



Theory for the Terascale

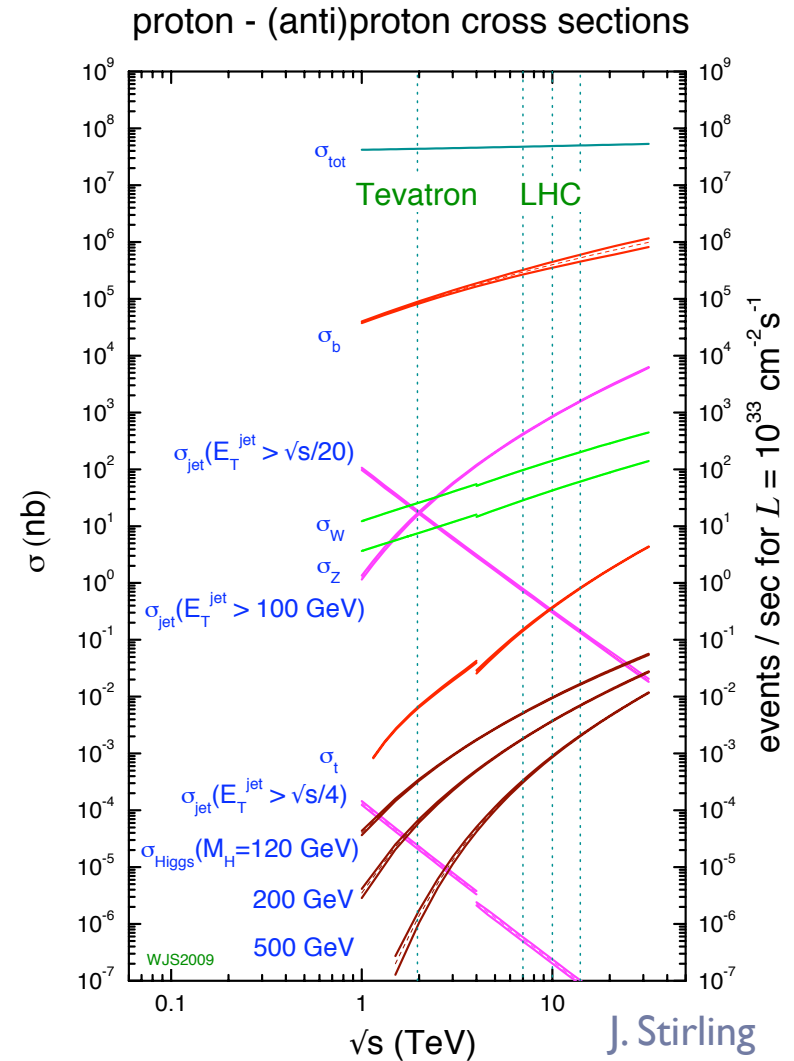
Thomas Gehrman

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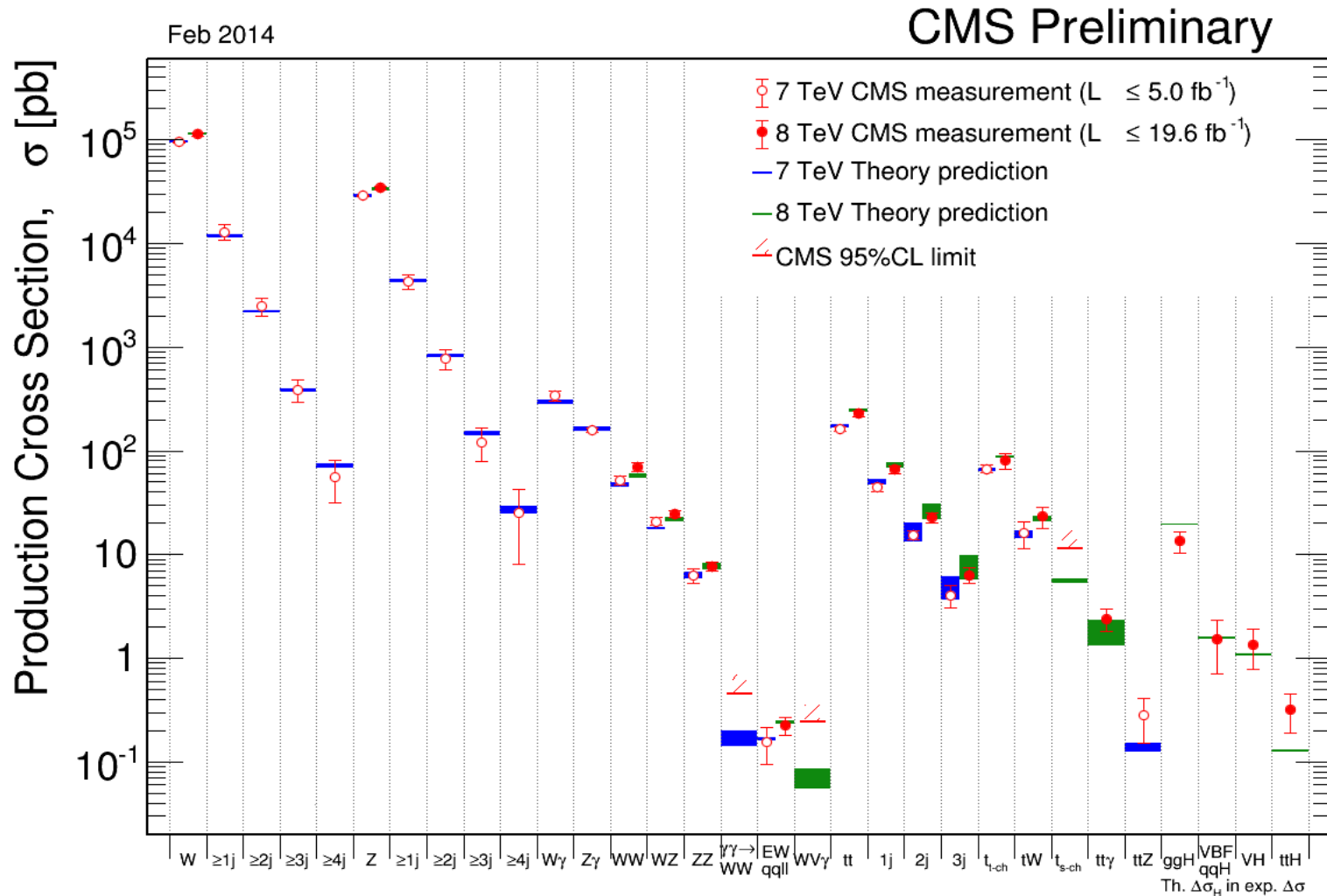
8th Annual Meeting of “Physics at the Terascale”, DESY, 01.12.2014

Benchmark processes at LHC

- ▶ Large production rates for Standard Model processes
 - ▶ jets
 - ▶ top quark pairs
 - ▶ vector bosons
- ▶ Allow precision measurements
 - ▶ masses
 - ▶ couplings
 - ▶ parton distributions
- ▶ Require precise theory

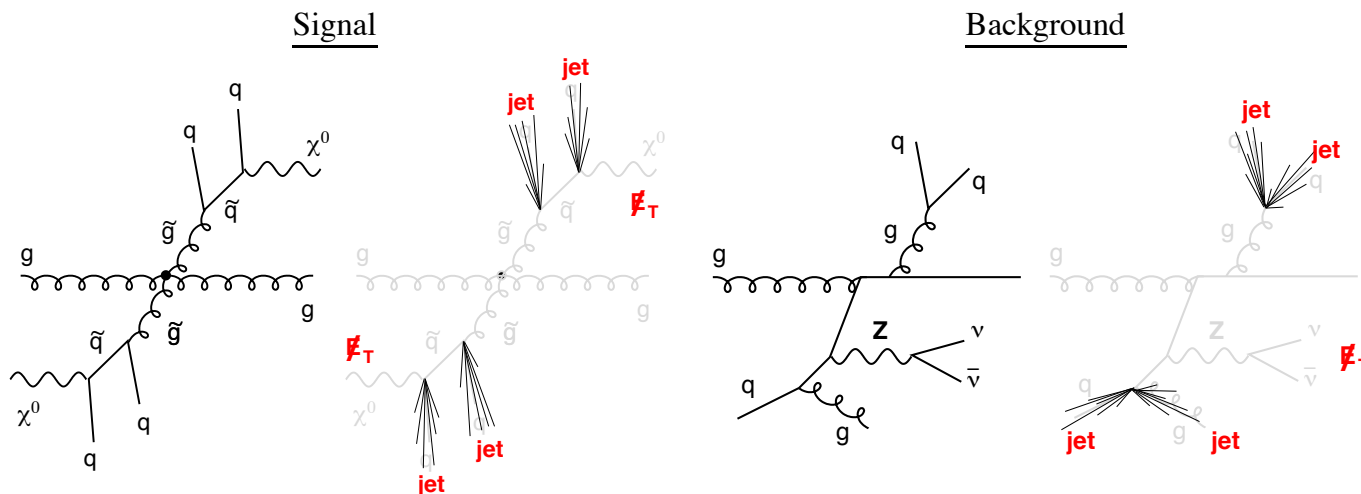


Benchmark processes at LHC



Multi-particle production at LHC

- ▶ LHC brings new frontiers in energy and luminosity
- ▶ Production of short-lived heavy states (Higgs, top, SUSY...)
 - ▶ detected through their decay products
 - ▶ yield multi-particle final states involving jets, leptons, γ , \cancel{E}_T
- ▶ Search for new effects in multi-particle final states
- ▶ Need precise predictions for hard scattering processes



