Spacetime curvature and the Higgs stability during inflation

arXiv:1407.3141

Tommi Markkanen[1,2] Matti Herranen[3] Sami Nurmi[1,2]
Arttu Rajantie[4]

[1]University of Helsinki
[2]Helsinki Institute of Physics
[3]Niels Bohr International Academy, Copenhagen
[4]Blackett Laboratory, Imperial College, London

Particle Cosmology after Planck,
DESY Theory Workshop
2014

Outline

1. Introduction
2. Higgs stability during inflation (QFT in Minkowski)
3. Higgs stability during inflation (QFT in curved space)
4. Conclusions

1 Introduction

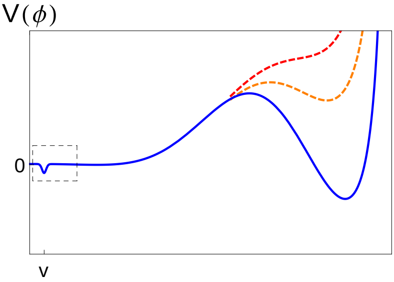
2 Higgs stability during inflation (QFT in Minkowski)

3 Higgs stability during inflation (QFT in curved space)

4 Conclusions

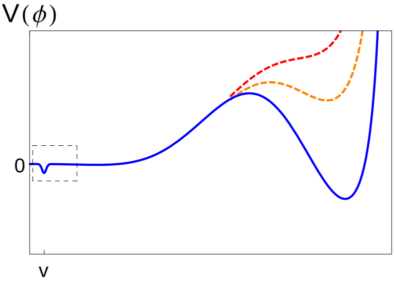
Standard Model Higgs potential

- $V(\phi)$ has a minimum at $\phi = v$
- Very sensitive to M_h and M_t
- A vacuum at $\phi \neq v$ incompatible with observations



Standard Model Higgs potential

- $V(\phi)$ has a minimum at $\phi = v$
- Very sensitive to M_h and M_t
- A vacuum at $\phi \neq v$ incompatible with observations

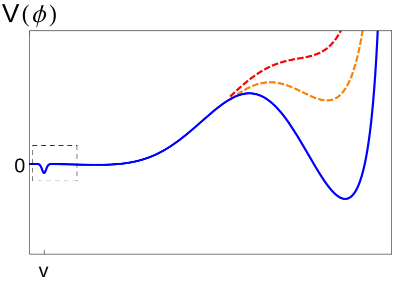


- *Meta* stable at 99% CL [1]
  - Lifetime much longer than 13.8×10^9 years
- Is this also true for the early Universe (inflation)?

[1] Buttazzo et al. (2013); Spencer-Smith (2014)

Standard Model Higgs potential

- $V(\phi)$ has a minimum at $\phi = v$
- Very sensitive to M_h and M_t
- A vacuum at $\phi \neq v$ incompatible with observations



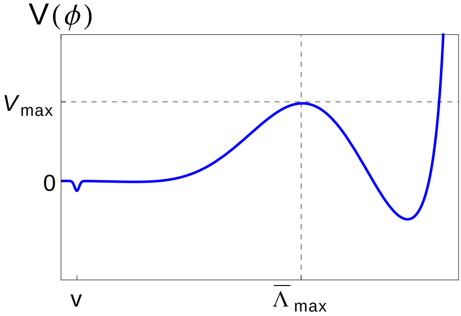
- *Meta* stable at 99% CL [1]
    - Lifetime much longer than 13.8×10^9 years
- Is this also true for the early Universe (inflation)?

New physics needed to stabilize the vacuum?

