CP Violation from Finite Groups

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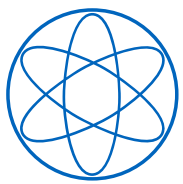
in collaboration with M.-C. Chen, K. T. Mahanthappa, M. Ratz and A. Trautner

Chen, MF, Mahanthappa, Ratz and Trautner, *Nucl. Phys.* B **883** (2014), arXiv: 1402.0507 [hep-ph].



Technische Universität München

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25 September 2014

DESY Theory Workshop 2014, Hamburg

CP violation and finite symmetries

I

Direct evidence of $\boldsymbol{CP}$ violation in flavour observables

$$\delta_{\boldsymbol{CP}} \neq 0,\ \pi$$

II

Powerful framework for flavour model building:

Finite non-abelian symmetries.

Eg: $\mathrm{A}_4, \mathrm{T}', \mathrm{S}_4, \Delta(27), \ldots$

Investigate interplay between CP and finite symmetries.

Discussion not limited to flavour symmetries.

Outline

1 Motivation

2 Generalised CP and finite groups

3 Three types of finite groups

4 Example

5 Conclusion

CP transformation

- Canonical $\boldsymbol{CP}$ transformation:

particle $\Leftrightarrow$ anti-particle

$$\vec{x} \Leftrightarrow -\vec{x}$$

$$\vec{p} \Leftrightarrow -\vec{p}$$

$$\phi(t, \vec{x}) \xmapsto{\boldsymbol{CP}} \eta_{\boldsymbol{CP}}\, \phi(t, -\vec{x})^*$$

Inconsistency of CP and finite symmetries?

Holthausen, Lindner and Schmidt (2013)
Feruglio, Hagedorn and Ziegler (2013)

- Setting: A_4 with x, y in $\mathbf{3}$, ϕ in $\mathbf{1}_2$ and $\omega = \mathrm{e}^{\frac{2\pi\mathrm{i}}{3}}$.

$$\Big[\phi_{\mathbf{1}_2} \otimes (x_{\mathbf{3}} \otimes y_{\mathbf{3}})_{\mathbf{1}_1}\Big]_{\mathbf{1}_0} \propto \phi \left(x_1\, y_1 + \omega^2\, x_2\, y_2 + \omega\, x_3\, y_3\right)$$

- Canonical $\boldsymbol{CP}$:

$$\Big[\phi_{\mathbf{1}_2} \otimes (x_{\mathbf{3}} \otimes y_{\mathbf{3}})_{\mathbf{1}_1}\Big]_{\mathbf{1}_0} \xmapsto{\boldsymbol{CP}} \phi^* \left(x_1^*\, y_1^* + \omega^2\, x_2^*\, y_2^* + \omega\, x_3^*\, y_3^*\right)$$

Not A_4 invariant.

- Naive reason: complex Clebsch–Gordan coefficients.

CP transformation

- Canonical $\boldsymbol{CP}$ transformation:

$$\text{particle} \Leftrightarrow \text{anti-particle}$$

$$\vec{x} \Leftrightarrow -\vec{x}$$

$$\vec{p} \Leftrightarrow -\vec{p}$$

$$\phi(t,\vec{x}) \xmapsto{\boldsymbol{CP}} \eta_{\boldsymbol{CP}}\, \phi(t,-\vec{x})^*$$

- Fields are multiplets:

Ecker, Grimus and Konetschny (1981)
Branco, Gerard and Grimus (1984)

$$\Phi = (\phi_1, \ldots, \phi_n).$$

- Generalised $\boldsymbol{CP}$ transformation with unitary matrix U_{CP}:

$$\Phi(t,\vec{x}) \xmapsto{\widetilde{\boldsymbol{CP}}} U_{CP}\, \Phi(t,-\vec{x})^*$$

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Not A_4 invariant.