Example: SUSY signature $4j + \cancel{E}_T$

The case for precision

▶ Implications of Higgs boson discovery at ATLAS and CMS

▶ Higgs mechanism established H^0

$$J = 0$$

▶ Higgs boson mass measured

$$\text{Mass } m = 125.7 \pm 0.4 \text{ GeV}$$

▶ Standard Model of particle physics complete

H^0 Signal Strengths in Different Channels

$$\text{Combined Final States} = 1.17 \pm 0.17 \quad (S = 1.2)$$

$$W W^* = 0.87^{+0.24}_{-0.22}$$

$$Z Z^* = 1.11^{+0.34}_{-0.28} \quad (S = 1.3)$$

$$\gamma\gamma = 1.58^{+0.27}_{-0.23}$$

$$b\bar{b} = 1.1 \pm 0.5$$

$$\tau^+\tau^- = 0.4 \pm 0.6$$

$$Z\gamma < 9.5, \text{ CL} = 95\%$$

▶ Beyond the Standard Model

▶ Planck mass sets fundamental limit: $M_p \simeq 10^{19} \text{ GeV}$

▶ Internal consistency of Standard Model

▶ Hierarchy problem

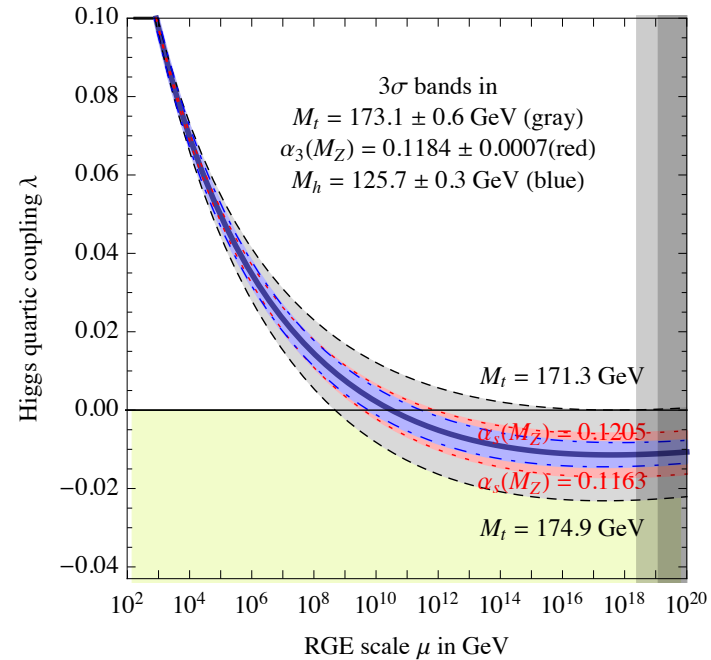
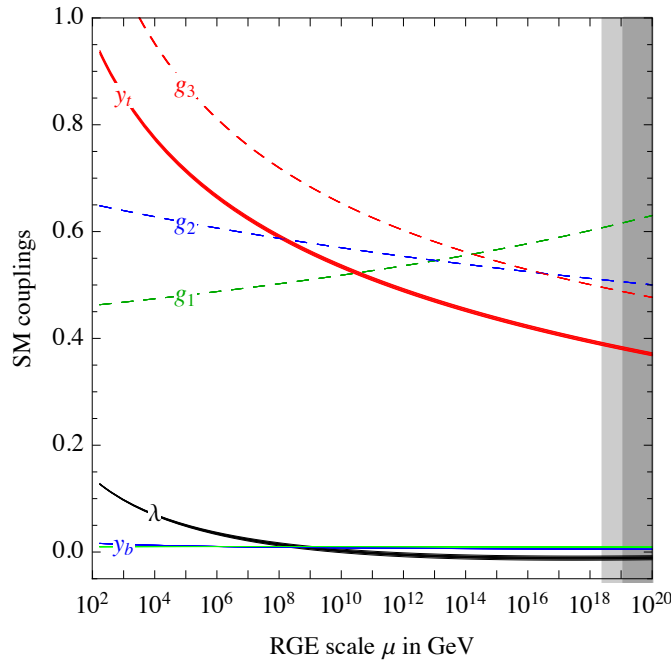
▶ Extrapolation to high energies

▶ Stability of the Higgs potential

PDG 2014

Stability of the Higgs potential

► Renormalization group evolution of quartic coupling

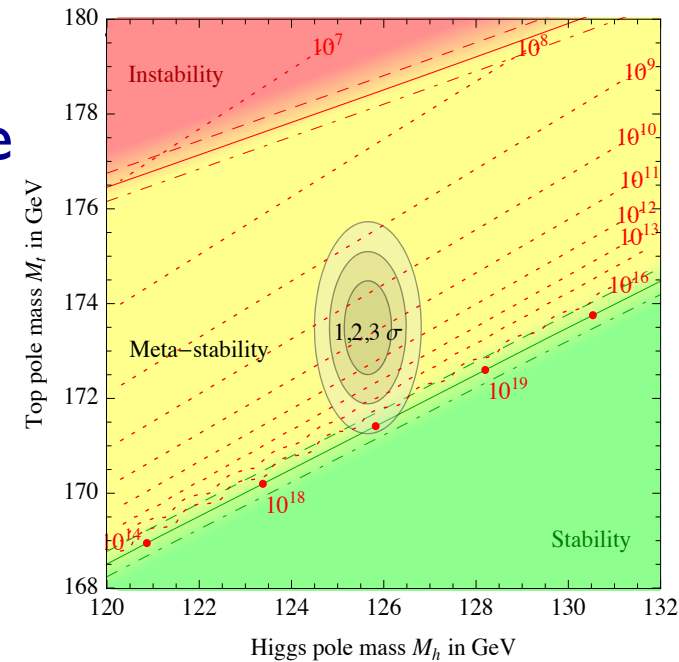
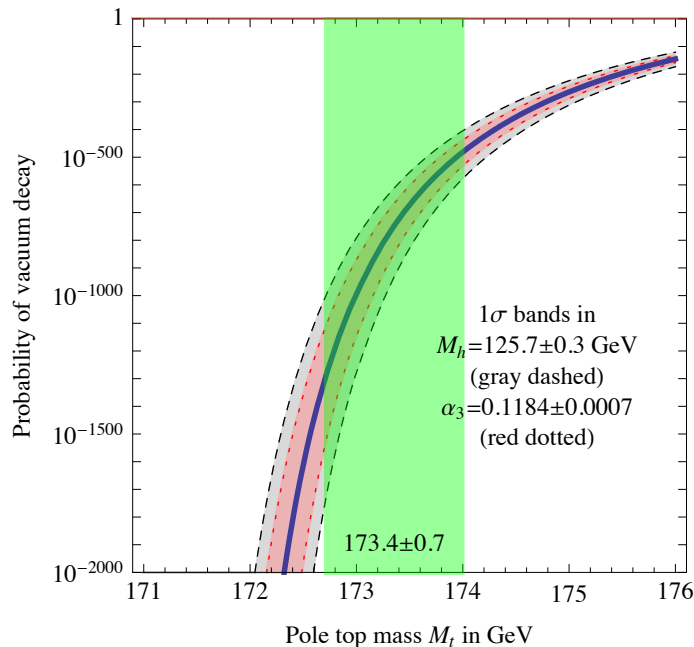


G. Degrandi et al., K. Chetyrkin, M. Zoller

► Propagation of errors on Standard Model parameters

Stability of the Higgs potential

- ▶ Determines vacuum stability
- ▶ Current data indicate metastable state
- ▶ Precision on parameters and for RGE evolution and matching crucial



F. Bezrukov, M. Kalmykov, B. Kniehl,
M. Shaposhnikov;
J. Elias-Miro et al, G. Degrandi et al.;
F. Jegerlehner

QCD: precision physics at LHC

- ▶ NLO: methods and directions
- ▶ Parton showers, resummation, matching
- ▶ NNLO: precision QCD
- ▶ Precision frontier: aims and ideas

▶ **NLO: methods, results, directions**

NLO multi-particle production

▶ Why NLO?

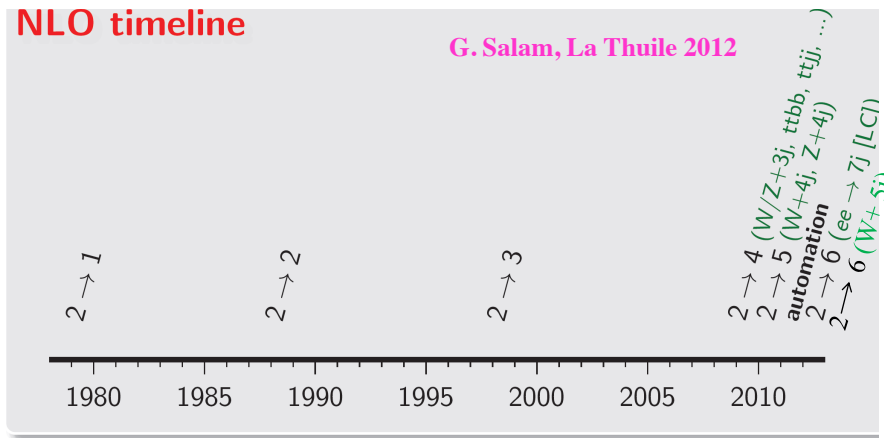
- ▶ reduce scale uncertainty of LO theory prediction
- ▶ reliable normalization and shape
- ▶ accounts for effects of extra radiation
- ▶ jet algorithm dependence

▶ Typical observations

- ▶ sizable NLO corrections
- ▶ corrections not constant, but kinematics-dependent
- ▶ remaining uncertainty at NLO typically 10-20%

NLO multi-parton production

- ▶ Enormous progress in getting NLO predictions for $2 \rightarrow (4,5,6!)$ processes over the last years



