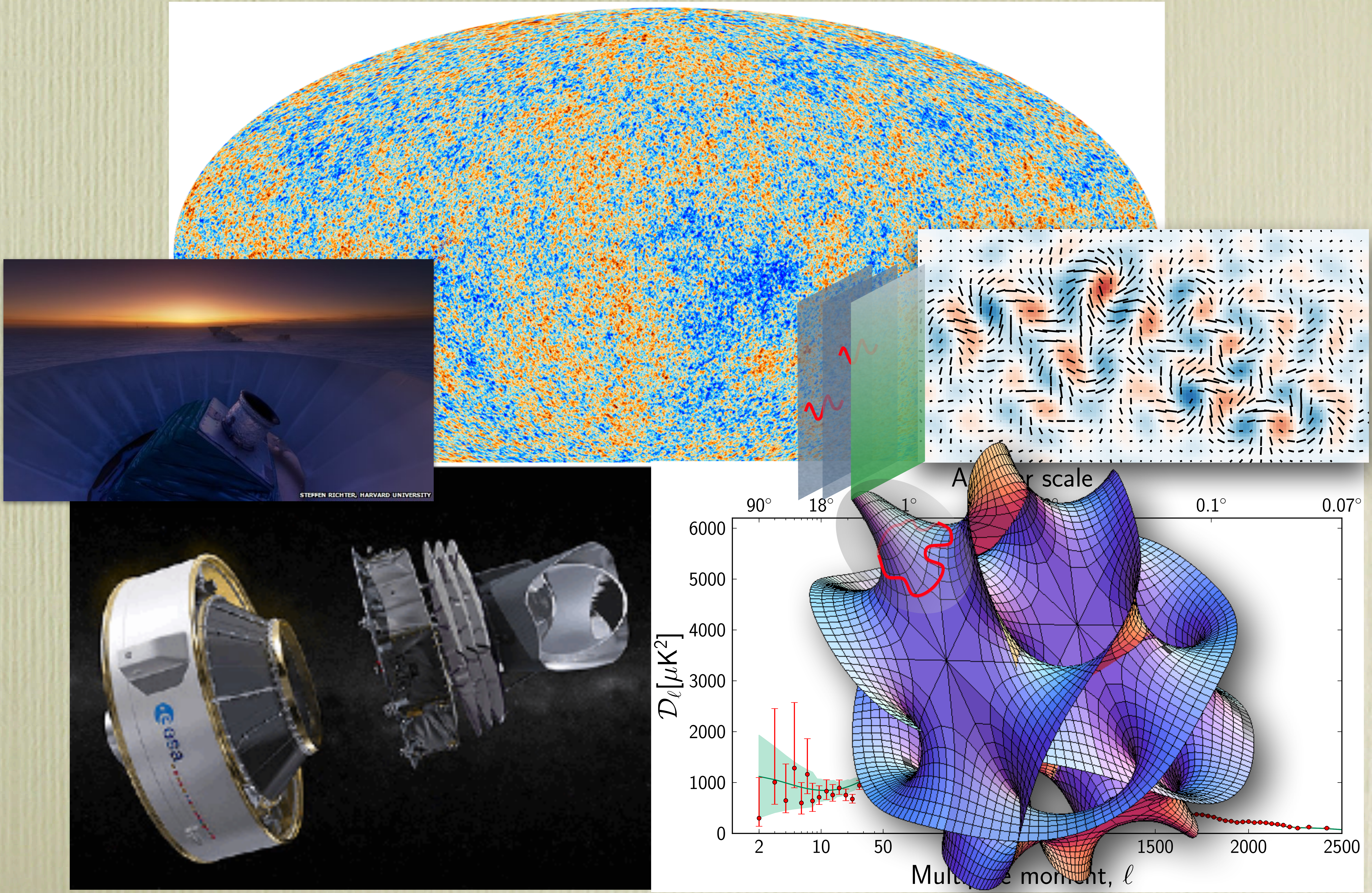
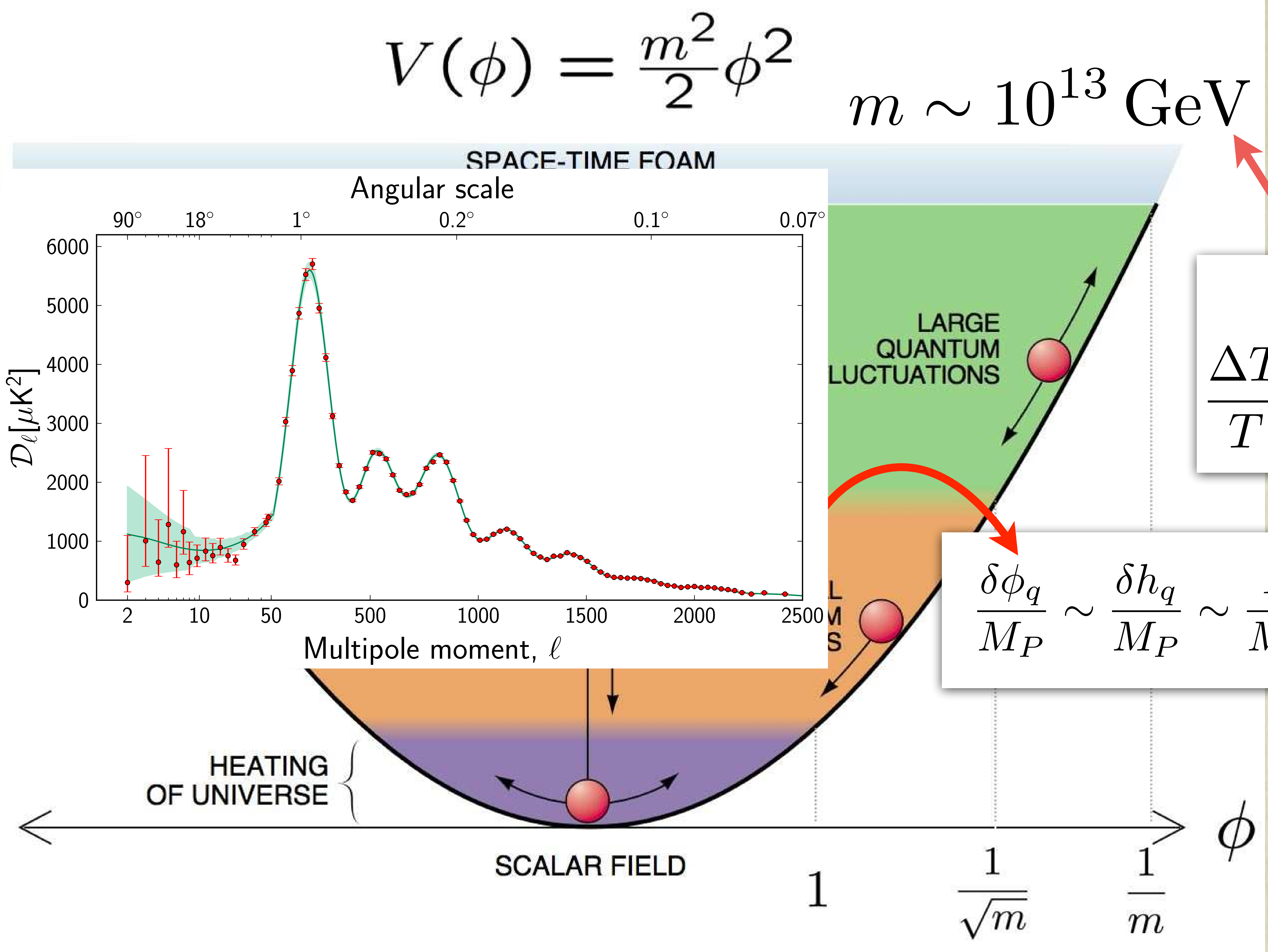
Inflation in String Theory - from Small to Large Fields