[1] Buttazzo et al. (2013); Spencer-Smith (2014)

Inflation and the Standard Model

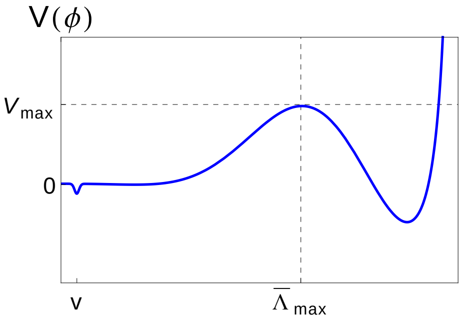
- In principle we can assume the SM to be valid
  - Energy-density is dominated by decoupled physics
- Inflation induces fluctuations to the Higgs field $\Delta\phi \sim H$
  - Important if $\overline{\Lambda}_{\rm max} \lesssim H$
  - State of the art calculations [2]: $\overline{\Lambda}_{\rm max} \sim 10^{11}{\rm GeV}$



[2] Degrazzi et. al.(2013); Buttazzo et. al. (2013)

Inflation and the Standard Model

- In principle we can assume the SM to be valid
  - Energy-density is dominated by decoupled physics
- Inflation induces fluctuations to the Higgs field $\Delta\phi \sim H$
  - Important if $\overline{\Lambda}_{\max} \lesssim H$
  - State of the art calculations [2]: $\overline{\Lambda}_{\max} \sim 10^{11}\mathrm{GeV}$



$r = \mathcal{P}_T / \mathcal{P}_{\mathcal{R}}$

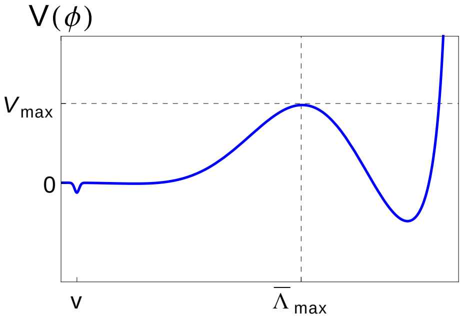
$\overline{\Lambda}_{\max} \sim H$

$\Rightarrow r \sim 2 \cdot 10^{-7}$

[2] Degrazzi et. al.(2013); Buttazzo et. al. (2013)

Inflation and the Standard Model

- In principle we can assume the SM to be valid
  - Energy-density is dominated by decoupled physics
- Inflation induces fluctuations to the Higgs field $\Delta\phi \sim H$
  - Important if $\overline{\Lambda}_{\rm max} \lesssim H$
  - State of the art calculations [2]: $\overline{\Lambda}_{\rm max} \sim 10^{11}\,\mathrm{GeV}$



$r = \mathcal{P}_T/\mathcal{P}_\mathcal{R}$

$\overline{\Lambda}_{\rm max} \sim H$

$\Rightarrow r \sim 2 \cdot 10^{-7}$

BICEP2:
$\overline{\Lambda}_{\rm max} \sim 10^{-3} H$

[2] Degrazzi et. al.(2013); Buttazzo et. al. (2013)

Outline

1. Introduction
2. Higgs stability during inflation (QFT in Minkowski)
3. Higgs stability during inflation (QFT in curved space)
4. Conclusions

Higgs stability during inflation

- Fluctuations induced by inflation may be treated as stochastic variables [3]

[3] Starobinsky (1986); Starobinsky & Yokoyama (1994)

Higgs stability during inflation

- Fluctuations induced by inflation may be treated as stochastic variables [3]
- Probability density $P(t,\phi)$ from the Fokker-Planck equation

$$\dot{P}(t,\phi) = \frac{1}{3H}\frac{\partial}{\partial\phi}\left[P(t,\phi)V_{\rm eff}(\phi)\right] + \frac{H^3}{8\pi^2}\frac{\partial^2}{\partial\phi^2}P(t,\phi)$$

[3] Starobinsky (1986); Starobinsky & Yokoyama (1994)

Higgs stability during inflation

- Fluctuations induced by inflation may be treated as stochastic variables [3]
- Probability density $P(t,\phi)$ from the Fokker-Planck equation

$$\dot{P}(t,\phi) = \frac{1}{3H}\frac{\partial}{\partial\phi}\left[P(t,\phi)V_{\rm eff}(\phi)\right] + \frac{H^3}{8\pi^2}\frac{\partial^2}{\partial\phi^2}P(t,\phi)$$

- Importantly, $V_{\rm eff}(\phi)$ is the renormalization group (RG) improved *effective potential*

[3] Starobinsky (1986); Starobinsky & Yokoyama (1994)