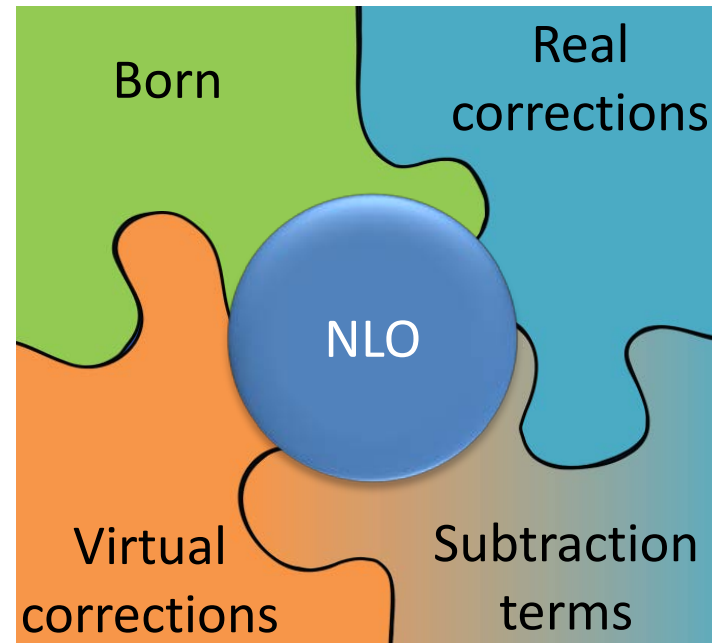
- ▶ Made possible by
 - ▶ Improved techniques for loop amplitudes
 - ▶ **Crucial:** a high level of automation

Process ($V \in \{Z, W, \gamma\}$)	Comments
Calculations completed since Les Houches 2005	
1. $pp \rightarrow VV$ jet	WW jet completed by Dittmaier/Kalweit/Uwer [27, 28]; Campbell/Ellis/Zanderighi [29]. ZZ jet completed by Binoth/Gleisberg/Karg/Kauer/Sanguinetti [30]
2. $pp \rightarrow \text{Higgs}+2\text{jets}$	NLO QCD to the gg channel completed by Campbell/Ellis/Zanderighi [31]; NLO QCD+EW to the VBF channel completed by Ciccolini/Denner/Dittmaier [32, 33]
3. $pp \rightarrow VVV$	Interference QCD-EW in VBF channel [34, 35] ZZZ completed by Lazopoulos/Melnikov/Petriello [36] and WWZ by Hankele/Zeppenfeld [37]. see also Binoth/Ossola/Papadopoulos/Pitau [38] VBFNLO [39, 40] meanwhile also contains $WWW, ZZW, WW\gamma, ZZ\gamma, WZ\gamma, W\gamma\gamma, \gamma\gamma WZj, W\gamma j, \gamma j j$
4. $pp \rightarrow t\bar{t}b\bar{b}$	relevant for $t\bar{t}H$, computed by Breckenridge/Denner/Dittmaier/Pozzorini [41, 42] and Bevilacqua/Czakon/Papadopoulos/Pitau/Worek [43]
5. $pp \rightarrow V+3\text{jets}$	$W+3\text{jets}$ completed by the Blackhat/Sherpa [44] and Bevilacqua [45] collaborations $Z+3\text{jets}$ by Blackhat/Sherpa [46]
Calculations remaining from Les Houches 2005	
6. $pp \rightarrow t\bar{t}+2\text{jets}$	relevant for $t\bar{t}H$, computed by Bevilacqua/Czakon/Papadopoulos/Worek [47, 48]
7. $pp \rightarrow VV b\bar{b}$	Pozzorini et al [25], Bevilacqua et al [23]
8. $pp \rightarrow VV+2\text{jets}$	$W+W+2\text{jets}$ [49], $W+W^-+2\text{jets}$ [50], VBF contributions calculated by (Bozzi/Jäger/Oleari/Zeppenfeld [51, 52, 53])
NLO calculations added to list in 2007	
9. $pp \rightarrow b\bar{b}b\bar{b}$	Binoth et al. [54, 55]
NLO calculations added to list in 2009	
10. $pp \rightarrow V+4\text{jets}$	top pair production, various new physics signatures Blackhat/Sherpa: $W+4\text{jets}$ [22], $Z+4\text{jets}$ [20] see also HEJ [56] for $W+4\text{jets}$
11. $pp \rightarrow Wbbj$	top, new physics signatures, Reina/Schutzmeier [11]
12. $pp \rightarrow t\bar{t}t\bar{t}$	various new physics signatures
also: $pp \rightarrow 4\text{jets}$	Blackhat/Sherpa [19]

K. Melnikov, MITP, 2013

NLO automation

- ▶ **Well-defined interfaces (Binoth Les Houches accord)**
 - ▶ combine different ingredients from different codes
- ▶ **One-loop amplitudes**
 - ▶ **BlackHat** (Z. Bern, L. Dixon, F. Febres Cordero, S. Höche, H. Ita, D. Kosower, D. Maitre, K. Ozeren)
 - ▶ **GoSam** (G. Cullen, N. Greiner, G. Heinrich, G. Luisoni, P. Mastrolia, G. Ossola, T. Reiter, F. Tramontano)
 - ▶ **OpenLoops** (F. Cascioli, P. Maierhöfer, S. Pozzorini)
 - ▶ **NJet** (S. Badger, B. Biedermann, P. Uwer, V. Yundin)
 - ▶ **MadLoop/aMC@NLO** (R. Frederix et al.)
 - ▶ **CutTools** (G. Ossola, C. Papadopoulos, R. Pittau)
- ▶ **Real radiation, subtraction terms and phase space (infrastructure)**
 - ▶ **Sherpa** (F. Kraus et al.)
 - ▶ **Madgraph/MadEvent** (F. Maltoni et al.)
 - ▶ **HelacNLO** (G. Bevilacqua, C. Papadopoulos et al.)
 - ▶ **MCFM** (J. Campbell, K. Ellis, C. Williams)
 - ▶ **VBFNLO** (D. Zeppenfeld et al.)



Automation in NLO computations

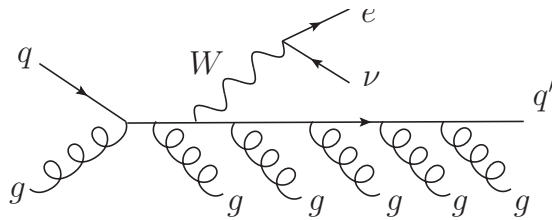
- ▶ **Impressive list of recent results:**
 - ▶ multiple jets (up to 4) (Blackhat + Sherpa; Njet)
 - ▶ gauge boson and up to 5 jets (Blackhat + Sherpa)
 - ▶ two gauge bosons with up to 2 jets (T. Melia et al.; VBFNLO: F. Campanario, M. Kerner, L.D. Ninh, D. Zeppenfeld; GoSam + MadEvent)
 - ▶ Three gauge bosons (VBFNLO: G. Bozzi, F. Campanario, C. Englert, M. Rauch, D. Zeppenfeld)
 - ▶ Top quarks with jets (up to 2) (A. Denner, S. Dittmaier, S. Kallweit, S. Pozzorini; G. Bevilacqua, M. Czakon, C. Papadopoulos, M. Worek)
 - ▶ Top quarks with a gauge boson (A. Lazopoulos, K. Melnikov, F. Petriello; K. Melnikov, M. Schulze, A. Scharf; HelacNLO: A. Kardos, Z. Trocsanyi, C. Papadopoulos; MCFM: J. Campbell, K. Ellis)
 - ▶ Higgs with a top quark pair and one jet (GoSam + Sherpa + MadEvent: H. van Deurzen, G. Luisoni, P. Mastrolia, E. Mirabella, G. Ossola, T. Peraro)
 - ▶ Higgs and up to 3 jets (GoSam + Sherpa + MadEvent: G. Cullen, H. van Deurzen, N. Greiner, G. Luisoni, P. Mastrolia, E. Mirabella, G. Ossola, T. Peraro, F. Tramontano)
- ▶ **Broad implications for precision phenomenology**

W+5 jets at NLO

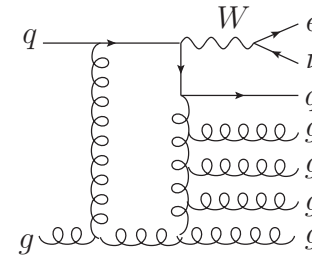
- ▶ First **2→6** NLO calculation at a hadron collider
- ▶ Using Blackhat + Sherpa

(Z. Bern, L. Dixon, F. Febres Cordero, S. Höche, H. Ita, D. Kosower, D. Maitre, K. Ozeren)

- ▶ Blackhat: virtual one-loop corrections using on-shell methods
- ▶ Sherpa: real emission, subtraction, phase space integration



Example diagram for real emission
(2→8) at tree level



Example diagram for virtual emission
(2→7) at one-loop (octagon)

- ▶ Computation at the actual frontier of NLO complexity
 - ▶ Considered impossible until few years ago

W+5 jets at NLO

▶ Distribution in H_T^{jets} (sum of jet transverse energies)

▶ Dynamical scale choice

$$\mu_R = \mu_F = \hat{H}'_T / 2$$

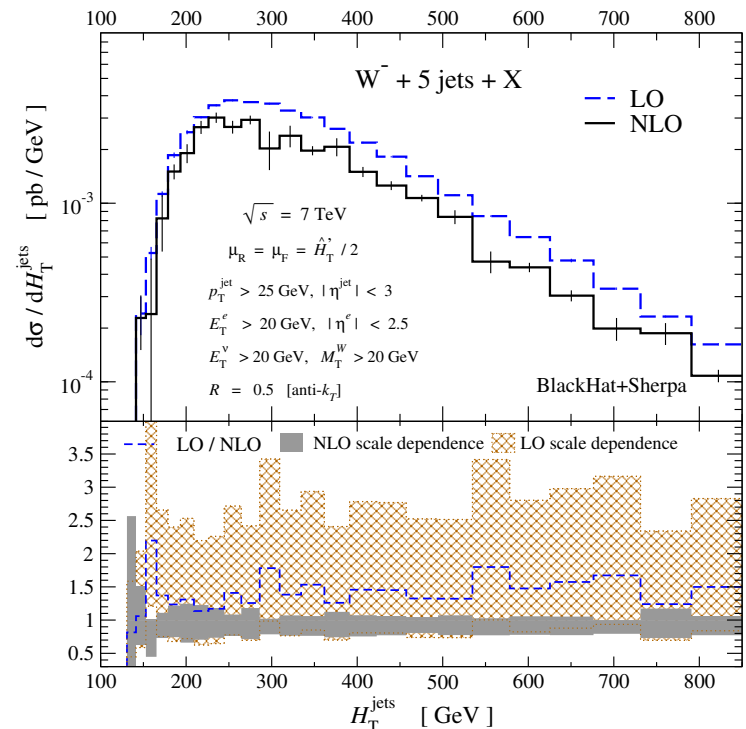
$$\hat{H}'_T \equiv \sum_m p_T^m + E_T^W$$

