Basic Considerations on Spatial Resolution in Patterned Detectors

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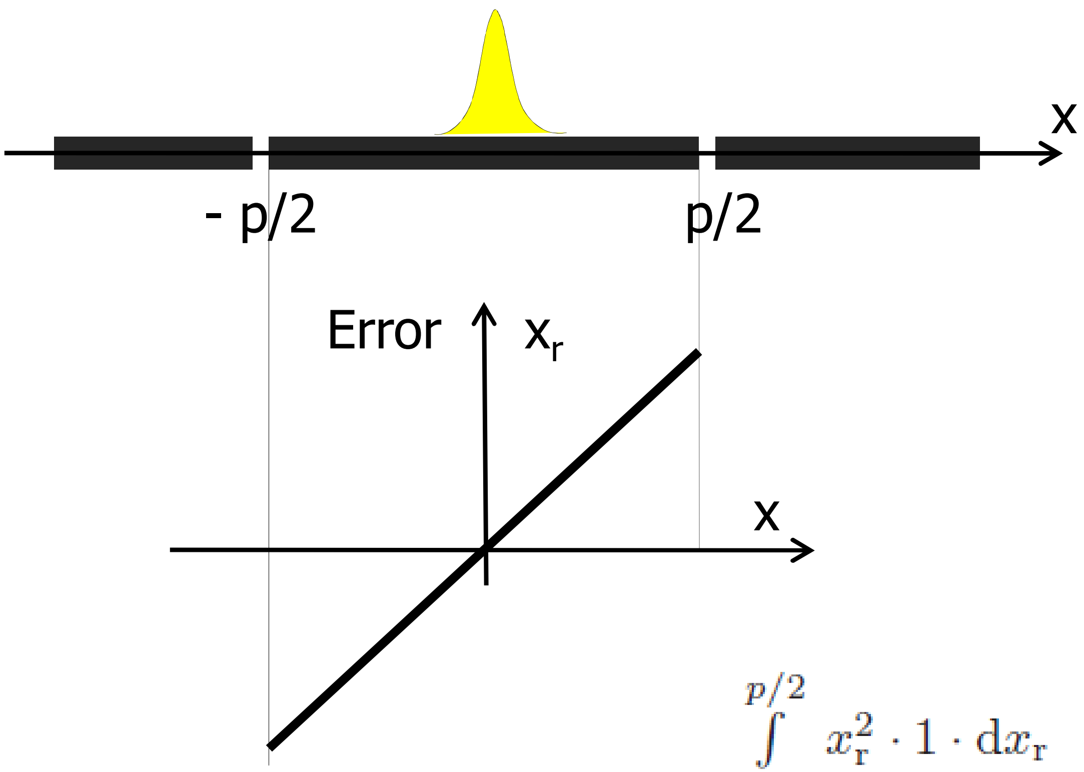
Overview

- Warmup:
  Resolution with binary readout – optimal signal width
- Error of Center-of-Gravity: When do we need a fit?
- Influence of noise on spatial resolution
  - Higher Moments
  - Correlated Noise
  - 2D structures
  - Wide signals
- Error when doing ‘Eta-reconstruction’
  - Search for ‘best’ response function

BINARY READOUT OF BOX SIGNALS

Spatial Resolution of Narrow Signals

- Consider very narrow signal
- Only **one** strip is hit
- Reconstructed position = strip center, error = offset in strip

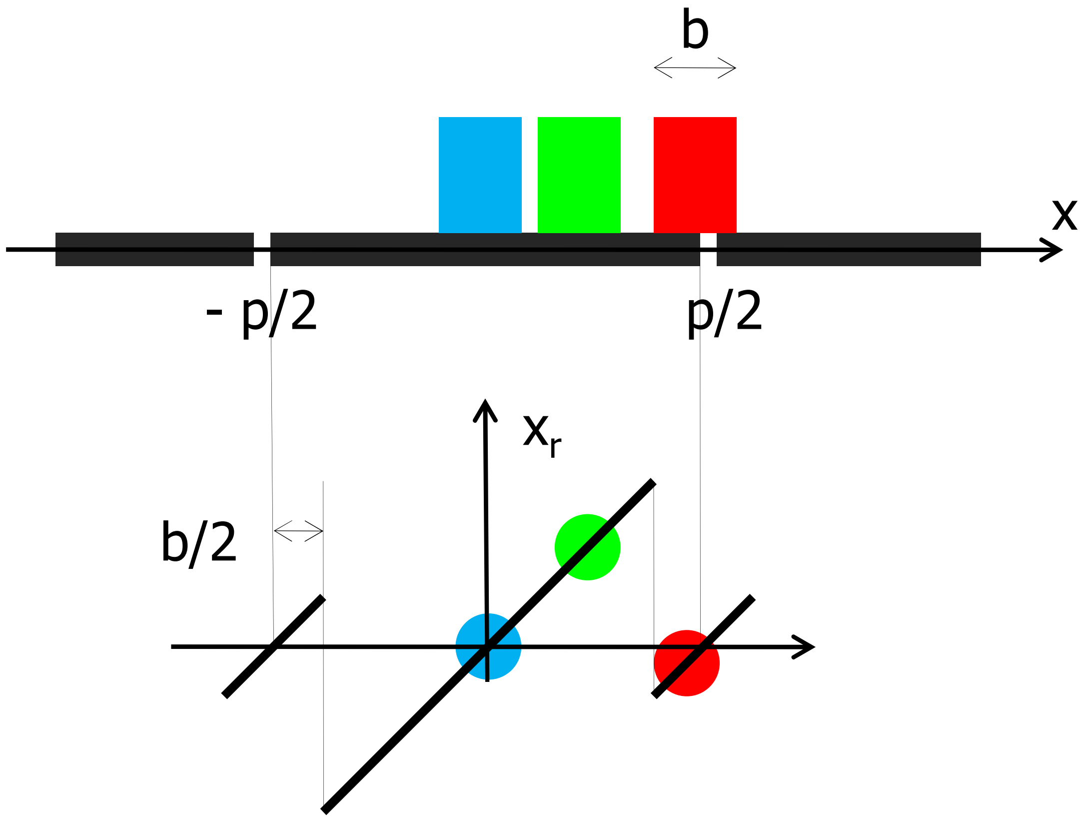


- Sigma of Error $\sigma^2_{\mathrm{position}} = \dfrac{\int_{-p/2}^{p/2} x_r^2 \cdot 1 \cdot \mathrm{d}x_r}{\int_{-p/2}^{p/2} 1 \cdot \mathrm{d}x_r} = \dfrac{p^2}{12}$

$$\boxed{\sigma_{\mathrm{position}} = \frac{p}{\sqrt{12}}}$$

Resolution with wider Signals (Binary Readout!)

- Consider 'Box' Signals for simplicity
- When 2 strips are hit → reconstruct at edge → small error

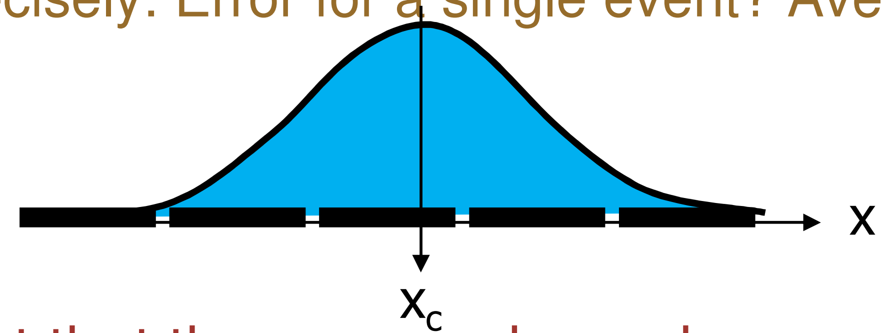


- Minimum Error for b = p/2. Error becomes half: $\sigma = ½\, p/\sqrt{12}$
- Note that we have 50% single / 50% double hits

CENTER-OF-GRAVITY RECONSTRUCTION:
WHEN IS IT SUFFICIENT - OR -
WHEN DO WE NEED A FIT?

The question:

- A 1D signal with (spatial) shape f(x) falls onto a strip structure with pitch *a*
- We assume **∫f(x)dx = 1** *and* **f(x) symmetric**.
- This generates (analogue) signals on several strips.
- We assume for now that **noise = 0**.
- Question:
  What is the reconstruction error for GoG reconstruction ?
  - More precisely: Error for a single event? Average error? Sigma?