- Naive reason: complex Clebsch–Gordan coefficients.
- Generalised $\boldsymbol{CP}$: Exchange second and third components: $2 \leftrightarrow 3$.

$$\left[\phi_{\mathbf{1}_2} \otimes (x_{\mathbf{3}} \otimes y_{\mathbf{3}})_{\mathbf{1}_1}\right]_{\mathbf{1}_0} \xmapsto{\widetilde{\boldsymbol{CP}}} \phi^* \left(x_1^*\, y_1^* + \omega^2\, x_3^*\, y_3^* + \omega\, x_2^*\, y_2^*\right)$$

Generalised CP (?) transformations

Holthausen, Lindner and Schmidt (2013)
Feruglio, Hagedorn and Ziegler (2013)

Generalised CP

$$\Phi(t, \vec{x}) \xmapsto{\widetilde{CP}} U_{CP}\, \Phi(t, -\vec{x})^*$$

Finite group G

$$\Phi(x) \xmapsto{G} \rho(g)\, \Phi(x)$$

Consistency condition

Consistent if there is an automorphism u of G such that

$$\rho(u(g)) = U_{CP}\, \rho(g)^*\, {U_{CP}}^{-1}\,, \qquad \forall\, g \in G.$$

Does every automorphism yield a CP transformation?

Problems with this prescription

Holthausen, Lindner and Schmidt (2013)

- In the original proposal:

$$\Phi \text{ actually } (\Phi, \Phi^*)\,.$$

- Solutions to consistency condition include

$$\Phi \xmapsto{\widetilde{CP}} U_{CP}\,\Phi\,.$$

- No conjugation.

  $\Rightarrow$ No inversion of quantum numbers.

  $\Rightarrow$ No connection to δ_{CP} or baryogenesis.

Generalised CP transformations

Chen, MF, Mahanthappa, Ratz and Trautner (2014)

Generalised CP

$$\phi_i(t,\vec{x}) \xmapsto{\widetilde{CP}} U_i\,\phi_i(t,-\vec{x})^*\,,$$

$$U_i = U(\boldsymbol{R}_i)$$

Finite group G

$$\phi_i(x) \xmapsto{G} \rho_i(g)\,\phi_i(x)\,,$$

ϕ_i in $\boldsymbol{R}_i$ irreducible

Consistency condition

Consistent if and only if
there is a class-inverting automorphism u of G such that

