Chiral Gravity Waves & Leptogenesis during Inflation with non-Abelian Gauge Fields

Azadeh Maleknejad
IPM, Tehran

Particle Cosmology after Planck, DESY Theory Workshop
25 September 2014

Outline

- Non-Abelian Gauge fields and Inflation
- Tensor pert. in the presence of non-Abelian gauge fields
- Chiral Gravity Waves and Leptogenesis
- Conclusion

Non-Abelian Gauge fields and Inflation

Based on

Phys.Lett. B723 (2013) 224-228 [arXiv:1102.1513]

Phys.Rev. D84 (2011) 043515[arXiv:1102.1932]

with M. M. Sheikh-Jabbari

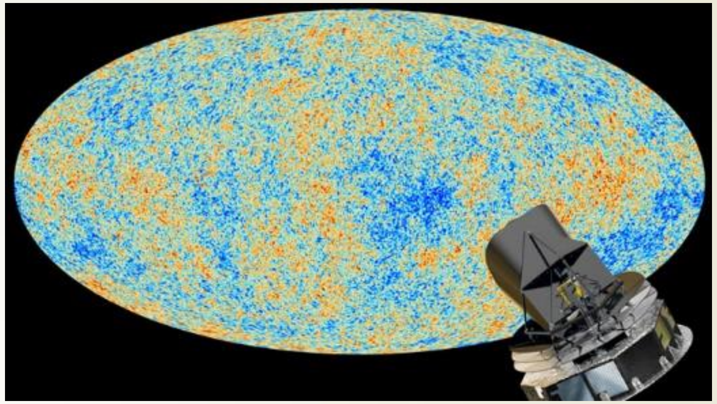
&

Gauge fields and Inflation

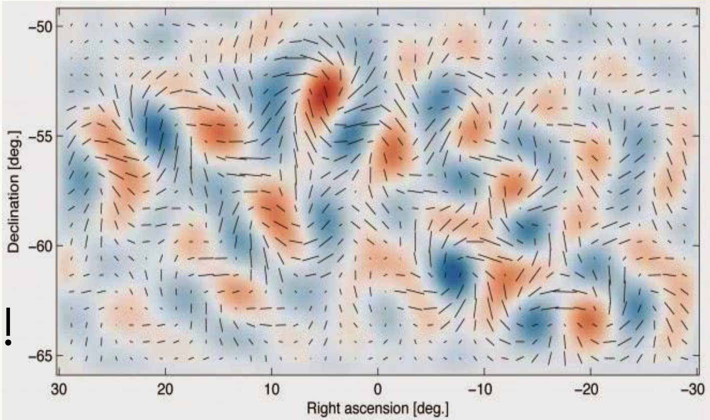
Phys. Rept. 528 (2013) 161-261[arXiv:1212.2921]

with M. M. Sheikh-Jabbari and Jiro Soda

Exciting time for Cosmology!



- Precision cosmological measurements of CMB and LSS increase our understanding of fundamental physics and the early universe.



○ BICEPII detected the B-modes in the CMB polarization, which If proved to be due to the primordial gravitational wave, will offer a good deal of information about the early universe.

○ We are expecting the Planck polarization data in November?!

○ Current measurements of CMB and LSS are in great agreement with the concept of inflation. The next generation of experience will improve our understanding about the physics behind inflation…

Inflation has many variants...

- assisted brane inflation
- anomaly-induced inflation
- assisted inflation
- assisted chaotic inflation
- B-inflation
- boundary inflation
- brane inflation
- brane-assisted inflation
- brane gas inflation
- brane-antibrane inflation
- braneworld inflation
- Brans-Dicke chaotic inflation
- Brans-Dicke inflation
- bulky brane inflation
- chaotic inflation
- chaotic hybrid inflation
- chaotic new inflation
- D-brane inflation
- D-term inflation
- dilaton-driven inflation
- dilaton-driven brane inflation
- double inflation
- double D-term inflation
- dual inflation
- dynamical inflation
- dynamical SUSY inflation
- S-dimensional assisted inflation
- eternal inflation
- extended inflation
- extended open inflation
- extended warm inflation
- extra dimensional inflation
- F-term inflation

- F-term hybrid inflation
- false-vacuum inflation
- false-vacuum chaotic inflation
- fast-roll inflation
- first-order inflation
- gauged inflation
- Ghost inflation
- Hagedorn inflation

- higher curvature inflation
- hybrid inflation
- Hyper-extended inflation
- induced gravity inflation
- intermediate inflation
- inverted hybrid inflation
- Power-law inflation
- K-inflation
- Super symmetric inflation
- Roulette inflation

- Quintessential inflation
- curvature inflation
- Natural inflation
- Warm natural inflation
- Super inflation
- Super natural inflation
- Thermal inflation
- Discrete inflation
- Polarcap inflation
- Open inflation
- Topological inflation
- Multiple inflation
- Warm inflation
- Stochastic inflation
- Generalised assisted inflation
- Self-sustained inflation
- Graduated inflation
- Local inflation
- Singular inflation
- Slinky inflation
- Locked inflation
- Elastic inflation
- Mixed inflation
- Phantom inflation
- Non-commutative inflation
- Tachyonic inflation
- Tsunami inflation
- Lambda inflation
- Steep inflation
- Oscillating inflation
- Mutated hybrid inflation
- Inhomogeneous inflation

F. R. Bouchet: "CMB anisotropies: Status & Prospects" ICGC11, Goa, Dec 18th 2011 53

- In most of the inflationary models, inflation is driven by the coupling of one or more scalar fields to gravity:
- $\frac{1}{2}(\partial_\mu \varphi_I)^2 \ll V(\varphi_I)$, $(\frac{V_{\varphi_I}}{V})^2 \ll 1$ and $\frac{V^2_{\varphi_I}}{V} \ll 1$
- Models of particle physics are gauge field theories including gauge fields, fermions & scalar (Higgs-type) fields.

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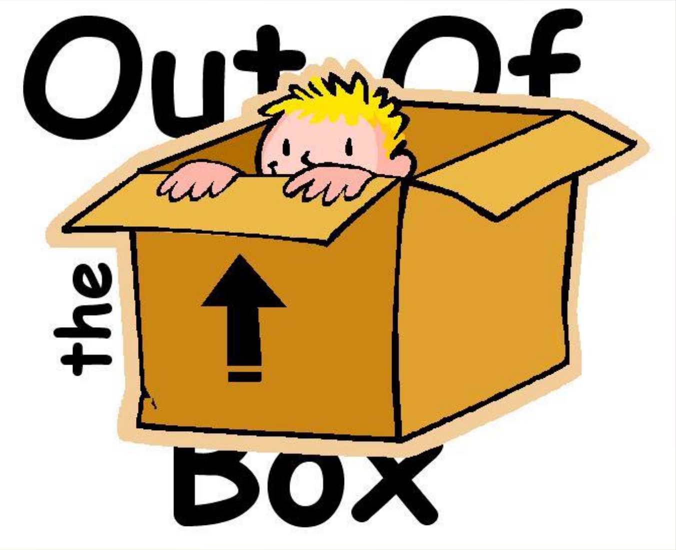
- Models of particle physics are gauge field theories including gauge fields, fermions & scalar (Higgs-type) fields.
- What is the rule of $\overset{\text{Non-Abelian}}{\text{gauge}}$ fields during inflation??