Renormalization group improvement

- The perturbative $V_{\rm eff}(\phi)$ suffers from large logarithms

Example: $V(\phi) = (1/2)m^2\phi^2 + (\lambda/4!)\phi^4$

$$V_{\rm eff}(\phi) = \frac{1}{2}m^2\phi^2 + \frac{\lambda}{4!}\phi^4 + \frac{\overbrace{M(\phi)^4}^{M(\phi)^2 \equiv m^2 + \frac{\lambda}{2}\phi^2}}{64\pi^2}\left[\log\left(\frac{M(\phi)^2}{\mu^2}\right) - \frac{3}{2}\right]$$

Renormalization group improvement

- The perturbative $V_{\text{eff}}(\phi)$ suffers from large logarithms

Example: $V(\phi) = (1/2)m^2\phi^2 + (\lambda/4!)\phi^4$

$$V_{\text{eff}}(\phi) = \frac{1}{2}m^2\phi^2 + \frac{\lambda}{4!}\phi^4 + \frac{\overbrace{M(\phi)^4}^{M(\phi)^2 \equiv m^2 + \frac{\lambda}{2}\phi^2}}{64\pi^2}\left[\log\left(\frac{M(\phi)^2}{\mu^2}\right) - \frac{3}{2}\right]$$

- The physical result must not depend on μ:

$$\frac{d}{d\mu}V_{\text{eff}}(\phi) = 0 \quad \Leftrightarrow \quad \left\{\mu\frac{\partial}{\partial\mu} + \beta_\lambda\frac{\partial}{\partial\lambda} + \gamma_\phi\phi\frac{\partial}{\partial\phi}\right\}V_{\text{eff}}(\phi) = 0$$

Renormalization group improvement

- The perturbative $V_{\text{eff}}(\phi)$ suffers from large logarithms

Example: $V(\phi) = (1/2)m^2\phi^2 + (\lambda/4!)\phi^4$

$$V_{\text{eff}}(\phi) = \frac{1}{2}m^2\phi^2 + \frac{\lambda}{4!}\phi^4 + \frac{\overbrace{M(\phi)^4}^{M(\phi)^2 \equiv m^2 + \frac{\lambda}{2}\phi^2}}{64\pi^2}\left[\log\left(\frac{M(\phi)^2}{\mu^2}\right) - \frac{3}{2}\right]$$

- The physical result must not depend on μ:

$$\frac{d}{d\mu}V_{\text{eff}}(\phi) = 0 \quad \Leftrightarrow \quad \left\{\mu\frac{\partial}{\partial\mu} + \beta_\lambda\frac{\partial}{\partial\lambda} + \gamma_\phi\phi\frac{\partial}{\partial\phi}\right\}V_{\text{eff}}(\phi) = 0$$

$\Rightarrow$ can be used to improve the perturbative result [4]

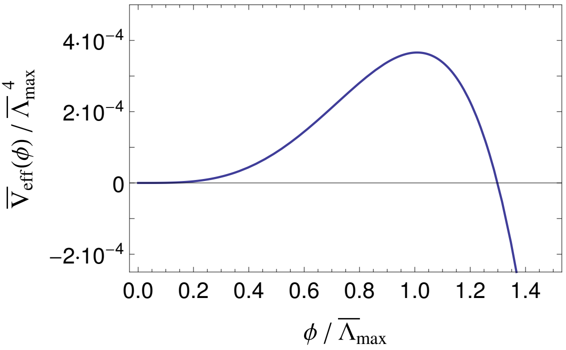
The optimal choice

$$\mu(t) \sim \phi(t)$$

$\Rightarrow$ No large logarithms!