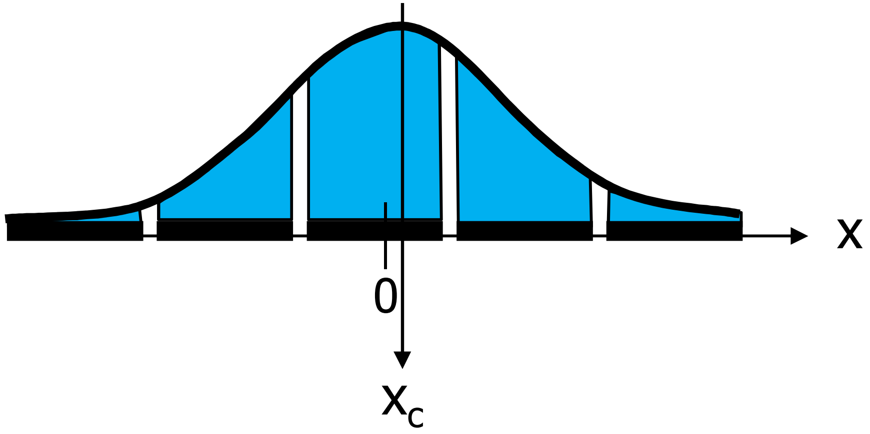
- We expect that the answer depends on
  - signal shape
  - Strip pitch a
  - signal position (for single events)

Remark

- The following calculation involves partial integrals over arbitrary function.
- Normally we must give up soon analytically (consider Gaussians..)
- But it turns out that we can…

- I could only show the result, but I like the fact that so many ‘simple’ aspects of basic Analysis are required…

1. Signal on Strips



We **assume** the signal on a strip is the integral of f(x)

Strip centers are at m·a

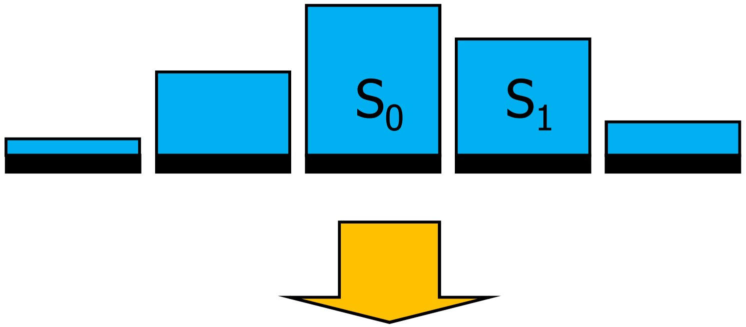
$$S_m(x_c) = \int_{(m-1/2)a}^{(m+1/2)a} f(x - x_c)\, dx$$

This is the signal in strip m when the charge cloud is centered around x_c

$$= \int_{-\infty}^{\infty} f(x) \cdot \mathrm{Rect}_a(x - (ma - x_c))\, dx$$

Box of width a centered at m a-x_c

2. Reconstructed Position



x_{rek} ?

position reconstructed by CoG
(we assumed a normalized signal)

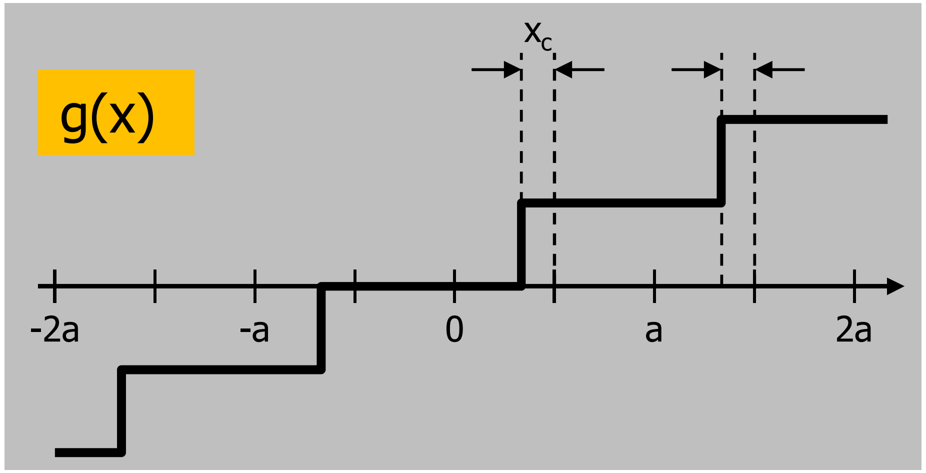
$$x_{rek}(x_c) = \sum_m ma \cdot S_m(x_c)$$

$$= a \int f(x) \underbrace{\sum_m m \cdot \mathrm{Box_a}(x + x_c - ma)\, dx}$$

$$= a \int f(x)\, g(x)\, dx$$

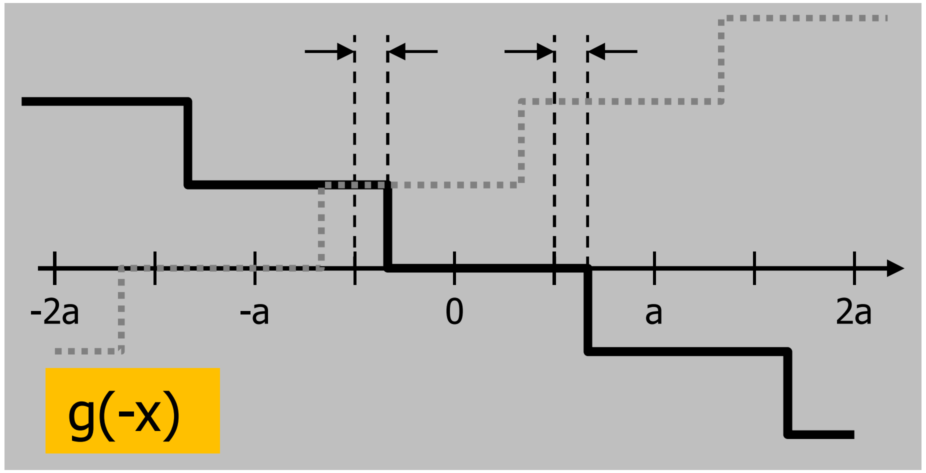
Staircase = g(x)

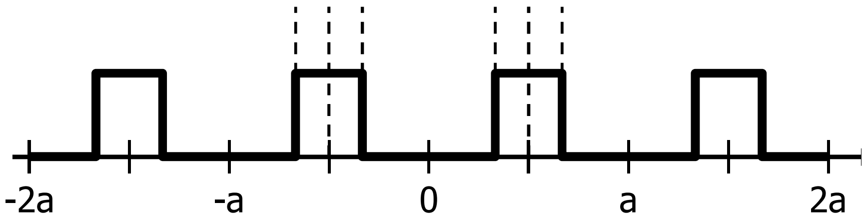
3. Divide Staircase in sym. / antisym. parts



$$g(x) = g_{\mathrm{sym}}(x) + g_{\mathrm{antisym}}(x)$$

$$g_{\mathrm{sym}}(x) = \frac{g(x) + g(-x)}{2}$$





$$g_{\mathrm{sym}}(x) = \frac{1}{2} - \frac{1}{2} \sum_m \mathrm{Box}_{\mathrm{a-2x_c}}(x - ma)$$

4. Simplify the integrals, Move to Fourier Space

- Integral of $g_{antisym}(x)$ is zero because f is symmetric
- We are left with

$$
\begin{aligned}
x_{rek}(x_c) &= a \int f(x)\, g_{\mathrm{sym}}(x)\, dx \\
&= \frac{a}{2} - \frac{a}{2} \int f(x) \sum_m \mathrm{Box}_{\mathrm{a-2x_c}}(x - ma)\, dx \\
&= \frac{a}{2} - \frac{a}{2} \int f(x) \left[\mathrm{Box}_{\mathrm{a-2x_c}}(x) \star \mathrm{Comb}_{\mathrm{a}}(x) \right] dx
\end{aligned}
$$

