Spontaneous CP violation from discrete symmetries

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DESY Seminar

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Outline

- **Introduction**
- The quark sector and strong CP problem
- A non-trivial case: T'
- Summary and Conclusions

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  - CP and discrete symmetries
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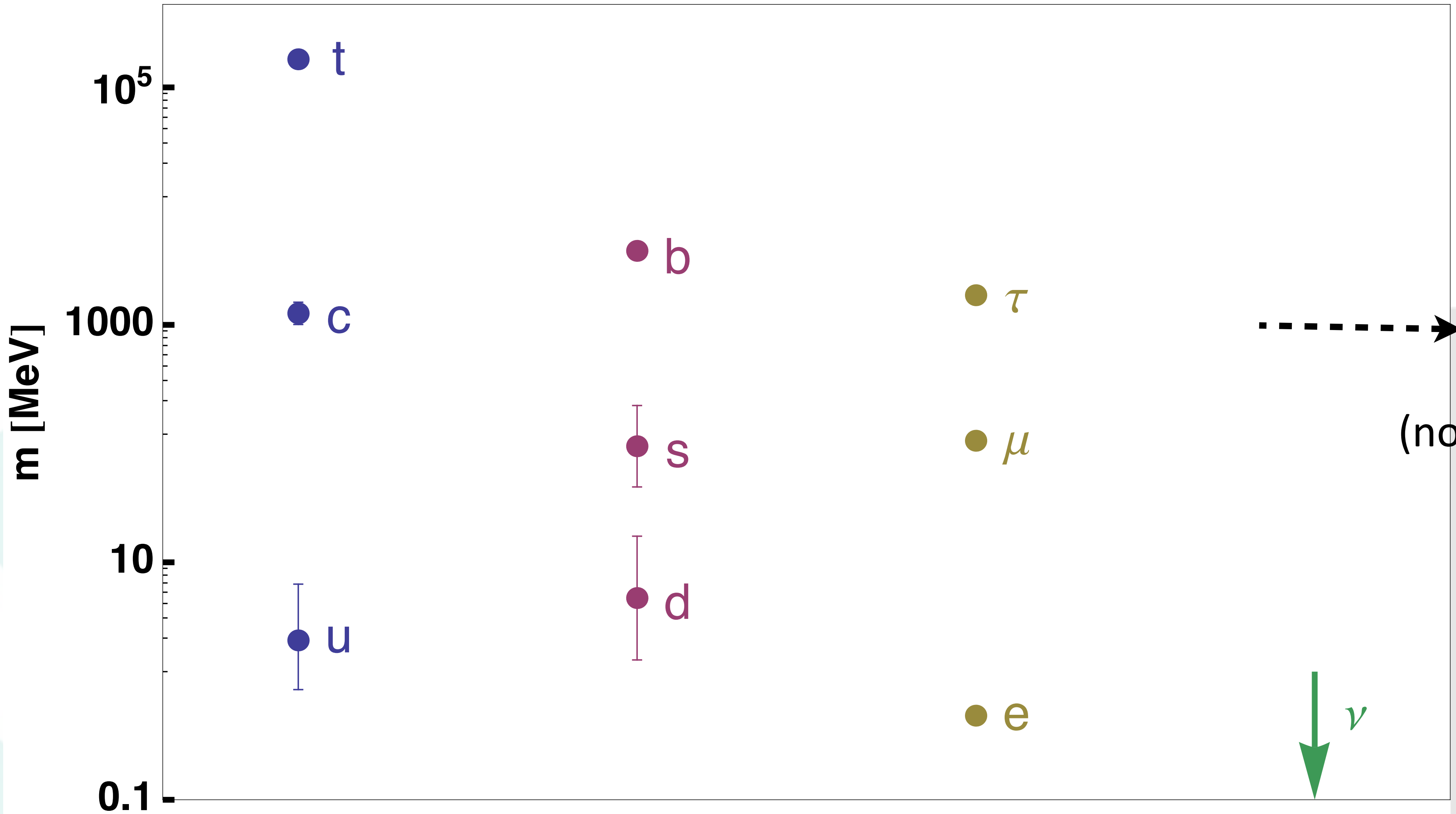
The Flavour Puzzle I

The flavour sector has many (13+7+2) parameters with peculiar patterns:



GUTs!

(not part of this talk)

Fermion masses (errors are from 2009 and scaled by a factor of 3)

The Flavour Puzzle II

	Quarks [PDG '08]	Leptons [NuFIT 1.2, NO]
θ_{12} in $^\circ$	12.9	34.0
θ_{23} in $^\circ$	2.4	41.8
θ_{13} in $^\circ$	0.2	9.0
δ in $^\circ$	68.9	(270)
α_1 in $^\circ$	-	?
α_2 in $^\circ$	-	?

Mixing parameters

Great Recent Progress in the Lepton Sector

	March 2011, NH	Jun 2012, NH
$\sin^2\theta_{12}$	$0.312^{+0.017}_{-0.015}$	$0.307^{+0.018}_{-0.016}$
$\sin^2\theta_{23}$	0.51 ± 0.06	$0.386^{+0.024}_{-0.021}$
$\sin^2\theta_{13}$	$0.010^{+0.009}_{-0.006}$	0.0241 ± 0.0025
δ	?	$(1.08^{+0.28}_{-0.31})\pi$

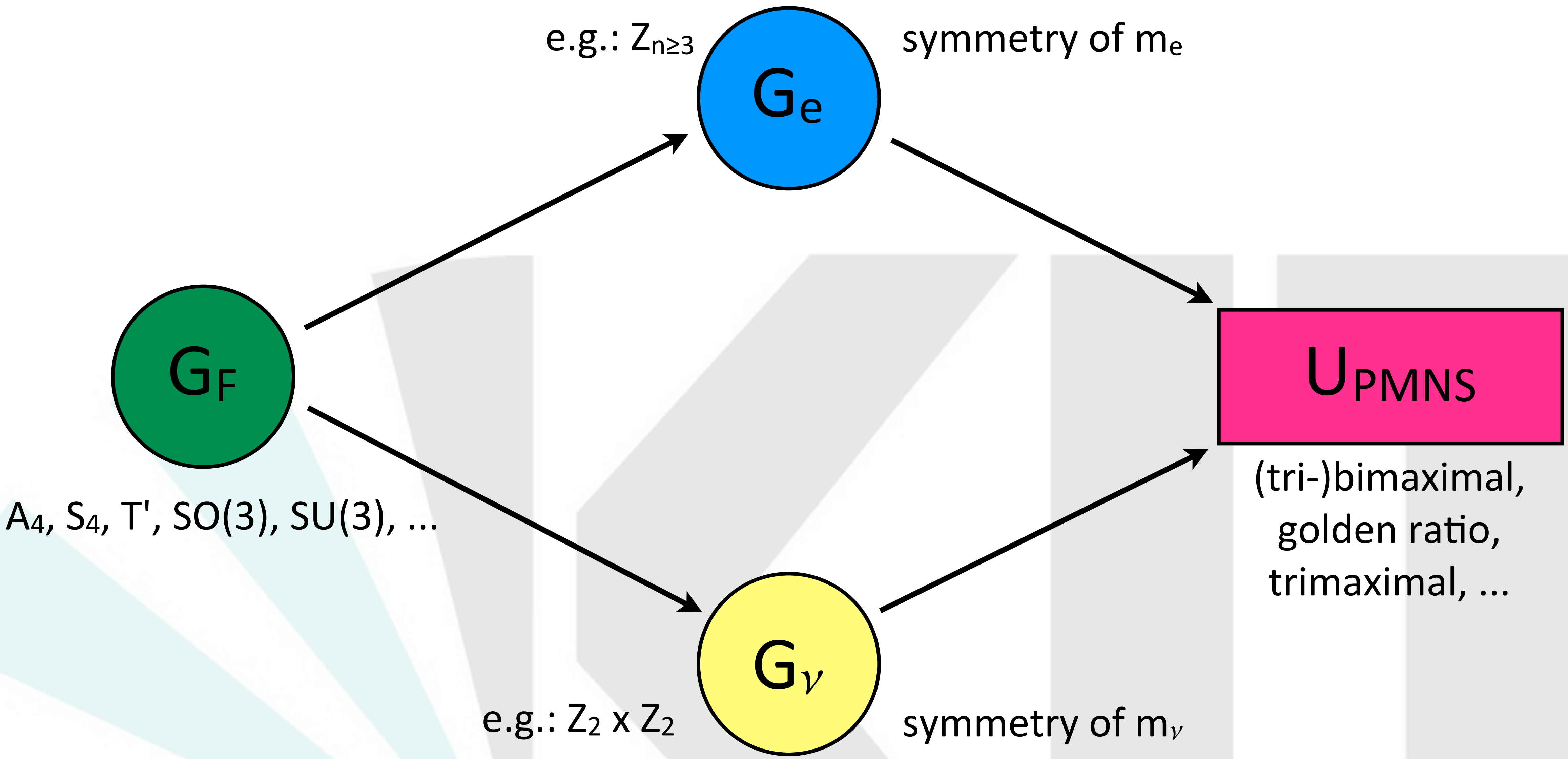
[Schwetz, Tortola, Valle 2011] **[Fogli, Lisi *et al.* 2012]**