[4] Ford et. al. (1993)

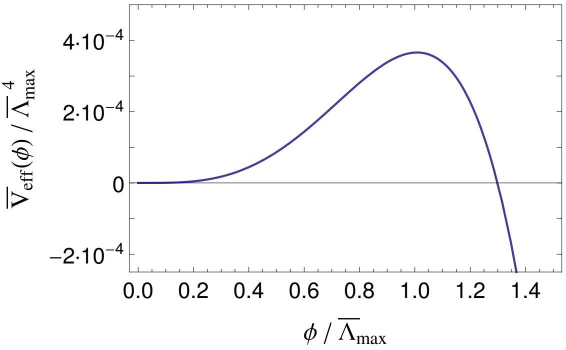
Stability results (Minkowski)



- For large H $(\sim 10^3\overline{\Lambda}_{\rm max})$, the SM is not stable [5]

[5] Kobakhidze & Spencer-Smith (2014); Hook et. al. (2014); Fairbairn & Hogan (2014); Enqvist, Meriniemi & Nurmi (2014)

Stability results (Minkowski)



- For large H $(\sim 10^3\overline{\Lambda}_{\rm max})$, the SM is not stable [5]

> Does including spacetime curvature in the quantum calculation change this?

[5] Kobakhidze & Spencer-Smith (2014); Hook et. al. (2014); Fairbairn & Hogan (2014); Enqvist, Meriniemi & Nurmi (2014)

Outline

1. Introduction
2. Higgs stability during inflation (QFT in Minkowski)
3. Higgs stability during inflation (QFT in curved space)
4. Conclusions

Curved space effective action

- What is usually meant by "curved space QFT"?
  - No graviton loops

$$Z[J, g^{\mu\nu}] = \int \mathcal{D}\varphi e^{\,iS[\varphi, g^{\mu\nu}] + i\int J\varphi}$$

- Loops also generate a term $\propto R\phi^2$

Curved space effective action

- What is usually meant by "curved space QFT"?
  - No graviton loops

$$Z[J, g^{\mu\nu}] = \int \mathcal{D}\varphi e^{\,iS[\varphi, g^{\mu\nu}] + i\int J\varphi}$$

- Loops also generate a term $\propto R\phi^2$
- We must solve Klein-Gordon equation in FRW space

$$\left[-\Box + m^2\right]\hat{\phi} = 0; \qquad \hat{\phi} = \int \frac{d^3k}{a(t)^{3/2}}\left[\hat{a}_{\mathbf{k}} u_{\mathbf{k}} + \hat{a}^{\dagger}_{\mathbf{k}} u^*_{\mathbf{k}}\right],$$

$$u_{\mathbf{k}} = \frac{1}{\sqrt{W}} e^{-i\int^t W dt'} e^{i\mathbf{k}\cdot\mathbf{x}}$$

Curved space effective action

- What is usually meant by "curved space QFT"?
  - No graviton loops

$$Z[J, g^{\mu\nu}] = \int \mathcal{D}\varphi e^{\,iS[\varphi, g^{\mu\nu}] + i\int J\varphi}$$

- Loops also generate a term $\propto R\phi^2$
- We must solve Klein-Gordon equation in FRW space

$$\left[-\Box + m^2\right]\hat{\phi} = 0; \qquad \hat{\phi} = \int \frac{d^3k}{a(t)^{3/2}}\left[\hat{a}_{\mathbf{k}} u_{\mathbf{k}} + \hat{a}^{\dagger}_{\mathbf{k}} u^{*}_{\mathbf{k}}\right],$$

$$u_{\mathbf{k}} = \frac{1}{\sqrt{W}} e^{-i\int^t W dt'} e^{i\mathbf{k}\cdot\mathbf{x}}$$

$$\Rightarrow \quad W^2 = \frac{k^2}{a(t)^2} + m^2 - \frac{R}{6} + \mathcal{O}(k^{-2})$$

Curved space effective action

Mass shift $\propto R$ (scalar curvature)

Curved space effective action

Mass shift $\propto$ R (scalar curvature)

- Works also for spinors and gauge fields [6]

[6] Our method coincides with Jack & Parker (1985)

Curved space effective action

Mass shift $\propto R$ (scalar curvature)

- Works also for spinors and gauge fields [6]
- Rederive $V_{\rm eff}(\phi)$ with curved space modes
  - Also include a non-minimal ξ-term [7]

[6] Our method coincides with Jack & Parker (1985)
[7] $\xi_{\rm EW} < 10^{15}$, Atkins & Calmet (2012)

Curved space effective action

Mass shift $\propto R$ (scalar curvature)