Write Sum of Boxes as convolution of a single Box with Dirac Comb

- To solve this, move to Fourier Space with

$$\widetilde{f}(k) := \int f(x)\, e^{-2\pi i k x} dx$$

- We can use $\widetilde{a \star b} = \widetilde{a} \cdot \widetilde{b}$ and $\int a(x)\, b(x)\, \mathrm{d}x = \int \widetilde{a}(k)\, \widetilde{b}(k)\, \mathrm{d}k$ (for symmetrical a,b)

5. Get rid of the Integral

$$x_{rek}(x_c) = \frac{a}{2} - \frac{a}{2}\int f(x)\left[\mathrm{Box}_{a-2x_c}(x) \star \mathrm{Comb}_a(x)\right] dx$$

$$= \frac{a}{2} - \frac{a}{2}\int \tilde{f}(k)\cdot \widetilde{\mathrm{Box}}_{a-2x_c}(x)\cdot \widetilde{\mathrm{Comb}}_a(x)\, dk$$

This is again a Dirac Comb, i.e. a **sum** of peaks at distances 1/a

sinc

integral can be carried out. Sum is left

$$= \frac{a}{2} - \frac{a}{2}\sum_{m=-\infty}^{\infty} \tilde{f}\left(\frac{m}{a}\right)\frac{\sin\left(m\pi - \frac{2\pi m x_c}{a}\right)}{m\pi}$$

6. Use Symmetry, Simplify the Sin() function

$$x_{rek} = \frac{a}{2} - \frac{a}{2} \sum_{m=-\infty}^{\infty} \tilde{f}\left(\frac{m}{a}\right) \frac{\sin\left(m\pi - \frac{2\pi m x_c}{a}\right)}{m\pi}$$

Use symmetry in m.

Treat m=0:
~f(0) = 1,
Sin(m k)/m ->k

$$= x_c - \frac{a}{\pi} \sum_{m=1}^{\infty} \tilde{f}\left(\frac{m}{a}\right) \frac{\sin\left(m\pi - \frac{2\pi m x_c}{a}\right)}{m}$$

Center position x_c shows up !

$$\sin(m\pi - x) = \sin(m\pi)\cos(x) - \cos(m\pi)\sin(x)$$
$$= -(-1)^m \sin(x)$$

$$\boxed{x_{err}(x_c) = \frac{a}{\pi} \sum_{m=1}^{\infty} \frac{(-1)^m}{m} \tilde{f}\left(\frac{m}{a}\right) \sin\left(\frac{2\pi m x_c}{a}\right)}$$

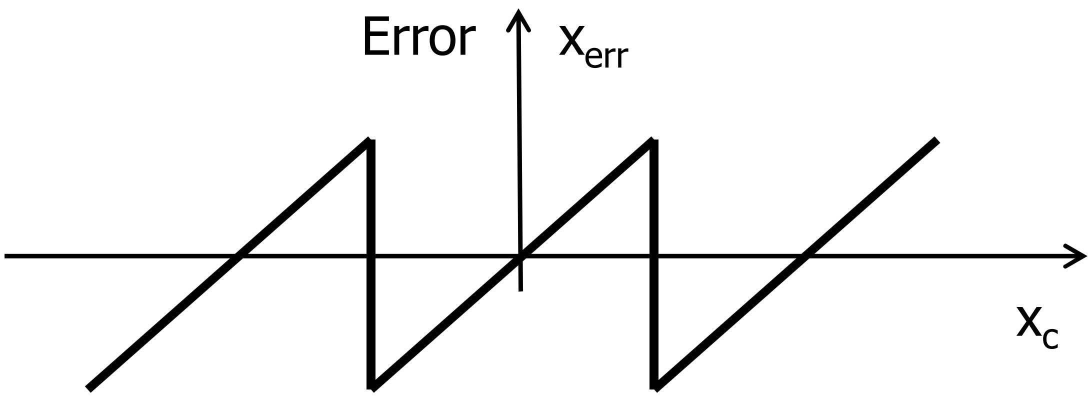
A first check

$$x_{err}(x_c) = \frac{a}{\pi} \sum_{m=1}^{\infty} \frac{(-1)^m}{m} \tilde{f}\left(\frac{m}{a}\right) \sin\left(\frac{2\pi m x_c}{a}\right)$$

- For very narrow f(x), we have $\tilde{f}(k) \to 1$ so that

$$x_{err}(x_c) = \frac{a}{\pi} \sum_{m=1}^{\infty} \frac{(-1)^m}{m} \sin\left(m\pi \frac{2x_c}{a}\right)$$

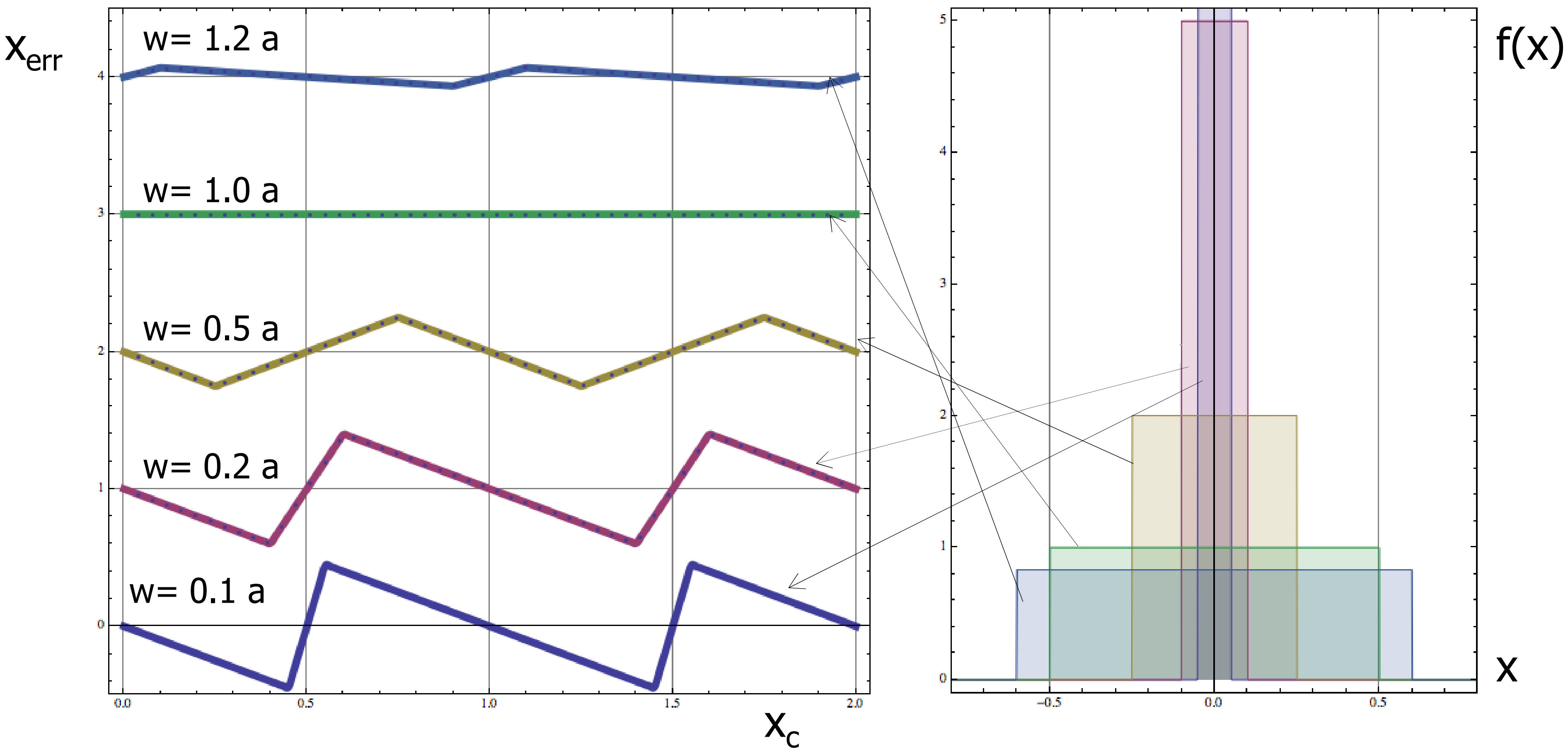
- This is the Fourier Series of a Saw-Tooth, as expected!