The Flavour Puzzle

- Why do we have three generations?
- Why are the SM fermion masses so vastly different?
- What is the origin of CP violation?
- Why are the quark mixing angles rather small and the leptonic mixing angles rather large?

Non-Abelian (discrete) family symmetries

[for a recent review see King, Luhn 2013]



The Strong CP Problem

- QCD violates CP via

$$\mathcal{L} \supset \frac{\bar{\theta}}{32\pi^2} \tilde{G}_{\mu\nu} G^{\mu\nu}$$

- This term has two contributions

$$\bar{\theta} = \theta + \arg\det(M_u M_d)$$

- Experiment tells us

$$\bar{\theta} \lesssim 10^{-11}$$

[PDG '13]

- Why is the cancellation so perfect?

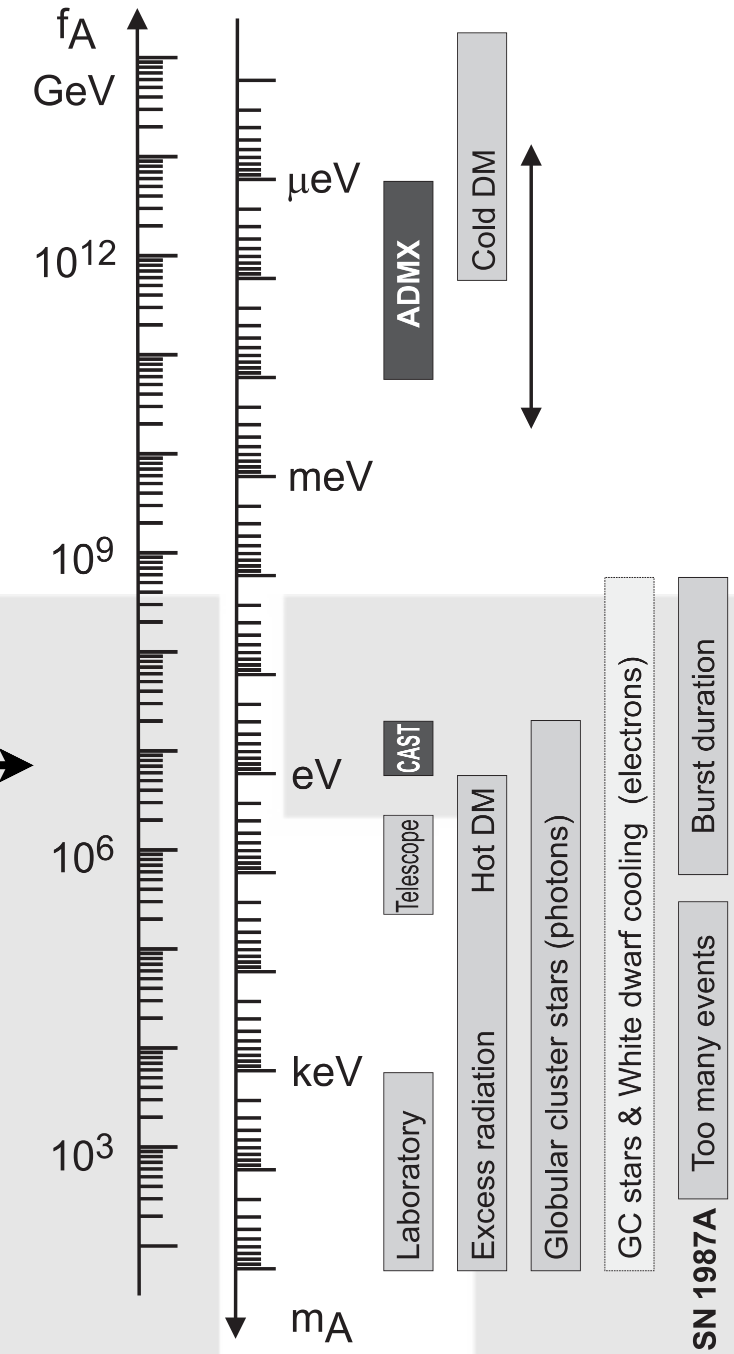
3 Popular Solutions

1. Massless up-quark, but:

$$m_u(2\ \text{GeV}) = 2.3^{+0.7}_{-0.5}\ \text{MeV}$$

[PDG '13]

