Boltzmann hierarchy for interacting neutrinos

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DESY theory workshop

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Standard Model of Particle Physics: only left handed neutrinos exist

→ massless neutrinos ↯ observation of neutrino oscillations

Extension of right-handed neutrinos:

$$-m_D(\bar{\nu}_R \nu_L + \bar{\nu}_L \nu_R)$$ → arbitrary...?

Break B-L → include Majorana mass term:

$$-\frac{1}{2} m_R(\bar{\nu}_{RC} \nu_R + \bar{\nu}_R \nu_{RC})$$

→ diagonalize mass matrix

→ new mass scale: “See-Saw-Mechanism”

→ heavy sterile neutrinos

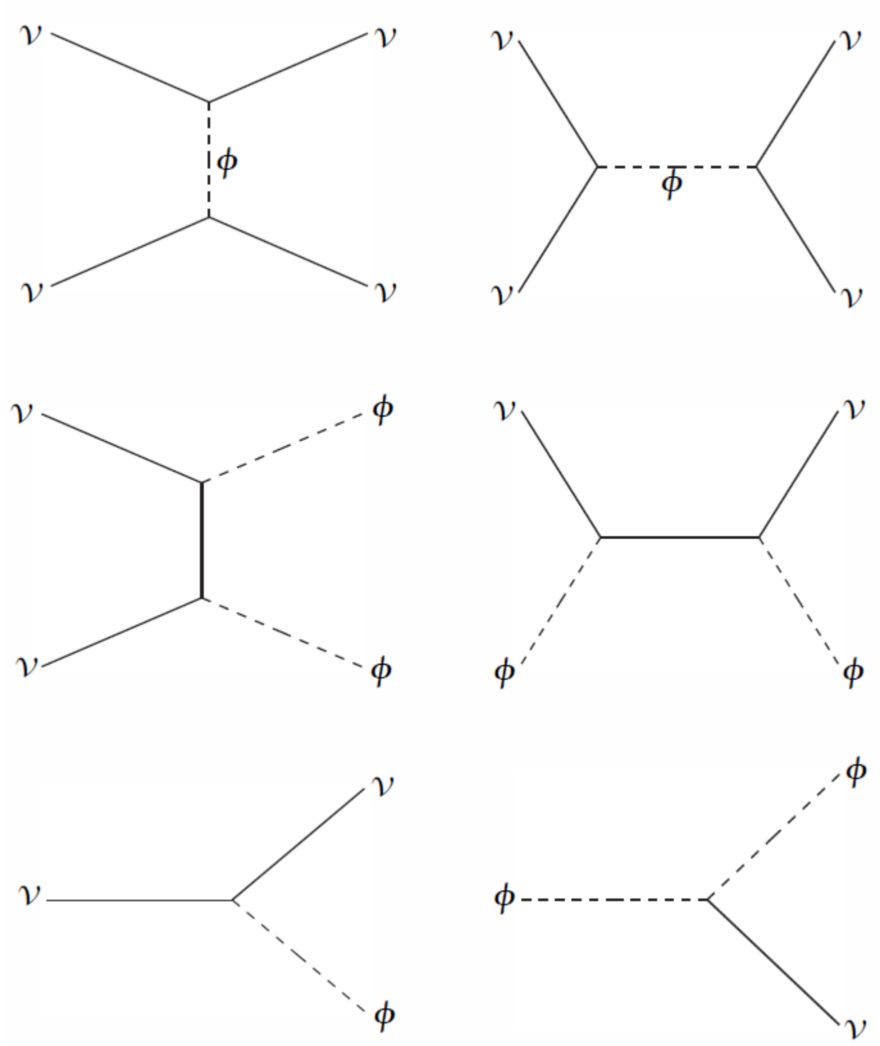
<u>B-L spontanously broken?</u>

→ Majoron models

- New Goldstone Boson: Majoron
- Non-standard neutrino interactions

$$\mathcal{L}_{\text{int}} = \mathfrak{g}_{ij}\bar{\nu}_i\nu_j\phi + \mathfrak{h}_{ij}\bar{\nu}_i\gamma_5\nu_j\phi$$

→ cosmological signatures???