- Works also for spinors and gauge fields [6]
- Rederive $V_{\text{eff}}(\phi)$ with curved space modes
  - Also include a non-minimal ξ-term [7]

The optimal scale in curved space

$$\mu(t)^2 \sim \phi(t)^2 + R$$

[6] Our method coincides with Jack & Parker (1985)
[7] $\xi_{\text{EW}} < 10^{15}$, Atkins & Calmet (2012)

1-loop Effective potential in curved space

$$V_{\text{eff}}(\phi, R) = -\frac{1}{2}m^2(t)\phi(t)^2 + \frac{1}{2}\xi(t)R\phi(t)^2 + \frac{1}{4}\lambda(t)\phi(t)^4$$

$$+\sum_{i=1}^{9}\frac{n_i}{64\pi^2}M_i^4(t)\left[\log\frac{|M_i^2(t)|}{\mu^2(t)} - c_i\right] \qquad ; M_i^2(t) = \kappa_i\phi(t)^2 - \kappa_i' + \theta_i R$$

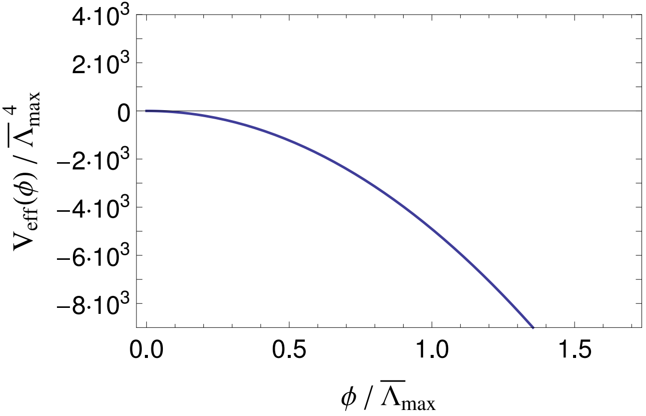
Φ	i	n_i	κ_i	κ_i'	θ_i	c_i
$W^\pm$	1	2	$g^2/4$	0	$1/12$	$3/2$
	2	6	$g^2/4$	0	$-1/6$	$5/6$
	3	-2	$g^2/4$	0	$-1/6$	$3/2$
Z^0	4	1	$(g^2+g'^2)/4$	0	$1/12$	$3/2$
	5	3	$(g^2+g'^2)/4$	0	$-1/6$	$5/6$
	6	-1	$(g^2+g'^2)/4$	0	$-1/6$	$3/2$
t	7	-12	$y_t^2/2$	0	$1/12$	$3/2$
ϕ	8	1	3λ	m^2	$\xi - 1/6$	$3/2$
χ_i	9	3	λ	m^2	$\xi - 1/6$	$3/2$

Stability results (curved space) I

- First attempt, set $\xi_{\rm EW} = 0$ and $H \sim 10^{10}\rm GeV$ ($\sim 10^3\overline{\Lambda}_{\rm max}$)

Stability results (curved space) I

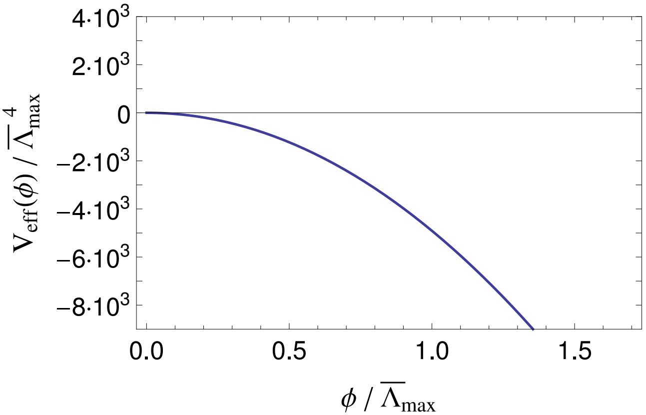
- First attempt, set $\xi_{\rm EW} = 0$ and $H \sim 10^{10}\text{GeV}$ ($\sim 10^3 \overline{\Lambda}_{\rm max}$)