▶ scale variation $\mu/2 \dots 2\mu$

▶ Observe:

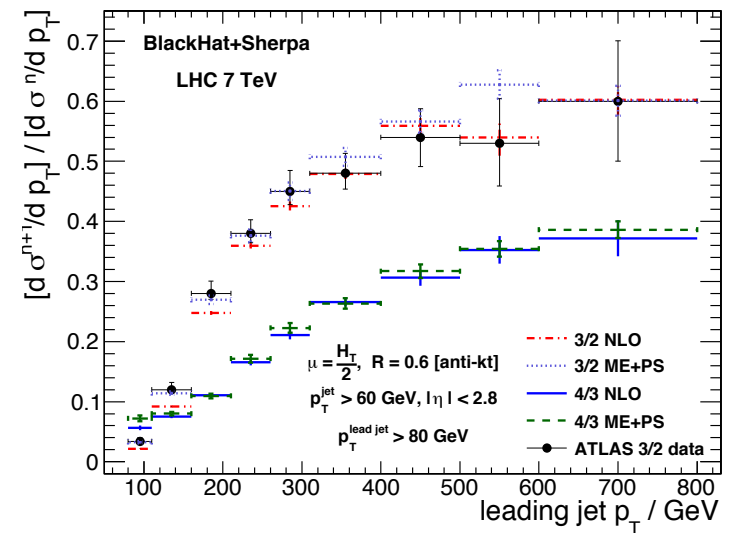
- ▶ Scale dependence reduced at NLO
- ▶ ratio NLO/LO constant over full kinematical range

▶ NLO helps to motivate the scale choice



Jet ratios at NLO

- ▶ Systematic uncertainties (th. and exp.) cancel in ratios
 - ▶ Predictions more reliable
 - ▶ Can be used in data-driven background estimation
- ▶ Jet ratio as function of leading jet p_T
 - ▶ NLO and parton shower both agree with data for large p_T
 - ▶ Parton shower (multiple emission) better at low p_T
 - Large uncertainty on parton shower not shown



Observe: 3/2 ratio below the data at small p_T

▶ Parton showers, resummation, matching

Fixed order versus parton shower

▶ Fixed order calculations

- ▶ Expansion in powers of the coupling constant
- ▶ Correctly describes hard radiation pattern
- ▶ Final states are described by single hard particles
- ▶ NLO: up to two particles in a jet, NNLO: up to three..
- ▶ Soft radiation poorly described

▶ Parton shower

- ▶ Exponentiates multiple soft radiation (leading logarithms)
- ▶ Describes multi-particle dynamics and jet substructure
- ▶ Allows generation of full events (interface to hadronization)
- ▶ Basis of multi-purpose generators (SHERPA, HERWIG, PYTHIA)
- ▶ Fails to account for hard emissions

▶ Ideally: combine virtues of both approaches

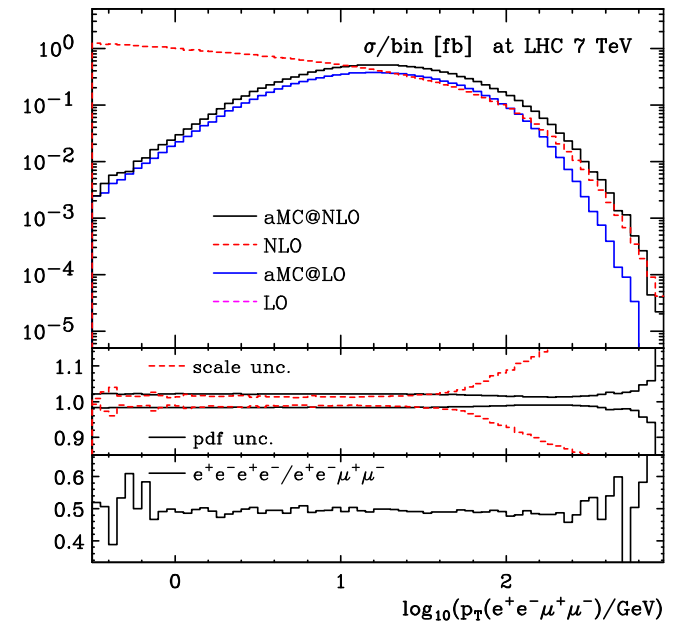
Merging of fixed order and parton shower

▶ Merging multiplicities

- ▶ Combine fixed-order matrix elements at different multiplicity with vetoed shower
- ▶ Leading order prescriptions: CKKW (S. Catani, F. Krauss, R. Kuhn, B. Webber) and MLM (M. Mangano)
- ▶ Has become standard for parton shower simulations

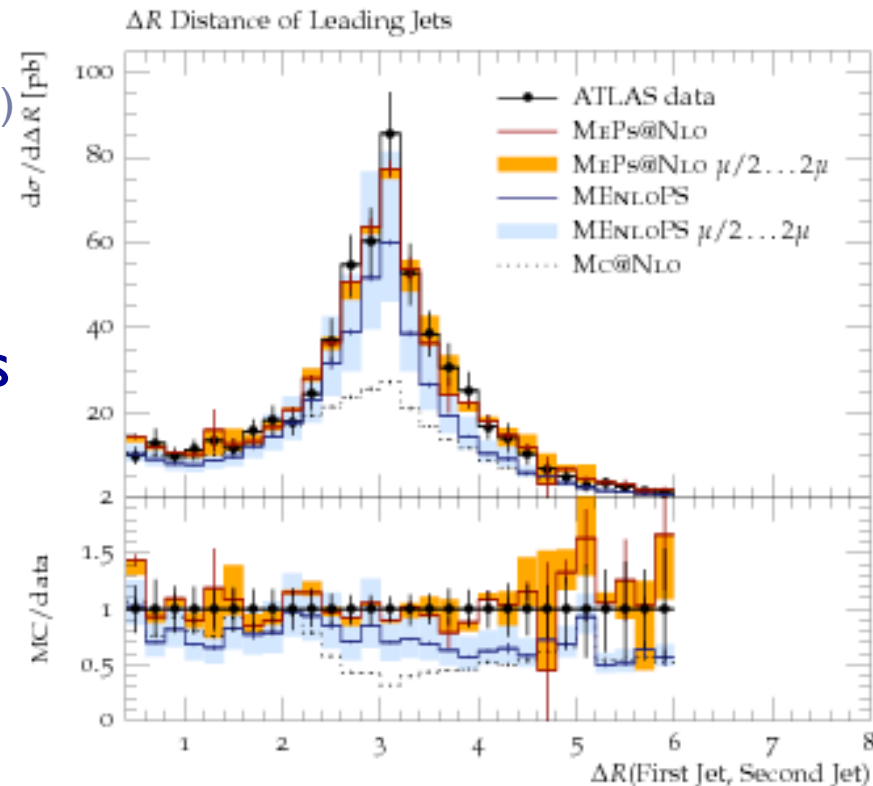
▶ Merging NLO with parton shower

- ▶ Combine fixed-multiplicity NLO calculation with parton shower
- ▶ Accomplished for many processes (MC@NLO: S. Frixione, B. Webber; POWHEG: P. Nason, C. Oleari et al.)
- ▶ Automation: aMC@NLO (R. Frederix, S. Frixione, V. Hirschi, F. Maltioni, R. Pittau, P. Torrielli)



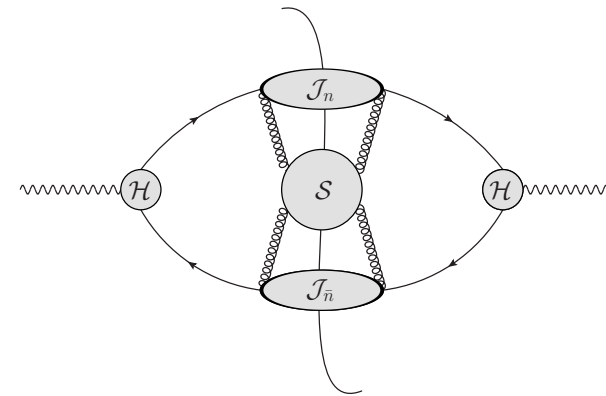
Merging of fixed order and parton shower

- ▶ Combining NLO computations for different multiplicities and interfacing with parton showers (proof-of-principle)
 - ▶ SHERPA (S. Höche, F. Krauss, M. Schönherr, F. Siegert)
 - ▶ MINLO (K. Hamilton, P. Nason, C. Oleari, G. Zanderighi)
 - ▶ UNLOPS (L. Lönnblad, S. Prestel)
 - ▶ FxFx (S. Frixione, R. Frederix)
- ▶ Yields combined event samples
- ▶ Improves especially jet-jet correlations
- ▶ Work in progress



Resummation

- ▶ Parton shower: leading logarithmic accuracy (LL)
- ▶ Resummation of higher-order logarithms
 - ▶ NLL: largely automated (CAESAR: A. Banfi, G. Salam, G. Zanderighi)
 - ▶ NNLL and beyond: process-by-process calculations
- ▶ **Methods**
 - ▶ Laplace-space resummation (CSS: J. Collins, D. Soper, G. Sterman)
 - ▶ Soft-collinear effective theory (SCET: C. Bauer, S. Fleming, D. Pirjol, I. Rothstein, I. Stewart; M. Beneke, A. Chapovsky, M. Diehl, T. Feldmann)
 - ▶ Systematic extension beyond NLL