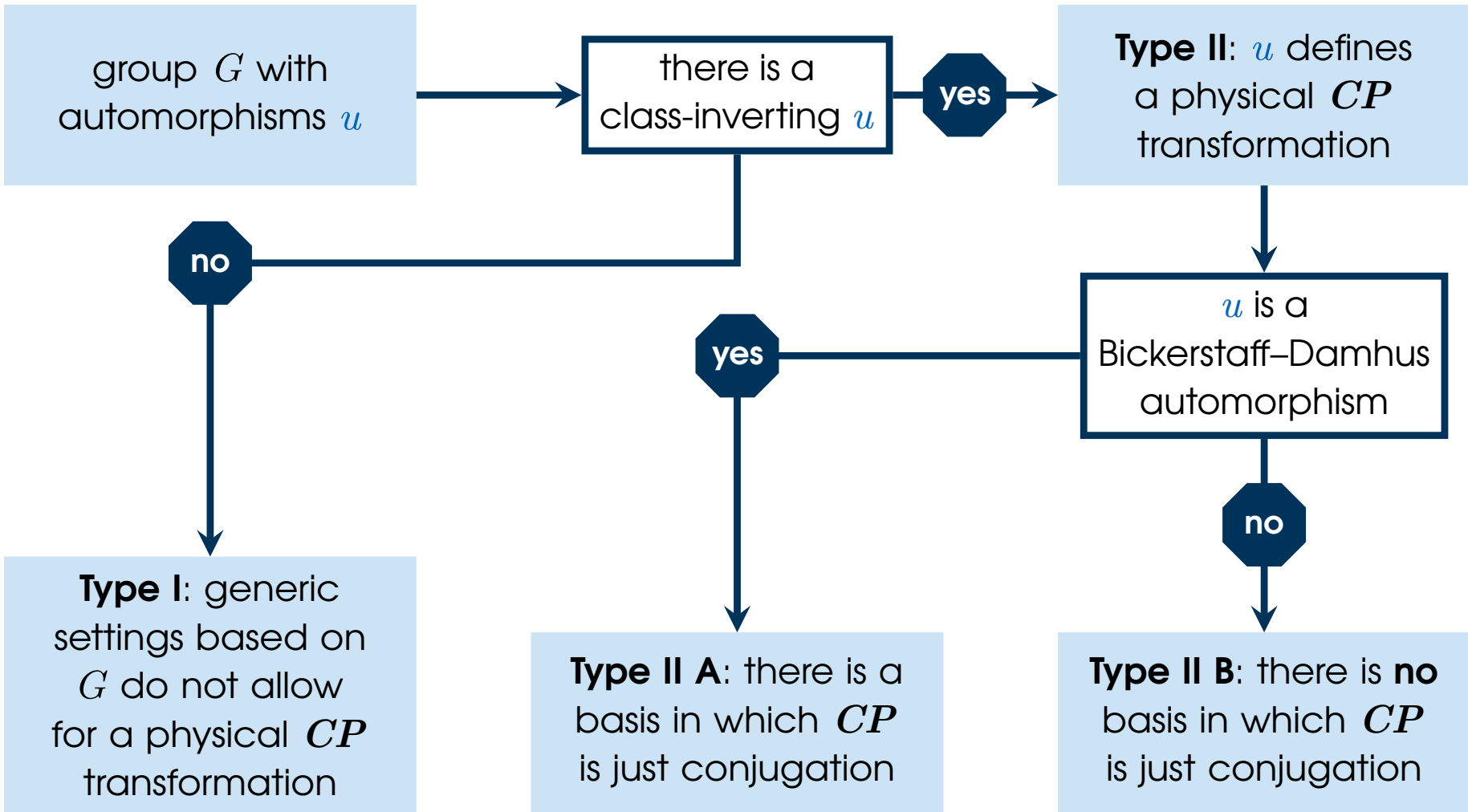
$$\rho_i(u(g)) = U_i\,\rho_i(g)^*\,U_i^{-1}\,, \qquad \forall\, g \in G,\ i \in \boldsymbol{R}_i.$$

class-inverting = sends each element to the conjugacy class of its inverse element

Three types of finite groups

Do all finite groups G have a proper $\boldsymbol{CP}$ transformation?

Three types of finite groups



Examples for the three types of finite groups

SG=SmallGroup ID of GAP

- **Type I: No CP in generic settings**

G	$\mathbb{Z}_5 \rtimes \mathbb{Z}_4$	T_7	$\Delta(27)$	$\mathbb{Z}_9 \rtimes \mathbb{Z}_3$
SG	(20,3)	(21,1)	(27,3)	(27,4)

$+$ (almost?) all odd order, non-abelian groups

- **Type II A: CP in all settings**

G	Q_8	A_4	$\mathbb{Z}_3 \rtimes \mathbb{Z}_8$	T'	S_4	A_5
SG	(8,4)	(12,3)	(24,1)	(24,3)	(24,12)	(60,5)

- **Type II B: CP in all settings but with additional parity**

G	$\Sigma(72)$	$((\mathbb{Z}_3 \times \mathbb{Z}_3) \rtimes \mathbb{Z}_4) \rtimes \mathbb{Z}_4$
SG	(72,41)	(144,120)

Example for type I: $\Delta(27)$

Type I

No class-inverting automorphism

$=$

No physical CP transformation in generic settings

Example for type I: $\Delta(27)$

- Eleven irreducible representations

$$\mathbf{1}_0 \text{ to } \mathbf{1}_8\,,\ \mathbf{3} \text{ and } \mathbf{3}^*\,.$$

