Three-loop Heavy-Flavour Corrections to Deep-Inelastic Scattering

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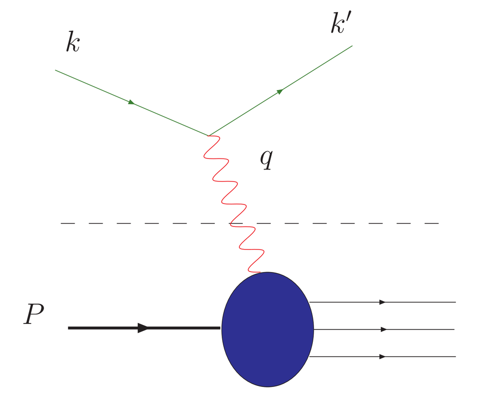
Unpolarized Deep-Inelastic Scattering (DIS)



$$Q^2 := -q^2, \quad x := \frac{Q^2}{2P.q} \quad \text{Bjorken-x}$$

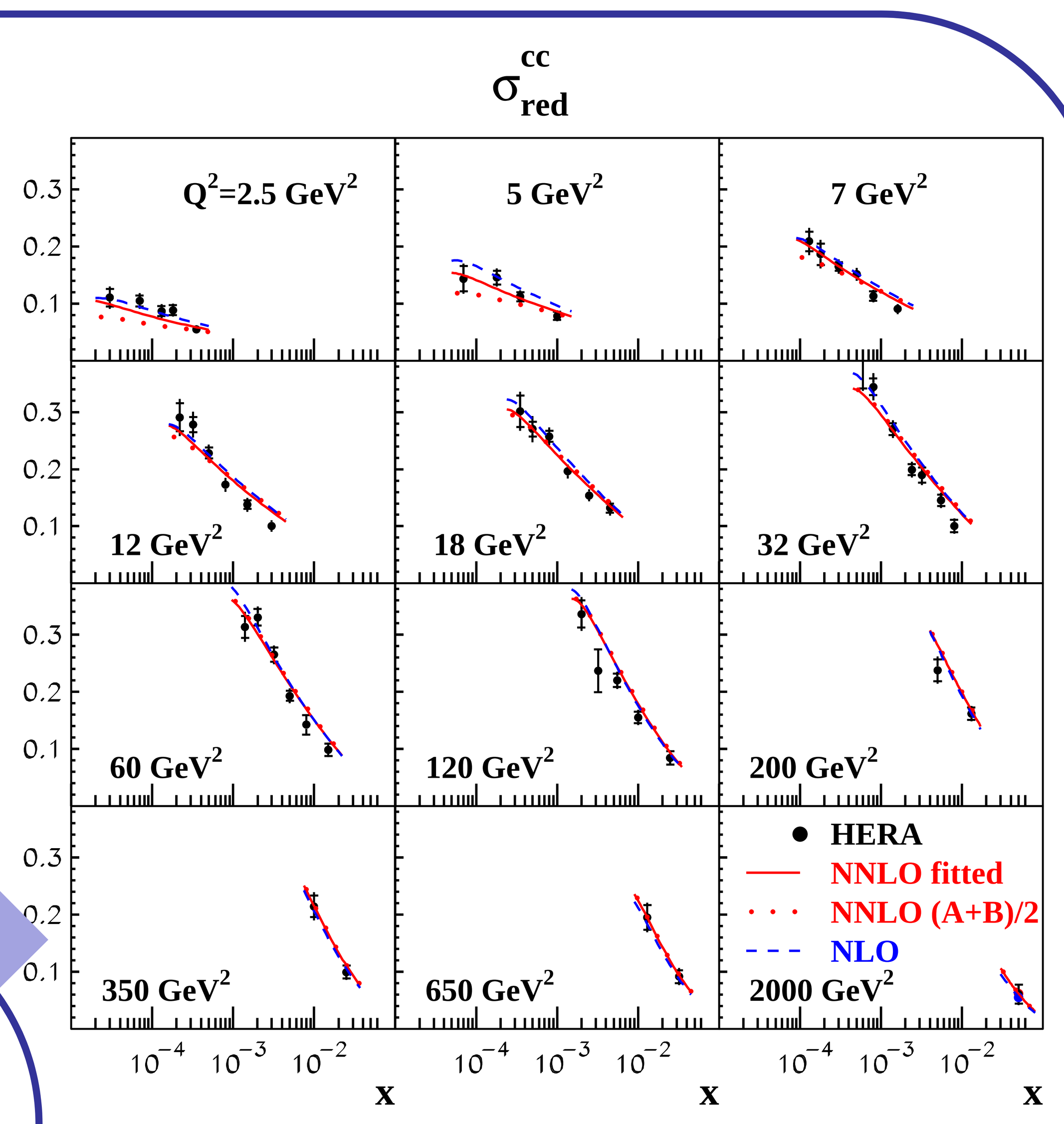
$$\frac{d\sigma}{dQ^2\,dx} \sim W_{\mu\nu} L^{\mu\nu}$$