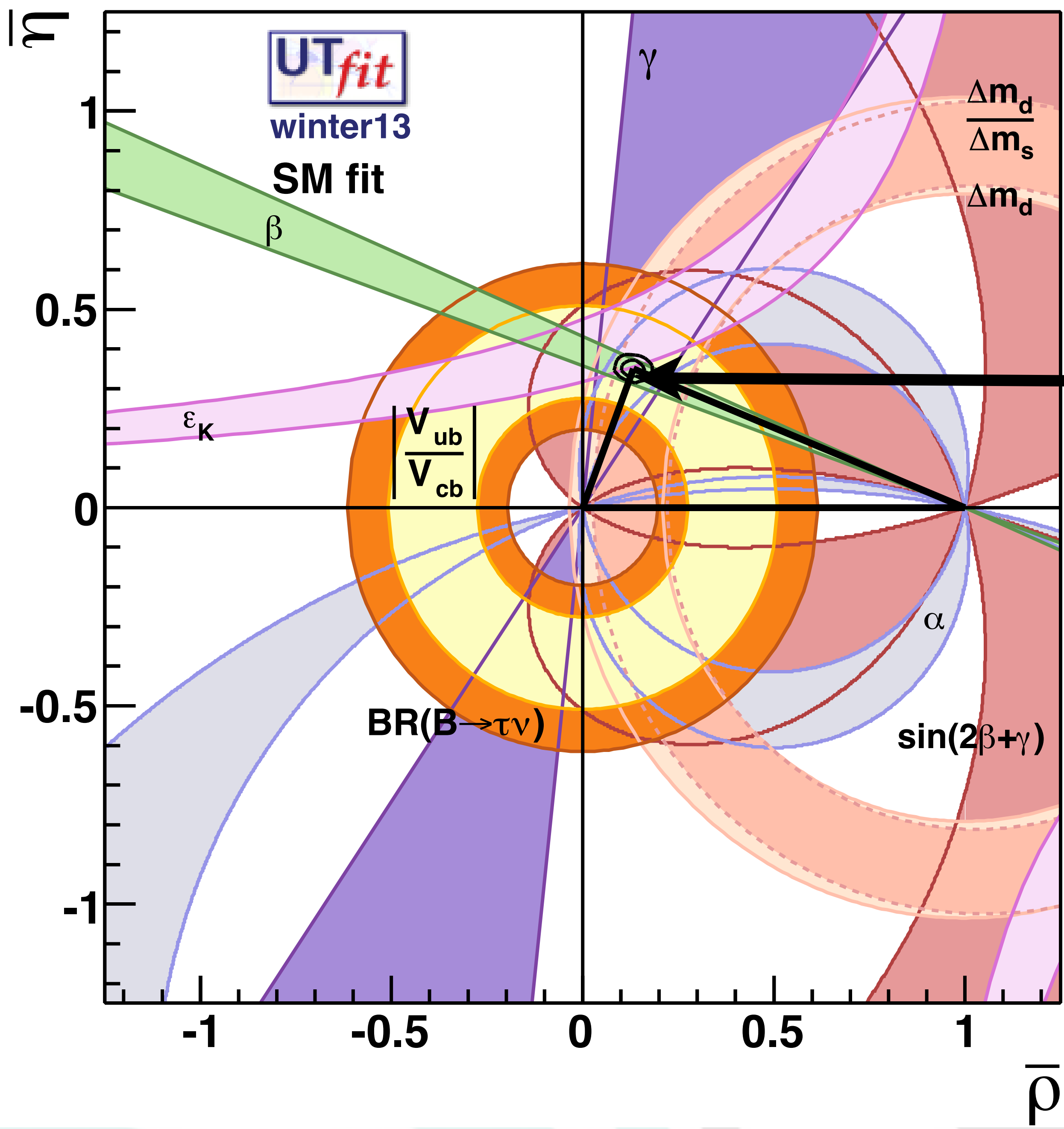
2. Axion solution →



[Plot taken from the PDG Axion Review '13]

3. Spontaneous CP violation:
   Topic of this talk

The CKM Unitarity Triangle



$$(V_{\rm CKM}^\dagger V_{\rm CKM})_{bd} = 0$$

Fit result:

$$\alpha = (88.7 \pm 3.1)^\circ$$

[UTFit Winter '13 SM Fit]

Accident or sign of spontaneous CP violation?

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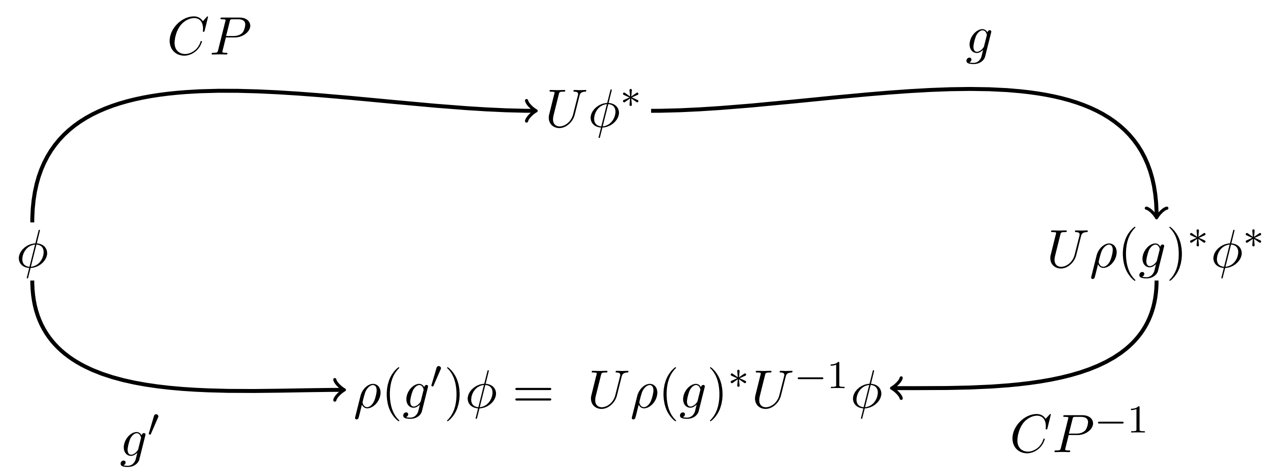
Generalised CP I

- In the Standard Model CP commutes with internal symmetries
- This is in general not the case for arbitrary symmetry groups
- First discovered in the 80's, recently regained attention

[Ecker, Grimus, Neufeld '87, '88; ... Feruglio, Hagedorn, Ziegler '12; Holthausen, Lindner, Schmidt '12; Chen, Fallbacher, Mahanthappa, Ratz, Trautner '14]

Consistency condition

[following the discussion in Holthausen, Lindner, Schmidt '12]



- A field transforms as

$$\phi \xrightarrow{G} \rho(g)\phi$$

$$\phi \xrightarrow{\text{CP}} U\phi^*$$

- Implying the consistency condition

$$U\rho(g)^*U^{-1} = \rho(u(g))\ , u \in \text{Out}(G)$$

In terms of mass matrices

- For the charged leptons

$$\rho^\dagger(g_{e_i})\, M_e^\dagger\, M_e\, \rho(g_{e_i}) = M_e^\dagger\, M_e \text{ with } g_{e_i} \in G_e < T' \rtimes H_{\text{CP}} \;,$$

$$U_{e,\text{CP}}^\dagger\, \left(M_e^\dagger\, M_e\right) U_{e,\text{CP}} = \left(M_e^\dagger\, M_e\right)^* \text{ with } U_{e,\text{CP}} \in G_e < T' \rtimes H_{\text{CP}} \;.$$

- For the (Majorana) neutrinos

$$\rho^T(g_{\nu_i})\, M_\nu\, \rho(g_{\nu_i}) = M_\nu \text{ with } g_{\nu_i} \in G_\nu < T' \rtimes H_{\text{CP}} \;,$$

$$U_{\nu,\text{CP}}^T\, M_\nu\, U_{\nu,\text{CP}} = M_\nu^* \text{ with } U_{\nu,\text{CP}} \in G_\nu < T' \rtimes H_{\text{CP}} \;.$$

Generalised CP II

[following the discussion in Holthausen, Lindner, Schmidt '12]