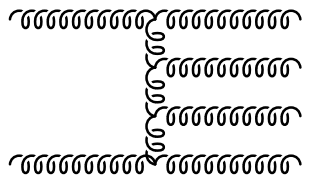
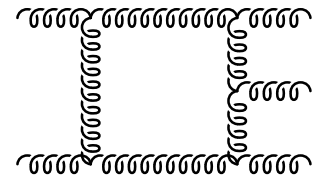
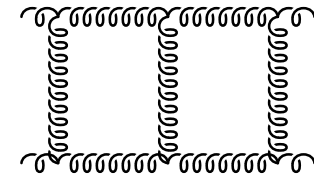
▶ **NNLO: towards precision QCD**

NNLO observables at hadron colliders

- ▶ NNLO predictions
 - ▶ expected to have a per-cent level accuracy
 - ▶ yielding first reliable estimate of theoretical uncertainty
- ▶ For processes measured to few per cent accuracy
 - ▶ jet production
 - ▶ vector boson (+jet) production
 - ▶ top quark pair production
- ▶ For processes with potentially large perturbative corrections
 - ▶ New channels and/or phase space regions open up
 - ▶ Higgs or vector boson production

NNLO calculations

- ▶ Require three principal ingredients (here: $pp \rightarrow 2j$)
 - ▶ two-loop matrix elements
 - ▶ explicit infrared poles from loop integral
 - known for all massless $2 \rightarrow 2$ processes
 - ▶ one-loop matrix elements
 - ▶ explicit infrared poles from loop integral
 - ▶ and implicit poles from single real emission
 - usually known from NLO calculations
 - ▶ tree-level matrix elements
 - ▶ implicit poles from double real emission
 - known from LO calculations
- ▶ Infrared poles cancel in the sum
- ▶ **Challenge:** combine contributions into parton-level generator
 - ▶ Need a method to extract implicit infrared poles



Real radiation at NNLO: methods

▶ Sector decomposition

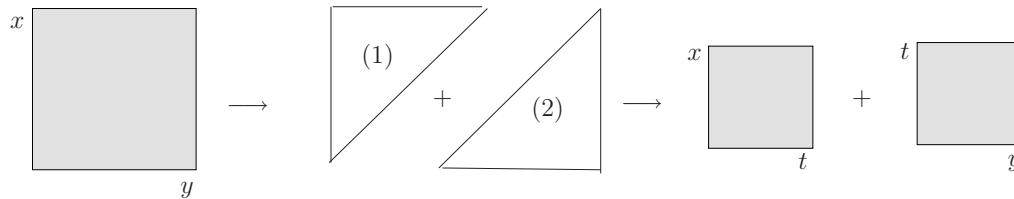
(T. Binoth, G. Heinrich; C. Anastasiou, K. Melnikov, F. Petriello)

▶ $pp \rightarrow H, pp \rightarrow V$, including decays

(C. Anastasiou, K. Melnikov, F. Petriello; S. Bühler, F. Herzog, A. Lazopoulos, R. Müller)

▶ Split final state phase space into different singular sectors

$$I = \int_0^1 dx \int_0^1 dy x^{-1-a\epsilon} y^{-b\epsilon} \left(x + (1-x)y \right)^{-1}$$



▶ Expand phase space integral in distributions

$$I_j = -\frac{1}{b_j\epsilon} \mathcal{I}_j(0, \{t_{i \neq j}\}, \epsilon) + \int_0^1 dt_j t_j^{-1-b_j\epsilon} \left(\mathcal{I}(t_j, \{t_{i \neq j}\}, \epsilon) - \mathcal{I}_j(0, \{t_{i \neq j}\}, \epsilon) \right)$$

Real radiation at NNLO: methods

▶ Sector-improved subtraction schemes

(M. Czakon; R. Boughezal, K. Melnikov, F. Petriello)

▶ $pp \rightarrow t\bar{t}$ (M. Czakon, P. Fiedler, A. Mitov)

▶ $pp \rightarrow H+j$ (R. Boughezal, F. Caola, K. Melnikov, F. Petriello, M. Schulze)

▶ $pp \rightarrow t+j$ (M. Brucherseifer, F. Caola, K. Melnikov)

▶ Construct subtraction term in each unresolved sector

▶ Using universal factorization properties of QCD matrix elements

▶ Fully local subtraction terms

▶ Expand subtraction terms in distributions

▶ Numerically integrate subtraction terms

Real radiation at NNLO: methods

▶ q_T -subtraction (S. Catani, M. Grazzini)

▶ $pp \rightarrow H, pp \rightarrow V, pp \rightarrow \gamma \gamma, pp \rightarrow VH$

(S. Catani, L. Cieri, D. de Florian, G. Ferrera M. Grazzini, F. Tramontano)

▶ $pp \rightarrow Z \gamma$ (M. Grazzini, S. Kallweit, D. Rathlev, A. Torre)

▶ $pp \rightarrow ZZ, pp \rightarrow VV$ (F. Cascioli, M. Grazzini, S. Kallweit, P. Maierhöfer, A. von Manteuffel, S. Pozzorini, D. Rathlev, L. Tancredi, E. Weihs, TG)

▶ Production of colourless final states at hadron colliders

▶ Universal behaviour in the limit of small transverse momentum, known from resummation

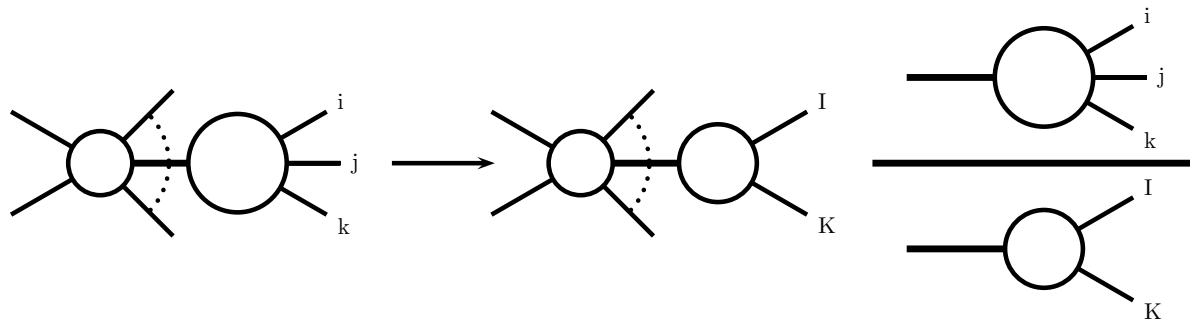
▶ Use small- q_T limit to construct subtraction term (non-local)

$$d\sigma_{NNLO}^F = \mathcal{H}_{NNLO}^F \otimes d\sigma_{LO}^F + \left[d\sigma_{NLO}^{F+jet} - d\sigma_{NLO}^{CT} \right]$$

▶ Implementation based on NLO calculation for **F+jet**

Real radiation at NNLO: methods

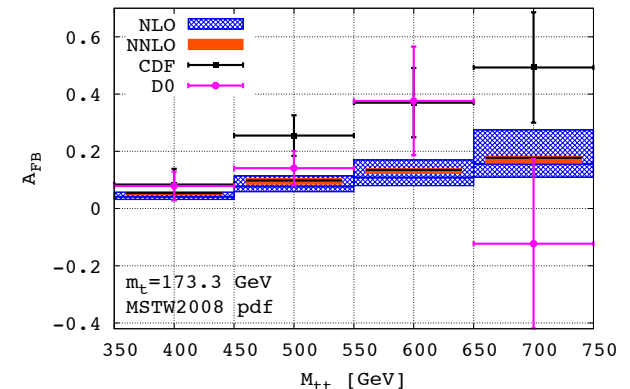
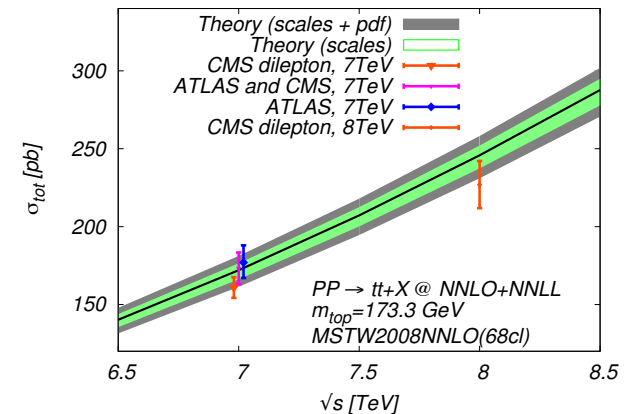
- ▶ **Antenna subtraction** (A. Gehrmann-De Ridder, E.W.N. Glover, TG)
 - ▶ $e^+e^- \rightarrow 3j$ (A. Gehrmann-De Ridder, E.W.N. Glover, G. Heinrich, TG; S. Weinzierl)
 - ▶ $pp \rightarrow 2j$ (J. Currie, A. Gehrmann-De Ridder, E.W.N. Glover, J. Pires, TG)
 - ▶ $pp \rightarrow H+j$ (X. Chen, E.W.N. Glover, M. Jaquier, TG)
 - ▶ $pp \rightarrow t\bar{t}$ (G. Abelof, A. Gehrmann-De Ridder, P. Maierhöfer, S. Pozzorini)
- ▶ **Construct subtraction terms from antenna functions**
 - ▶ Encapsulate all unresolved limits between a pair of hard partons



- ▶ Ensure analytical cancellation of poles

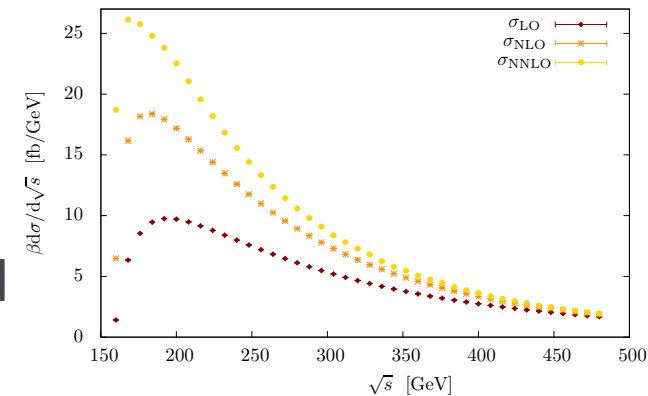
Top quark pair production at LHC

- ▶ Large production cross section at the LHC ($\sim 250\text{pb}$ at 8TeV)
 - ▶ Expected experimental error of $\sim 5\%$ for $\sigma_{t\bar{t}}$
 - ▶ NLO+NLL predictions yield an uncertainty of $\sim 10\%$
- ▶ NNLO accuracy of theory needed
- ▶ Calculation for the total cross section completed (M. Czakon, P. Fiedler, A. Mitov)
 - ▶ based on sector-improved subtraction
 - ▶ numerical cancellation of infrared poles
 - ▶ Observe: theoretical and experimental uncertainties comparable (% level)
- ▶ Differential distributions in progress
 - ▶ Forward-backward asymmetry at the Tevatron explained