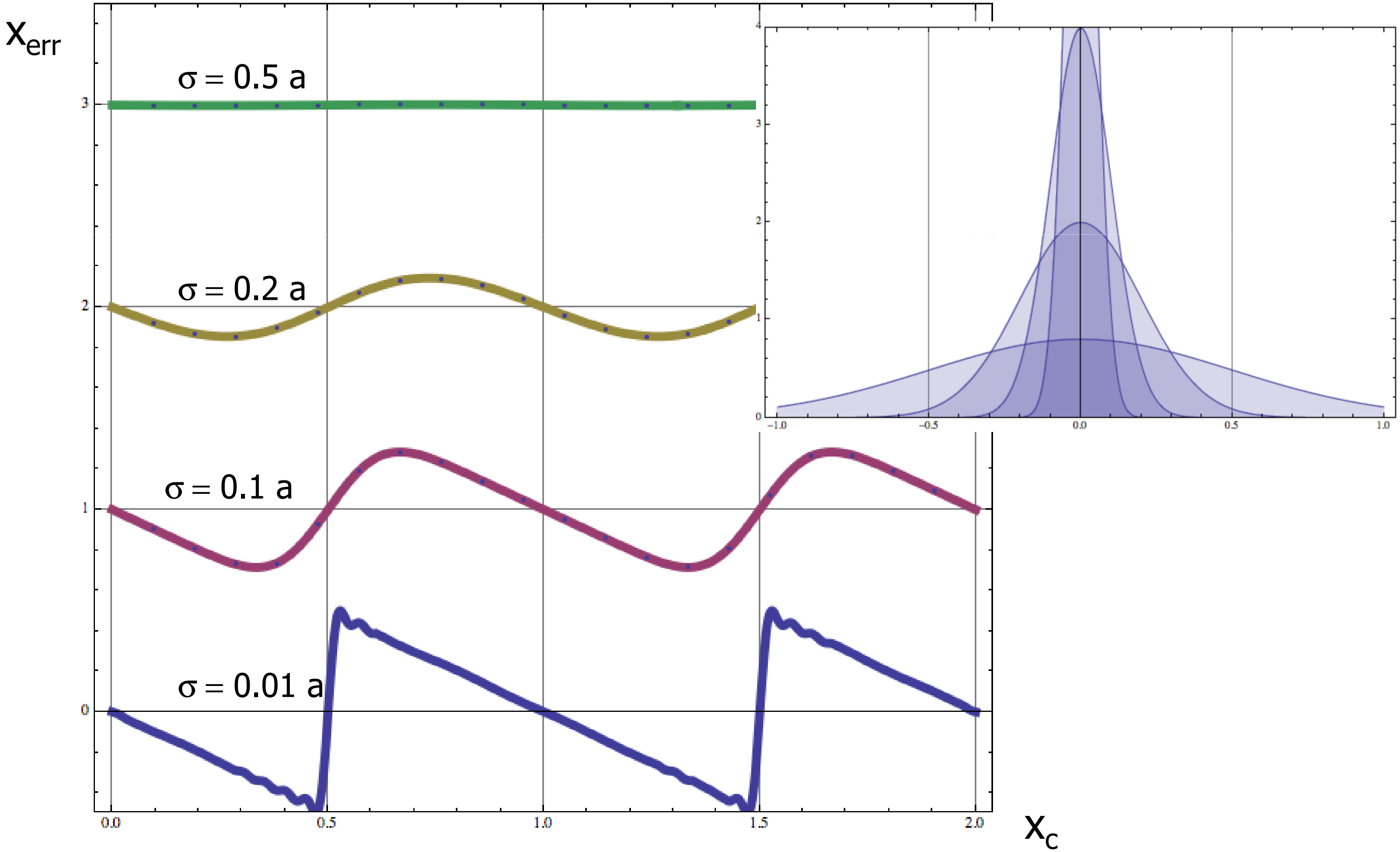
Check with Box f(x)

- For Box of width a, $\tilde{f}\left(\frac{m}{a}\right)$ = sin(mπ)/mπ is zero for integer m.
  → reconstruction is perfect..

$$x_{err}(x_c) = \frac{a}{\pi}\sum_{m=1}^{\infty}\frac{(-1)^m}{m}\tilde{f}\left(\frac{m}{a}\right)\sin\left(\frac{2\pi m x_c}{a}\right)$$



Check with Gaussians



- Error already very small for $\sigma = 0.5a$

Going Further: **Sigma** of x_{err}?

$$x_{err}(x_c) = \frac{a}{\pi} \sum_{m=1}^{\infty} \frac{(-1)^m}{m} \tilde{f}\left(\frac{m}{a}\right) \sin\left(\frac{2\pi m x_c}{a}\right)$$

$$\begin{aligned}
\sigma^2_{\mathrm{rec}} &= \frac{1}{a}\int_{-a/2}^{a/2} x^2_{\mathrm{err}}(x_{\mathrm{c}})\, dx_{\mathrm{c}} \\
&= \frac{a}{\pi^2} \sum_{n,m=1}^{\infty} \frac{(-1)^{n+m}}{nm} \tilde{f}\left(\frac{n}{a}\right) \tilde{f}\left(\frac{m}{a}\right) \underbrace{\int_{-a/2}^{a/2} \sin\frac{2\pi n x_{\mathrm{c}}}{a} \sin\frac{2\pi m x_{\mathrm{c}}}{a}\, dx_{\mathrm{c}}}_{} \\
&= \frac{a}{\pi^2} \sum_{n,m=1}^{\infty} \frac{\cancel{(-1)^{n+m}}}{nm} \tilde{f}\left(\frac{n}{a}\right) \tilde{f}\left(\frac{m}{a}\right) \frac{a}{2} \boxed{\delta_{n,m}}
\end{aligned}$$

sin() are orthorgonal!

$$\boxed{\sigma^2_{\mathrm{rec}} = \frac{a^2}{2\pi^2} \sum_{m=1}^{\infty} \frac{1}{m^2} \tilde{f}^2\left(\frac{m}{a}\right).}$$

Another Check

$$\sigma_{\mathrm{rec}}^2 = \frac{a^2}{2\pi^2} \sum_{m=1}^{\infty} \frac{1}{m^2} \tilde{f}^2\left(\frac{m}{a}\right).$$

- For very narrow signals, we have again $\tilde{f}(k) \to 1$ so that

$$\left(\frac{\sigma_{\mathrm{err}}}{a}\right)^2 = \frac{1}{2\pi^2} \sum_{m=1}^{\infty} \frac{1}{m^2} = \frac{1}{12}$$

as expected….

$\pi^2/6$

- This is probably the most complicated way to get the 1/12…

f(x) = Gaussian / Box

- For a Gaussian signal with width σ

$$G(x) = \frac{1}{\sqrt{2\pi}\sigma_s} \exp\left(-\frac{x^2}{2\sigma_s^2}\right) \qquad \text{with} \qquad \tilde{G}(k) = \exp\left(-2\pi^2 k^2 \sigma_s^2\right)$$

we get

$$\boxed{\left(\frac{\sigma_{\text{err}}}{a}\right)^2 = \frac{1}{2\pi^2} \sum_{m=1}^{\infty} \frac{\exp\left(-\frac{4m^2\pi^2\sigma_s^2}{a^2}\right)}{m^2}}$$