- A CP symmetry is an outer automorphism of the other symmetries
- Some examples:
  - For continuous symmetries outer automorphisms trivial or Z_2 apart from SO(8) with Out(SO(8)) = S_3
  - Out(A_4) = Z_2 (interchanges complex 1-dim. irreps)
  - Out(T') = Z_2 (will be discussed later)
  - Out($\Delta(27)$) = GL(2, 3)

Some recent approaches to Family Symmetries and CP

[taken partially from Hagedorn (BeNe 2012)]

- $\mu\tau$ reflection symmetry [Harrison, Scott '02, '04; Grimus, Lavoura '03; also Babu, Ma, Valle '02]
- triχmaximal mixing [Harrison, Scott '02, '04; Harrison (ICHEP12)]
- SU(3) x CP [Ross, Velasco-Sevilla, Vives '04]
- $\Delta(27)$ [Ferreira et al. '12]/$\Delta(6\ n^2)$ [King *et al*.]
- S_4 with CP [Mohapatra, Nishi '12]
- Geometrical CP violation [Branco et al. '83; recently revived by de Medeiros Varzielas et al.]
- T' with "Geometrical CP violation" [Chen, Mahanthappa '09, ...; Meroni, Petcov, MS '12; Girardi, Meroni, Petcov, MS '13]
- S_2 and accidental CP [Babu, Kubo '04, '11]

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  - Strategy
  - The Model
  - Relation to Nelson Barr Models
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How to suppress the strong CP phase?

[Antusch, Holthausen, Schmidt, MS '13]

- Step 1: Promote CP to be fundamental

$$\bar{\theta} = 0$$

- Step 2: Break CP such that arg det (M_u M_d) = 0
  - Step 2a: Make M_u real with vanishing 1-3 element
  - Step 2b: For M_d use ('*' real elements)

$$M_d = \begin{pmatrix} 0 & * & 0 \\ * & \mathrm{i}\,* & * \\ 0 & 0 & * \end{pmatrix}$$

The Phase Sum Rule

[Antusch, King, Malinsky, MS '10]

- We parameterise the mixing matrices as, e.g.

$$U_d^\dagger = U_{23}^d U_{13}^d U_{12}^d \text{ with } U_{12}^d = \begin{pmatrix} c_{12}^d & s_{12}^d \mathrm{e}^{-\mathrm{i}\,\delta_{12}^d} & 0 \\ -s_{12}^d \mathrm{e}^{\mathrm{i}\,\delta_{12}^d} & c_{12}^d & 0 \\ 0 & 0 & 1 \end{pmatrix} \, (U_d M_d M_d^\dagger U_d^\dagger = \text{diag.})$$

- For mass matrices with vanishing 1-3 element:

$$\alpha \approx \delta_{12}^d - \delta_{12}^u \overset{!}{\approx} 90^\circ$$

[Antusch, King, Malinsky, MS '09] see also [Fritzsch and Xing; Masina and Savoy '06; Harrison, Dallison, Roythorne, Scott '09]

- Simple solution (ignoring signs), e.g.

$$M_d = \begin{pmatrix} 0 & * & 0 \\ * & \mathrm{i}\,* & * \\ 0 & 0 & * \end{pmatrix} \,, \; M_u \text{ real} \; \Rightarrow \; \delta_{12}^d = 90^\circ \,, \; \delta_{12}^u = 0$$

Discrete Vacuum Alignment

[Antusch, King, Luhn, MS '11]

- Yukawas proportional to flavon vevs, e.g.

$$\langle \phi \rangle \propto (0,0,x)^T \quad \text{or} \quad \langle \phi \rangle \propto (x,x,x)^T$$

- Add term to W compatible with Z_n symmetry

$$\mathcal{W} \supset P \left(\kappa \frac{\phi^n}{\Lambda^{n-2}} \mp \lambda M^2 \right)$$

- Solve F-term conditions ($|F_P|=0$) + CP symmetry*

$$\arg(\langle \phi \rangle) = \arg(x) = \begin{cases} \frac{2\pi}{n} q \,, \quad q = 1, \dots, n & \text{for ``$-$''} \\ \frac{2\pi}{n} q + \frac{\pi}{n} \,, \quad q = 1, \dots, n & \text{for ``$+$''} \end{cases}$$

* We assumed here for simplicity that κ and λ are positive.

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Overview

[Antusch, Holthausen, Schmidt, MS '13]

- Symmetry of the Model

$$\underbrace{SU(3)_C \times SU(2)_L \times U(1)_Y}_{\text{gauge sym.}} \times \underbrace{A_4 \times Z_2 \times Z_4^5}_{\text{discrete family/shaping sym.}} \times \underbrace{U(1)_R}_{R \text{ sym.}}$$

- Flavour model for quarks only (d_R is A_4 triplet)!
- 5 singlet flavons ξ_i with real vevs, 4 triplets:

$$\langle \phi_1 \rangle \sim \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} , \quad \langle \phi_2 \rangle \sim \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} , \quad \langle \phi_3 \rangle \sim \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} , \quad \langle \tilde{\phi}_2 \rangle \sim \mathrm{i} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

Flavon Alignment

[Antusch, Holthausen, Schmidt, MS '13]

Study only one example for simplicity:

$$\mathcal{W} = \tilde{A}_2 \cdot (\tilde{\phi}_2 \star \tilde{\phi}_2) + \tilde{O}_{1;2}(\phi_1 \cdot \tilde{\phi}_2) + \tilde{O}_{2;3}(\tilde{\phi}_2 \cdot \phi_3) + \frac{P}{\Lambda^2}\left(\tilde{\phi}_2^4 \pm M_F^4\right)$$

1. Fix the possible vev direction in flavour space

$$F_{\tilde{A}_i} = 2(\tilde{\phi}_2)_j(\tilde{\phi}_2)_k = 0, \text{ where } i \neq j \neq k \neq i \Rightarrow \langle\tilde{\phi}_2\rangle \sim (0,1,0)$$

2. Fix the direction in relation to other vevs

$$F_{\tilde{O}_{1;2}} = 0 \Rightarrow \langle\phi_1\rangle \perp \langle\tilde{\phi}_2\rangle \text{ and } F_{\tilde{O}_{2;3}} = 0 \Rightarrow \langle\phi_3\rangle \perp \langle\tilde{\phi}_2\rangle$$

3. Fix the phase

$$F_P = 0 \Rightarrow \arg\langle\tilde{\phi}_2\rangle = 0, \frac{\pi}{2}, \pi, \frac{\pi}{2}$$

Coupling to Matter

[Antusch, Holthausen, Schmidt, MS '13]

- The effective superpotential reads

$$\mathcal{W}_d = Q_1 \bar{d} H_d \frac{\phi_2 \xi_d}{\Lambda^2} + Q_2 \bar{d} H_d \frac{\phi_1 \xi_d + \tilde{\phi}_2 \xi_s + \phi_3 \xi_t}{\Lambda^2} + Q_3 \bar{d} H_d \frac{\phi_3}{\Lambda}$$

$$\mathcal{W}_u = Q_1 \bar{u}_1 H_u \frac{\xi_u^2}{\Lambda^2} + Q_1 \bar{u}_2 H_u \frac{\xi_u \xi_c}{\Lambda^2} + Q_2 \bar{u}_2 H_u \left(\frac{\xi_c}{\Lambda} + \frac{\xi_t^2}{\Lambda^2} \right)$$

$$+ (Q_2 \bar{u}_3 + Q_3 \bar{u}_2) H_u \frac{\xi_t}{\Lambda} + Q_3 \bar{u}_3 H_u$$