Einstein equation: $$\bar{G}_{\mu\nu} + \delta G_{\mu\nu} = 8\pi G(\bar{T}_{\mu\nu} + \delta T_{\mu\nu})$$

$$ds^2 = a(\tau)^2 \left(-d\tau^2 + (\delta_{ij} + h_{ij})dx^i dx^j\right)$$

Boltzmann equation: $$P^\alpha \frac{\partial f}{\partial x^\alpha} - \Gamma^\gamma_{\alpha\beta} P^\alpha P^\beta \frac{\partial f}{\partial P^\gamma} = \left(\frac{\partial f}{\partial \tau}\right)_{\text{coll}}$$

- Perturb $f = \bar{f} + F$
- Fourier space

$$\dot{F}(\boldsymbol{k}, \boldsymbol{q}, \eta) + \mathrm{i}\frac{|\boldsymbol{q}||\boldsymbol{k}|}{\epsilon}(\hat{k}\cdot\hat{q})\, F(\boldsymbol{k}, \boldsymbol{q}, \eta) + \frac{\partial \bar{f}(|\boldsymbol{q}|)}{\partial \ln |\boldsymbol{q}|}\left[\dot{\tilde{\eta}} - (\hat{k}\cdot\hat{q})^2\, \frac{\dot{h} + 6\dot{\tilde{\eta}}}{2}\right] = \left(\frac{\partial f}{\partial \eta}\right)^{(1)}_{\text{coll}}$$

Apply on all relevant particle species:

	interacting	non-interacting
relativistic	photons	Neutrinos ???
non-relativistic	baryons	CDM

$$F(|\boldsymbol{k}|,|\boldsymbol{q}|,\cos\epsilon) = \sum_{\ell=0}^{\infty}(-\mathrm{i})^{\ell}(2\ell+1)\,F_{\ell}(|\boldsymbol{k}|,|\boldsymbol{q}|)\,\mathrm{P}_{\ell}(\cos\epsilon)$$

→ Taking moments

→ Boltzmann hierarchy:

$$\begin{aligned}
\dot{\delta}_{\nu} &= -\frac{4}{3}\theta_{\nu} - \frac{2}{3}\dot{h},\\
\dot{\theta}_{\nu} &= k^{2}\left(\frac{1}{4}\delta_{\nu} - \sigma_{\nu}\right),\\
\dot{F}_{\nu 2} &= 2\dot{\sigma}_{\nu} = \frac{8}{15}\theta_{\nu} - \frac{3}{5}kF_{\nu 3} + \frac{4}{15}\dot{h} + \frac{8}{5}\dot{\eta},\\
\dot{F}_{\nu l} &= \frac{k}{2l+1}\left[lF_{\nu(l-1)} - (l+1)F_{\nu(l+1)}\right],\ l \geq 3
\end{aligned}$$

Neutrino interactions???

1.) Tightly coupled limit: $F_{\nu l\geq 2} = 0$

$$\begin{aligned}
\dot{\delta}_{\nu} &= -\tfrac{4}{3}\theta_{\nu} - \tfrac{2}{3}\dot{h}\\
\dot{\theta}_{\nu} &= \tfrac{1}{4}k^{2}\delta_{\nu}
\end{aligned}$$

→ only valid for sufficiently strong couplings

2.) Adding damping term:

$$\dot{\mathcal{F}}_{\nu 2} = \frac{8}{15}\theta_\nu - \frac{3}{5}k\mathcal{F}_{\nu 3} + \frac{4}{15}\dot{h} + \frac{8}{5}\dot{\tilde{\eta}} + \frac{9}{10}\alpha_2\dot{\tau}_\nu\mathcal{F}_{\nu 2} \ ,$$

$$\dot{\mathcal{F}}_{\nu\ell} = \frac{k}{2\ell+1}\left[\ell\mathcal{F}_{\nu(\ell-1)} - (\ell+1)\mathcal{F}_{\nu(\ell+1)}\right] + \alpha_\ell\dot{\tau}_\nu\mathcal{F}_{\nu\ell} \ , \quad \ell \geq 3$$