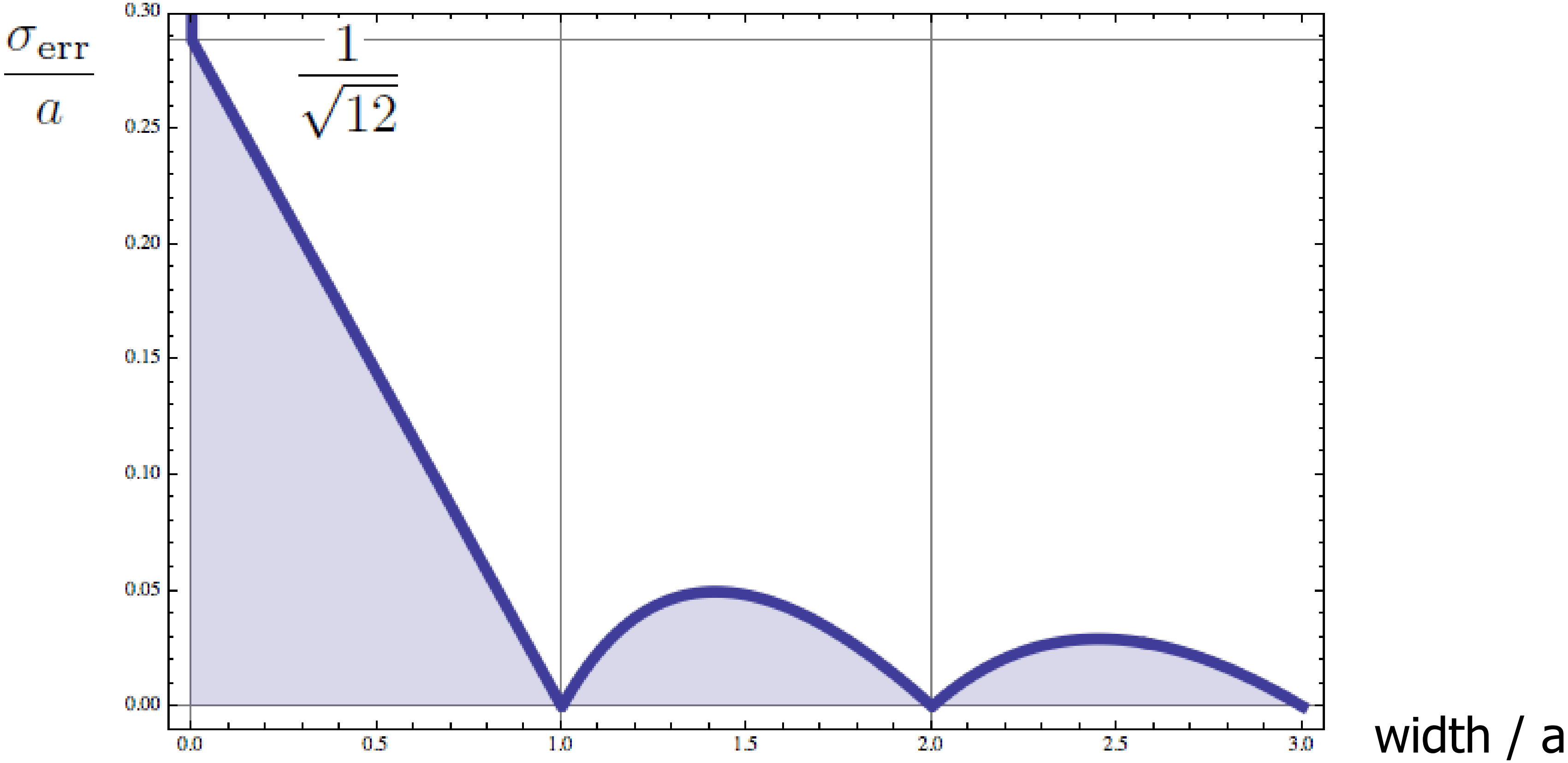
- For a Box of width s a

$$\left(\frac{\sigma_{\text{err}}}{a}\right)^2 = \frac{1}{360 s^2} - \frac{1}{4\pi^4 s^2} \sum_{m=1}^{\infty} \frac{\cos(2\pi m s)}{m^4}$$

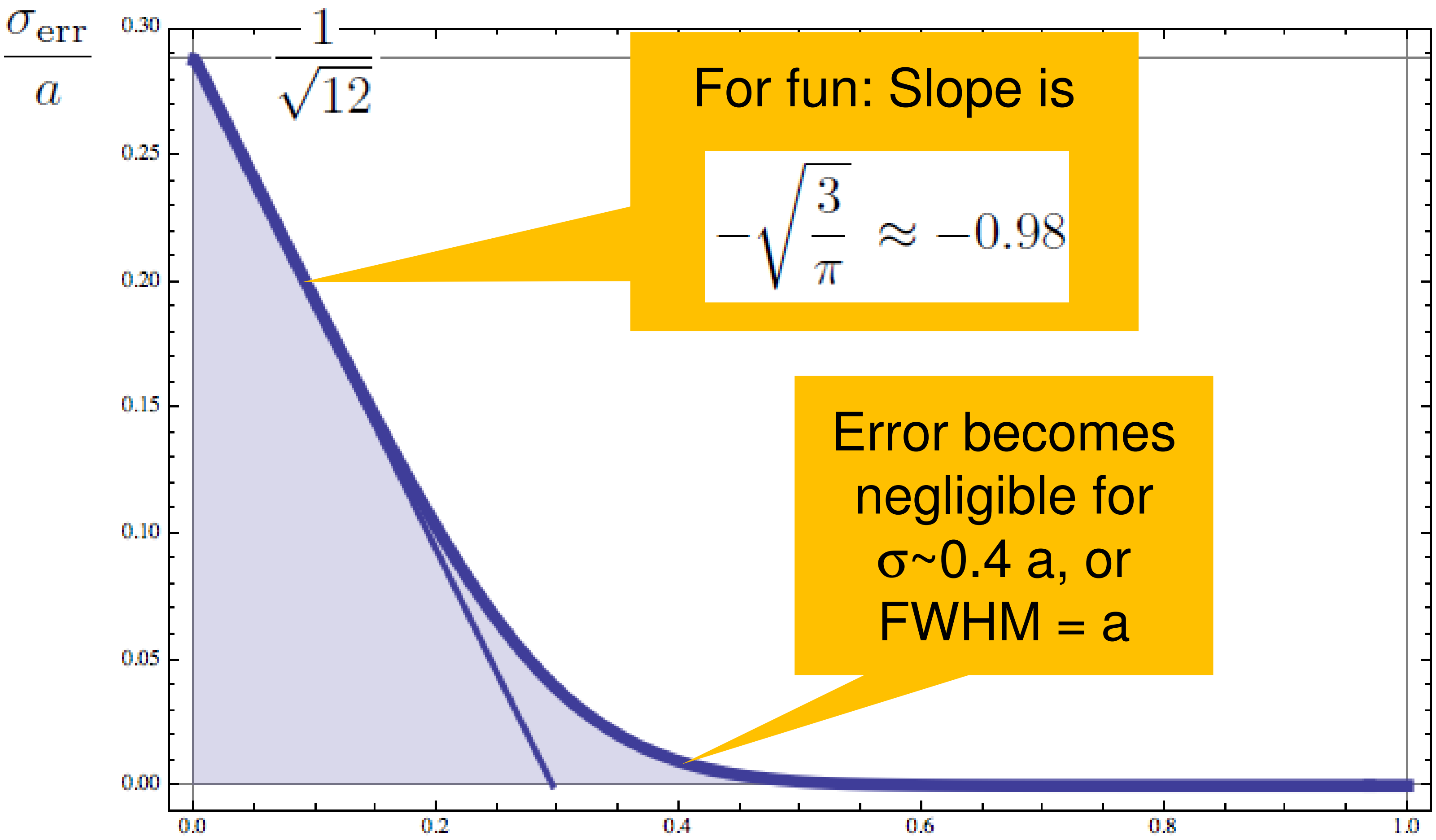
which is zero for integer s thanks to $\Sigma(1/n^4) = \pi^4/90$..

Plot this for f(x) = Box

- Error zero for integer box width.
- Behavior in between not trivial…



Plot this for f(x) = Gauss



The result 'Error ≈ 0 for FWHM ≈ a' can be found for many pulse shapes. We knew this… but now we know *for sure*…

LIMIT OF SPATIAL RESOLUTION FROM NOISE

The Question

- How is spatial resolution degraded by noise?
- We all know $\sigma_{\mathrm{err}} = \kappa \cdot \sigma_{\mathrm{n}} = \frac{\kappa}{SNR}$

  This states, that the resolution degrades with noise 'linearly to first order'.
- The proportionality κ is empirical. We want to calculate it
- We also want to check what happens with correlated noise
- We want to see what happens to higher order
  - What is this here? It is the distribution of the noise…
- We assume we can reconstruct with CoG (more later…)
- We restrict on a 1D treatment, but 2D is straight forward

Write down x_{rek} with noise

- A Signal at $\vec{x}$ is distributed over N strips at positions $\vec{x}_i$
- Signal on i-th strip is $S_i(\vec{x})$
- The sum of all signals shall be normalized to 1:

$$\sum S_i = 1 \qquad (1)$$

- Assume we can perfectly reconstruct position as center of gravity:

$$\vec{x} = \frac{\sum S_i \vec{x}_i}{\sum S_i} = \sum S_i \vec{x}_i \qquad (2)$$

- Now assume noise n_i on all strips
- the reconstructed position is:

$$\vec{x}_{\mathrm{rek}}(\vec{x}) = \frac{\sum (S_i + n_i)\vec{x}_i}{\sum (S_i + n_i)} = \frac{\vec{x} + \sum n_i \vec{x}_i}{1 + \sum n_i}$$

Assume noise is small. Get the standard dev.