- Potential negative *everywhere*

Stability results (curved space) I

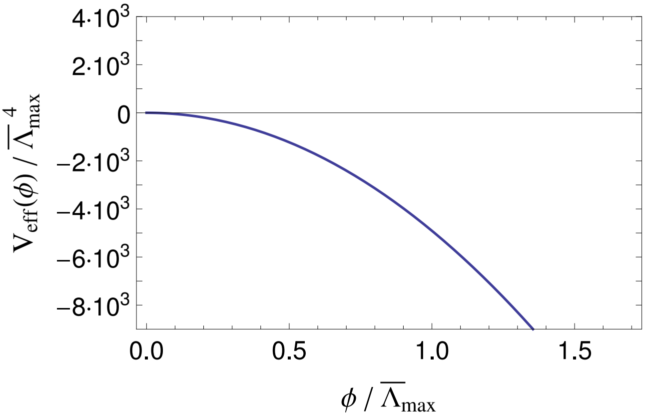
- First attempt, set $\xi_{\rm EW} = 0$ and $H \sim 10^{10}\text{GeV}$ ($\sim 10^3\overline{\Lambda}_{\rm max}$)



- Potential negative *everywhere*
  - In curved space $\lambda(\mu) < 0$ since $\mu^2 \sim \phi^2 + R$

Stability results (curved space) I

- First attempt, set $\xi_{\rm EW} = 0$ and $H \sim 10^{10}\mathrm{GeV}$ ($\sim 10^{3}\overline{\Lambda}_{\rm max}$)



- Potential negative *everywhere*
  - In curved space $\lambda(\mu) < 0$ since $\mu^2 \sim \phi^2 + R$
  - ξ Can become positive or negative depending on $\xi_{\rm EW}$

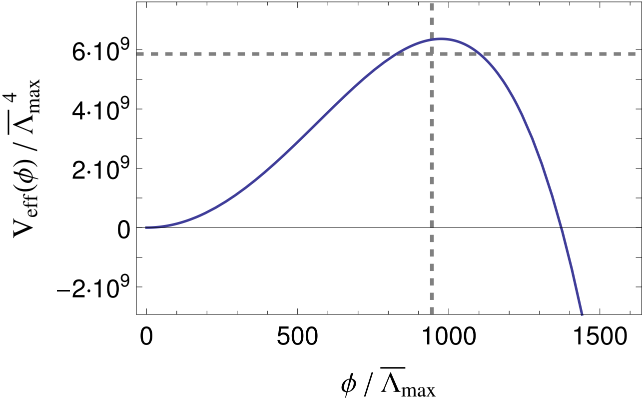
Stability results (curved space) II

- Now choosing $\xi_{\rm EW} = 0.1$ [8]

[8] Espinosa, Giudice & Riotto (2008)

Stability results (curved space) II

- Now choosing $\xi_{\rm EW} = 0.1$ [8]



$$\Lambda_{\rm max} \simeq H\left(\frac{12\xi}{|\lambda|}\right)^{1/2}$$

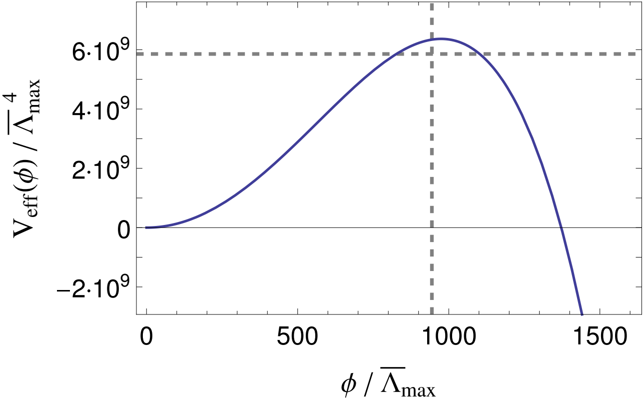
$$V_{\rm max}^{1/4} \simeq H\frac{(6\xi)^{1/2}}{|\lambda|^{1/4}}$$

- $V_{\rm max}(\text{curved}) \gg V_{\rm max}(\text{flat})$ (and at a higher scale)