Because of the over-present of gauge fields



Part 1 Non-Abelian Gauge fields and Inflation

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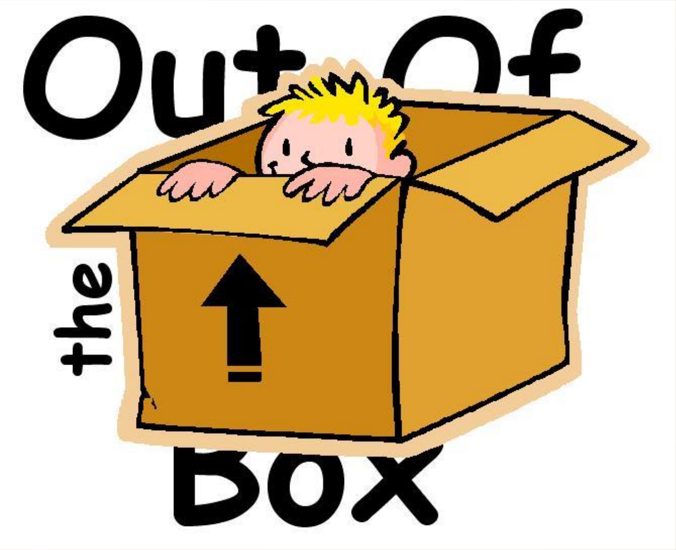
- What is the rule of Non-Abelian gauge fields during inflation??
- how to preserve the spatial isotropy?
- how to break the conformal invariance?



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Part 1 Non-Abelian Gauge fields and Inflation

- the role and consequences of non-Abelian gauge fields, theoretical and observational during inflationary era

- within Einstein GR with minimally coupled fields in 4D

$$ds^2 = -dt^2 + a^2(t)\delta_{ij}dx^i dx^j \qquad \mathrm{L} = -\frac{1}{2}R + \mathrm{L}_{\mathrm{m}}(\phi_I, \mathrm{F}_{\mu\nu})$$

- having a vector without the gauge symmetry, we expect to have ghost instability □, so we will respect gauge symmetry!

(NOT the vector inflation models)

□ B. Himmetoglu, C. R. Contaldi and M. Peloso, Phys. Rev. Lett. 102 (2009) 111301

1^{st} Q: How to get an isotropic model?

- In the isotropic and homogenous *FRW* background, with a Lagrangian of the form $$\mathrm{L} = -\frac{1}{2}R + \mathrm{L}_{\mathrm{m}}(\phi_I, \mathrm{F}_{\mu\nu})$$
- The non-Abelian gauge field in the background

$$\boxed{\mathrm{A}_\mu^a = \begin{cases} 0 & \mu = 0 \\ a(t)\psi(t)\delta_i^a & \underset{\nearrow}{\mu = i} \end{cases}} \quad \text{and} \quad \phi_I = \phi_I(t)$$

- we have a homogenous and isotropic solution

A. M. and M. M. Sheikh-Jabbari, Phys.Lett. B723 (2013) 224-228 [arXiv:1102.1513 [hep-ph]].
A. M. and M. M. Sheikh-Jabbari, Phys. Rev. D 84, 043515 (2011) [arXiv:1102.1932 [hep-ph]].

2nd Q: How to prevent the damping of the vector fields?

- The gauge-flation model

$$S=\int d^4x\sqrt{-g}\left(-\frac{R}{2}-\frac{1}{4}F^a_{\mu\nu}F_a^{\mu\nu}+\frac{\kappa}{384}\left(\varepsilon^{\mu\nu\lambda\sigma}F^a_{\mu\nu}F^a_{\lambda\sigma}\right)^2\right)$$

$$Tr\left(F\wedge F\right)^2$$

$$\rho_{YM}=3P_{YM} \qquad \rho_\kappa=-P_\kappa$$

- Slow-roll parameter $\varepsilon=\dfrac{2\rho_{YM}}{\rho_{YM}+\rho_\kappa}$

Reduced Lagrangian $L_{red}=\frac{3}{2}((\dot{\psi}+H\psi)^2-g^2\psi^4+\kappa g^2\psi^4(\dot{\psi}+H\psi)^2)$

$$\varepsilon=(1+\frac{g^2\psi^2}{H^2})\psi^2$$

$$\eta=\psi^2$$

A. M. and M. M. Sheikh-Jabbari, Phys.Lett. B723 (2013) 224-228 [arXiv:1102.1513 [hep-ph]].
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2nd Q: How to prevent the damping of the vector fields?

- The chromo-natural model

$$S=\int d^4x\sqrt{-g}\left(-\frac{R}{2}-\frac{1}{4}F^a_{\mu\nu}F_a^{\mu\nu}-\frac{1}{2}(\partial_\mu\chi)^2-\mu^4(1+\cos(\frac{\chi}{f}))+\frac{\lambda}{8f}\chi(\varepsilon^{\mu\nu\gamma\lambda}F^a_{\mu\nu}F^a_{\gamma\lambda})\right)$$

$\chi\in(0,f\pi)$

$\rho_{YM}=3P_{YM}$

$\rho_\chi=-P_\chi+\dot{\chi}^2$

topological term

- Slow-roll parameter

$$\varepsilon=\frac{2\rho_{YM}+\frac{3}{2}\dot{\chi}^2}{\rho_{YM}+\rho_\chi}$$

$$V(\chi)=\mu^4(1+\cos(\frac{\chi}{f}))>>\dot{\chi}^2,\rho_{YM}$$

Reduced Lagrangian

$$\mathrm{L}_{red}=\frac{3}{2}((\dot{\psi}+H\psi)^2-g^2\psi^4)+\frac{1}{2}\dot{\chi}^2-\mu^4(1+\cos(\frac{\chi}{f}))-\frac{3\lambda g}{f}\chi\psi^2(\dot{\psi}+H\psi)$$

$H^2\approx\frac{1}{3}\mu^4(1+\cos(\frac{\chi}{f}))$

$\psi^3\approx\frac{\mu^4}{3\lambda gH}\sin(\frac{\chi}{f})$

P. Adshead, M. Wyman, Phys. Rev. Lett. 108, 261302 (2012).

Generic setup

Consider the following general expansion for the Lagrangian density

$$\begin{aligned}\mathcal{L}_m(F^a_{\mu\nu},\Phi_I) &= -\frac{1}{2}\sum_{I=1}^{m}(\partial_\mu\Phi_I)^2 - V(\Phi_I) - \frac{1}{4}f_1(\Phi_I)F^a_{\ \mu\nu}F_a^{\ \mu\nu} + \frac{1}{8}f_2(\Phi_I)\epsilon^{\mu\nu\lambda\sigma}F^a_{\ \mu\nu}F_{a\lambda\sigma} \\ &+ \frac{1}{6}f_3(\Phi_I)\epsilon_{abc}F^a_{\ \mu\nu}F^{b\nu}_{\ \ \lambda}F^{c\lambda\mu} + \frac{1}{12}f_4(\Phi_I)\epsilon_{abc}\epsilon^{\mu\nu\lambda\sigma}F^a_{\ \mu\nu}F^b_{\ \lambda}{}^{\xi}F^c_{\ \sigma\xi} + \dots ,\end{aligned}$$

In the tensor sector, the generic model has some specific, robust and unique features!