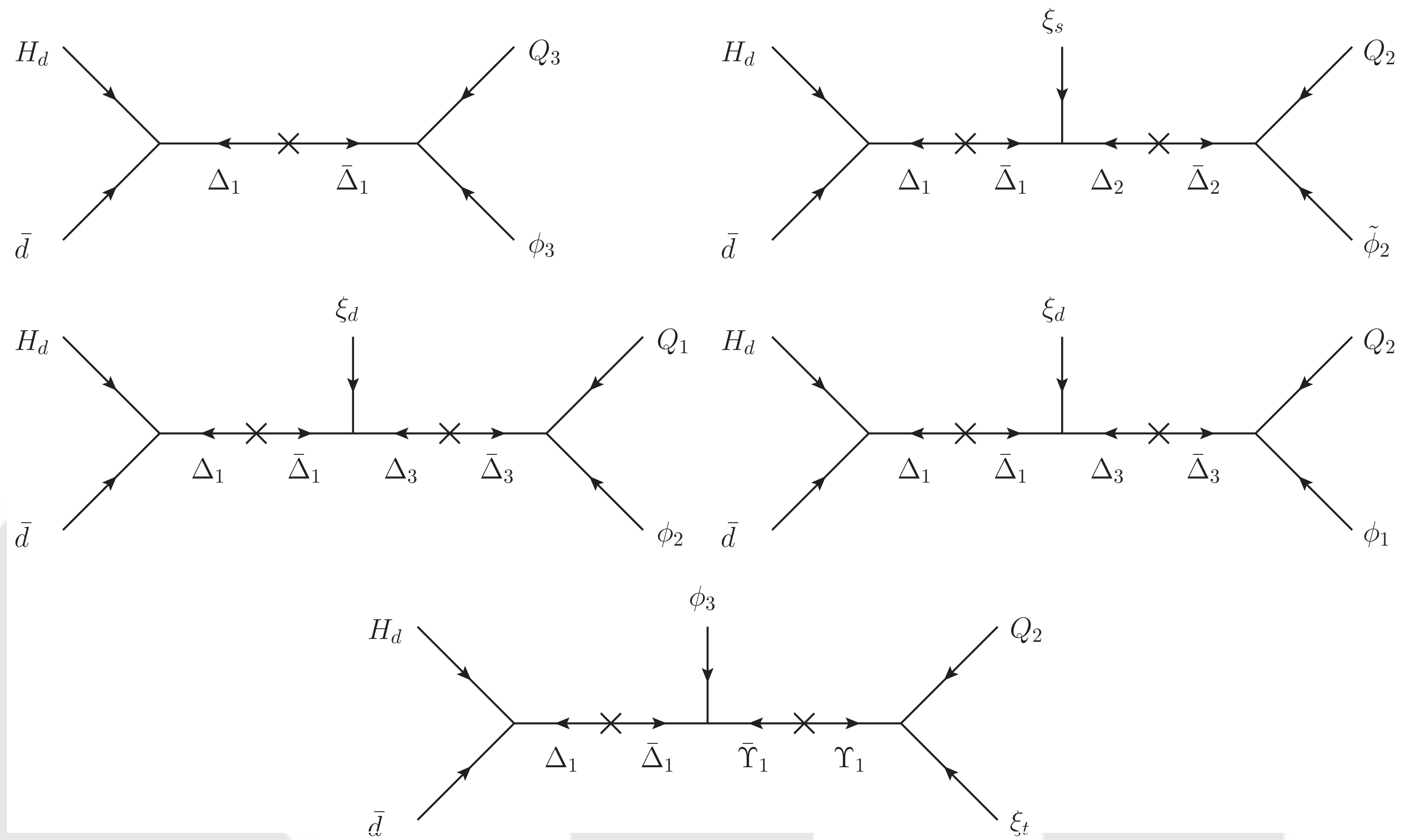
- Giving the mass matrix structure

$$M_d = \begin{pmatrix} 0 & b_d & 0 \\ b'_d & \mathrm{i} c_d & d_d \\ 0 & 0 & e_d \end{pmatrix} \text{ and } M_u = \begin{pmatrix} a_u & b_u & 0 \\ 0 & c_u & d_u \\ 0 & d'_u & e_u \end{pmatrix}$$

Higher Dimensional Operators I

[Antusch, Holthausen, Schmidt, MS '13]

We give a "UV completion" of the model giving full control over the effective operators!



Higher Dimensional Operators II

[Antusch, Holthausen, Schmidt, MS '13]

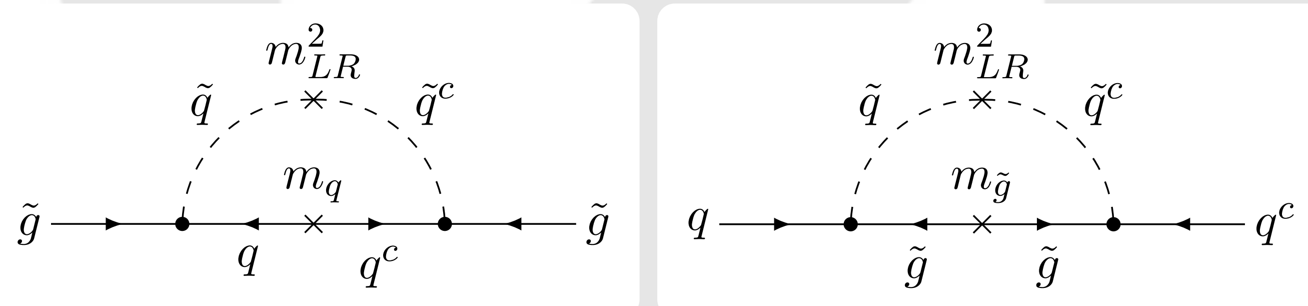
1. Corrections to the flavon sector:
   At least dimension seven, do not change the flavon directions and phases
2. Corrections to the up-quark sector:
   Suppressed (real) corrections to the 1-1, 1-2, 2-2 elements of M_u
3. Corrections to the down-quark sector:
   No corrections from higher-dim. operators!

Corrections from SUSY breaking

- SUSY breaking might give corrections, e.g.

$$\delta\bar{\theta} = 3\arg(m_{\tilde{g}})$$

- Also LR sfermion mixing gives corrections



- If SUSY breaking conserves CP and is close to MFV, corrections are potentially small enough.

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Nelson-Barr Models

[Nelson '84; Barr '84]

Nelson-Barr models defined by two conditions:

1. At the tree level there are no Yukawa or mass terms coupling *F* fermions to C^* fermions, or *C* fermions to *C* fermions.

2. The CP-violating phases appear at the tree level only in those Yukawa terms that couple *F* fermions to $R = (C + C^*)$ fermions.