[8] Espinosa, Giudice & Riotto (2008)

Stability results (curved space) II

- Now choosing $\xi_{\rm EW} = 0.1$ [8]



$$\Lambda_{\max} \simeq H \left(\frac{12\xi}{|\lambda|} \right)^{1/2}$$

$$V_{\max}^{1/4} \simeq H \frac{(6\xi)^{1/2}}{|\lambda|^{1/4}}$$

- $V_{\max}(\text{curved}) \gg V_{\max}(\text{flat})$ (and at a higher scale)

$$P \sim \exp\left[-8\pi^2 \left(V_{\max}/3H^4 \right) \right] \quad \Rightarrow \quad \text{Stable!}$$

[8] Espinosa, Giudice & Riotto (2008)

Outline

1. Introduction
2. Higgs stability during inflation (QFT in Minkowski)
3. Higgs stability during inflation (QFT in curved space)
4. Conclusions

Work for the future

- More loops
  - In flat space Λ_{max} changes by orders of magnitude
- Smaller H

Work for the future

- More loops
  - In flat space $\Lambda_{\rm max}$ changes by orders of magnitude
- Smaller H

Conclusions

- SM with $\xi_{EW} = 0$ is unstable during inflation for large H
- Curvature changes the quantum calculation significantly
- Having $\xi_{EW} \gtrsim 6 \times 10^{-2}$ stabilizes the vacuum

Work for the future

- More loops
  - In flat space $\Lambda_{\rm max}$ changes by orders of magnitude
- Smaller H

Conclusions

- SM with $\xi_{EW} = 0$ is unstable during inflation for large H
- Curvature changes the quantum calculation significantly
- Having $\xi_{EW} \gtrsim 6 \times 10^{-2}$ stabilizes the vacuum

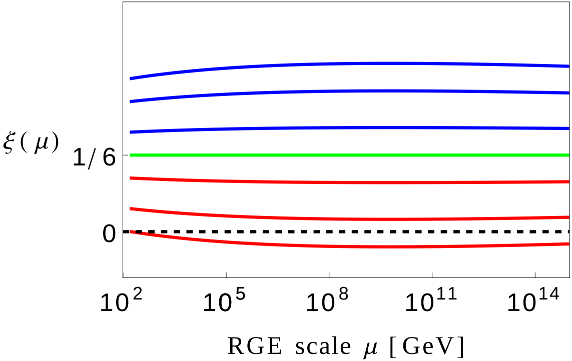
Thank You!

Outline

1. Introduction
2. Higgs stability during inflation (QFT in Minkowski)
3. Higgs stability during inflation (QFT in curved space)
4. Conclusions

Running of ξ

- ξ Can become positive or negative depending on $\xi_{\rm EW}$

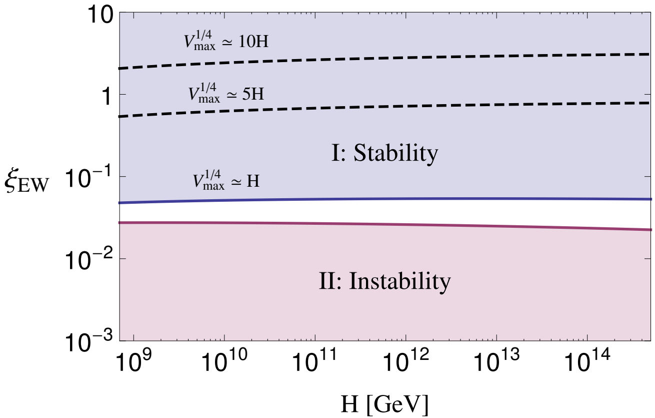


ξ_{EW}

0, 0.05, 0.12, 1/6,
0.22, 0.28, 0.33

Stability regions

- The (in)stability of the potential is determined by ξ_{EW}



Sensitivity to the choice of μ

- A loop calculation is never fully scale invariant
- How dependent is the result on the choice $\mu(t)^2 = \phi(t)^2 + R$?

$$\mu(t)^2 = \alpha\phi(t)^2 + \beta R \qquad \alpha, \beta \in \{0.1 \cdots 10\}$$