# The MSSM at $\tan\beta = \infty$ and $(g-2)_\mu$

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# Muon Anomalous Magnetic Moment

- defined by: $a_\mu := \frac{(g-2)_\mu}{2} = -2\, m_\mu\, F_M(0)$
- experiment: $a_\mu^{\mathsf{exp}} = (11\,659\,208.9 \pm 6.3) \cdot 10^{-10}$ (E821 at BNL)

  $\Rightarrow$ new measurements in preparation (Fermilab, J-PARC)
- SM prediction: $a_\mu^{\mathsf{SM}} = (11\,659\,180.2 \pm 4.9) \cdot 10^{-10}$ [arXiv:1010.4180]
- discrepancy: $\Delta a_\mu = a_\mu^{\mathsf{exp}} - a_\mu^{\mathsf{SM}} = (28.7 \pm 8.0) \cdot 10^{-10} \mathrel{\widehat{=}} 3.6\,\sigma$

new physics?

# Contributions from New Physics

- $a_\mu$ not only points to BSM physics but also restricts it
- close connection between $a_\mu$ und muon self-energy:

$$a_\mu^{\mathsf{NP}} = \mathcal{O}(1) \times \left(\frac{m_\mu}{M_{\mathsf{NP}}}\right)^2 \times \left(\frac{\Delta m_\mu(\mathsf{NP})}{m_\mu}\right)$$

[arXiv:hep-ph/0102122]

$\Rightarrow \Delta m_\mu(\mathsf{NP}) \approx m_\mu$ promising

- MSSM one-loop result for common SUSY mass $M_S$:

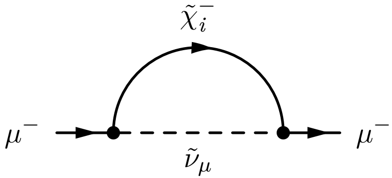
$$a_\mu^{\mathsf{SUSY,1L}} \approx 1.3 \cdot 10^{-9} \; \mathrm{sgn}(\mu M_2) \; \tan\beta \; \left(\frac{100 \text{ GeV}}{M_S}\right)^2$$

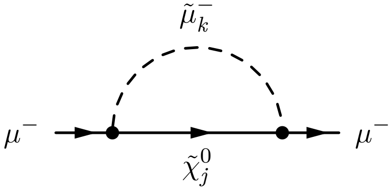
with ratio of Higgs VEVs: $\tan\beta := v_u / v_d$

What happens for $\tan\beta \to \infty$?

# Radiative Muon Mass Generation

- tree-level: $m_\mu = m_\mu^0 \equiv y_\mu \, v_d \quad \Rightarrow \tan\beta \lesssim 50$ for perturbative $y_\mu$
- $\tan\beta \gg 50$ possible if $\tan\beta$-enhanced loop corrections are considered
$$\Rightarrow m_\mu = m_\mu^0 - \Sigma_\mu = m_\mu^0 \, (1 + \Delta_\mu)$$
- $\tan\beta = \infty$ implies: $\quad v_d = 0 \, , \quad v_u = \sqrt{2} \, M_W / g_2$
$$\Rightarrow m_\mu^0 = 0 \quad \Rightarrow m_\mu = -\Sigma_\mu$$
- MSSM one-loop diagrams contributing to $\Sigma_\mu$:



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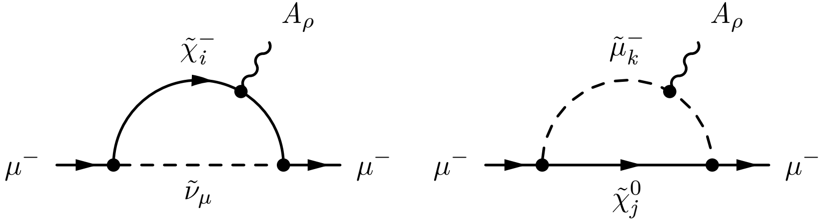