In terms of a mass matrix including heavy vector-like fields:

$$M_D \sim \begin{pmatrix} Y v_d & 0 \\ \langle \phi \rangle & M_\Upsilon \end{pmatrix}$$

Yv_d and M_Υ real, $<\phi>$ complex

Relation to our Model Class

[Antusch, Holthausen, Schmidt, MS '13]

In our model we have:

$$M_D \sim \begin{pmatrix} 0 & 0 & 0 & \langle \phi_2^T \rangle & 0 & 0 \\ 0 & 0 & \langle \tilde{\phi}_2^T \rangle & \langle \phi_1^T \rangle & \langle \xi_t \rangle & \langle \xi_c \rangle \\ 0 & \langle \phi_3^T \rangle & 0 & 0 & 0 & \langle \xi_t \rangle \\ v_d & M_{\Delta_1} & 0 & 0 & 0 & 0 \\ 0 & \langle \xi_s \rangle & M_{\Delta_2} & 0 & 0 & 0 \\ 0 & \langle \xi_d \rangle & 0 & M_{\Delta_3} & 0 & 0 \\ 0 & \langle \phi_3^T \rangle & 0 & 0 & M_{\Upsilon_1} & \langle \xi_t \rangle \\ 0 & 0 & 0 & 0 & 0 & M_{\Upsilon_2} \end{pmatrix}$$

with a real determinant as well:

$$\det M \sim v_d^3 M_{\Delta_2}^3 M_{\Delta_3}^3 M_{\Upsilon_1} M_{\Upsilon_2} \langle \xi_d^2 \rangle \langle \phi_1 \rangle \langle \phi_2 \rangle \langle \phi_3 \rangle$$

In Words

Nelson-Barr	Our Class of Models
Real Yukawa Couplings	Effective Yukawa Couplings (apart from top)
Real "Messenger" Masses	Real Messenger Masses
Complex Couplings between light & heavy Fields	Real couplings but complex Flavon vevs
Structure of Couplings	Alignment of Flavon vevs

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What is T'?

- T' is the double covering of A_4:

$$T' =< R, S, T | S^2 = R, R^2 = T^3 = (ST)^3 = 1, RT = TR >$$

- Seven representations:

$$\mathbf{1}\ ,\ \ \mathbf{1}'\ ,\ \ \mathbf{1}''\ ,\ \ \mathbf{2}\ ,\ \ \mathbf{2}'\ ,\ \ \mathbf{2}''\ ,\ \ \mathbf{3}$$

- In our basis, complex Clebsch-Gordan coefficients, e.g.

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}_{\mathbf{2}} \otimes \begin{pmatrix} x'_1 \\ x'_2 \end{pmatrix}_{\mathbf{2}} = \left(\frac{x_1 x'_2 - x_2 x'_1}{\sqrt{2}} \right)_{\mathbf{1}} \oplus \begin{pmatrix} \frac{(1-\mathrm{i})}{2} (x_1 x'_2 + x_2 x'_1) \\ \mathrm{i}\ x_1 x'_1 \\ x_2 x'_2 \end{pmatrix}_{\mathbf{3}}$$

CP Transformations

[Girardi, Meroni, Petcov, MS '13]

- In our basis, we find the CP transformations

$$\mathbf{1} \to \mathbf{1}^* \,, \quad \mathbf{1}' \to \mathbf{1}'^* \,, \quad \mathbf{1}'' \to \mathbf{1}''^* \,, \quad \mathbf{3} \to \mathbf{3}^* \,,$$

$$\mathbf{2} \to \begin{pmatrix} \mathrm{e}^{\mathrm{i}\pi/4} & 0 \\ 0 & \mathrm{e}^{-\mathrm{i}\pi/4} \end{pmatrix} \mathbf{2}^* \,, \quad \mathbf{2}' \to \begin{pmatrix} 1 & 0 \\ 0 & -\mathrm{i} \end{pmatrix} \mathbf{2}'^* \,, \quad \mathbf{2}'' \to \begin{pmatrix} \mathrm{i} & 0 \\ 0 & 1 \end{pmatrix} \mathbf{2}''^*$$

- In this basis couplings can be forced to be complex, e.g.

$$\lambda \mathcal{O} = \lambda \left[(\mathbf{2}'' \times \mathbf{2}'')_{\mathbf{1}'} \times \mathbf{1}'' \right]$$
$$\Rightarrow \mathrm{CP}[\mathcal{O}]/\mathcal{O}^* = \mathrm{i} \Rightarrow \arg(\lambda) = \pi/4$$

- In a different basis this might change, but physics will not change!

What is physical?

- There are three sources for CP violation
  - Complex Clebsch-Gordan coefficients of T'
  - Complex coupling constants
  - Complex (or real) vacuum expectation values
- Only the combination of the three is physical!

Vevs and phases in the Lagrangian

- Take an operator invariant under CP
- Add two flavon fields to this operator, e.g.

$$\mathcal{L} \supset \lambda \mathcal{O}_{\mathbf{3}} (\psi' \psi'')_{\mathbf{3}}$$

- This product remains real if

$$\psi' = \begin{pmatrix} X_1 \, \mathrm{e}^{\mathrm{i}(\alpha - \pi/2)} \\ X_2 \, \mathrm{e}^{\mathrm{i}(\alpha + \pi/4)} \end{pmatrix} \, , \quad \psi'' = \begin{pmatrix} Y_1 \, \mathrm{e}^{-\mathrm{i}(\alpha - \pi/4)} \\ Y_2 \, \mathrm{e}^{-\mathrm{i}\alpha} \end{pmatrix}$$

- In this basis real vevs can introduce phases!

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Symmetries and field content

[Girardi, Meroni, Petcov, MS '13]

- Symmetries:

$$G_{\mathrm{SM}} \times (T' \rtimes H_{\mathrm{CP}}) \times Z_8 \times Z_4^2 \times Z_3^2 \times Z_2 \times U(1)_R$$

- Flavour model for leptons only

	L	$\bar{E}$	$\bar{E}_3$	N_R	H_d	H_u
T'	$\mathbf{3}$	$\mathbf{2}''$	$\mathbf{1}''$	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}$

- Flavon fields without singlets:

$$\langle\phi\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \phi_0 \,, \quad \langle\tilde{\phi}\rangle = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \tilde{\phi}_0 \,, \quad \langle\hat{\phi}\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \hat{\phi}_0 \,, \quad \langle\xi\rangle = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \xi_0 \,,$$

$$\langle\psi'\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \psi'_0 \,, \quad \langle\psi''\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \psi''_0 \,, \quad \langle\tilde{\psi}''\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tilde{\psi}''_0$$

Yukawa and Mass Matrices

[Girardi, Meroni, Petcov, MS '13]

- The effective superpotentials read

$$\begin{aligned}\mathcal{W}_{\mathrm{Y}_e} =& \frac{y_{33}^{(e)}}{\Lambda}\left(\bar{E}_3\,H_d\right)_{\mathbf{1}''}(L\,\phi)_{\mathbf{1}'} + \frac{y_{32}^{(e)}}{\Lambda^2}\left(\bar{E}_3\,H_d\right)_{\mathbf{1}''}\left(L\,\hat{\phi}\right)_{\mathbf{1}'}\zeta + \frac{\hat{y}_{32}^{(e)}}{\Lambda^2}\,\bar{\Omega}\,\left(\bar{E}_3\,H_d\right)_{\mathbf{1}''}\left[L\,(\psi'\,\psi'')_{\mathbf{3}}\right]_{\mathbf{1}'} \\ &+ \frac{y_{22}^{(e)}}{\Lambda^2}\left(\bar{E}\,\psi'\right)_{\mathbf{1}} H_d\,(L\,\phi)_{\mathbf{1}} + \frac{y_{21}^{(e)}}{\Lambda^3}\,\bar{\Omega}\,\left(\bar{E}\,\psi'\right)_{\mathbf{1}} H_d\left[(\psi'\,\psi'')_{\mathbf{3}}\;L\right]_{\mathbf{1}} \\ &+ \frac{y_{11}^{(e)}}{\Lambda^3}\,\Omega\,\left(\bar{E}\,\tilde{\psi}''\right)_{\mathbf{1}'} H_d\,\tilde{\zeta}'\left(L\,\tilde{\phi}\right)_{\mathbf{1}'}\;, \\ \mathcal{W}_{\mathrm{Y}_\nu} =& \lambda_1 N\,N\,\xi + N\,N\,(\lambda_2\rho + \lambda_3\tilde{\rho}) + \frac{y_\nu}{\Lambda}(NL)_1(H_u\rho)_1 + \frac{\tilde{y}_\nu}{\Lambda}(NL)_1(H_u\tilde{\rho})_1\end{aligned}$$