$$W_{\mu\nu}(q,P,s) = \frac{1}{4\pi}\int d^4\xi \exp(iq\xi)\langle P,s \mid [J_\mu^{em}(\xi), J_\nu^{em}(0)] \mid P,s\rangle =$$

$$\frac{1}{2x}\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) F_L(x,Q^2) + \frac{2x}{Q^2}\left(P_\mu P_\nu + \frac{q_\mu P_\nu + q_\nu P_\mu}{2x} - \frac{Q^2}{4x^2} g_{\mu\nu}\right) F_2(x,Q^2)$$

Structure functions $F_{2,L}$ contain light and heavy quark contributions

Precision data from ep-scattering at HERA



Fit theory prediction to data and obtain α_s

$\alpha_s(M_Z^2)$ from NNLO DIS(+) analyses [from ABM13]

	$\alpha_s(M_Z^2)$	
BBG	$0.1134\ ^{+0.0019}_{-0.0021}$	valence analysis, NNLO
GRS	0.112	valence analysis, NNLO
ABKM	0.1135 ± 0.0014	HQ: FFNS $N_f = 3$
JR	0.1128 ± 0.0010	dynamical approach
JR	0.1140 ± 0.0006	including NLO-jets
MSTW	0.1171 ± 0.0014	
MSTW	$0.1155 - 0.1175$	(2013)
ABM11_J	$0.1134 - 0.1149 \pm 0.0012$	Tevatron jets (NLO) incl.
ABM13	0.1133 ± 0.0011	
ABM13	0.1132 ± 0.0011	(without jets)
CTEQ	0.1159..0.1162	
CTEQ	0.1140	(without jets)
NN21	$0.1174 \pm 0.0006 \pm 0.0001$	
Gehrmann et al.	$0.1131\ ^{+0.0028}_{-0.0022}$	e^+e^- thrust
Abbate et al.	0.1140 ± 0.0015	e^+e^- thrust
BBG	$0.1141\ ^{+0.0020}_{-0.0022}$	valence analysis, N^3LO

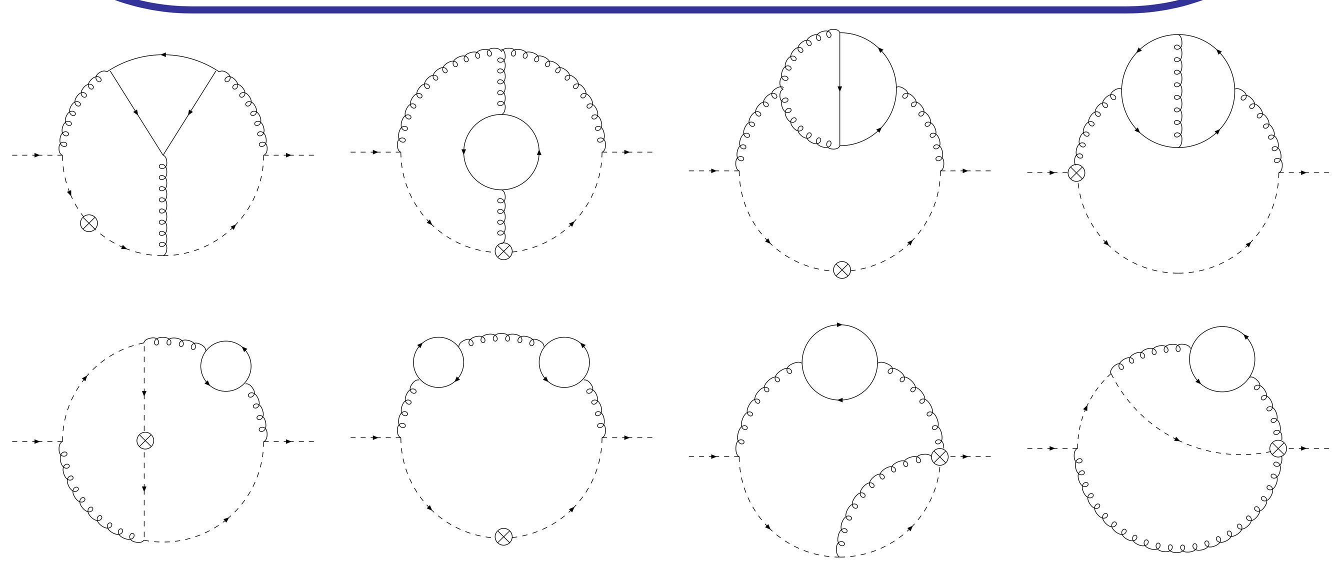
$$\Delta_{\text{Th}}\alpha_s = \alpha_s(\text{N}^3\text{LO})\text{-}\alpha_s(\text{NNLO}) + \Delta_{HQ}$$