slow-roll inflation ... [Linde '82]

$$V(\phi) = \frac{m^2}{2}\phi^2$$

$$m \sim 10^{13}\,\mathrm{GeV}$$



CMB:

$$\frac{\Delta T}{T} \sim 10^{-5}$$

$$\frac{\delta\phi_q}{M_P} \sim \frac{\delta h_q}{M_P} \sim \frac{H}{M_P} \sim 10^{-5}$$

[picture from lecture notes: Linde '07]

why strings?

- We need to understand generic dim ≥ 6 operators

$$\mathcal{O}_{p\geq 6} \quad \sim \quad V(\phi)\left(\frac{\phi}{M_{\mathrm{P}}}\right)^{p-4}$$

$$\Rightarrow \quad \Delta\eta \quad \sim \quad \left(\frac{\phi}{M_{\mathrm{P}}}\right)^{p-6} \gtrsim 1 \quad \forall p \geq 6 \quad \text{if} \quad \phi > M_{\mathrm{P}}$$

- requires UV-completion, e.g. string theory: need to know string and α'-corrections, backreaction effects, ...
- detailed information about moduli stabilization necessary!
- string theory manifestation of the supergravity eta problem

- string compactification produces **moduli**:

  - *need* moduli stabilization - spectrum of massive scalars:

    i) fluxes & orientifold planes fix the shape moduli
    ii) perturbative and/or non-perturbative corrections
    stabilize radii / volume moduli
    -> in IIB on a warped Calabi-Yau supergravity description
    possible - e.g. KKLT or LVS models **& Renata's talk!**

  - ***some radii & brane positions stay light -- inflaton candidates***

- and light **axions**:

  - ***axions are light -- inflaton candidates***

shades of difficulty ...

- tensor-to-scalar ratio links levels of difficulty:

$$r \equiv \frac{\mathcal{P}_T}{\mathcal{P}_R} = 16\epsilon \leq 0.003 \left(\frac{50}{N_e}\right)^2 \left(\frac{\Delta\phi}{M_{\rm P}}\right)^2$$

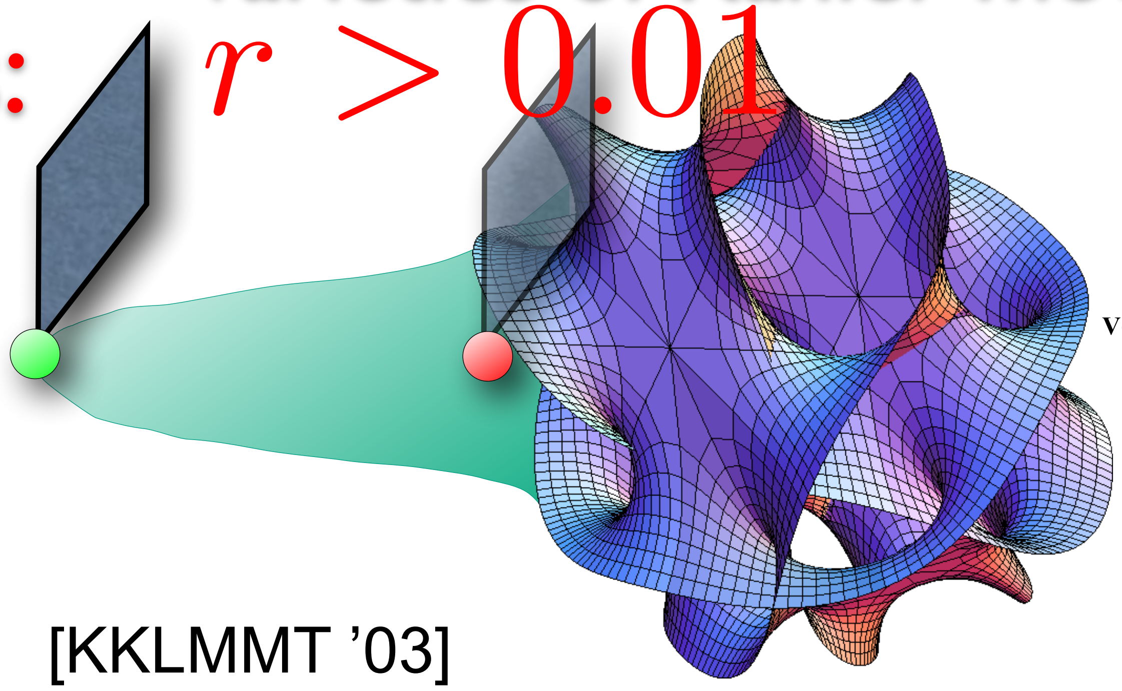
[Lyth '97]

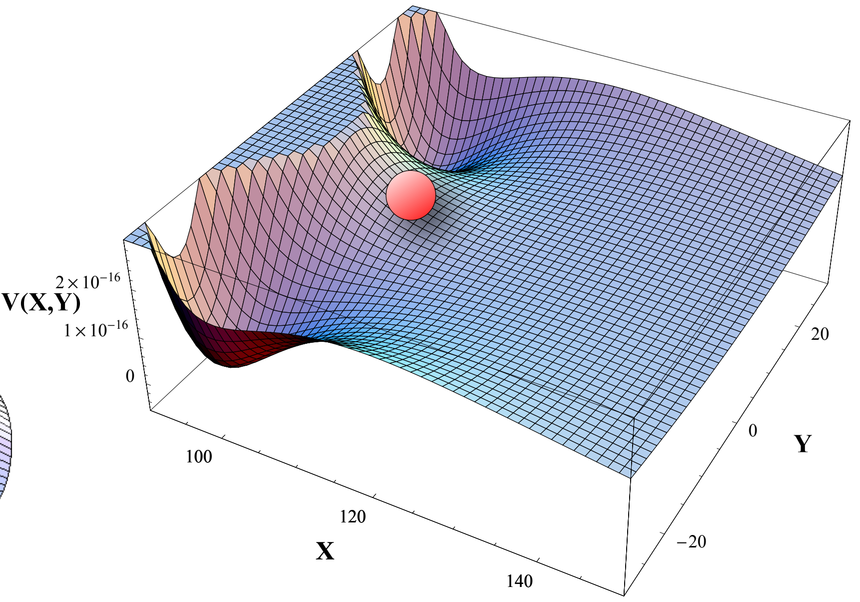
- r << $O(1/N_e^2)$ models:

$\Delta\phi \ll \mathcal{O}(M_P) \quad \Rightarrow$

observable tensors: $r > 0.01$

warped D-brane inflation & DBI;
varieties of Kähler moduli inflation





[KKLMMT '03]
[Baumann, Dymarsky, Klebanov, McAllister & Steinhardt '07]

- r = $O(1/N_e^2)$ models:

$\Delta\phi \sim \mathcal{O}(M_P) \quad \Rightarrow$

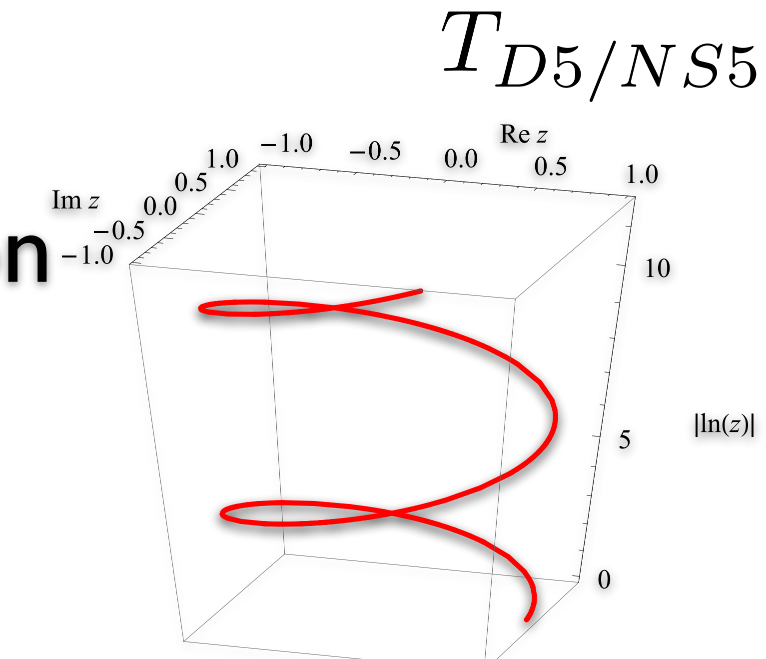
fibre inflation in LARGE volume
scenarios (LVS)

[Cicoli, Burgess & Quevedo '08]

- r = $O(1/N_e)$ models:

$\Delta\phi \sim \sqrt{N_e} M_P \gg M_P \quad \Rightarrow$

axion monodromy inflation
2-axion inflation
N-flation



small-field string inflation ...

- Brane-Antibrane Dvali & Tye; Alexander; Dvali, Shafi & Solganik; Burgess, Majumdar, Nolte, Quevedo, Rajesh & Zhang.

no mod. stab.

- D3-D7 Dasgupta, Herdeiro, Hirano & Kallosh; Hsu, Kallosh & Prokushkin; Hsu & Kallosh; Aspinwall & Kallosh; Haack, Kallosh, Krause, Linde, Lüst & Zagermann; ...

➡ • warped brane-antibrane Kachru, Kallosh, Linde, Maldacena, McAllister & Trivedi; Firouzjahi & Tye; Burgess, Cline, Stoica & Quevedo; Iizuka & Trivedi; Krause & Pajer; Baumann, Dymarsky, Klebanov, McAllister & Steinhardt; Baumann, Dymarsky, Kachru, Klebanov & McAllister; ...

- DBI Silverstein & Tong; Alishahiha, Silverstein & Tong; Chen; Chen; Shiu & Underwood; Leblond & Shandera; …
- Racetrack Blanco-Pillado, Burgess, Cline, Escoda, Gomez-Reino, Kallosh, Linde & Quevedo; Linde & AW; ...
- Kähler moduli Conlon & Quevedo; AW; Bond, Kofman, Prokushkin & Vaudrevange; Ben-Dayan, Jing, AW & Zarate …

$r = 0$

---

large-field string inflation ...

➡ • Fibre inflation ($r < 0.01$) Cicoli, Burgess & Quevedo

$r \sim 0.001$

- Single-Axion inflation with $f > M_P$ Grimm; Blumenhagen & Plauschinn;

➡ • 2-Axion inflation Kim, Nilles & Peloso; Berg, Pajer & Sjors; Kappl, Krippendorf & Nilles; Long, McAllister & McGuirk;Tye & Wong; Ben-Dayan, Pedro & AW; Gao, Li & Shukla ...

- N-flation Dimopoulos, Kachru, McGreevy, Wacker; Easther & McAllister; Grimm; Cicoli, Dutta & Maharana; Choi, Kim & Yun; Bachlechner, Dias, Frazer & McAllister

➡ • axion monodromy Silverstein & AW; McAllister, Silverstein & AW; Flauger, McAllister, Pajer, AW & Xu; Dong, Horn, Silverstein & AW; Shlaer; Palti & Weigand; Marchesano, Shiu & Uranga; Blumenhagen & Plauschinn; Hebecker, Kraus & Witkowski; Ibanez & Valenzuela; Kaloper, Lawrence & Sorbo; McAllister, Silverstein, AW & Wrase; Franco, Galloni, Retolaza & Uranga;

$r \sim 0.1$

inflaton candidates & field range in string theory

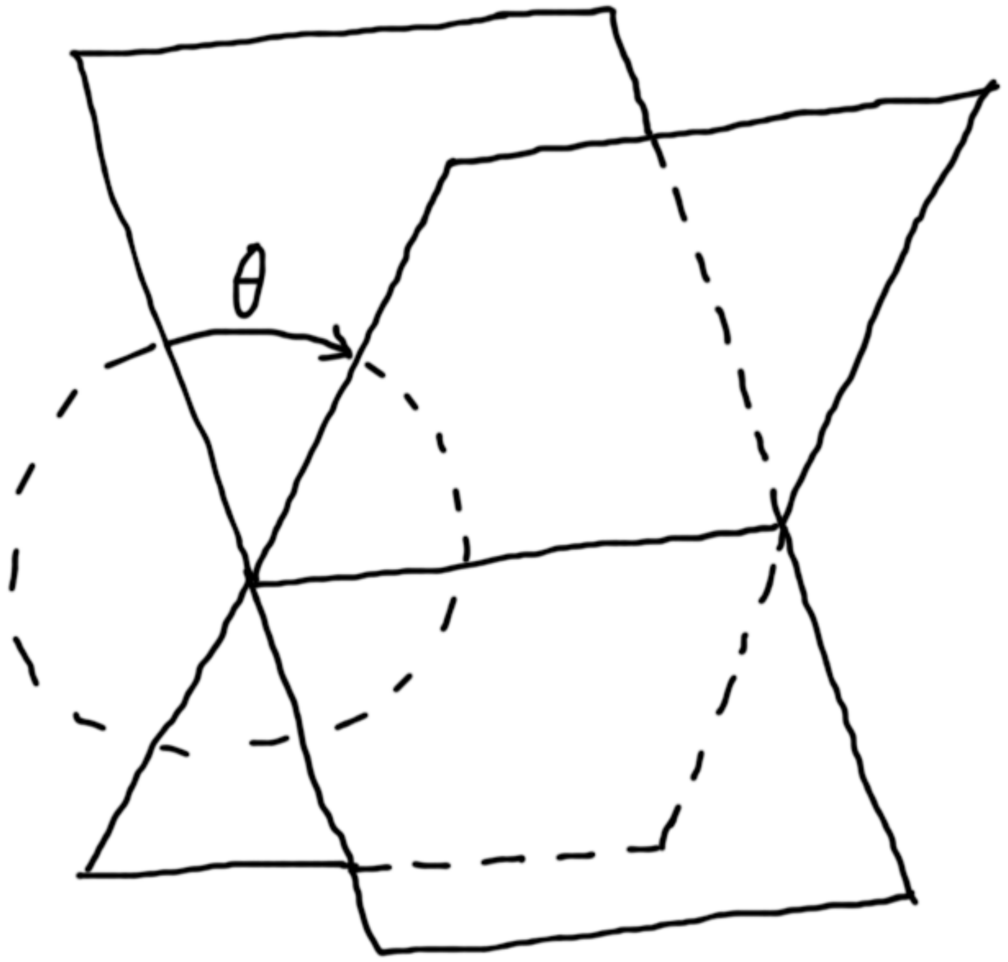
inflatons should be light ...