Tensor perturbations in the presence of non-Abelian gauge fields:

Perturbing the metric around FRW, we have the tensor fluctuations of the metric as

$$\underset{T}{\delta} g_{ij} = a^2(t) h_{ij}, \qquad \underset{T}{\delta} g_{00} = \underset{T}{\delta} g_{0i} = 0.$$

Perturbing the gauge field around its homogeneous and isotropic configuration, we obtain

$$\underset{T}{\delta} A^{a}_{\mu} = \begin{cases} 0 & \mu = 0 \\ a(t)\delta^{aj} X_{ij} & \mu = i \end{cases} \quad \text{and} \quad \underset{T}{\delta} \phi_I = 0$$

Here X_{ij} represents the tensor fluctuations of the non-Abelian gauge field.

The tensor part of the Einstein equations is

$$h''_{ij} + 2\mathcal{H} h'_{ij} - \nabla^2 h_{ij} = 2a^2 \pi^T_{ij},$$

A. M. and M. M. Sheikh-Jabbari, Phys.Lett. B723 (2013) 224-228 [arXiv:1102.1513 [hep-ph]].
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How non-Abelian gauge fields modifies the dynamics of GWs?

The tensor part of the Einstein equations is

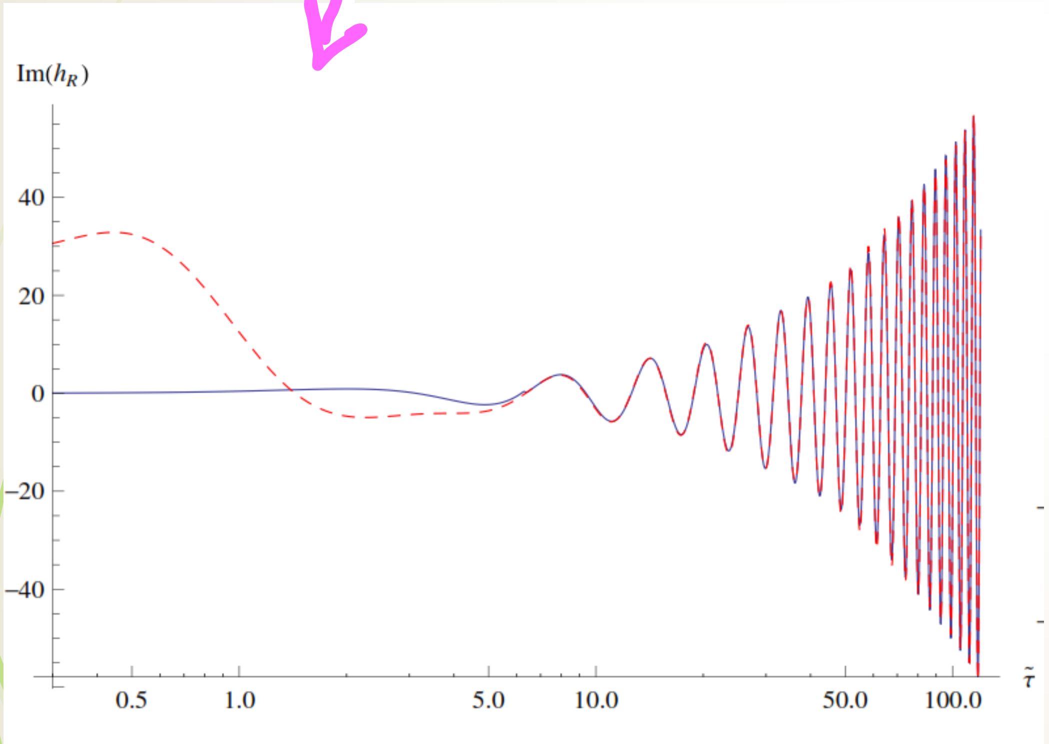
$$h''_{ij} + 2\mathcal{H}h'_{ij} - \nabla^2 h_{ij} = 2a^2\pi^T_{ij},$$

anisotropic inertia

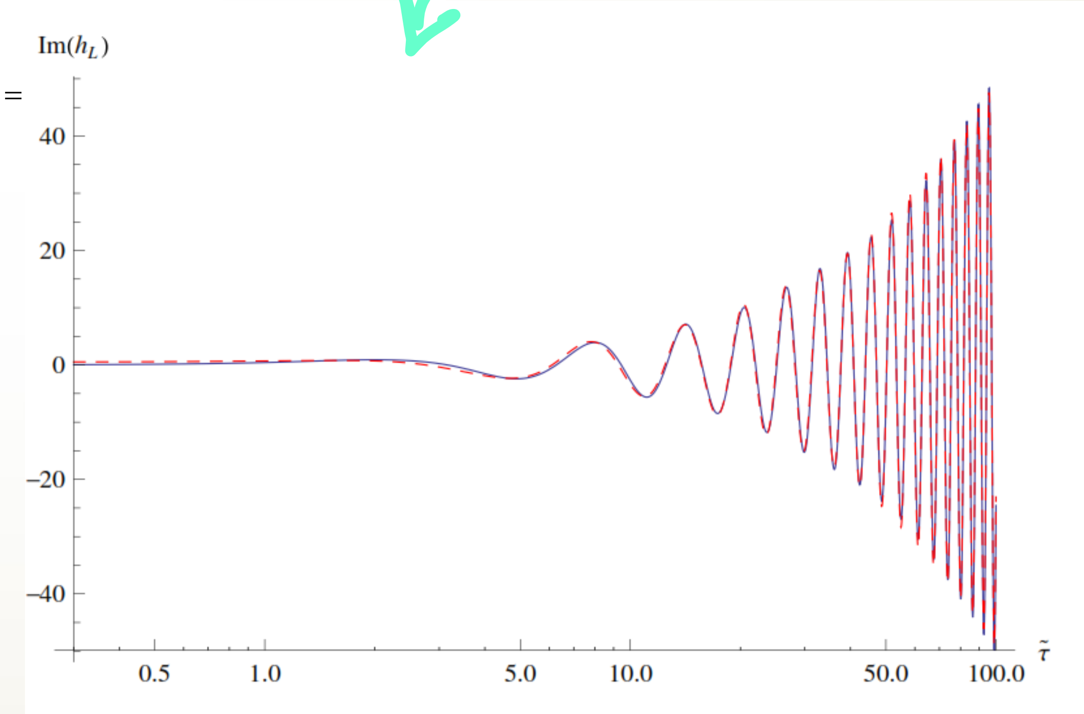
- Sizable tensor-to-scalar ratio $= r \sim 0.1$,
- Chiral gravitational waves and parity odd correlations $\langle TB\rangle$ and $\langle EB\rangle$,
- Violation of the Lyth bound, and
- Violation of the consistency relation (red or blue n_T).

How non-Abelian gauge fields modifies the dynamics of GWs?