3.) Parametrisation used to fit cosmologocal data:

$$(c^2_{\text{eff}}, c^2_{\text{vis}}) = (\tfrac{1}{3}, \tfrac{1}{3})$$

→ standard case

$$\dot{\delta}_\nu = -\frac{4}{3}\theta_\nu - \frac{2}{3}\dot{h} + \frac{\dot{a}}{a}(1 - 3c^2_{\text{eff}})\left(\delta_\nu + 4\frac{\dot{a}}{a}\frac{\theta_\nu}{k^2}\right) \ ,$$

$$\dot{\theta}_\nu = k^2\left(\frac{1}{4}\delta_\nu - \sigma_\nu\right) - \frac{k^2}{4}(1 - 3c^2_{\text{eff}})\left(\delta_\nu + 4\frac{\dot{a}}{a}\frac{\theta_\nu}{k^2}\right) \ ,$$

$$\dot{\mathcal{F}}_{\nu 2} = 2\dot{\sigma}_\nu = \frac{8}{15}\theta_\nu - \frac{3}{5}k\mathcal{F}_{\nu 3} + \frac{4}{15}\dot{h} + \frac{8}{5}\dot{\tilde{\eta}} - (1 - 3c^2_{\text{vis}})\left(\frac{8}{15}\theta_\nu + \frac{4}{15}\dot{h} + \frac{8}{5}\dot{\tilde{\eta}}\right) \ ,$$

$$\dot{\mathcal{F}}_{\nu\ell} = \frac{k}{2\ell+1}\left[\ell\mathcal{F}_{\nu(\ell-1)} - (\ell+1)\mathcal{F}_{\nu(\ell+1)}\right] \ , \quad \ell \geq 3 \ ,$$

$c^2_{\text{vis}} = 0 \quad c^2_{\text{eff}} < \frac{1}{3}$ → tightly coupled limit...?

!

- No proof.
- Accurate Treatment of interacting neutrinos includes calculation of the collision integral.

$$\left(\frac{\partial f_i}{\partial t}\right)_{\rm coll} = \frac{1}{2E(\boldsymbol{p})}\int \frac{\mathrm{d}^3\boldsymbol{n}}{(2\pi)^3 2E(\boldsymbol{n})}\int \frac{\mathrm{d}^3\boldsymbol{p}'}{(2\pi)^3 2E(\boldsymbol{p}')}\int \frac{\mathrm{d}^3\boldsymbol{n}'}{(2\pi)^3 2E(\boldsymbol{n}')}(2\pi)^4\delta_{\rm D}^{(4)}(p+n-p'-n')$$
$$\times |\mathcal{M}|^2 \left(f_k(p')f_l(n')\left[1\pm f_i(p)\right]\left[1\pm f_j(n)\right] - f_i(p)f_j(n)\left[1\pm f_k(p')\right]\left[1\pm f_l(n')\right]\right)$$

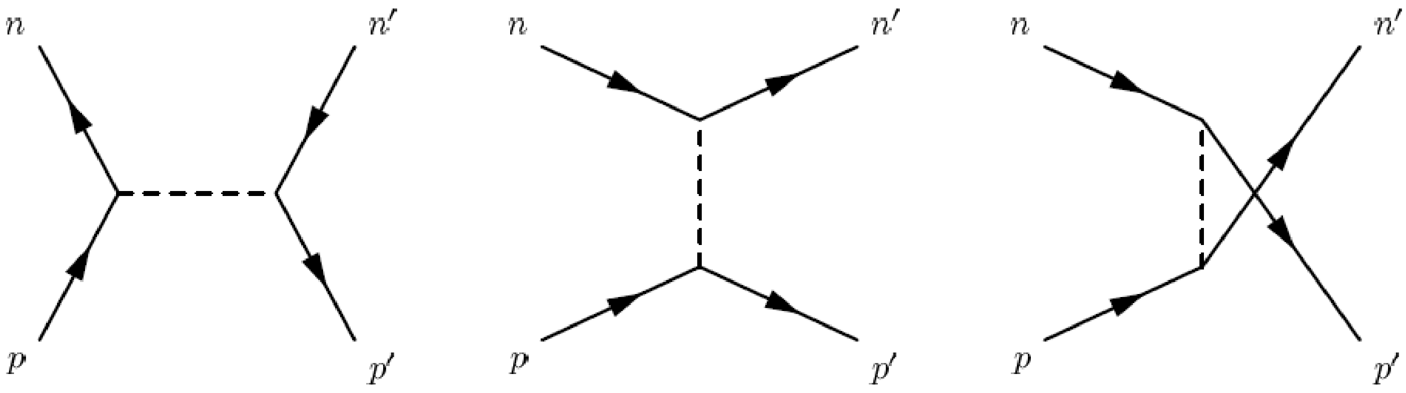
$$f(\boldsymbol{x},\boldsymbol{P},\eta) = \bar{f}(|\boldsymbol{q}|) + F(\boldsymbol{x},\boldsymbol{q},\eta)$$