- Giving the Yukawa and Mass matrices

$$\mathrm{Y}_e = \begin{pmatrix} \Omega\,a & 0 & 0 \\ \mathrm{i}b & c & 0 \\ 0 & d+\mathrm{i}k & e \end{pmatrix}\;,\quad \mathrm{Y}_\nu = y_\nu \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}\;,\quad M_R = \begin{pmatrix} 2Z+X & -Z & -Z \\ -Z & 2Z & -Z+X \\ -Z & -Z+X & 2Z \end{pmatrix}$$

Neutrino Spectra

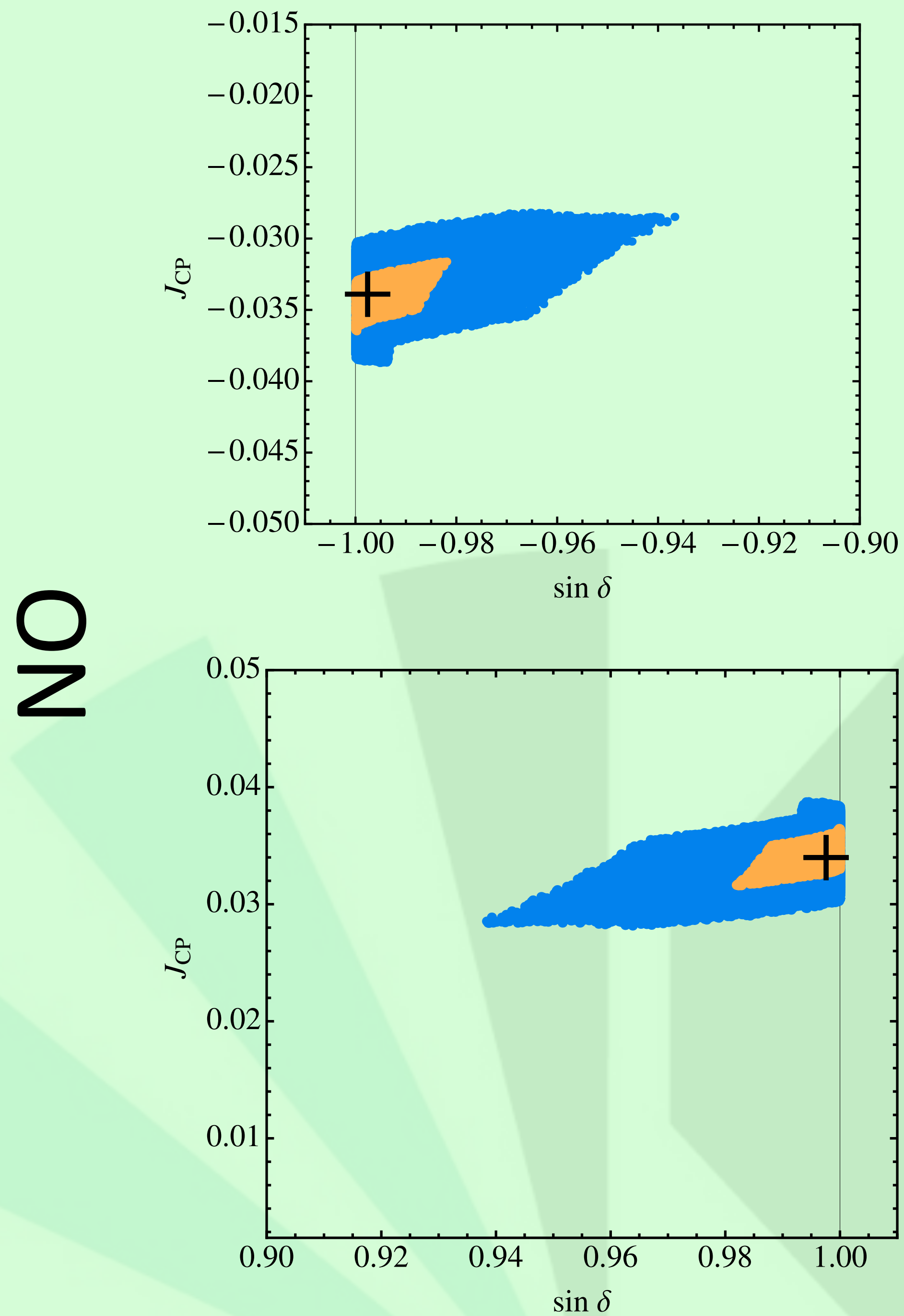
[Girardi, Meroni, Petcov, MS '13]