- This becomes (Taylor Expansion of Denominator):

$$\vec{x}_{\text{rek}}(\vec{x}) = \frac{\vec{x} + \sum n_i \vec{x}_i}{1 + \sum n_i} = \left(\vec{x} + \sum n_i \vec{x}_i\right)\left(1 - \sum n_i + \mathcal{O}(n^2)\right)$$

- The reconstruction *error* is:

$$\vec{x}_{\text{err}}(\vec{x}) = \sum_i n_i (\vec{x}_i - \vec{x}) + \mathcal{O}(n^2).$$

- We need the standard deviation:

$$\sigma_{\text{err}}^2 = \langle \vec{x}_{\text{err}}^2 \rangle - \langle \vec{x}_{\text{err}} \rangle^2$$

Average error is zero!

We need to average over
- ALL possible positions
- ALL noises

Do the averaging

$$\vec{x}_{\mathrm{err}}(\vec{x}) = \sum_i n_{\mathrm{i}}(\vec{x}_{\mathrm{i}} - \vec{x}) + \mathcal{O}(n^2).$$

$$\begin{aligned}\sigma_{\mathrm{err}}^2 &= \langle \vec{x}_{\mathrm{err}}^{\,2} \rangle \\ &= \sum_{i,j} \langle n_{\mathrm{i}} n_{\mathrm{j}} \rangle \langle (\vec{x}_{\mathrm{i}} - \vec{x})(\vec{x}_{\mathrm{j}} - \vec{x}) \rangle + \langle \mathcal{O}(n^3) \rangle \\ &= \sigma_{\mathrm{n}}^2 \cdot \sum_i \langle (\vec{x}_{\mathrm{i}} - \vec{x})^2 \rangle + \mathcal{O}(\sigma_{\mathrm{n}}^3)\end{aligned}$$

δ_{ij}

For **un**correlated noise

$\langle n_{\mathrm{i}} n_{\mathrm{j}} \rangle = \delta_{ij} \cdot \sigma_{\mathrm{n}}^2$

- We chose the origin such that

$$\sum_i \vec{x}_{\mathrm{i}} = \vec{0}$$

3

then this simplifies to:

$$\boxed{\sigma_{\mathrm{err}}^2 = \sigma_{\mathrm{n}}^2 \left(\sum_{i=1}^{N} \vec{x}_{\mathrm{i}}^{\,2} + N \langle \vec{x}^{\,2} \rangle \right) + \mathcal{O}(\sigma_{\mathrm{n}}^3).}$$

This is κ^2 !

Example: Strips

- Consider two strips at $x_1 = -a/2$ and $x_2 = +a/2$ (N = 2)
- Signals for a hit at x are

$$S_1(x) = (x_2 - x)/a \text{ and } S_2(x) = (x + x_2)/a$$

- 1 , 2 and 3 are fulfilled:

$S_1 + S_2 = 1$; $x_1 S_1 + x_2 S_2 = x$; $x_1 + x_2 = 0$

- We get $$\left(\frac{\sigma_{\mathrm{err}}}{\sigma_{\mathrm{n}}}\right)^2 \approx x_1^2 + x_2^2 + \frac{2}{a}\int_{x_1}^{x_2} x^2 dx = \frac{2}{3}a^2$$

- Or $$\boxed{\sigma_{\mathrm{err}} = 0.816 \cdot a \cdot \sigma_{\mathrm{n}}}$$

- For σ_n **= 0.1** (Signal/Noise = 10), **resolution = 8% · a**
- Resolution is better than optimal binary readout for S/N>5.6

Correlated Noise ?

- For FULLY correlated noise, $n_i = n_j$ and $\langle n_i n_j \rangle = \sigma_n^2$
- We get $\sigma_{err}^2 \approx \sigma_n^2 N^2 \langle \vec{x}^2 \rangle$
- For the strip example

$$\sigma_{err} = a\,\sigma_n / \sqrt{3} = 0.57\,a\,\sigma_n$$

- Correlated noise is less harmful than 'normal' noise
- Note: For mixed noise, superimpose both components)
- Note: If the Amplitude of the signal is *KNOWN* (X-ray), noise becomes correlated and resolution improves!

Higher Orders (in noise)

- Noise can have different distributions.
- They have different higher moments:

$$\begin{aligned} \langle n_i^2 \rangle &= \sigma_{\mathrm{n}}^2, \\ \langle n_i^4 \rangle &= \beta \cdot \sigma_{\mathrm{n}}^4. \end{aligned}$$

- They are $\beta := \dfrac{\int n^4\, p(n)\, dn}{\left(\int n^2\, p(n)\, dn\right)^2} = \begin{cases} 3 : \text{Gauss} \\ 9/5 : \text{Box} \\ 1 : \text{Peaks} \end{cases}$

- We need then higher order correlations (not trivial..):

$$\begin{aligned} \langle n_{\mathrm{i}} \rangle &= 0 \\ \langle n_{\mathrm{i}} n_{\mathrm{j}} \rangle &= \delta_{ij} \sigma_{\mathrm{n}}^2 \\ \langle n_{\mathrm{i}} n_{\mathrm{j}} n_{\mathrm{k}} \rangle &= 0 \\ \langle n_{\mathrm{i}} n_{\mathrm{j}} n_{\mathrm{k}} n_{\mathrm{l}} \rangle &= \delta_{ij}\delta_{jk}\delta_{kl}\,(\beta - 3)\,\sigma_{\mathrm{n}}^4 \quad + (\delta_{ij}\delta_{kl} + \delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk})\,\sigma_{\mathrm{n}}^4 \end{aligned}$$

Higher Orders

- Repeating the derivation yields

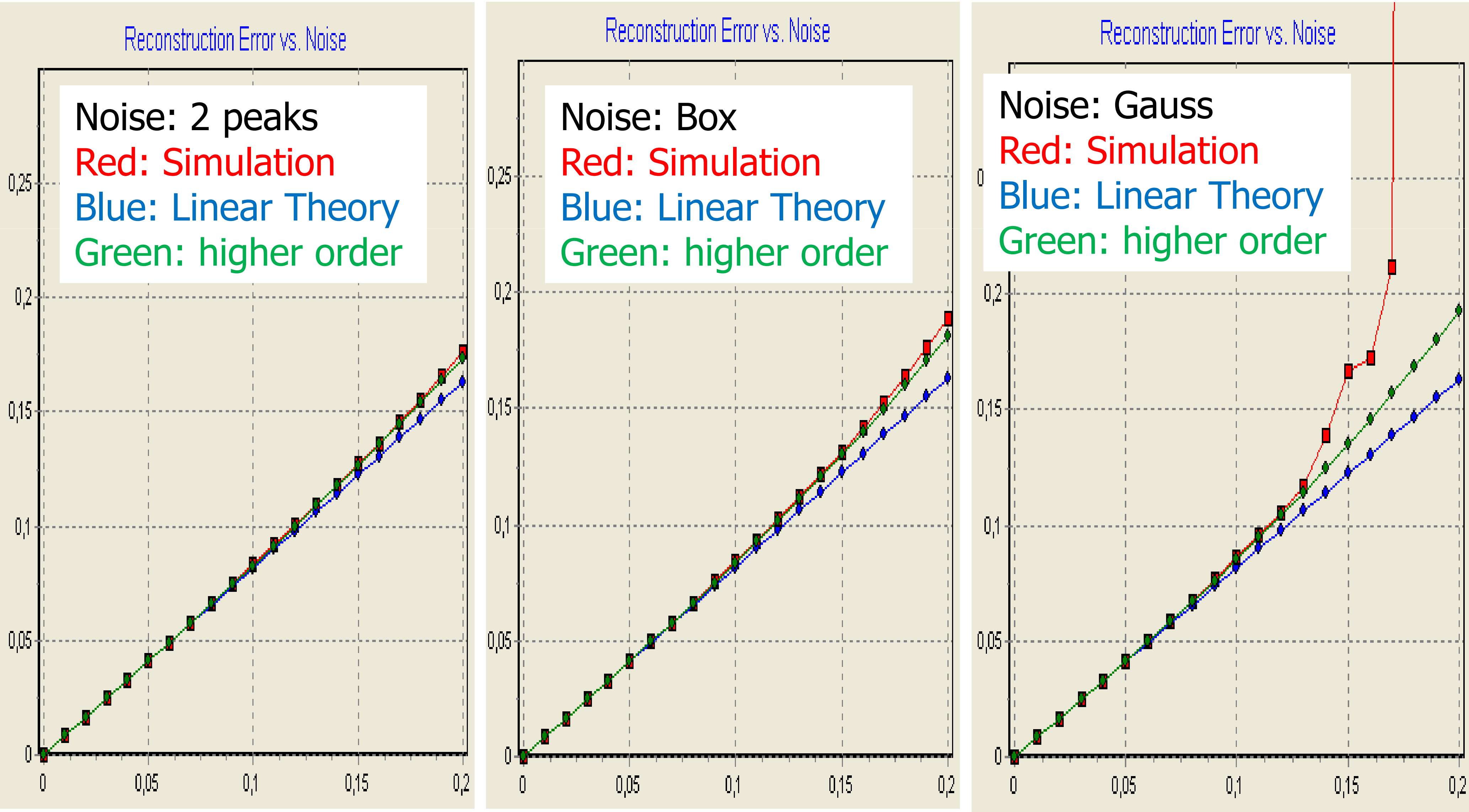
$$\begin{aligned}\sigma_{\mathrm{err}}^2 &= \sigma_{\mathrm{n}}^2 \cdot (A + N\,B) \\ &\quad \cdot \left(1 + 3\left[\beta - 3 + N\frac{A + 3\,N\,B}{A + N\,B}\right]\sigma_{\mathrm{n}}^2\right)\end{aligned}$$