• **supersymmetry/warping**		
◦ D-brane positions	**no symmetry protection**	**bounded by volume**
• **no-scale structure**		
◦ 'cycle' radii	**no symmetry protection**	**bounded by volume**
◦ overall volume modulus		**unbounded**
• **gauge/shift symmetries**		
◦ axions	**good symmetry protection**	**periodically bounded** by non-pert. effects

- *exclude* dilaton $S = 1/g_s$ -- *pervasive couplings to all sectors*

stringy large field inflation - why so difficult ?

- many periodic inflaton candidates e.g. brane positions or angles θ_a between branes



$$a < (2\pi)^2 \quad \text{or} \quad a < L^q$$

- $E_{internal} \longrightarrow 0$ for L large, all internal sources die at large volume

$$\mathcal{L}_{kin} \sim \frac{M_{\rm P}^2}{L^p} (\partial_\mu a)^2 \quad , \quad p > q$$

- field range is limited to < $M_{\rm P}$

[Banks, Dine, Fox & Gorbatov '03; Srvcek & Witten '06]
[Baumann, McAllister '06]

small fields ...

[KKLMMT '03]

warped D3-brane inflation ...

$$W = \int G \wedge \Omega + A(z_i) \cdot e^{-aT}$$

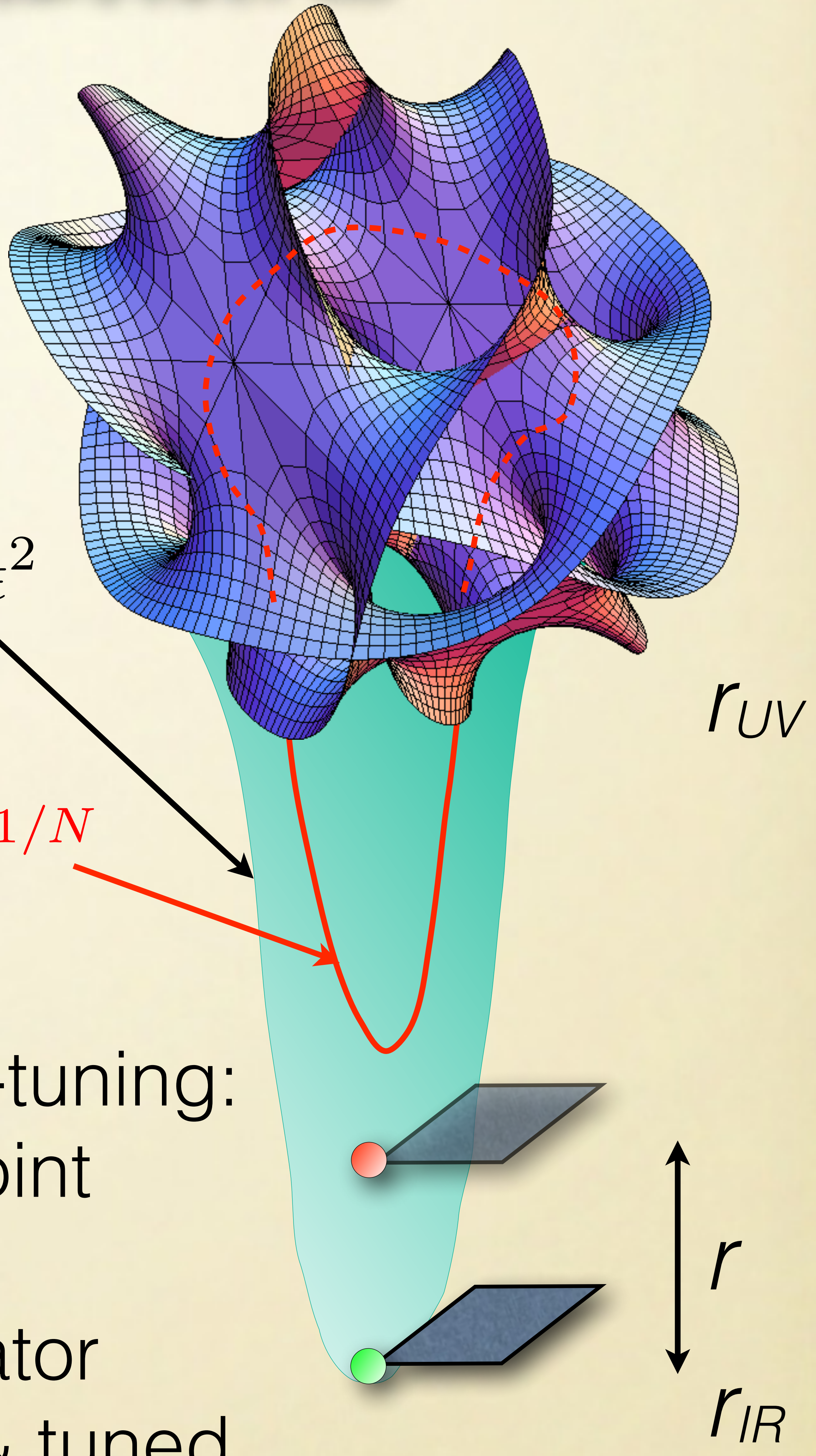
$$K = -3 \ln[T + \bar{T} - k(z_i, \bar{z}_i)]$$