Right-handed GW



Left-handed GW



Chiral Gravity Waves and Leptogenesis

Based on

Phys. Rev. D 90 (2014) 023542 [arXiv:1401.7628]

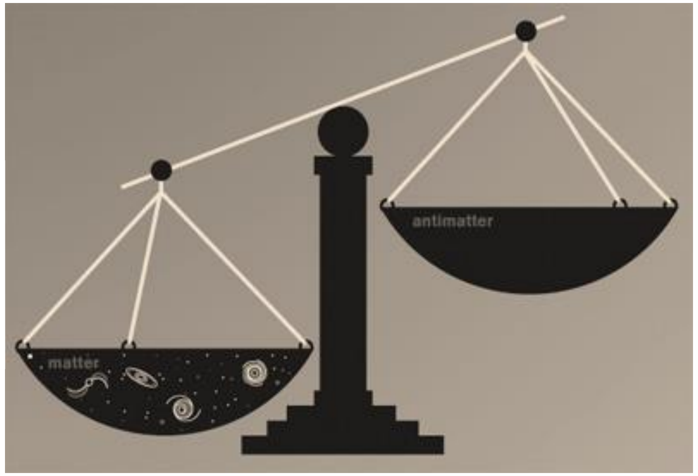
&

arXiv:1208.2807

A.M., Mahdiyar Noorbala and M. M. Sheikh-Jabbari

- The observable Universe is highly matter-antimatter asymmetric.

This asymmetry can be quantified as



$$\eta = \frac{n_B - \bar{n}_B}{n_\gamma}\Big|_0 \approx \frac{n_B}{n_\gamma} = 6.19 \pm 0.15 \times 10^{-10}$$

- n_B = # density of barions
- $\bar{n}_B$ = # density of antibarions
- n_γ = # density of photons

P. A. R. Ade et al. Planck Collaboration, ``Planck 2013 results. XVI. Cosmological parameters, Astron. Astrophys. (2014) arXiv:1303.5076 [astro-ph.CO].
`` Seven-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations'', Astrophys.J.Suppl.192:18,2011.

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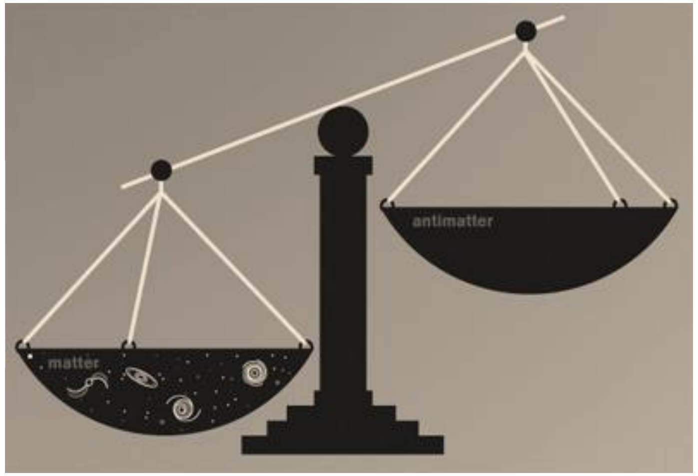
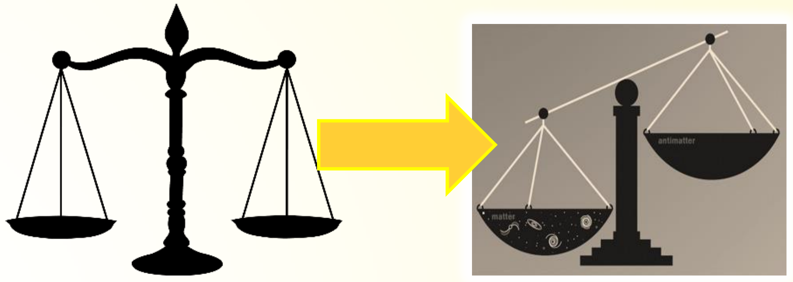


- The big bang should have produced equal amount of matter and antimatter.

- Leptogenesis: the cosmic baryon asymmetry is originated from an initial lepton number asymmetry in the early universe.



- In the standard approach of leptogenesis, the SM is extended by massive right handed neutrinos which (the source of CP violation in the model) decay and generate the initial lepton asymmetry.

- *the standard scenarios associate the matter-antimatter asymmetry to*

- the physics beyond the SM,

- after the inflationary era.

M. Fukugita, T. Yanagida, Phys. Lett. B 174 (1986) 45.
C. S. Fong, E. Nardi and A. Riotto, Adv. High Energy Phys. 2012, 158303 (2012) [arXiv:1301.3062 [hep-ph]].

- **Inflato- Leptogenesis:** a leptogenesis model during inflation and within the SM of particle physics.

The chiral gravitational waves produced during inflation through the gravitational anomaly in the SM leads to a net lepton number density.

Chiral Gravitational Waves

Gravitational Anomaly in SM

Net Lepton Number Density

Total lepton number density

- Setting $N_L - N_R = 3$ and assuming $n_0 = 0$ at the beginning of inflation, we have

$$n(\tau) = \frac{3}{16a^3(\tau)\pi^2}\int \sqrt{-g}\left\langle \tilde{R}R \right\rangle d\tau' d^3x$$

Perturbing the metric around FRW, we obtain

$$\sqrt{-g}\left\langle \tilde{R}R \right\rangle = \frac{2}{\pi^2}\int_{k_{IR}}^{k_{UV}} k^3 dk \frac{d}{d\tau}(h_R'(\tau,k)h_R^{*}{}'(\tau,k) - k^2 h_R(\tau,k)h_R^{*}(\tau,k) - R \leftrightarrow L)$$

Related to the existence of an imbalance between the left- and right-handed GWs! ($h_R \neq h_L$)

h_R and h_L = right- & left-handed polarizations of gravity waves,

k_{UV} and k_{IR} = UV and IR cut-offs

Total lepton number density

- Setting $N_L - N_R = 3$ and assuming $n_0 = 0$ at the beginning of inflation, we have

$$n(\tau) = \frac{3}{16a^3(\tau)\pi^2} \int \sqrt{-g} \left\langle \tilde{R}R \right\rangle d\tau' d^3x$$

Chiral gravitational waves ($h_R \neq h_L$) generates a net lepton number density!

Inflationary models involving **non-Abelian gauge fields** produce intrinsic **chiral gravitational** waves ($\tilde{\tau} = -k\tau$)

$$\boxed{\bar{h}_{R,L}(\tilde{\tau}) \simeq \frac{\tilde{\tau}}{2} \left(1 - \frac{\psi}{M_{Pl}} \frac{\sqrt{\gamma} D \mp iB}{\tilde{N}\tilde{D}\sqrt{1 \mp \tilde{D}/\tilde{\tau}}} \exp(\mp i\tilde{D} \ln \tilde{\tau}) \right) \exp(i\tilde{\tau})}$$

$$\tilde{D} \simeq \sqrt{\gamma} + \frac{\frac{\dot{f}_2}{2H}}{(f_1 - H\psi(\sqrt{\gamma} f_3 + f_4))} \qquad \tilde{N} = \sqrt{f_1 - H\psi(\sqrt{\gamma} f_3 + f_4)} \qquad h_{R,L}(\tau, k) = \frac{H}{M_{\rm Pl}} k^{-\frac{3}{2}} \bar{h}_{R,L}(\tilde{\tau})$$

$$B = (f_1 + \sqrt{\gamma} H\psi(f_3 - \sqrt{\gamma} f_4)) \qquad D = (f_1 - \frac{H\psi}{\sqrt{\gamma}}(f_3 - \sqrt{\gamma} f_4))$$

lepton to photon number density ratio

- Performing the integral in $\Lambda \gg H$ limit, we obtain the lepton number density by the end of inflation as

$$n(\tau_{\rm inf}) \simeq \frac{\mathcal{C}\mathcal{A}}{24\pi^4}\frac{\psi}{M_{\rm Pl}}H^3 \times \left(\frac{H}{M_{\rm Pl}}\right)^2 \times \left(\frac{\Lambda}{H}\right)^4$$

where $\mathcal{C} \simeq \frac{4\alpha}{\tilde{N}(16+\tilde{D}^2)}\left(\tilde{\alpha}\cos(\tilde{D}\ln(\frac{\Lambda}{H})) - \sin(\tilde{D}\ln(\frac{\Lambda}{H}))\right)$, $\alpha = ((1+\tilde{D}^2/4)B + \frac{3}{4}\sqrt{\gamma}D\tilde{D})$ and $\tilde{\alpha} = (\frac{3}{4}B\tilde{D} - \sqrt{\gamma}(1+\tilde{D}^2/4)D)/((1+\tilde{D}^2/4)B + \frac{3}{4}\sqrt{\gamma}D\tilde{D})$.