Previous result

Correction

- Only the *correction* depends on the 'type' (shape) of noise.
- For small noise, there is *no need* to simulate Gaussian noise:
  Randomly adding or subtracting $\pm\,\sigma_n$ has the same effect!

Is this true? → Small Monte Carlo



- Reconstruction for Gauss noise fails in few cases

2D Structures

- Can be treated similarly
- Observations:
  - Small number of electrodes is good
  - Well confined acceptance 'circle' is good

$$\sigma_{\mathrm{err}}^2 = \sigma_{\mathrm{n}}^2 \left(\sum_{i=1}^{N} \vec{x}_{\mathrm{i}}^{\,2} + N \langle \vec{x}^{\,2} \rangle \right) + \mathcal{O}(\sigma_{\mathrm{n}}^3).$$

Geometry	$\sigma_{1\mathrm{D}}$ theory linear	$\sigma_{1\mathrm{D}}$ theory correction	numerical (A=p=1)
strips	$\sqrt{\frac{2}{3}} \cdot p$	$\sqrt{1 + 3\beta\sigma_{\mathrm{n}}^2}$	0.8165
square	$\frac{2}{\sqrt{3}} \cdot \sqrt{A}$	$\sqrt{1 + 3(3+\beta)\sigma_{\mathrm{n}}^2}$	1.1547
hexagon	$\frac{\sqrt{5}\cdot 3^{1/4}}{6} \cdot \sqrt{A}$	$\sqrt{1 + 3\left(\frac{6}{5} + \beta\right)\sigma_{\mathrm{n}}^2}$	0.4905

- Hexagons are best.