	S	X	Y	Ψ		Σ
$\Delta(27)$	$\mathbf{1}_0$	$\mathbf{1}_1$	$\mathbf{1}_3$	$\mathbf{3}$		$\mathbf{3}$
$\mathrm{U}(1)$	$q_\Psi - q_\Sigma$	$q_\Psi - q_\Sigma$	0	q_Ψ	$\neq$	q_Σ

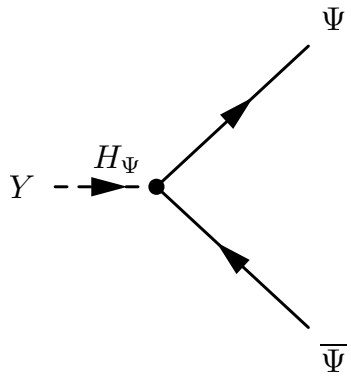
$$\mathcal{L} \supset g_S \left[S_{\mathbf{1}_0} \otimes \left(\overline{\Psi} \otimes \Sigma\right)_{\mathbf{1}_0}\right]_{\mathbf{1}_0} + g_X \left[X_{\mathbf{1}_1} \otimes \left(\overline{\Psi} \otimes \Sigma\right)_{\mathbf{1}_2}\right]_{\mathbf{1}_0} +$$
$$+ h_\Psi \left[Y_{\mathbf{1}_3} \otimes \left(\overline{\Psi} \otimes \Psi\right)_{\mathbf{1}_6}\right]_{\mathbf{1}_0} + h_\Sigma \left[Y_{\mathbf{1}_3} \otimes \left(\overline{\Sigma} \otimes \Sigma\right)_{\mathbf{1}_6}\right]_{\mathbf{1}_0} + \text{h. c.}$$

Y decay

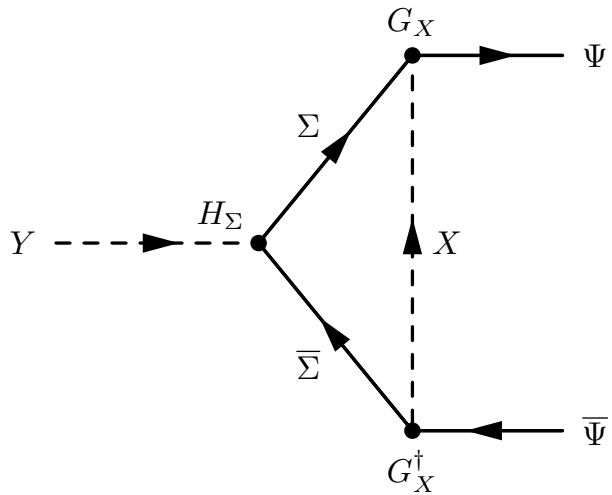
CP asymmetry

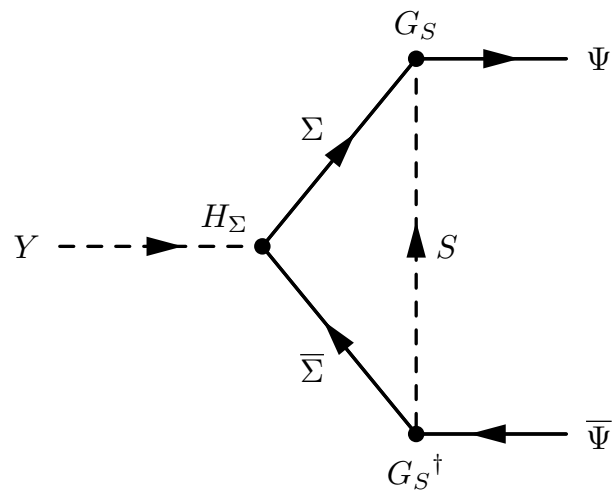
$$\varepsilon_{Y\to\overline{\Psi}\Psi} = \frac{\Gamma(Y\to\overline{\Psi}\Psi) - \Gamma(Y^*\to\overline{\Psi}\Psi)}{\Gamma(Y\to\overline{\Psi}\Psi) + \Gamma(Y^*\to\overline{\Psi}\Psi)}\,.$$