- Considering a phenomenological (slightly better instant reheating model), we have

$$\boxed{\eta \simeq 1.3 \times 10^{-3}\frac{\mathcal{A}\mathcal{C}}{g_*^{\frac{1}{4}}\sigma^{\frac{3}{4}}}\frac{\psi}{M_{\rm Pl}}\left(\frac{H}{M_{\rm Pl}}\right)^{\frac{7}{2}}\left(\frac{\Lambda}{H}\right)^4}$$

Summary and Outlook

- It is possible to have inflationary models involving non-Abelian gauge fields in the FRW background.
- Presence of non-Abelian gauge fields during inflation leads to the following robust predictions:
- Sizable tensor-to-scalar ratio $r \sim 0.1$,
- Chiral gravitational waves and parity odd correlations $\langle TB \rangle$ and $\langle EB \rangle$,
- Violation of the Lyth bound, and
- Violation of the consistency relation (red or blue n_T).
- This class of models provides a natural setting for leptogenesis within SM.

Thank You!

Extra slides….

- the role and consequences of non-Abelian gauge fields, theoretical and observational during inflationary era
- within Einstein GR with minimally coupled fields in 4D

$$ds^2 = -dt^2 + a^2(t)\delta_{ij}dx^i dx^j$$

$$\mathcal{L} = -\frac{1}{2}R + \mathcal{L}_m(\phi_I, F_{\mu\nu})$$

As any non-Abelian gauge group has a SU(2) subgroup, without loss of generality, we can choose G = SU(2), $f^{abc} = \epsilon^{abc}$ & a,b,c=1,2,3

1st Q: How to get an isotropic model?

- Turning on gauge (vector) fields in the background breaks rotational symmetry

$$\boxed{A_i^a \to R_i^j A_j^a}$$

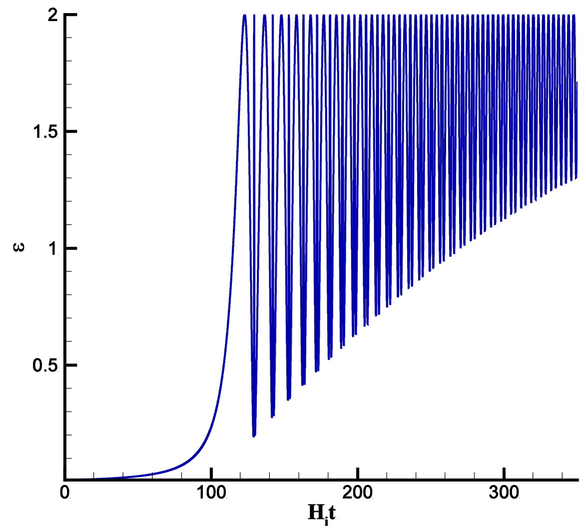
- Gauge fields are defined up to gauge transformations

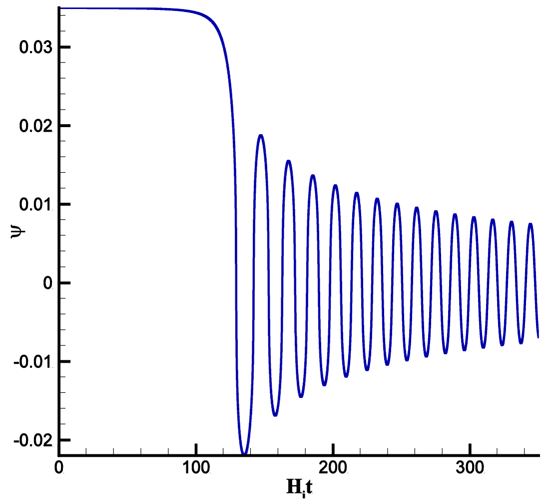
$$A_\mu^a \to \partial_\mu \lambda^a - \varepsilon_{bc}^a \lambda^b A_\mu^c$$

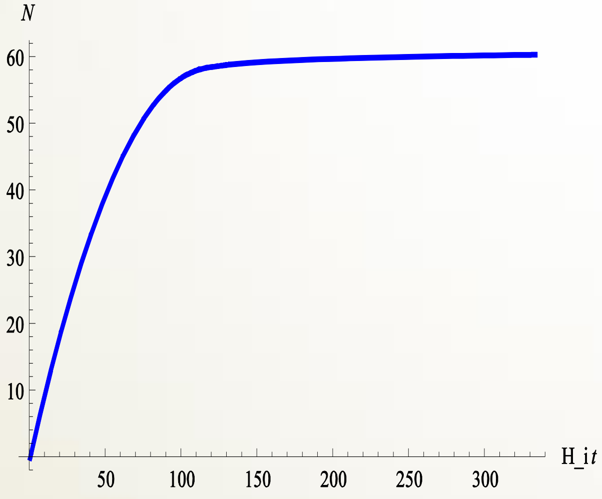
- In the temporal gauge $A_0^a = 0$, we still can make time independent gauge transformations of the form $\boxed{A_i^a \to \Lambda_b^a A_i^b}$

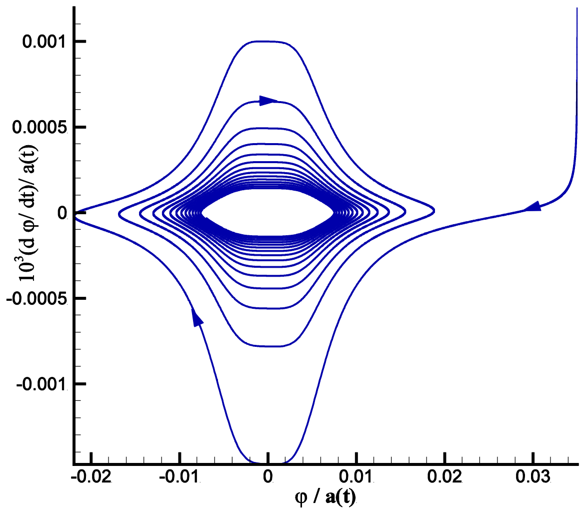
Rotational non-invariance is compensated by global time independent gauge transformation!

$$\psi_i = 3.5\times10^{-2}, \quad \dot{\psi}_i = -10^{-10}, \quad g = 2.5\times10^{-3}, \quad \kappa = 1.733\times10^{14}$$