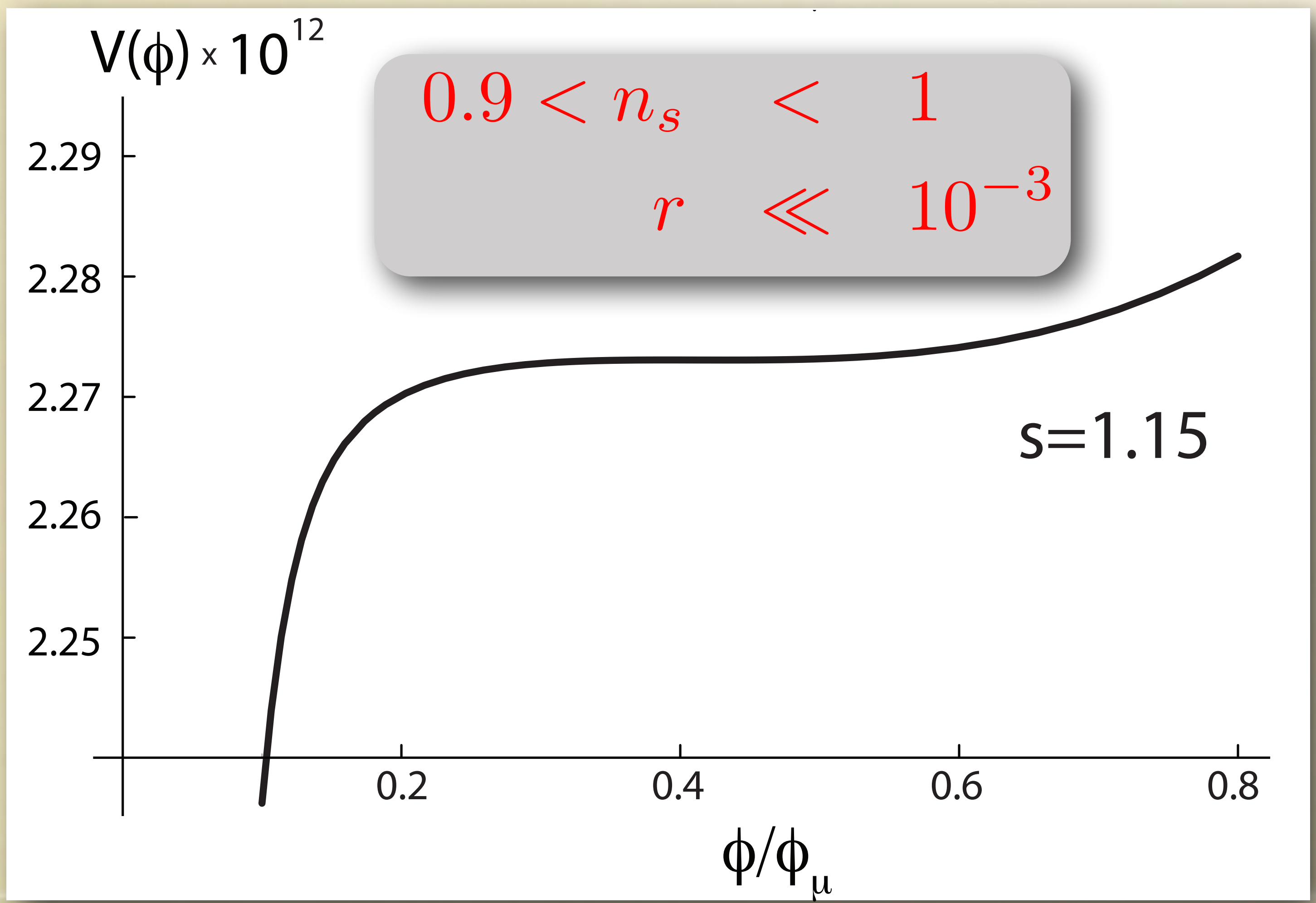
$$k(z_i, \bar{z}_i) \sim \sum_i |z_i|^2 \sim r^2$$

conifold
$$\sum_i z_i^2 = \epsilon^2$$

N embedded D7-branes (fix moduli)

if Kuperstein embedding: $A(z_i) = (z_1 - \mu)^{1/N}$





- explicit fine-tuning: inflection point
- dim-6 operator computed & tuned

[Baumann, Dymarsky, Klebanov, McAllister & Steinhardt '07]

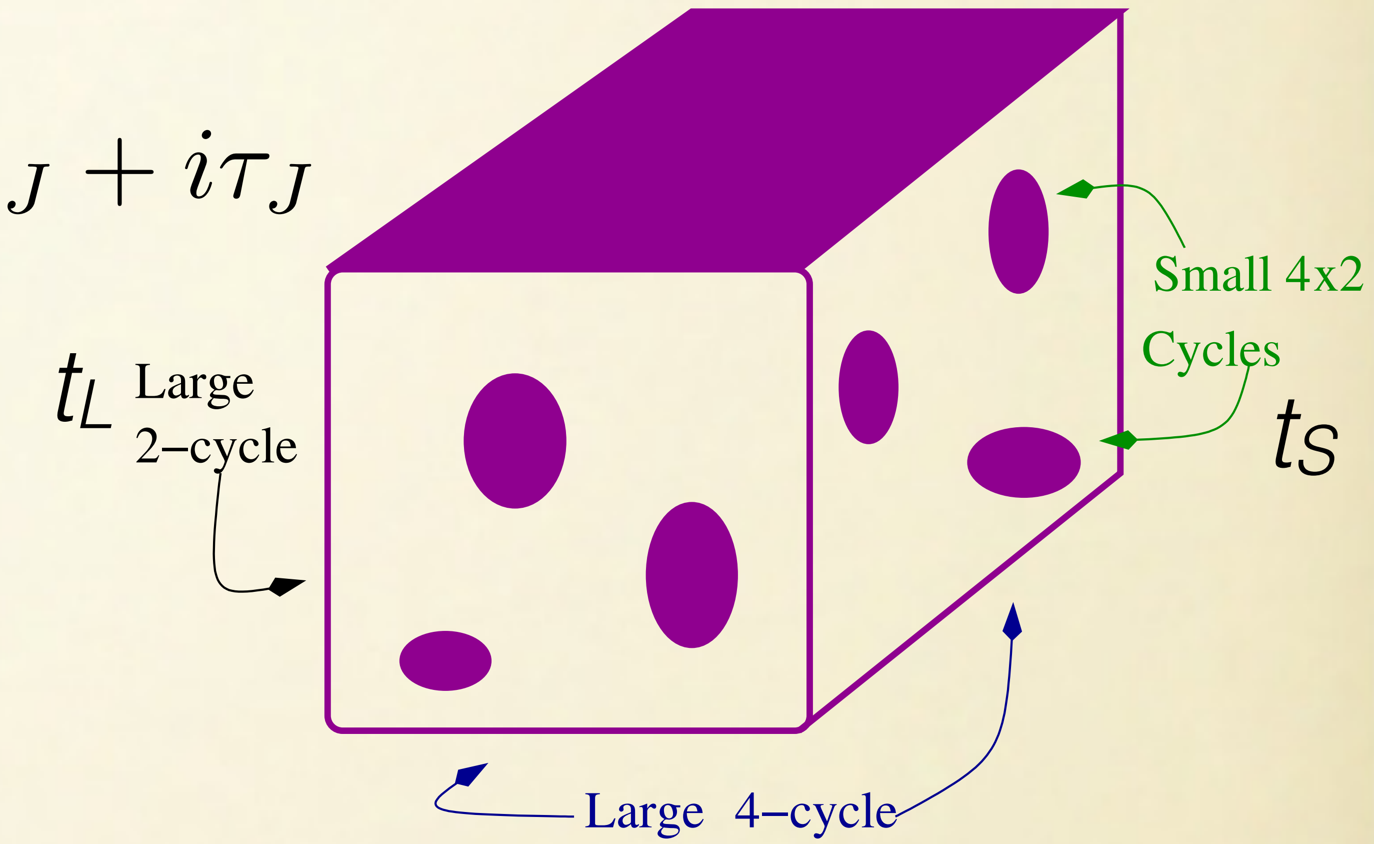
Fibre inflation

[Cicoli, Burgess & Quevedo ’08]