- Tree-level:



- Loop-level:





Y decay

CP asymmetry

$$\varepsilon_{Y\to\overline{\Psi}\Psi} = |g_S|^2\ \mathrm{Im}\left(I_S\right)\ \mathrm{Im}\left(h_\Psi\, h_\Sigma^*\right) + |g_X|^2\ \mathrm{Im}\left(I_X\right)\ \mathrm{Im}\left(\omega\, h_\Psi\, h_\Sigma^*\right).$$

- $I_{S/X}$ are loop integrals, $\omega = \mathrm{e}^{\frac{2\pi\mathrm{i}}{3}}$.
- Invariant under re-phasing of fields.
- Two possibilities for vanishing asymmetry:

1

$\boldsymbol{CP}$ trafo relating S and X.

2

Tuning of the couplings.

Y decay

CP asymmetry

$$\varepsilon_{Y\to\overline{\Psi}\Psi} = |g_S|^2\ \mathrm{Im}\left(I_S\right)\ \mathrm{Im}\left(h_\Psi\, h_\Sigma^*\right) + |g_X|^2\ \mathrm{Im}\left(I_X\right)\ \mathrm{Im}\left(\omega\, h_\Psi\, h_\Sigma^*\right).$$

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1

~~CP trafo relating S and X.~~

2

Tuning of the couplings.

No automorphism
can relate $\mathbf{1}_0$ and $\mathbf{1}_1$.

Y decay

CP asymmetry

$$\varepsilon_{Y\to\overline{\Psi}\Psi} = |g_S|^2\ \mathrm{Im}\left(I_S\right)\ \mathrm{Im}\left(h_\Psi\, h_\Sigma^*\right) + |g_X|^2\ \mathrm{Im}\left(I_X\right)\ \mathrm{Im}\left(\omega\, h_\Psi\, h_\Sigma^*\right).$$

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- Invariant under re-phasing of fields.
- Two possibilities for vanishing asymmetry:

1

~~CP trafo relating S and X.~~

No automorphism can relate $\mathbf{1}_0$ and $\mathbf{1}_1$.

2

~~Tuning of the couplings.~~

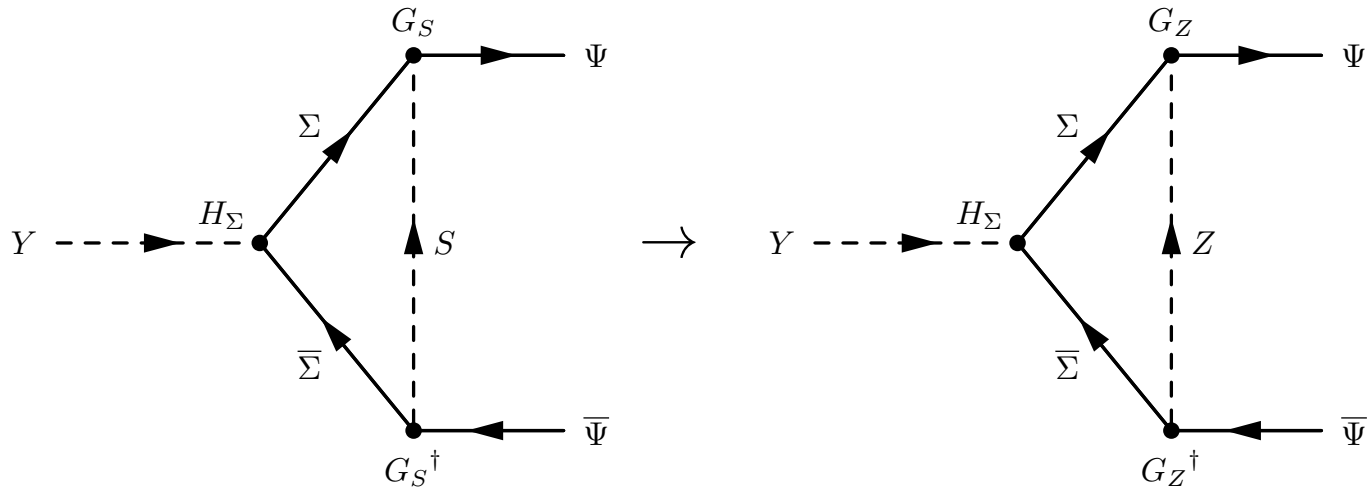
Not RGE stable.