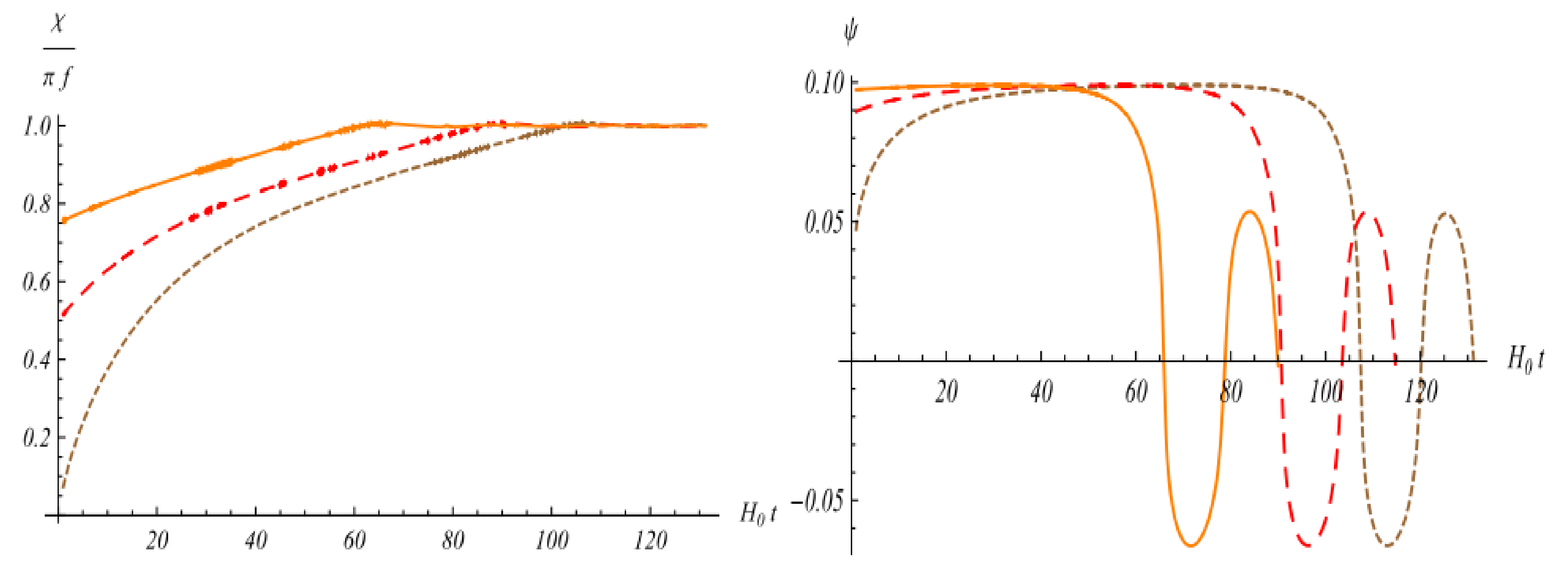








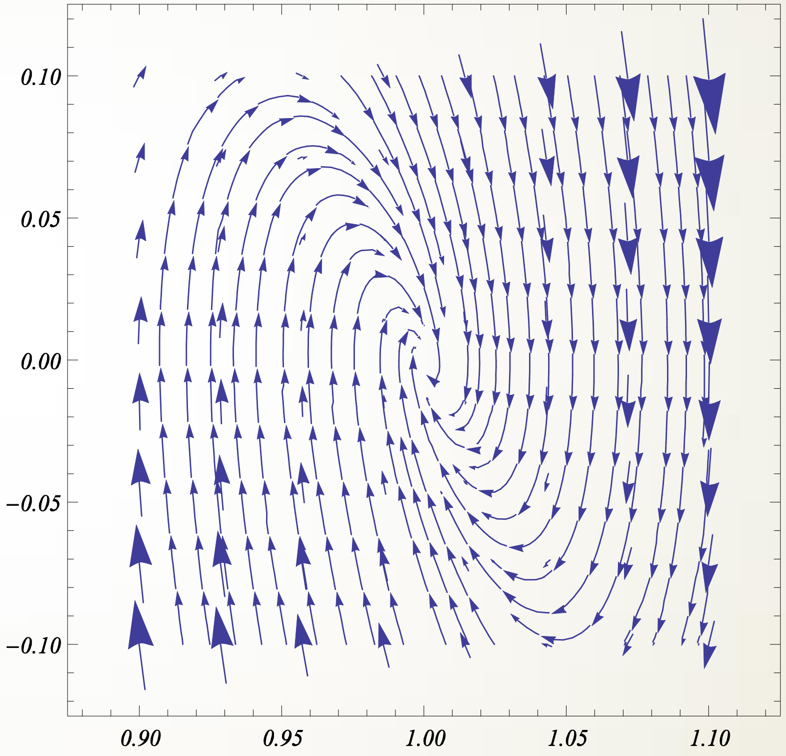
Chromo-natural Model



The classical trajectories with $g = 10^{-6}, \quad \lambda = 400, \quad \mu = 7 \times 10^{-4}, \quad f = 10^{-2}$ started from different axion initial values, χ0. In both panels, the solid (orange) lines, the dashed (red) lines and the dotted (brown) lines correspond to χ0/f values equal to 3π/4, π/2 and 0.01, respectively.

Stability of the Isotropic Solution I

- Bianchi type I $ds^2 = -dt^2 + e^{2\alpha(t)}(e^{-4\sigma(t)}dx^2 + e^{2\sigma(t)}(dy^2 + dz^2))$
- Anisotropic ansatz
- $A_i^a = \psi_i e_i^a, \psi_1 = \frac{\psi}{\lambda^2}, \psi_2 = \lambda\psi$



Result:

Isotropic FLRW cosmology is an attractor of the gauge-flation dynamics.