also: "Poly-Instanton Inflation"

[Cicoli, Pedro & Tasinato ’11]

‘swiss cheese Calabi-Yau’



$$K = -2\ln\left(\underbrace{t_L^{3/2} - t_S^{3/2}}_{\mathcal{V}} + \alpha'^3\xi\right) \qquad T_J = t_J + i\tau_J$$

$$W = W_0 + e^{-2\pi T_L} + e^{-2\pi T_S}$$

$$\downarrow$$

$$V = \frac{W_0^2}{\mathcal{V}^3}\left(\lambda\sqrt{\ln\mathcal{V}} - \mu\ln\mathcal{V} + \xi\nu\right) \quad \wedge \quad \mathcal{V} \sim e^{a_S t_S}$$

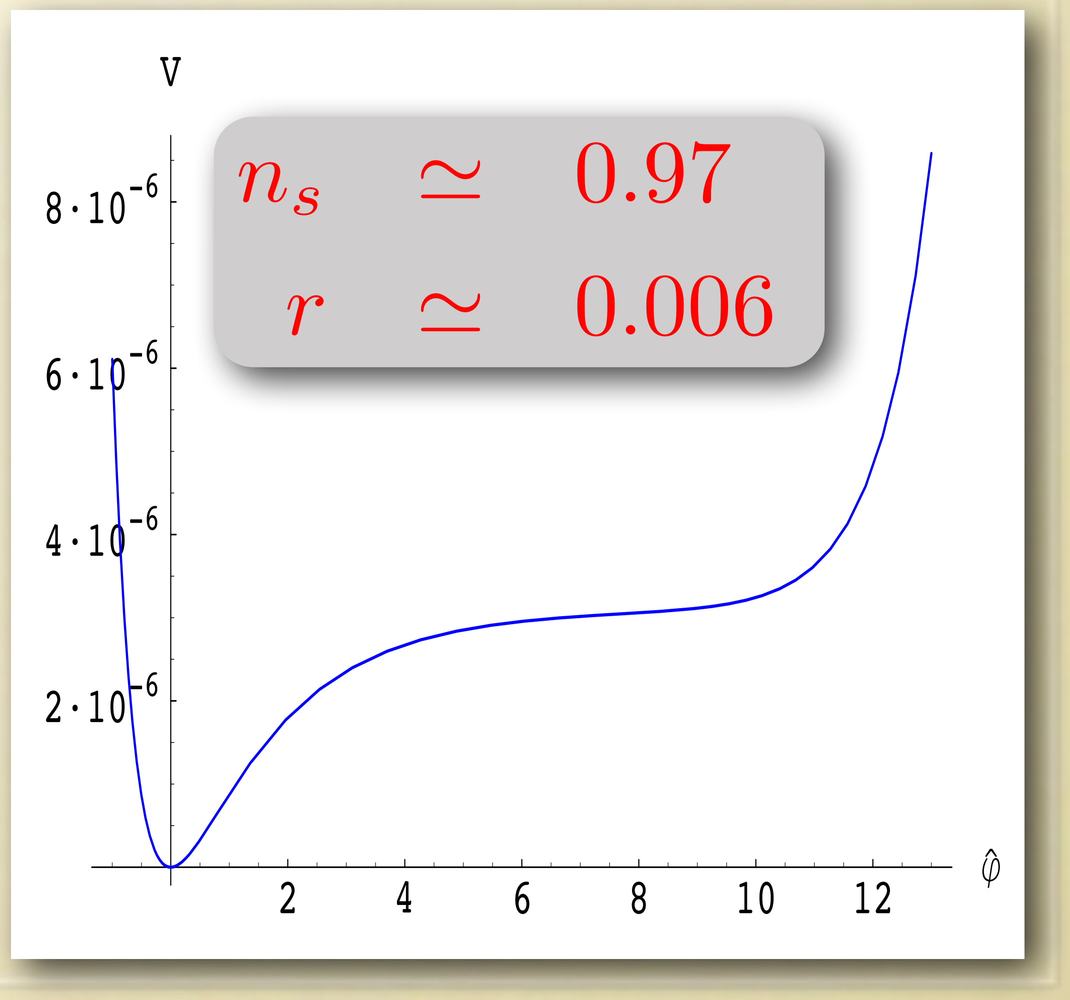
LARGE volume ...

modify *K*, take 3 Kahler moduli:

$$K = -2\ln\left(\sqrt{t_1}t_2 - t_S^{3/2} + \alpha'^3\xi\right)$$

flat in t_l at tree-level - add string loops:

$$V(\mathcal{V},\phi) \simeq \frac{C}{\mathcal{V}^{10/3}}\left(3 - 4e^{-\phi/\sqrt{3}}\right) \;,\; \phi = \frac{\sqrt{3}}{2}\ln t_1$$



$n_s \simeq 0.97$

$r \simeq 0.006$

large fields ...

shift symmetry

- effective theory of large-field inflation:

$$\mathcal{L} = \frac{1}{2}R + \frac{1}{2}(\partial_\mu \phi)^2 - \mu^{4-p}\phi^p$$

- the last term — the potential — spoils the shift symmetry …

- However, if: $V_0 = \mu^{4-p}\phi^p \ll M_{\mathrm{P}}^4$

- quantum GR only couples to $T_{\mu\nu}$:

$$\delta V^{(n)} \sim V_0 \left(\frac{V_0}{M_{\mathrm{P}}^4}\right)^n , \quad V_0 \left(\frac{V_0''}{M_{\mathrm{P}}^2}\right)^n \ll V_0 \qquad \textbf{\underline{not}} \qquad \delta V^{(n)} \sim c_n \frac{\phi^n}{M_{\mathrm{P}}^n}$$

shift symmetry

- while field fluctuation interactions:

$$\mu^{4-p}(\phi_\star+\delta\phi)^p \sim \mu^{4-p}\phi_\star^p\left(1+\sum_n c_n\frac{\delta\phi^n}{\phi_\star^n}\right)$$

die out with increasing field displacement …

- if the inflaton potential breaks the shift symmetry *weakly* & *smoothly* (means: with falling derivatives)

—

it does *not* matter, if the shift symmetry is periodically broken, or secularly

- a shift symmetry itself does not guarantee *smoothness* of breaking — need UV theory as input, for all models!

axion monodromy

axion inflation in string theory ...