Higgs+jet production at the LHC

- ▶ Essential to establish the properties of the newly discovered Higgs boson
- ▶ Experiments select events according to number of jets
 - ▶ Different backgrounds for different jet multiplicities
 - ▶ **H+0jet** and **H+1jet** samples of comparable sizes
 - ▶ **H+0jet** and inclusive **H** production known at NNLO
(C.Anastasiou, K. Melnikov, F. Petriello; S.Catani, M. Grazini)
 - ▶ **H+1jet** and **H+2jet** known at NLO
- ▶ NNLO for **H+1jet** needed
 - ▶ gluons-only total cross section completed
(R. Boughezal, F. Caola, K. Melnikov, F. Petriello, M. Schulze)
 - ▶ Full calculation and differential distributions in progress
(X. Chen, T. Gehrmann, E.W.N. Glover, M. Jaquier)

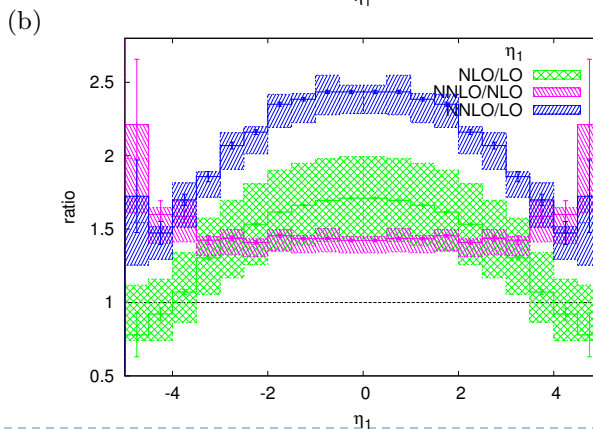
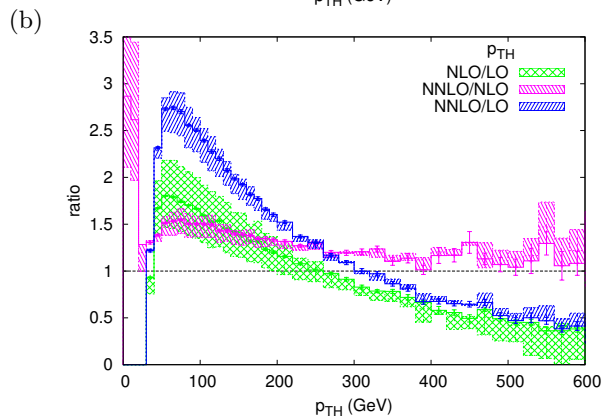
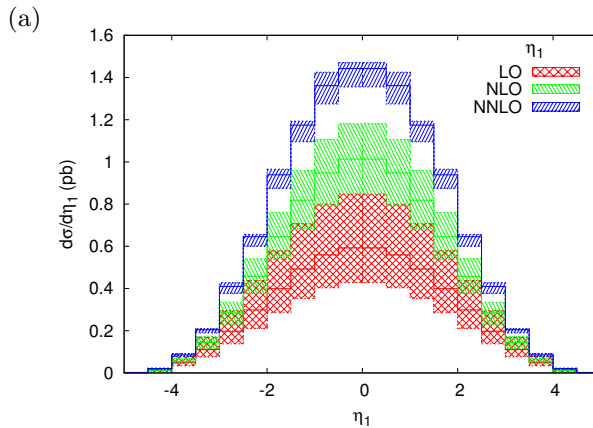
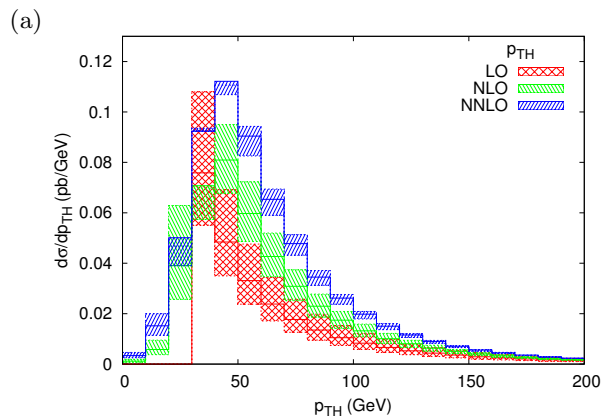


Higgs+jet production at NNLO

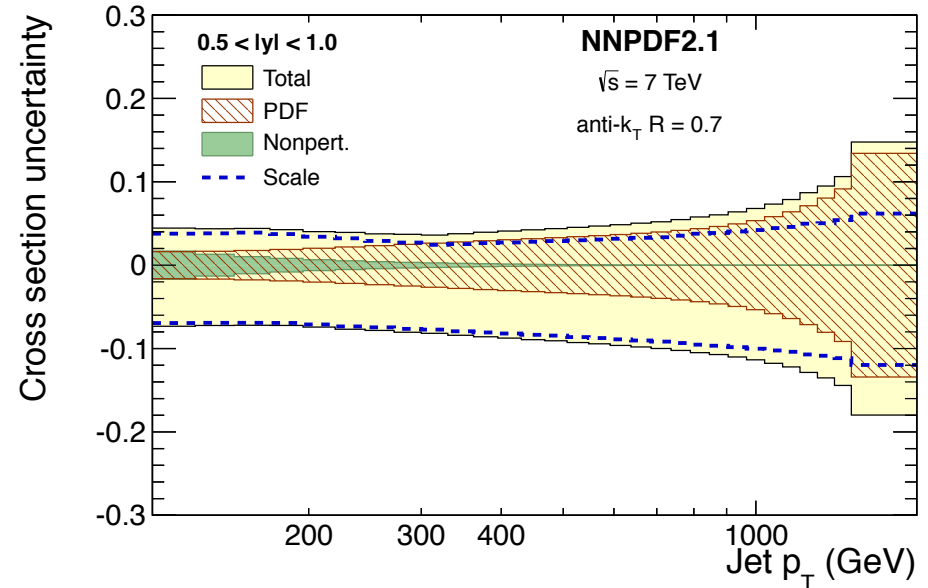
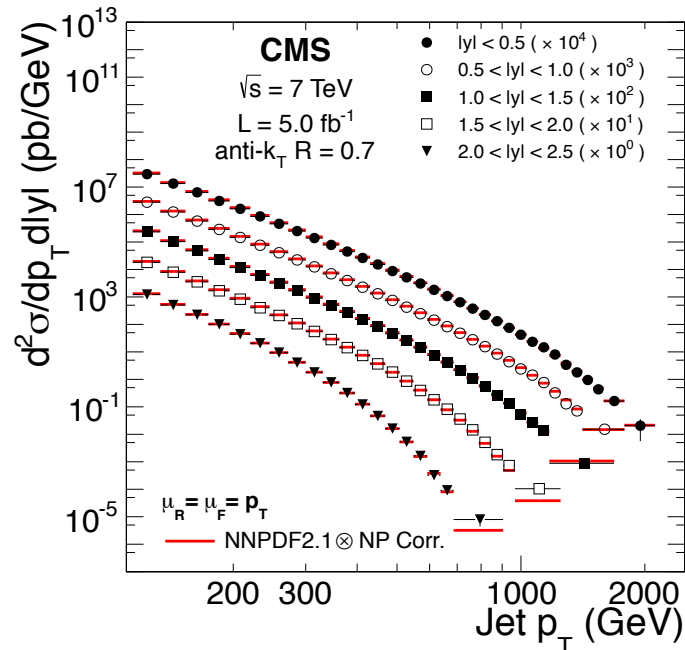
- ▶ Distributions for **H+jet** total cross section (gluons only)
(X. Chen, E.W.N. Glover, M. Jaquier, TG)

- ▶ Using antenna subtraction

$p_{tj} > 30$ GeV, k_T -alg., $R=0.5$



Jet cross sections at LHC



- ▶ Jet data can be used to constrain parton distributions
- ▶ Scale and PDF uncertainties on NLO prediction of comparable size
- ▶ Need improved theory (NNLO)

pp \rightarrow 2 jets at NNLO

▶ First results at NNLO available

- ▶ $gg \rightarrow gg$ and $q\bar{q} \rightarrow gg$ subprocess

(J. Currie, A. Gehrmann-De Ridder, E.W.N. Glover, J. Pires, TG)