The model violates CP.

Amended toy model

- Replace S (in $\mathbf{1}_0$) with Z in $\mathbf{1}_8$:

$$\mathcal{L} \supset g_Z \left[Z_{\mathbf{1}_8} \otimes \left(\overline{\Psi} \otimes \Sigma \right)_{\mathbf{1}_4} \right]_{\mathbf{1}_0} + g_X \left[X_{\mathbf{1}_1} \otimes \left(\overline{\Psi} \otimes \Sigma \right)_{\mathbf{1}_2} \right]_{\mathbf{1}_0} + \\ + h_\Psi \left[Y_{\mathbf{1}_3} \otimes \left(\overline{\Psi} \otimes \Psi \right)_{\mathbf{1}_6} \right]_{\mathbf{1}_0} + h_\Sigma \left[Y_{\mathbf{1}_3} \otimes \left(\overline{\Sigma} \otimes \Sigma \right)_{\mathbf{1}_6} \right]_{\mathbf{1}_0} + \text{h. c.}$$



& enlarge the flavour symmetry to $\Delta(27) \rtimes \mathbb{Z}_2$.

Calculable phases

- $\Delta(27) \rtimes \mathbb{Z}_2$ has a physical $\boldsymbol{CP}$ transformation (type II A).

$$\Rightarrow \boldsymbol{CP} \text{ conserved.}$$

- Break $\Delta(27) \rtimes \mathbb{Z}_2 \to \Delta(27)$ spontaneously.

$$\Rightarrow \text{Spontaneous } \boldsymbol{CP} \text{ violation.}$$

Phase predicted by group theory

$$\varepsilon_{Y \to \overline{\Psi}\Psi} \propto |g_X|^2 \, |h_\Psi|^2 \, \mathrm{Im}\,(\omega) \, [\mathrm{Im}\,(I_X) - \mathrm{Im}\,(I_Z)]$$

Conclusion

- Only class-inverting automorphisms define physical CP transformations.
- Three types of groups:

Type I	Type II A	Type II B
■ No physical CP transformation in a generic setting	■ Physical CP transformation in any setting ■ Canonical CP basis	■ Physical CP transformation in any setting ■ No canonical CP basis

Spontaneous CP violation with group theoretical phases.

Thank You!

Bibliography I

1. G. Branco, J.-M. Gerard and W. Grimus, 'Geometrical T-violation', *Phys. Lett.* B **136** (1984), 383–386, (inSPIRE).
2. M.-C. Chen, M. Fallbacher, K. T. Mahanthappa, M. Ratz and A. Trautner, 'CP violation from finite groups', *Nucl. Phys.* B **883** (2014), 267–305, arXiv: 1402.0507 [hep-ph], (inSPIRE).
3. G. Ecker, W. Grimus and W. Konetschny, 'Quark Mass Matrices in Left-right Symmetric Gauge Theories', *Nucl. Phys.* B **191** (1981), 465, (inSPIRE).
4. F. Feruglio, C. Hagedorn and R. Ziegler, 'Lepton mixing parameters from discrete and CP symmetries', *JHEP* **2013** (2013), 027, arXiv: 1211.5560 [hep-ph], (inSPIRE).
5. M. Holthausen, M. Lindner and M. A. Schmidt, 'CP and discrete flavour symmetries', *JHEP* **2013** (2013), 122, arXiv: 1211.6953 [hep-ph], (inSPIRE).