- shift symmetry dictates use of string theory axions for large-field inflation

  - periodic, e.g.

$$b = \int_{\Sigma_2} B_2 \quad , \quad b \to b + (2\pi)^2 \quad \text{since} \quad S_{string} \supset \frac{1}{2\pi\alpha'} \int B_2$$

  - field range from kinetic terms *f* < *M*P :

$$S \sim \int d^{10}x \sqrt{-g} |H_3|^2 \supset \int d^4x \sqrt{-g_4} \frac{1}{L^4} (\partial_\mu b)^2$$

$$B_2 = b\omega_2 \quad \Rightarrow \quad \phi = fb \quad , \quad f = \frac{M_{\rm P}}{L^2} < M_{\rm P}$$

[Banks, Dine, Fox & Gorbatov '03]

however, maybe not strict: [Grimm; Blumenhagen & Plauschinn; Kenton & Thomas '14]

axion inflation in string theory ...

- large field-range from assistance effects of many fields

  - N-flation ...

[Dimopoulos, Kachru, McGreevy & Wacker '05]
[Easther & McAllister '05]
[Grimm '07]
[Cicoli, Dutta & Maharana '14]

- or *monodromy*

  - *generic presence* from branes & fluxes !

[Silverstein & AW '08]
[McAllister, Silverstein & AW '08]
[Dong, Horn, Silverstein & AW '10]

  - cos-potential for 2 axions can align/tune for large-field direction

[Kim, Nilles & Peloso '04; Kappl, Krippendorf & Nilles '14]
[Berg, Pajer & Sjörs '09]
[Ben-Dayan, Pedro & AW '14]
[Tye & Wong; Long, McAllister & McGuirk '14]
[Gao, Li & Shukla; Higaki & Takahashi '14]

... see Hans-Peter's talk

axion monodromy — the general story

- EM Stueckelberg gauge symmetry:

$$S_{EM} = \int d^4x\sqrt{-g}\,\{F_{MN}F^{MN} - \rho^2(A_M + \partial_M C)^2 + \dots\}$$

$$A_M \rightarrow A + \partial_M\Lambda_0 \qquad \Rightarrow \qquad C \rightarrow C - \Lambda_0$$

- string theory contains analogous gauge symmetries for NSNS and RR axions - e.g. IIA:

$$\begin{aligned} H &= dB\,, \\ F_0 &= Q_0\,, \\ \tilde{F}_2 &= dC_1 + F_0 B\,, \\ \tilde{F}_4 &= dC_3 + C_1 \wedge H_3 + \frac{1}{2}F_0 B \wedge B \end{aligned} \qquad \Rightarrow \qquad \begin{aligned} \delta B &= d\Lambda_1\,, \\ \delta C_1 &= -F_0\Lambda_1\,, \\ \delta C_3 &= -F_0\Lambda_1 \wedge B \end{aligned}$$

- type IIB similar

flux monodromy

[Marchesano, Shiu & Uranga '14]
[Blumenhagen & Plauschinn '14]
[Hebecker, Kraus & Witkowski '14]
[McAllister, Silverstein, AW & Wrase '14]

- fluxes generate a potential for the axions:

$$-\frac{1}{\alpha'^4}\int d^{10}x\sqrt{-G}\left\{\frac{1}{g_s^2}|H|^2+|Q_0B|^2+|Q_0B\wedge B|^2+\gamma_4 g_s^2|Q_0B\wedge B|^4+\dots\right\}$$

- produces periodically spaced set of multiple branches of large-field potentials:

$$f(\chi,\dots)\,\frac{(Q^{(n)}a^n+Q^{(n-1)}a^{n-1}+\cdots+Q^{(0)})^2}{L^{2n'}}+\cdots\sim\tilde{f}(\chi,\dots)\,a^{p_0}\quad \text{for } a\gg 1$$

➡ for given flux quanta $Q^{(i)}$ potential is non-periodic – a given branch

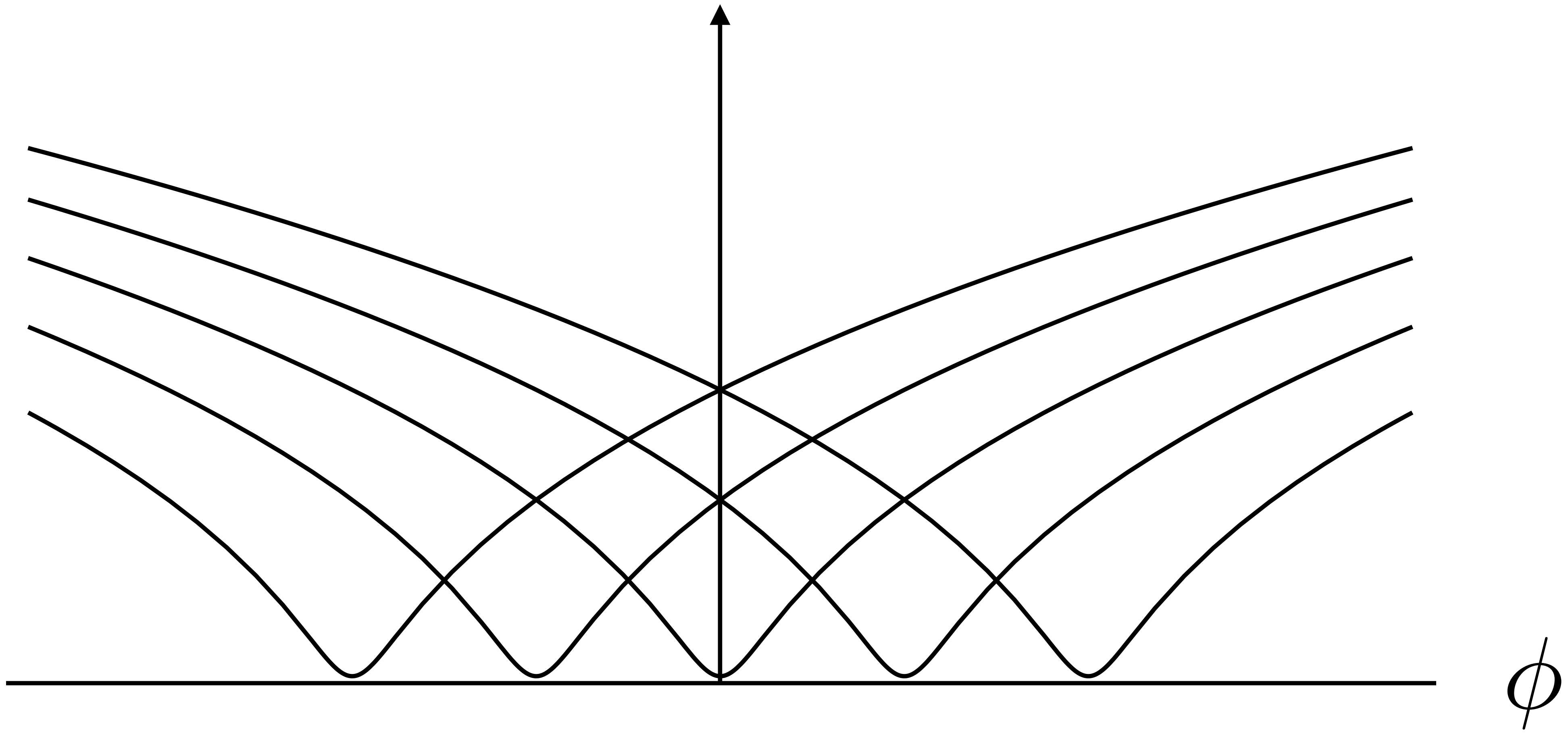
➡ $Q^{(i)}$ change by brane-flux tunneling – $Q^{(i)}$ shift absorbed by a-shift – many branches, full theory has periodicity