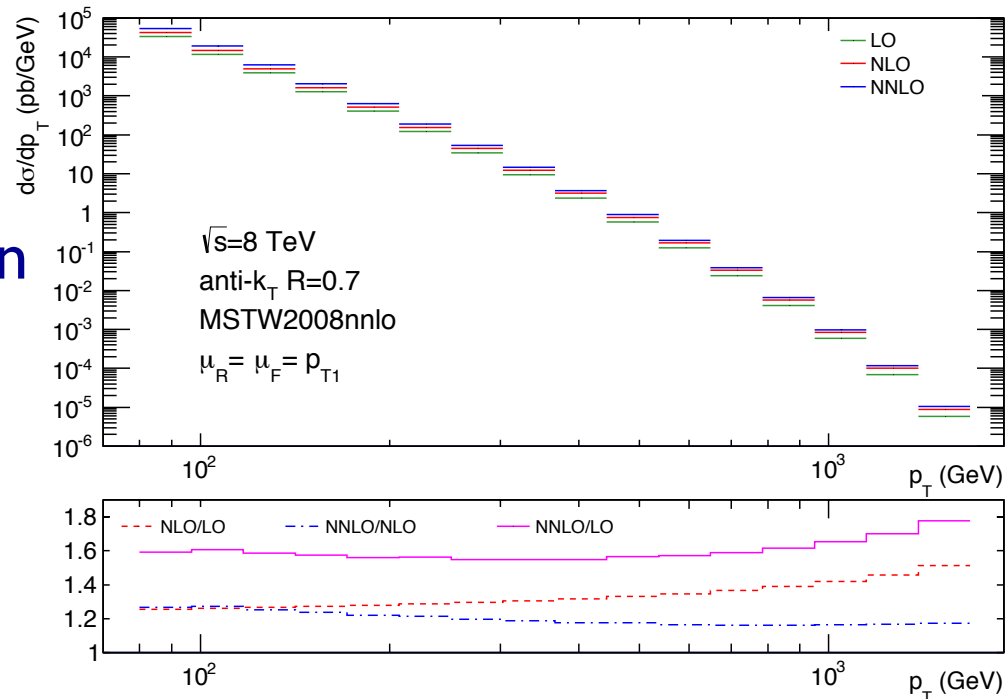
- ▶ Developed a new parton-level event generator NNLOJET

- ▶ using antenna subtraction

- ▶ analytic cancellation of infrared poles

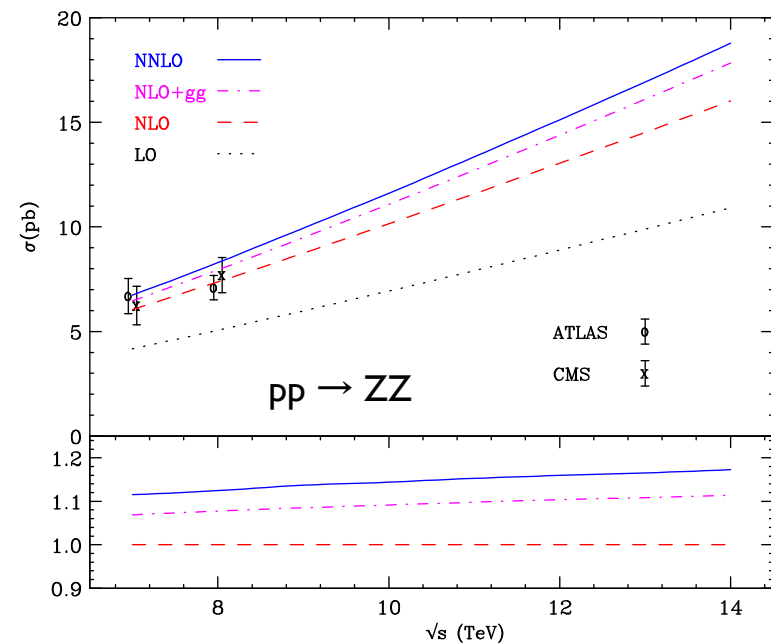
▶ Inclusive jet p_T distribution

- ▶ NNLO/NLO differential K-factor flat over the whole p_T range



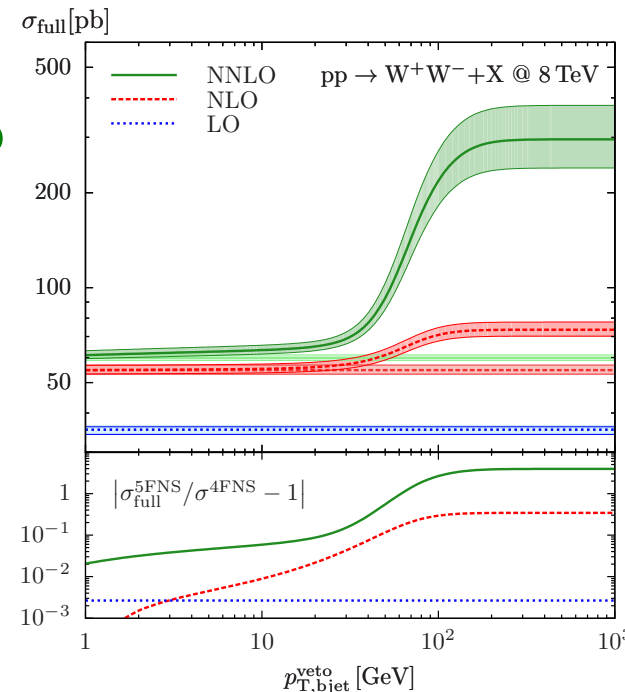
pp \rightarrow VV at NNLO

- ▶ Vector boson pair production
 - ▶ Test standard model coupling structure (anomalous couplings)
 - ▶ Final state configurations similar to beyond-SM signatures
- ▶ Recently completed using q_T -subtraction
 - ▶ $pp \rightarrow Z \gamma, pp \rightarrow W \gamma$
(M. Grazzini, S. Kallweit, D. Rathlev, A. Torre)
 - ▶ $pp \rightarrow ZZ$ (F. Cascioli et al.)
 - ▶ Moderate NNLO corrections, about half from $gg \rightarrow ZZ$



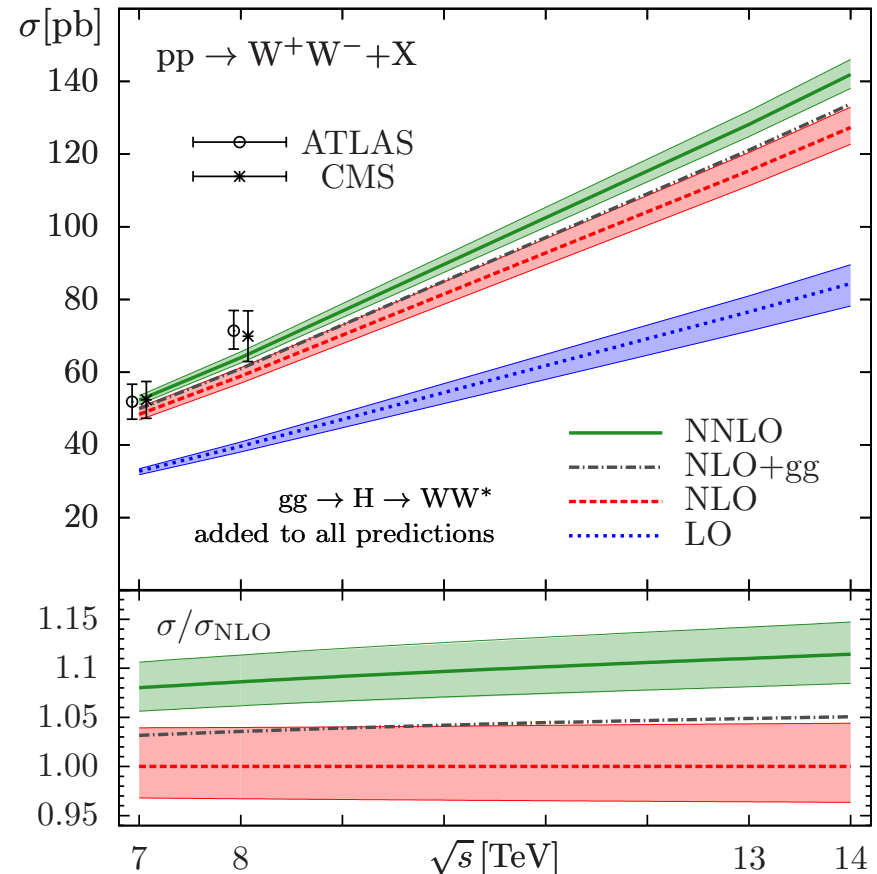
$pp \rightarrow W^+W^-$ at NNLO

- ▶ Total cross section for W pair production
 - ▶ $pp \rightarrow WW$ (M. Grazzini, S. Kallweit, P. Maierhöfer, A. von Manteuffel, S. Pozzorini, D. Rathlev, L. Tancredi, TG)
- ▶ At higher orders: $pp \rightarrow WW$ not well defined
 - ▶ NLO: contribution from $gb \rightarrow WWb$
 - ▶ NNLO: contribution from $q\bar{q}/gg \rightarrow WWb\bar{b}$
 - ▶ Can not be removed consistently in 5FNS
 - ▶ Define 5FNS contribution from scaling behaviour with top quark width
 - ▶ Good agreement of 4FNS and 5FNS



pp \rightarrow W^+W^- at NNLO

- ▶ Total cross section in 4FNS
 - ▶ Improved description of data
 - ▶ Data based on interpolation from fiducial region
 - ▶ Calls for fully differential description, including vector boson decays and off-shell effects



▶ **Precision frontier: aims and ideas**

Towards NNLO automation

- ▶ **Methods for real radiation at NNLO becoming mature**
 - ▶ q_T subtraction
 - ▶ Sector-improved schemes
 - ▶ Antenna subtraction
- ▶ **Issues**
 - ▶ Automation of code generation
 - ▶ Numerical efficiency and stability

Towards NNLO automation

- ▶ **Virtual two-loop amplitudes: analytically process-by-process**
 - ▶ **Current stockpile**
 - ▶ $pp \rightarrow 2j$ (C. Anastasiou, N. Glover, C. Oleari, M. Tejeda-Yeomans; Z. Bern, L. Dixon, A. De Freitas)
 - ▶ $pp \rightarrow V+j$ (L. Garland, N. Glover, A. Koukoutsakis, E. Remiddi, TG)
 - ▶ $pp \rightarrow V+ \gamma$ (L. Tancredi, E. Weihs, TG)
 - ▶ $pp \rightarrow H+j$ (N. Glover, M. Jaquier, A. Koukoutsakis, TG)
 - ▶ $pp \rightarrow tt$ (P. Bärnreuther, M. Czakon, P. Fiedler; R. Bonciani, A. Ferroglia, A. von Manteuffel, C. Studerus, TG)
 - ▶ $pp \rightarrow VV$ (L. Tancredi, E. Weihs, TG; F. Caola, J. Henn, K. Melnikov, V. Smirnov, A. Smirnov)
 - ▶ **Research directions: towards different masses and $2 \rightarrow 3$**
 - ▶ Systematic techniques to compute master integrals (J. Henn)
 - ▶ Semi-numerical approaches (P. Bärnreuther, M. Czakon, P. Fiedler)
 - ▶ Classification of integral basis (H. Johansson, D. Kosower, K. Larsen)
 - ▶ Unitarity-based methods (P. Mastrolia, E. Mirabella, G. Ossola, T. Peraro)