Stability of the Isotropic Solution II

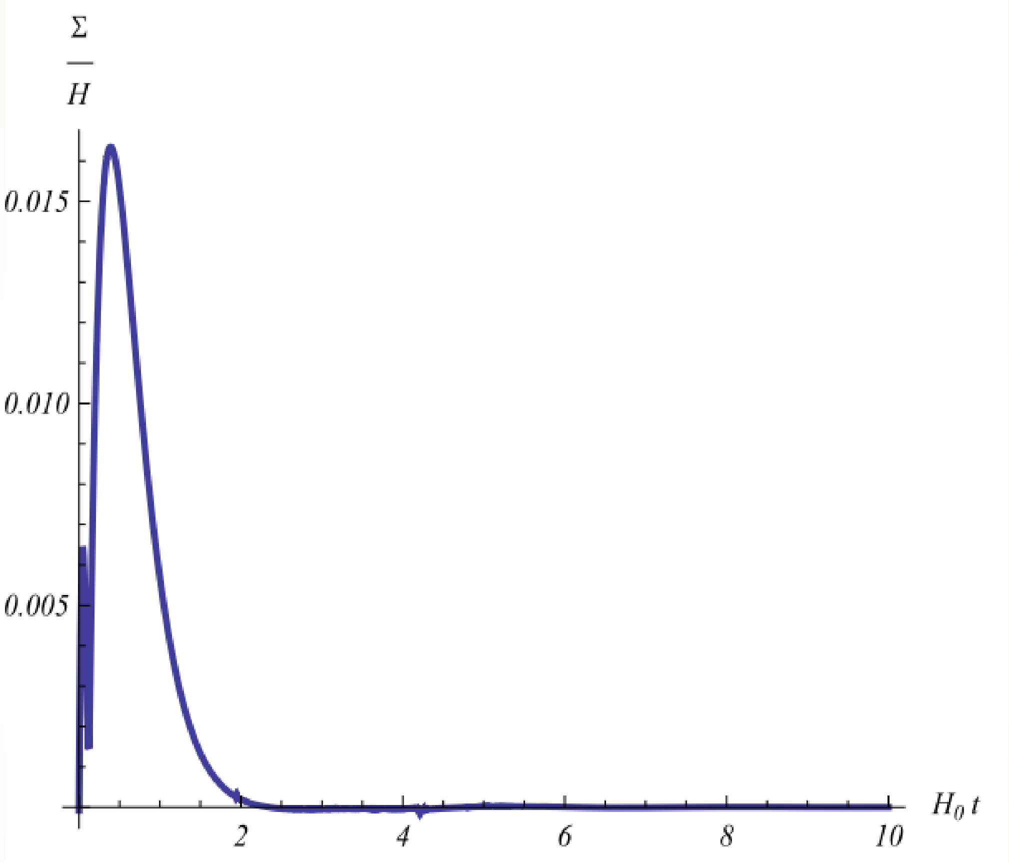


Figure 14: The classical trajectory for $\kappa = 3.77 \times 10^{15}$, $g = 10^{-1}$, $\psi_0 = 0.6 \times 10^{-3}$, $\dot{\psi}_0 = 10^{-10}$, $\lambda_0 = 10$, $\dot{\lambda}_0 = -3.6$. Here $\dot{\lambda} = -\dot{\alpha}\lambda$ which corresponds to $A_1 = 0$ case in (5.78). As expected from our analytical calculations there is a short period in which Σ/H is positive and rapidly increasing, but this lasts very short and quickly (in a couple of e-folds) decreases to become almost zero. These values lead to a trajectory with $\dot{\alpha}_0 = 4 \times 10^{-4}$, $\epsilon_0 = 0.24$ and $(\sigma/H)_0 = -5 \times 10^{-7}$. The initial value of ϵ is rather large, but with in one number of e-folds it decreases and reaches 10^{-2}. Note that value of ϵ at the point of maximum Σ/H is equal to 0.05 ($\Sigma/H \simeq \frac{1}{3}\epsilon$), almost saturating our upper bound for anisotropy Σ/H.

How non-Abelian gauge fields modifies the dynamics of GWs?

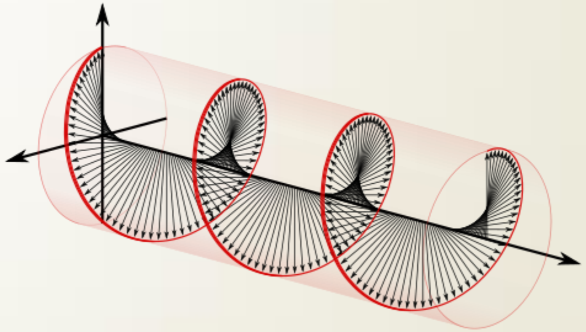
The tensor part of the Einstein equations is

$$h''_{ij} + 2\mathcal{H}h'_{ij} - \nabla^2 h_{ij} = 2a^2\pi^T_{ij},$$

We have some parity odd terms in the action $\varepsilon^{ijk} X_{kl}\partial_i X_{ij}$ and $\varepsilon^{ijk} h_{kl}\partial_i X_{ij}$

- Instead of + and × polarizations, we need Right-handed and Left-handed
- The R and L polarizations are not equal anymore!-→ chiral GWs!

For a vector k=(0,0,1), we have $h_{R,L} \equiv (h_{11} \pm ih_{12})/2$



Tensor perturbations in the presence of non-Abelian gauge fields:

Perturbing the metric around FRW, we have the tensor fluctuations of the metric as

$$\underset{T}{\delta} g_{ij} = a^2(t) h_{ij}, \qquad \underset{T}{\delta} g_{00} = \underset{T}{\delta} g_{0i} = 0.$$

Perturbing the gauge field around its homogeneous and isotropic configuration, we obtain

$$\underset{T}{\delta} \mathrm{A}^{\mathrm{a}}_{\mu} = \begin{cases} 0 & \mu = 0 \\ a(t)\delta^{aj} X_{ij} & \mu = i \end{cases} \qquad \text{and} \qquad \underset{T}{\delta} \phi_I = 0$$

Here X_{ij} represents the tensor fluctuations of the non-Abelian gauge field.

$$\underset{T}{\delta} \mathrm{F}^{\mathrm{a}}_{0i} = \delta^{aj} \dot{(aX_{ij})}, \qquad \underset{T}{\delta} \mathrm{F}^{\mathrm{a}}_{ij} = 2(a\delta^{ak}\partial_{[i} X_{j]k} - a^2 g \psi \varepsilon^{ak}{}_{[j} X_{i]k}).$$

A. M. and M. M. Sheikh-Jabbari, Phys.Lett. B723 (2013) 224-228 [arXiv:1102.1513 [hep-ph]].
A. M. and M. M. Sheikh-Jabbari, Phys. Rev. D 84, 043515 (2011) [arXiv:1102.1932 [hep-ph]].

How non-Abelian gauge fields modifies the dynamics of GWs?

The tensor part of the Einstein equations is

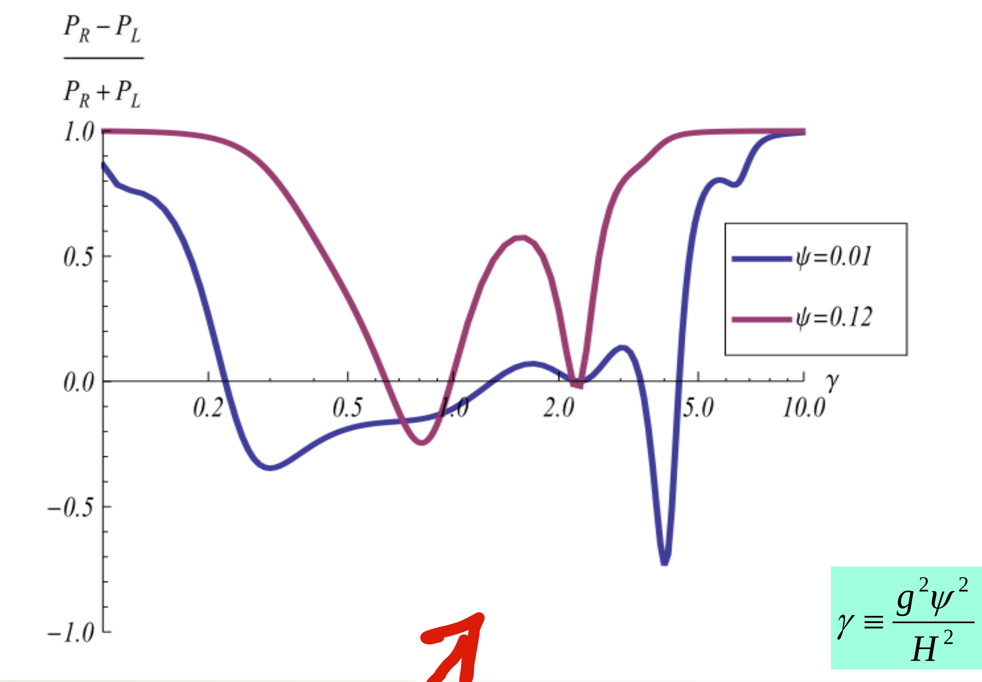
$$h''_{ij} + 2\mathcal{H}h'_{ij} - \nabla^2 h_{ij} = 2a^2\pi^T_{ij},$$

We have some parity odd terms in the action $\varepsilon^{ijk}X_{kl}\partial_i X_{ij}$ and $\varepsilon^{ijk}h_{kl}\partial_i X_{ij}$

- Instead of + and × polarizations, we need Right-handed and Left-handed
- The R and L polarizations are not equal anymore!-→ chiral GWs!