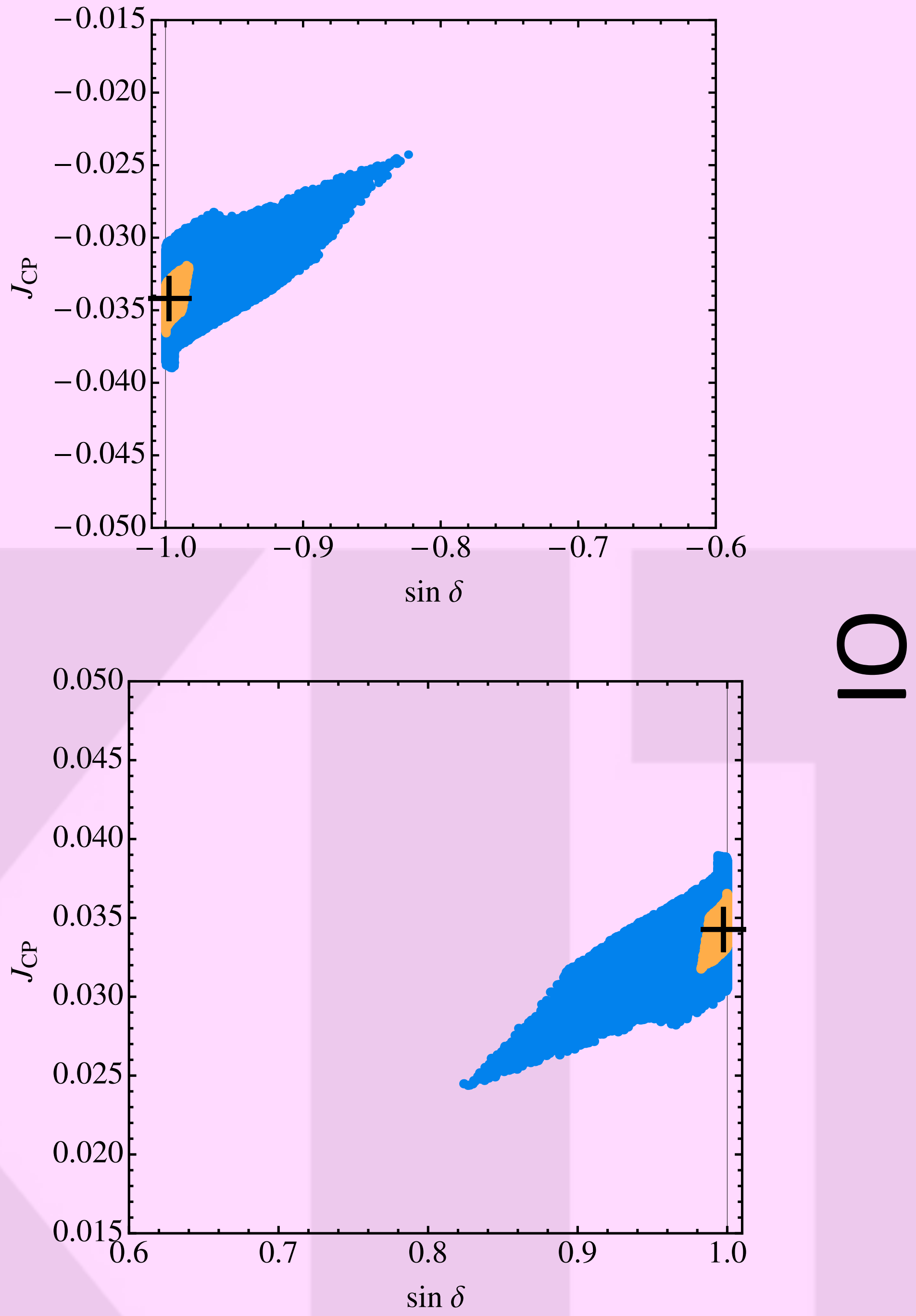
$@3\sigma$	NO A	NO B	IO A
m_1 in 10^{-3} eV	[4.23, 4.66]	[5.56, 6.20]	[45.7, 58,7]
m_2 in 10^{-3} eV	[9.23, 10.17]	[9.83, 11.20]	[46.3, 59.6]
m_3 in 10^{-2} eV	[4.28, 5.56]	[4.23, 5.74]	[1.53, 1.98]
$\sum m_i$ in 10^{-2} eV	[5.63, 7.04]	[5.77, 7.48]	[10.73, 13.81]

Only three possible spectra: two with normal ordering, one with inverted ordering

J_{CP} vs. sin δ

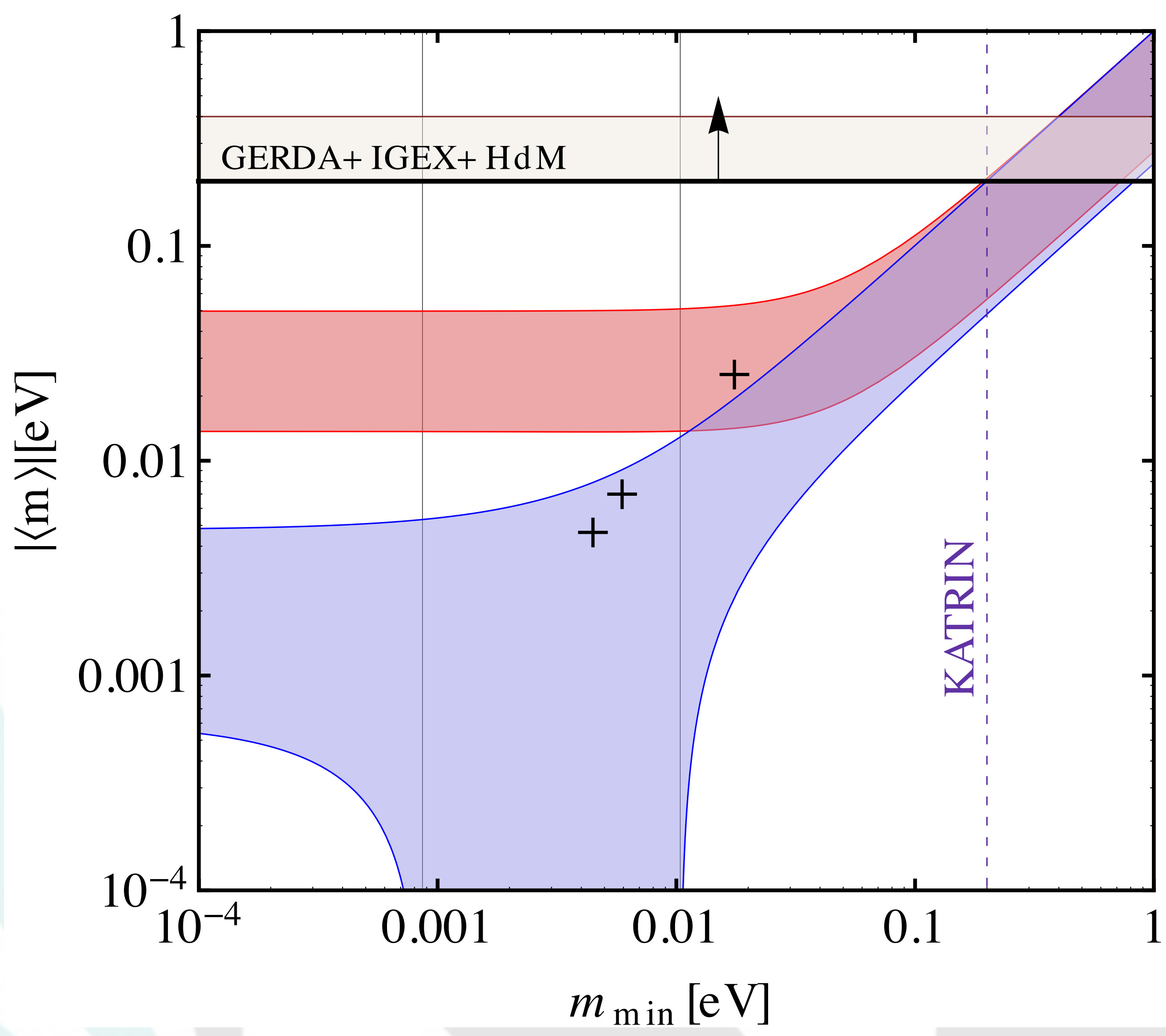
[Girardi, Meroni, Petcov, MS '13]





Neutrinoless Double Beta Decay

[Girardi, Meroni, Petcov, MS '13]



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- A non-trivial case: T'
- **Summary and Conclusions**

Summary and Conclusions I

- Solution to Strong CP problem with discrete Symmetries but without Axions
- Ingredients:
  - Fundamental CP symmetry
  - Phase sum rule (right CKM unitarity triangle)
  - Discrete symmetries to fix flavour directions and phases
  - Messenger sector to control higher-dimensional operators

Summary and Conclusions II

- Combining CP and discrete symmetries can be non-trivial
  - Consistent treatment needed
  - Generalised CP does not have to be physical CP
  - New phenomenology
  - Quark sector shows hints of spontaneous CP violation
- Experimental results for leptonic CP violation needed!

Thank you for your attention!