$$= 0.0009 \pm 0.0006_{\text{heavy quark}}$$

Precision of experimental data requires the NNLO calculation of massive quark contributions to structure functions

Goals

- NNLO variable flavor number scheme will be provided
- Improved values of α_s and m_c
- Better constraints on sea quarks and gluon pdfs



Calculation

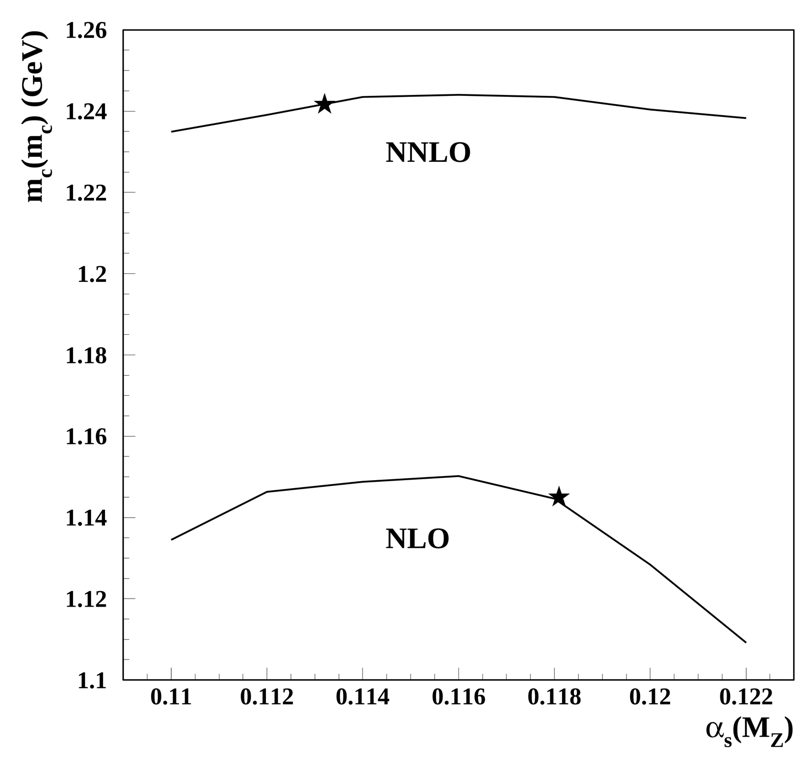
- Analytical methods and heavy use of computer algebra
- Feynman diagrams generated using QGRAF
- Reduction to master integrals using Reduze
- Master integrals calculated using
  - Differential equations
  - Advanced summation techniques using Sigma

High Complexity

- Thousands of diagrams
- Computation time several cpu years
- Large scale computer algebra needed (O(100 GB) RAM)
- Results given in analytic form → involved expressions, ready for efficient numerical use in experiment

First results for m_c

(Yet based on moment approximation)