difference to photon case: Thomson scattering = low energy transfer

Approximations:

- Neglect of quantum statistical effects → Boltzmann statistics $1-\bar{f}(q)\sim 1$
- Zero chemical potential
- Zero neutrino masses

Consider only neutrino self-interactions $\nu\nu \to \nu\nu$



Massless case: $|\mathcal{M}_0|^2 = \mathfrak{b}\mathfrak{g}^4$

→ recoupling

Very massive case: $|\mathcal{M}_\mathrm{m}|^2 = \frac{\mathfrak{c}\mathfrak{g}^4}{m_\phi^4}\left[s^2+t^2+u^2\right]$

→ delays time of free-streaming

arxiv: 1409.1577

$$\left(\frac{\partial f}{\partial \eta}\right)^{(1)}_{\mathrm{coll}}(\boldsymbol{k},\boldsymbol{q}) = \frac{1}{2|\boldsymbol{q}|(2\pi)^5}\int\frac{\mathrm{d}^3\boldsymbol{q}'}{2|\boldsymbol{q}'|}\int\frac{\mathrm{d}^3\boldsymbol{l}}{2|\boldsymbol{l}|}\int\frac{\mathrm{d}^3\boldsymbol{l}'}{2|\boldsymbol{l}'|}\,\delta_\mathrm{D}^{(4)}(q+l-q'-l')$$
$$\times\,|\mathcal{M}|^2\left[2\bar{f}(|\boldsymbol{l}'|)\,F(|\boldsymbol{k}|,\boldsymbol{q}') - \bar{f}(|\boldsymbol{q}|)\,F(|\boldsymbol{k}|,\boldsymbol{l}) - \bar{f}(|\boldsymbol{l}|)\,F(|\boldsymbol{k}|,\boldsymbol{q})\right]$$

$$\left(\frac{\partial f}{\partial \eta}\right)^{(1)}_{\rm coll} = -Y(|\boldsymbol{q}|)\,F(\boldsymbol{k},\boldsymbol{q}) + \int {\rm d}\cos\theta\,{\rm d}|\boldsymbol{q}'|\,Z(|\boldsymbol{q}|,|\boldsymbol{q}'|,\cos\theta)\,F(\boldsymbol{k},\boldsymbol{q}')$$

Massless case:

$$Y^0(|\boldsymbol{q}|) = \frac{\mathrm{N}\mathfrak{b}\mathfrak{g}^4}{8(2\pi)^3}\frac{T_{\nu,0}^2}{|\boldsymbol{q}|}$$

$$Z^0(|\boldsymbol{q}|,|\boldsymbol{q}'|,\cos\theta) = \frac{\mathrm{N}\mathfrak{b}\mathfrak{g}^4}{8(2\pi)^3}\frac{|\boldsymbol{q}'|}{|\boldsymbol{q}|}\left[T_{\nu,0}\frac{{\rm e}^{-\left(|\boldsymbol{q}|-|\boldsymbol{q}'|+\sqrt{|\boldsymbol{q}|^2-2|\boldsymbol{q}||\boldsymbol{q}'|\cos\theta+|\boldsymbol{q}'|^2}\right)/(2T_{\nu,0})}}{\sqrt{|\boldsymbol{q}|^2-2|\boldsymbol{q}||\boldsymbol{q}'|\cos\theta+|\boldsymbol{q}'|^2}} - \frac{1}{2}{\rm e}^{-|\boldsymbol{q}|/T_{\nu,0}}\right]$$