NNLO and beyond: techniques

- ▶ **Seemingly simple task: check equality of two expressions**
 - ▶ Becomes very tricky if complicated functions involved
 - ▶ e.g. Abel relation (1855)

$$\ln(1-x)\ln(1-y) = \text{Li}_2\left(\frac{x}{1-y}\right) + \text{Li}_2\left(\frac{y}{1-x}\right) - \text{Li}_2(x) - \text{Li}_2(y) - \text{Li}_2\left(\frac{xy}{(1-x)(1-y)}\right)$$

- ▶ **Systematic procedure for iterated rational integrals**
 - ▶ Symbol and coproduct (A. Goncharov, M. Spradlin, A. Volovich, C. Vergu; C. Duhr)
 - ▶ Often allows huge simplifications (many pages → few lines)
- ▶ **starts to get used for loop integrals**
 - ▶ simplification
 - ▶ analytical continuation
 - ▶ automated derivation of relations

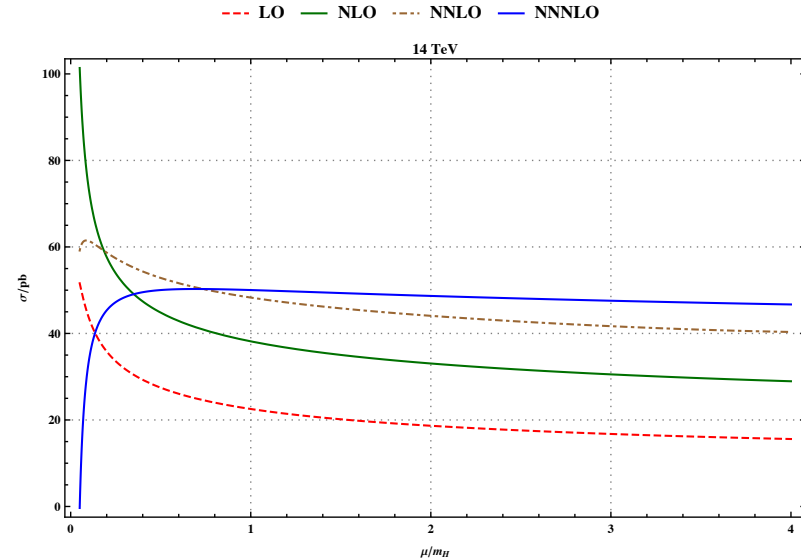
Beyond NNLO: observables

- ▶ **Hadronic R-ratio in e^+e^-**
 - ▶ Most precise QCD observable in Z and τ decays
 - ▶ Known to $\mathcal{O}(\alpha_s^4)$ (P. Baikov, K. Chetyrkin, H. Kühn, J. Rittinger)
 - ▶ Produces most precise $\alpha_s(M_Z) = 0.1198 \pm 0.0015$
- ▶ **Gluon-fusion Higgs cross section at hadron colliders**
 - ▶ Large NLO and NNLO corrections
 - ▶ Ultimate precision on Higgs couplings may require N³LO
 - ▶ Ingredients
 - ▶ Three-loop vertex functions (P. Baikov, K. Chetyrkin, A. Smirnov, V. Smirnov, M. Steinhauser; N. Glover, T. Huber, N. Ikizlerli, C. Studerus, TG)
 - ▶ Counterterms and lower-order expansions (C. Anastasiou, S. Bühler, C. Duhr, F. Herzog; M. Höschele, J. Hoff, A. Pak, M. Steinhauser, T. Ueda)
 - ▶ Triple real radiation (C. Anastasiou, C. Duhr, F. Dulat, B. Mistlberger)
 - ▶ Interplay of real and virtual corrections at N³LO (C. Duhr et al.)
 - ▶ Major work in progress

Towards $gg \rightarrow H$ at N³LO

- ▶ Expand coefficient function around production threshold
(C. Anastasiou, C. Duhr, F. Dulat, E. Furlan, F. Herzog, B. Mistlberger, TG)
- ▶ Reliable prediction: need further terms in threshold expansion

$$\begin{aligned}
 \hat{\eta}^{(3)}(z) = & \delta(1-z) \left\{ C_A^3 \left(-\frac{2003}{48} \zeta_6 + \frac{413}{6} \zeta_3^2 - \frac{7579}{144} \zeta_5 + \frac{979}{24} \zeta_2 \zeta_3 - \frac{15257}{864} \zeta_4 - \frac{819}{16} \zeta_3 + \frac{16151}{1296} \zeta_2 + \frac{215131}{5184} \right) \right. \\
 & + N_F \left[C_A^2 \left(\frac{869}{72} \zeta_5 - \frac{125}{12} \zeta_3 \zeta_2 + \frac{2629}{432} \zeta_4 + \frac{1231}{216} \zeta_3 - \frac{70}{81} \zeta_2 - \frac{98059}{5184} \right) \right. \\
 & \quad \left. + C_A C_F \left(\frac{5}{2} \zeta_5 + 3\zeta_3 \zeta_2 + \frac{11}{72} \zeta_4 + \frac{13}{2} \zeta_3 - \frac{71}{36} \zeta_2 - \frac{63991}{5184} \right) + C_F^2 \left(-5\zeta_5 + \frac{37}{12} \zeta_3 + \frac{19}{18} \right) \right] \\
 & + N_F^2 \left[C_A \left(-\frac{19}{36} \zeta_4 + \frac{43}{108} \zeta_3 - \frac{133}{324} \zeta_2 + \frac{2515}{1728} \right) + C_F \left(-\frac{1}{36} \zeta_4 - \frac{7}{6} \zeta_3 - \frac{23}{72} \zeta_2 + \frac{4481}{2592} \right) \right] \left. \right\} \\
 & + \left[\frac{1}{1-z} \right]_+ \left\{ C_A^3 \left(186 \zeta_5 - \frac{725}{6} \zeta_3 \zeta_2 + \frac{253}{24} \zeta_4 + \frac{8941}{108} \zeta_3 + \frac{8563}{324} \zeta_2 - \frac{297029}{23328} \right) + N_F^2 C_A \left(\frac{5}{27} \zeta_3 + \frac{10}{27} \zeta_2 - \frac{58}{729} \right) \right. \\
 & \left. + N_F \left[C_A^2 \left(-\frac{17}{12} \zeta_4 - \frac{475}{36} \zeta_3 - \frac{2173}{324} \zeta_2 + \frac{31313}{11664} \right) + C_A C_F \left(-\frac{1}{2} \zeta_4 - \frac{19}{18} \zeta_3 - \frac{1}{2} \zeta_2 + \frac{1711}{864} \right) \right] \right\} \\
 & + \left[\frac{\log(1-z)}{1-z} \right]_+ \left\{ C_A^3 \left(-77\zeta_4 - \frac{352}{3} \zeta_3 - \frac{152}{3} \zeta_2 + \frac{30569}{648} \right) + N_F^2 C_A \left(-\frac{4}{9} \zeta_2 + \frac{25}{81} \right) \right. \\
 & \left. + N_F \left[C_A^2 \left(\frac{46}{3} \zeta_3 + \frac{94}{9} \zeta_2 - \frac{4211}{324} \right) + C_A C_F \left(6\zeta_3 - \frac{63}{8} \right) \right] \right\} \\
 & + \left[\frac{\log^2(1-z)}{1-z} \right]_+ \left\{ C_A^3 \left(181\zeta_3 + \frac{187}{3} \zeta_2 - \frac{1051}{27} \right) + N_F \left[C_A^2 \left(-\frac{34}{3} \zeta_2 + \frac{457}{54} \right) + \frac{1}{2} C_A C_F \right] - \frac{10}{27} N_F^2 C_A \right\} \\
 & + \left[\frac{\log^3(1-z)}{1-z} \right]_+ \left\{ C_A^3 \left(-56\zeta_2 + \frac{925}{27} \right) - \frac{164}{27} N_F C_A^2 + \frac{4}{27} N_F^2 C_A \right\} \\
 & + \left[\frac{\log^4(1-z)}{1-z} \right]_+ \left(\frac{20}{9} N_F C_A^2 - \frac{110}{9} C_A^3 \right) + \left[\frac{\log^5(1-z)}{1-z} \right]_+ 8 C_A^3.
 \end{aligned}$$



▶ **Instead of a summary: Outlook**

Where do we stand?

- ▶ **Witnessed an NLO revolution**
 - ▶ Previously unthinkable NLO multi-particle calculations now feasible due to technological breakthroughs
 - ▶ High-level of automation
 - ▶ Standardization of interfaces: combine different codes (providers)
 - ▶ Interface to experiment (codes, ntuples, histograms,..)?
- ▶ **NLO and parton showers**
 - ▶ Matching of individual processes (MC@NLO, POWHEG)
- ▶ **Substantial progress on NNLO calculations**
 - ▶ Several different methods available
 - ▶ Calculations on process-by-process basis
 - ▶ Codes typically require HPC infrastructure

Future Directions

- ▶ **NLO+PS as new standard for event generation**
 - ▶ Fully automated public codes
 - ▶ Consistent matching to parton shower
 - ▶ Matching of different multiplicities at NLO
 - ▶ Monte Carlo with NLO-accurate event samples
- ▶ **NNLO automation**
 - ▶ Uncover analytical structures to organize calculation of real and virtual corrections
 - ▶ Develop standard interfaces
 - ▶ Interface to experiment ?
- ▶ **Beyond NNLO**
 - ▶ N^3 LO precision for benchmark processes

- ▶ Progress on precision physics on many frontiers
- ▶ Be prepared for exciting times at the Terascale

