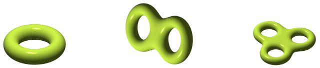

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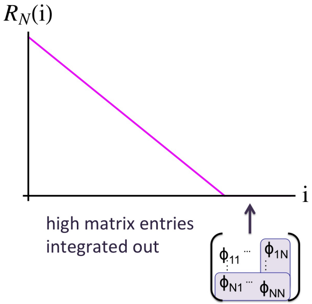
main advantage: can extend to d=4 tensor models!

Renormalization Group in matrix models

$$N\partial_N \Gamma_N = \frac{1}{2}\mathrm{tr}\left(\Gamma_N^2 + R_N\right)^{-1} N\partial_N R_N$$

A.E., T. Koslowski PRD 88, 084016 (2013))



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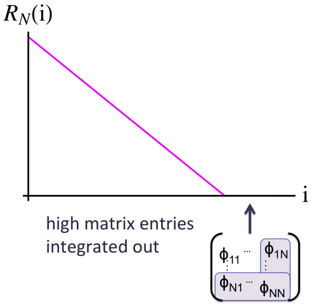
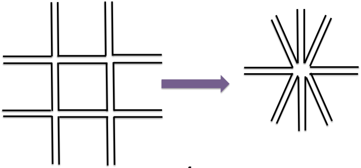


theory space:



$g_4^4 \to g_8$ etc.

$\to$ all random polygonizations generated! $\Gamma_N = \frac{Z}{2}\mathrm{tr}\phi^2 + \sum_i \frac{g_i}{i}\mathrm{tr}\phi^i + \ldots$

Results: Double scaling limit

Truncation: $Z, g_4, g_6, ..., g_{14}$ [A.E., T. Koslowski, PRD 88, 084016 (2013)]
fixed point with one relevant direction ($g \to g_c$ tuning)
RG works qualitatively in matrix models!

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What precision to expect? Wilson-Fisher fixed point in scalar ϕ^4 in $d = 3$:
leading order approximation ~ 20 %

optimization tools, extended truncations give quantitative precision

$\rightarrow$ route to quantitative precision in matrix models clear

Summary and Outlook: RG in matrix models

Matrix models: Gravitational path-integral from continuum limit in discretized setting

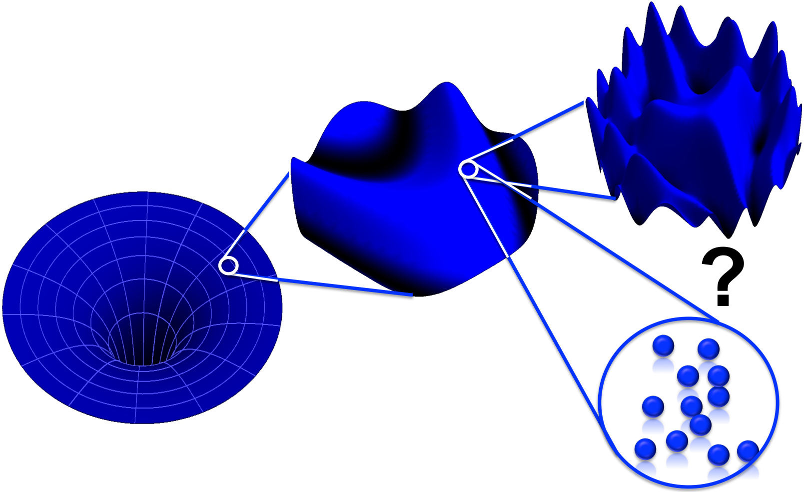
Double scaling limit accessible as fixed point of RG

Qualitative results ✓

Outlook: Development of optimization methods, extended truncations $\rightarrow$ towards quantitative precision!

Extension to higher dimensions $d = 4$
("recipe" in PRD 88, 084016 (2013))

The Renormalization Group in quantum gravity



Thank you for your attention!