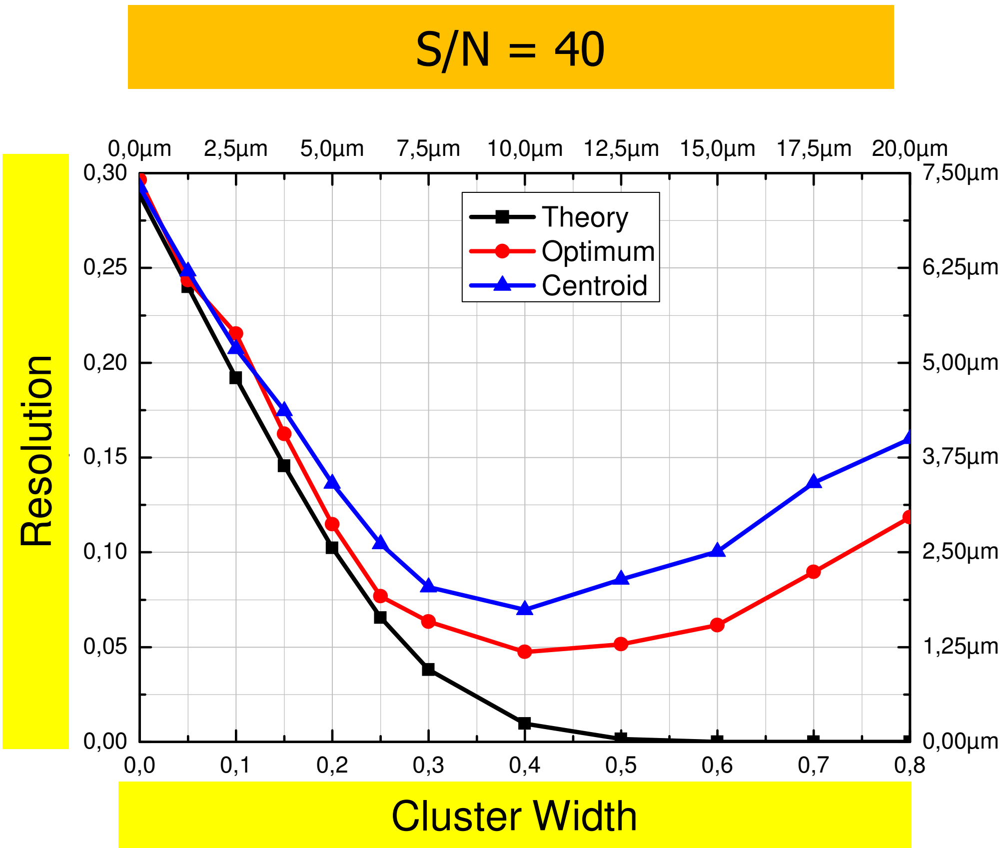
BACK TO GOG
NOW WITH NOISE

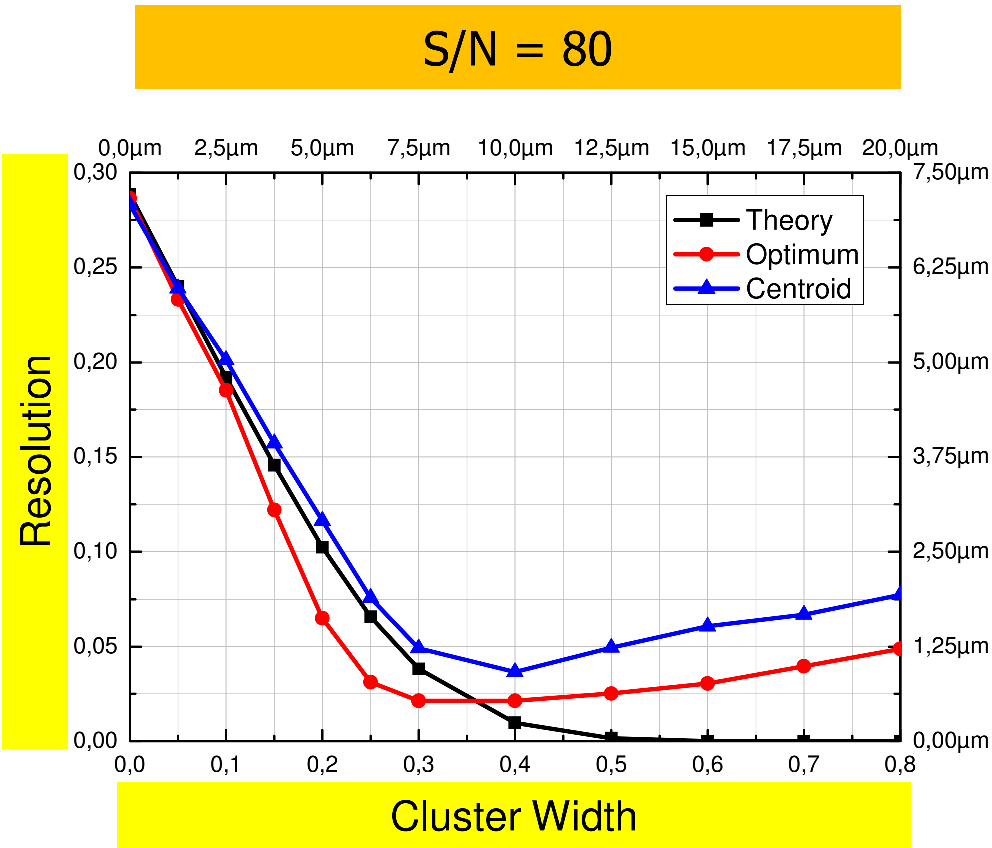
Problems with Centroid

- Resolution for small σ is bad → make f(x) wide
- BUT: The 'infinite' sum creates infinite noise (N high)
- Must chose N so that reconstruction is ok.
  - Obviously N ~ σ
- The choice is fairly arbitrary
- And:
  - In real system, there is often a **threshold** (hits below this are not read out)
  - The reconstructed amplitude is wrong (signals below threshold are lost)
  - Broken pixels need special treatment

Monte Carlo Simulation

- Many possibilities… I do not go in details



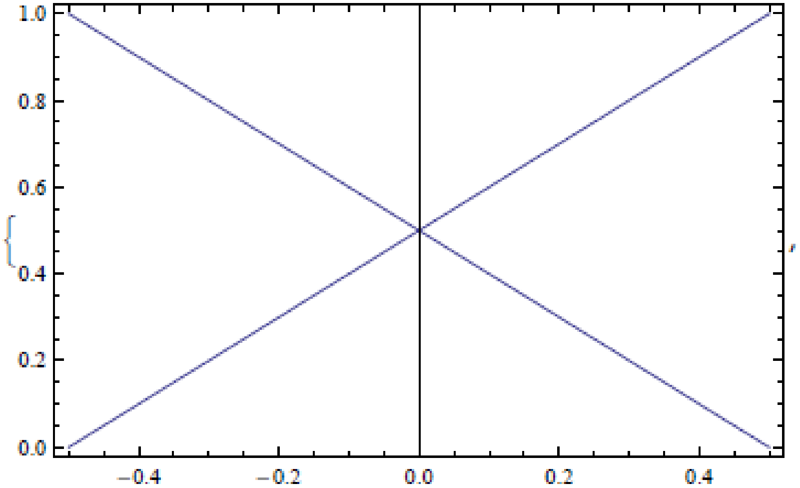


- The **optimum signal width is still close to FWHM = a!**

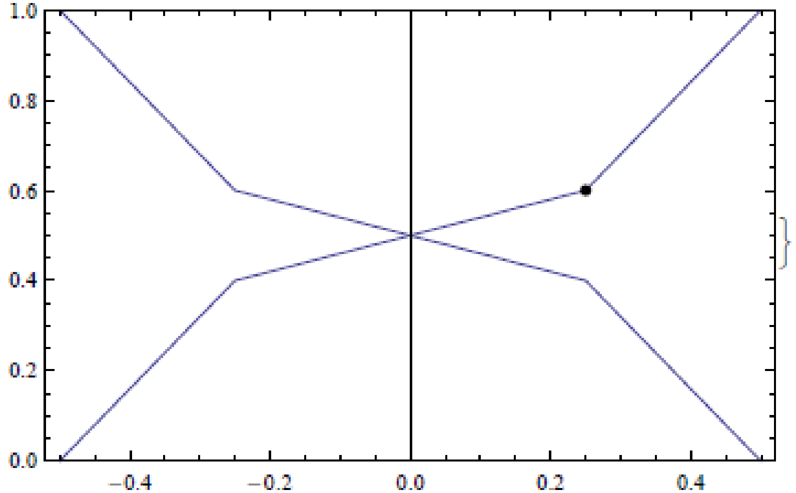
ETA FUNCTION

Motivation

- Often the Signals Distribution function (e.g. on 2 strips) is not linear.
- This is related to the 'famous' eta-function.



Linear Charge Sharing



Arbitrary function

- The position then cannot be calculated by GoG, but by using the inverse function (or the 'eta'-lookup table)
- Question: How does resolution depend on f(x)?

1D Case: Reconstruction with Inverse Function

- The signals on the two strips shall be

$$\begin{aligned} S_1(x) &= Q\, f(x) \\ S_2(x) &= Q - S_1(x) = Q\,(1 - f(x)) \end{aligned}$$

(we assume no signal is lost, i.e. we require $S_1+S_2 = Q$

- We require
  - f(x) is strictly monotonic (obvious)
  - f(x) shall be symmetric in x (may not always be the case)

- Obviously

$$x_{rek} = f^{-1}\left[\frac{S_1}{S_1+S_2}\right]$$

Adding Noise

- With Noise on S_1 and S_2 we get

$$
\begin{aligned}
x_{rek} &= f^{-1}\left[\frac{f(x)+n_1}{1+n_1+n_2}\right] \\
&\approx f^{-1}\left[(f(x)+n_1)(1-n_1-n_2)\right] \\
&\approx f^{-1}\left[f(x)+n_1(1-f(x))-n_2 f(x)\right] \\
&\approx x + \left.\frac{\mathrm{d}f^{-1}(s)}{\mathrm{d}s}\right|_{f(x)} \cdot \left[n_1(1-f(x))-n_2 f(x)\right]
\end{aligned}
$$

Add noise

Taylor (as before)

1^{st} order in noise

Taylor Series for f^{-1} around f(x)

$$x_{err} = \frac{n_1(1-f(x))-n_2 f(x)}{f'(x)}$$

A 'forgotten' math theorem: The derivative of the inverse function is the inverse of the derivative

Sigma – Averaging over Noise

- To get

$$x_{err} = \frac{n_1(1 - f(x)) - n_2 f(x)}{f'(x)}$$

$$\sigma^2_{err} = \langle x^2_{err} \rangle - \langle x_{err} \rangle^2$$

- we average first over noise. We get

$$\sigma^2_{err} = \sigma^2_n \left\langle \frac{1 - 2f + 2f^2}{f'^2} \right\rangle + 2 \langle n_1 n_2 \rangle \left\langle \frac{f^2 - f}{f'^2} \right\rangle$$

  - Coefficients depend on the signal shape
  - They are small where the response function is steep (obvious..)

- For uncorrelated noise, only the first term matters

$$\frac{\sigma^2_{err}}{\sigma^2_n} = \left\langle \frac{1 - 2f + 2f^2}{f'^2} \right\rangle$$

Average over position

Back to linear Interpolation

- What does this mean for linear interpolation, f(x) = x+0.5 ?
- Let us first look at the *position dependent* error

$$\frac{\sigma_{err}^2}{\sigma_n^2} = \frac{1-2f+2f^2}{f'^2} = \frac{1}{2} + 2x^2$$

- This is NOT constant. It doubles at the edges !!!

- The average error is

$$\frac{\sigma_{err}^2}{\sigma_n^2} = \int_{-1/2}^{1/2} \left(\frac{1}{2} + 2x^2\right) = \frac{2}{3}$$

(s. page 28)

as before.

Finding New Distribution functions

- Very exciting: Can we find a f(x) such that the integral is ***better*** than with linear interpolation
  - Probably not. But let's see…

- Can we find a distribution function so that the error is independent of position?
- One line of Mathematica is enough:

$$f_{flat}(x) = \frac{1}{2}\left(1 - \mathrm{Sinh}[2x\mathrm{ArcSinh}(s)]\right)$$

- The average σ^2 is 0.643, which is (a bit) ***better*** than 2/3 !!

- We found a distribution which is better than lin. interpolation!

Summary: What did we learn ?

- Basic Algebra is fun…..

- CoG is ‘perfect’ when signal width = strip width
- Larger signals are worse when there is noise
- Ideal κ for strips is 0.816
- Analogue readout for S/N<6 is useless.
- Noise shape does not matter for S/N > 10
- Correlated noise is less harmful
- Hexagons have better resolution
- Linear interpolation has more error at the edges.
- There is a better reconstruction function than linear
  - but the difference is negligible….

Thank you for your attention!