<u>Boltzmann hierarchy for interacting neutrinos:</u>

$$\dot{F}_0(q) = -kF_1(q) + \frac{1}{6}\frac{\partial\bar{f}}{\partial\ln q}\dot{h} - Y(q)\,F_0(q) + \int {\rm d}q'\,Z_0(q,q')\,F_0(q')\,,$$

$$\dot{F}_1(q) = -\frac{2}{3}kF_2(q) + \frac{1}{3}kF_0(q) - Y(q)\,F_1(q) + \int {\rm d}q'\,Z_1(q,q')\,F_1(q')\,,$$

$$\dot{F}_2(q) = -\frac{3}{5}kF_3(q) + \frac{2}{5}kF_1(q) - \frac{\partial\bar{f}}{\partial\ln q}\left(\frac{2}{5}\dot{\eta} + \frac{1}{15}\dot{h}\right) - Y(q)\,F_2(q) + \int {\rm d}q'\,Z_2(q,q')\,F_2(q')\,,$$

$$\dot{F}_\ell(q) = \frac{k}{2\ell+1}\left[\ell F_{\ell-1}(q) - (\ell+1)F_{\ell+1}(q)\right] - Y(q)\,F_\ell(q) + \int {\rm d}q'\,Z_\ell(q,q')\,F_\ell(q')\,,\quad \ell > 2\,,$$

$$\dot{F}_0(q) = -kF_1(q) + \frac{1}{6}\frac{\partial \bar{f}}{\partial \ln q}\dot{h} - Y(q)\,F_0(q) + \int \mathrm{d}q'\,Z_0(q,q')\,F_0(q')\,,$$
$$\dot{F}_1(q) = -\frac{2}{3}kF_2(q) + \frac{1}{3}kF_0(q) - Y(q)\,F_1(q) + \int \mathrm{d}q'\,Z_1(q,q')\,F_1(q')\,,$$
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$$\dot{F}_\ell(q) = \frac{k}{2\ell+1}\left[\ell F_{\ell-1}(q) - (\ell+1)F_{\ell+1}(q)\right] - Y(q)\,F_\ell(q) + \int \mathrm{d}q'\,Z_\ell(q,q')\,F_\ell(q')\,, \quad \ell > 2\,,$$

→ Momentum dependence reflects non-negligible energy transfer

Integrated over momentum space

$$\dot{\delta}_\nu = -\tfrac{4}{3}\theta_\nu - \tfrac{2}{3}\dot{h}$$
$$\dot{\theta}_\nu = \tfrac{1}{4}k^2(\delta_\nu - \sigma)$$
$$\dot{\sigma}_\nu = ???$$

→ $c^2_{\mathrm{eff}} = \frac{1}{3}$ but $c^2_{\mathrm{vis}} = ???$

→ **solve numerically!!!**

- Majoron Models: → mechanism of neutrino mass generation

  → non-standard neutrino interactions
- Literature Review: → tightly coupled limit

  → parametrization $(c^2_{\rm eff}, c^2_{\rm vis})$
- Full treatment: → calculation of collision integral of interacting neutrinos

  → Boltzmann hierarchy

  → $c^2_{\rm eff} = \frac{1}{3}$ $\quad c^2_{\rm vis} = ???$

<u>Outlook:</u>

- Implement resulting hierarchy in Boltzmann Code

  → cosmological imprint?

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  → non-standard neutrino interactions

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- Full treatment: → calculation of collision integral of interacting neutrinos

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  → $c_{\rm eff}^2 = \frac{1}{3} \quad c_{\rm vis}^2 = ???$

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Thank you for your attention!!!