flux monodromy

- p-form axions get **non-periodic** potentials from coupling to **branes** or **fluxes/field-strengths**

- produces periodically spaces set of multiple branches of large-field potentials:

$$V(\phi) \sim \mu^{4-p}\phi^{p} + \Lambda^{4}\cos(\phi/f)$$



flux monodromy - 4D effective picture

[Kaloper & Sorbo '08] [Dubovsky, Lawrence & Roberts '11]
[Lawrence, Kaloper & Sorbo '11] [Kaloper & Lawrence '14]

- 4D effective action: F_4 field strength, axion ϕ :

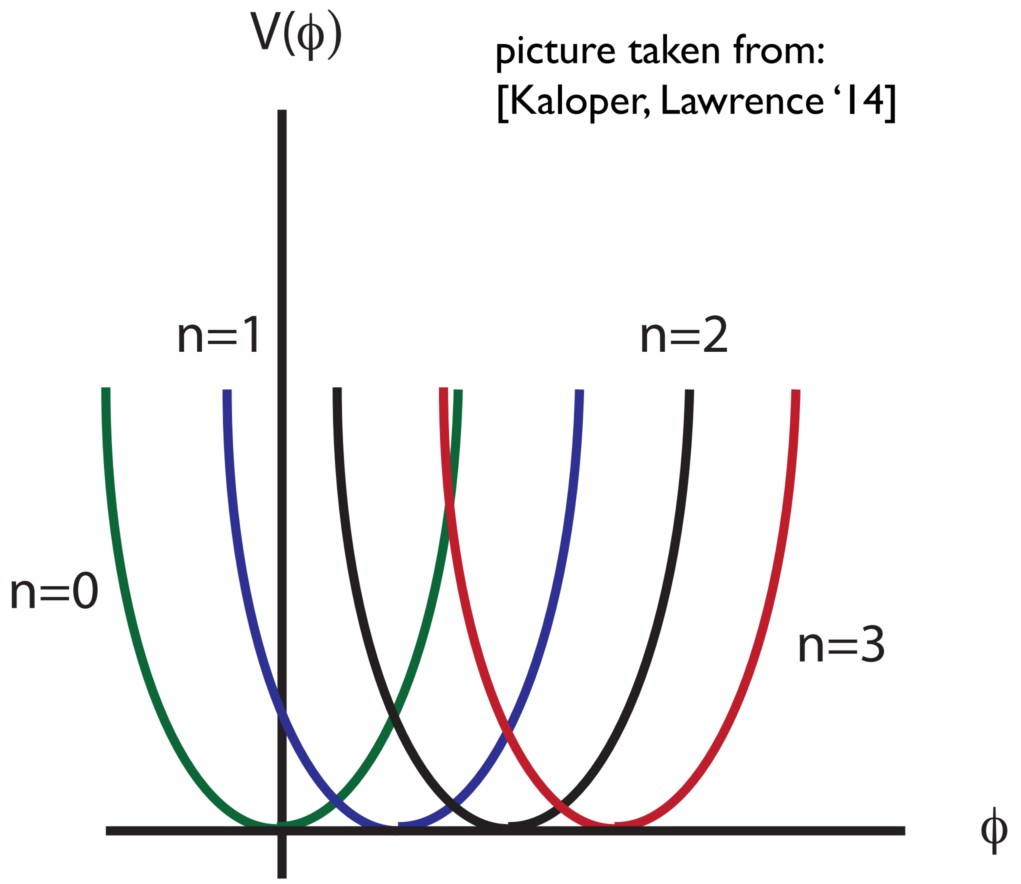
$$\mathcal{L} = \frac{1}{2} m_{pl}^2 R - \frac{1}{2} (\partial\phi)^2 - \frac{1}{48} F_{(4)}^2 + \frac{\mu}{24} \phi^* F_{(4)}$$

- 4D effective Hamiltonian:

$$H = \frac{1}{2} (p_\phi)^2 + \frac{1}{2} (\nabla\phi)^2 + \frac{1}{2} \left(ne^2 + \mu\phi\right)^2$$

$$\phi \rightarrow \phi + f \qquad \Rightarrow \qquad \mu f = e^2 \;\; , \;\; n \rightarrow n+1$$

- again:
  - axion *unwound into multiple branches*
  - *n jumps by flux tunneling*, *e^{-S}-suppressed*
  - periodicity by summing over branches



picture taken from:
[Kaloper, Lawrence '14]

flux monodromy with F-term supergravity description

- flux-induces potentials for B_2- or C_2-axions & D7-brane position moduli, the T-duals of Wilson lines

[Marchesano, Shiu & Uranga '14] **... see Gary's talk**

IIA F-term axion monodromy of B_2- or C_2-axions or Wilson lines from flux

ϕ^2 , ϕ^4 potentials

[Hebecker, Kraus & Witkowski '14]

IIB F-term D7-position axion monodromy from F-theory G_4 flux

ϕ^2 potential

-beautiful F-term realizations — but open question: moduli stabilization!!

-addressed in supergravity model:

> the D7-position proposal

[Hebecker, Kraus & Witkowski '14]

-and non-supergravity model:

> IIB on Riemann surfaces: $\phi^{4/3}$, ϕ^2 , ϕ^3

[McAllister, Silverstein, AW & Wrase '14]

[Blumenhagen & Plauschinn '14]

type IIB with dilaton-axion monodromy from type IIB G_3 flux

ϕ^2 potential

summary ...

- moduli stabilization essential for string inflation! There is no meaningful way to talk about string inflation in presence of massless moduli ...
- first constructions: many <u>*small-field*</u> models, $r = 0$
- field-range bounds, overcome by *monodromy* - many primary power-law <u>*large-field*</u> potentials $\phi^{2/3} \ \ldots \ \phi^4$
- *flattened* powers from moduli stabilization, so again crucial!

⇒ if BICEP2 validated with r ~ 0.1 — need large-field