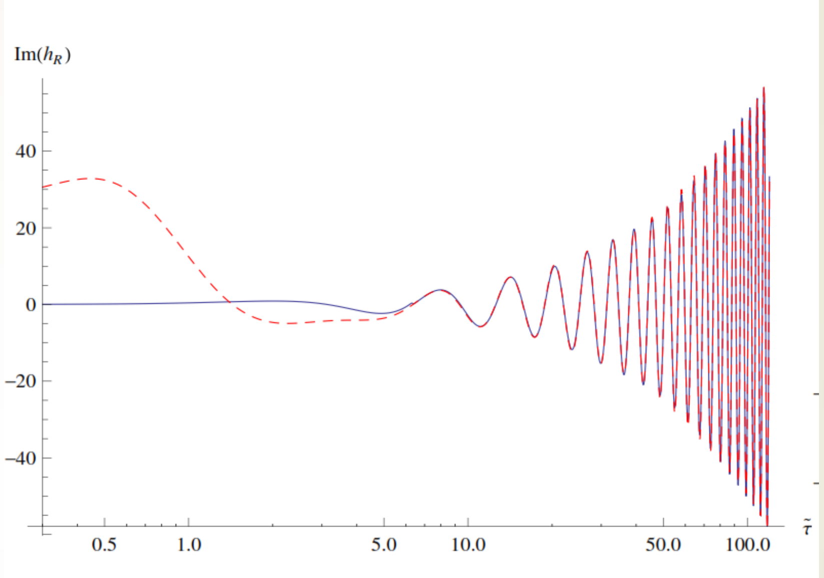
$$\begin{aligned}
a^2\pi^T_{R,L}|_{YM} &\simeq 2f_1(\Phi_I)\psi\left(\frac{\psi}{2}(1-\gamma)\mathcal{H}^2 h_{R,L} + (\gamma-1)\mathcal{H}^2 X_{R,L} - \mathcal{H}X'_{R,L} \mp \sqrt{\gamma}k\mathcal{H}X_{R,L}\right)\\
a^2\pi^T_{R,L}|_{F^3} &\simeq 2\sqrt{\gamma}f_3(\Phi_I)H\psi^2\left(\psi\mathcal{H}^2 h_{R,L} - 2\mathcal{H}^2 X_{R,L} - \mathcal{H}X'_{R,L} \pm \frac{\mathcal{H}}{\sqrt{\gamma}}kX_{R,L}\right),\\
a^2\pi^T_{R,L}|_{W} &\simeq 2\gamma f_4(\Phi_I)H\psi^2\left(-\psi\mathcal{H}^2 h_{R,L} + 2\mathcal{H}^2 X_{R,L} + \mathcal{H}X'_{R,L} \mp \frac{\mathcal{H}}{\sqrt{\gamma}}kX_{R,L}\right),
\end{aligned}$$

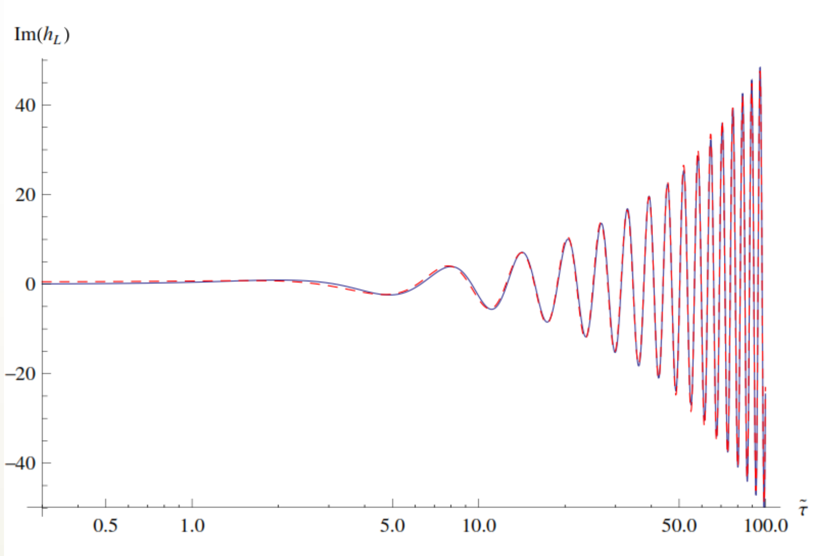
How non-Abelian gauge fields modifies the dynamics of GWs?



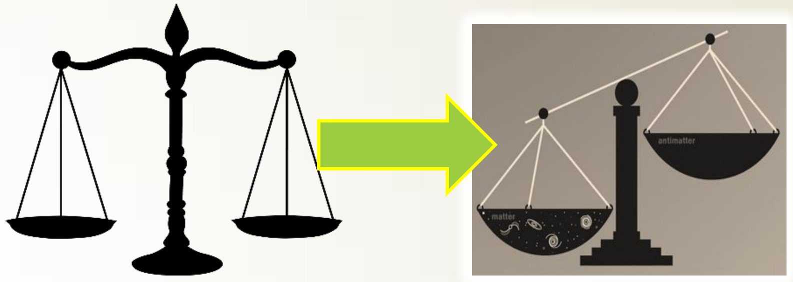
$$\gamma \equiv \frac{g^2\psi^2}{H^2}$$

Parameter of Parity violation





- The observed asymmetry must have been generated dynamically (baryogenesis)



- Sakharov conditions:
- the necessary & sufficient conditions to create a baryon-antibaryon asymmetry from symmetric initial conditions:
- *baryon number violating interactions,*
- *C and CP violation,*
- *Out of equilibrium state*

A. D. Sakharov, Pisma Zh.Eksp.Teor.Fiz. 5 (1967) 32-35 [JETP Lett. 5 (1967) 24-27].

Gravitational anomaly in the SM of particle physics

$$\nabla_\mu J^\mu = \frac{(N_L - N_R)}{16\pi^2}\tilde{R}R \xrightarrow{\text{integrating}} n(\tau) = n_0 + \frac{(N_L - N_R)}{16a^3(\tau)\pi^2}\int\sqrt{-g}\langle\tilde{R}R\rangle d\tau' d^3x$$

total lepton number density

- J^μ = lepton current,
- $N_{L,R}$ = #left/right-handed fermion degrees of freedom,
- $\tilde{R}R = \frac{1}{2}\varepsilon^{\lambda\mu\nu\xi}R_{\mu\nu\rho\sigma}R_{\nu\xi}{}^{\rho\sigma}$
- In the standard model of particle physics $N_L - N_R = 3$,
- in the beyond SM with right-handed neutrinos $N_L - N_R < 3$.