$m_c = 1.24^{+0.04}_{-0.08}$ GeV

Operator Matrix Element

$$\begin{aligned}
a_{Qq}^{(3),\mathrm{PS}}(N) = {} & C_F^2 T_F \Bigg\{ \frac{64(N^2+N+2)^2}{(N-1)N^2(N+1)^2(N+2)} S_{2,2}(2,\tfrac{1}{2}) - \frac{64(N^2+N+2)^2}{(N-1)N^2(N+1)^2(N+2)} S_{3,1}(2,\tfrac{1}{2}) \\
& + 2^N \Bigg[-\frac{32P_3 S_{2,1}(1,\frac{1}{2},N)}{(N-1)^2N^3(N+1)^2(N+2)} - \frac{32P_3 S_{1,1,1}(\frac{1}{2},1,1,N)}{(N-1)^2N^3(N+1)^2(N+2)} + \frac{32P_4 S_{1,1}(1,\frac{1}{2},N)}{(N-1)^3N^4(N+1)^2(N+2)} + \cdots \Bigg] \\
& + 2^{-N} \Bigg[-\frac{64(N^2+N+2)^2 S_{1,2,1}(2,\frac{1}{2},1,N)}{(N-1)N^2(N+1)^2(N+2)} + \frac{64(N^2+N+2)^2 S_{1,2,1}(2,1,\frac{1}{2},N)}{(N-1)N^2(N+1)^2(N+2)} + \cdots \Bigg] + \cdots \Bigg\} \\
& + C_F T_F^2 N_F \Bigg\{ -\frac{16(N^2+N+2)^2 S_1(N)^3}{27(N-1)N^2(N+1)^2(N+2)} + \frac{16P_9 S_1(N)^2}{27(N-1)N^3(N+1)^3(N+2)^2} \\
& + \Bigg[-\frac{208(N^2+N+2)^2}{9(N-1)N^2(N+1)^2(N+2)} S_2 - \frac{32P_{23}}{81(N-1)N^4(N+1)^4(N+2)^3} \Bigg] S_1 \\
& + \frac{32P_{31}}{243(N-1)N^5(N+1)^5(N+2)^4} + \frac{224(N^2+N+2)^2}{9(N-1)N^2(N+1)^2(N+2)} \zeta_3 + \cdots \Bigg\} \\
& + C_F C_A T_F \Bigg\{ \frac{2(N^2+N+2)^2 S_1(N)^4}{9(N-1)N^2(N+1)^2(N+2)} + \frac{4(N^2+N+2)P_6 S_1(N)^3}{27(N-1)^2N^3(N+1)^3(N+2)^2} \\
& + 2^{-N} \Bigg[\frac{16P_2 S_3(2,N)}{(N-1)N^3(N+1)^2} - \frac{16P_2 S_{1,2}(2,1,N)}{(N-1)N^3(N+1)^2} + \frac{16P_2 S_{2,1}(2,1,N)}{(N-1)N^3(N+1)^2} - \frac{16P_2 S_{1,1,1}(2,1,1,N)}{(N-1)N^3(N+1)^2} \Bigg] \\
& - \frac{32(N^2+N+2)^2 S_{1,1,2}(2,\frac{1}{2},1,N)}{(N-1)N^2(N+1)^2(N+2)} + \frac{32(N^2+N+2)^2 S_{1,1,2}(2,1,\frac{1}{2},N)}{(N-1)N^2(N+1)^2(N+2)} \\
& + \frac{32(N^2+N+2)^2 S_{1,2,1}(2,\frac{1}{2},1,N)}{(N-1)N^2(N+1)^2(N+2)} - \frac{32(N^2+N+2)^2 S_{1,2,1}(2,1,\frac{1}{2},N)}{(N-1)N^2(N+1)^2(N+2)} \\
& - \frac{32(N^2+N+2)^2 S_{1,1,1,1}(2,\frac{1}{2},1,1,N)}{(N-1)N^2(N+1)^2(N+2)} - \frac{32(N^2+N+2)^2 S_{1,1,1,1}(2,1,\frac{1}{2},1,N)}{(N-1)N^2(N+1)^2(N+2)} + \cdots \Bigg\} + \cdots
\end{aligned}$$

$$S_{3,1}\left(2,\frac{1}{2},N\right) = \sum_{i=1}^{N}\sum_{j=1}^{i} \frac{2^i}{i^3}\frac{(\frac{1}{2})^j}{j^1}$$