# Properties of the Analytic Results for $\Sigma_\mu^{\text{MSSM}}$ and $a_\mu^{\text{SUSY}}$

- relevant free MSSM parameters: $y_\mu, \mu, M_1, M_2, m_L, m_R$ (real)
- $m_\mu = -\Sigma_\mu^{\text{MSSM}}$ can be used to determine $y_\mu$ numerically
  $\Rightarrow$ $a_\mu^{\text{SUSY}}$ only depends on the five mass parameters
- $a_\mu^{\text{SUSY}}$ invariant under $\mu \to -\mu$ and $(M_1, M_2) \to (-M_1, -M_2)$
  $\Rightarrow$ choose $\mu$ and $M_1$ to be positive; sign of $M_2$ remains arbitrary

**Which sign is typical for $a_\mu^{\text{SUSY}}$?**

# Approximating the Results

- approximation: $M_Z \ll \mu, M_1, |M_2|, m_L, m_R$

  $\Rightarrow$ SUSY mass eigenstates equivalent to interaction eigenstates

- $\Sigma_\mu^{\text{MSSM}} \approx y_\mu \, \Sigma_{\mu,\text{red}}^{\text{MSSM}}$ , $a_\mu^{\text{SUSY}} \approx y_\mu \, a_{\mu,\text{red}}^{\text{SUSY}} \approx -m_\mu \, a_{\mu,\text{red}}^{\text{SUSY}} / \Sigma_{\mu,\text{red}}^{\text{MSSM}}$

- scaling behavior for $k > 0$:

$$a_\mu^{\text{SUSY}}(k\,\mu, \ldots, k\,m_R) \approx \frac{1}{k^2}\, a_\mu^{\text{SUSY}}(\mu, \ldots, m_R)$$

- result for common SUSY mass $M_S$:

$$a_\mu^{\text{SUSY}} \approx \frac{(1000\ \text{GeV})^2}{M_S^2} \begin{cases} -7.25 \cdot 10^{-9} & \text{for } M_2 > 0 \\ -5.34 \cdot 10^{-9} & \text{for } M_2 < 0 \end{cases}$$

discrepancy: $\Delta a_\mu = a_\mu^{\text{exp}} - a_\mu^{\text{SM}} \approx 3 \cdot 10^{-9}$

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How can a positive sign be achieved?

# Approximate Formulas

$$\Sigma_\mu^{\mathsf{MSSM}} \approx y_\mu \Bigg[ \operatorname{sgn}(M_2)\, 0.705\,\text{GeV}\ \hat{I}\left(\frac{|M_2|}{m_L}, \frac{\mu}{m_L}\right) - 0.067\,\text{GeV}\ \hat{I}\left(\frac{M_1}{m_L}, \frac{\mu}{m_L}\right) + 0.135\,\text{GeV}\ \hat{I}\left(\frac{M_1}{m_R}, \frac{\mu}{m_R}\right) - 0.135\,\text{GeV}\ \hat{I}\left(\frac{M_1}{m_R}, \frac{m_L}{m_R}\right) \frac{\mu}{m_L} \Bigg]$$

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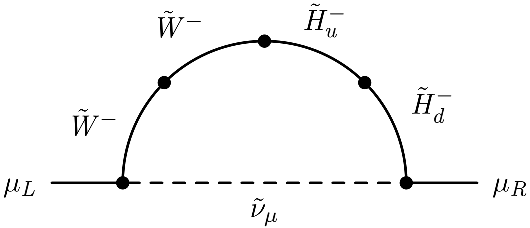
$\tilde{W}$–$\tilde{H}, \tilde{\nu}_\mu$ :



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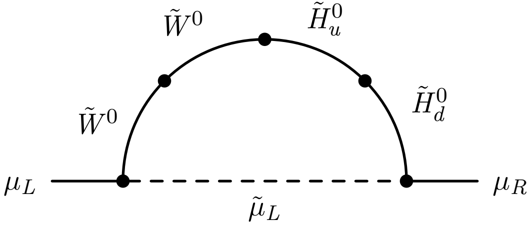

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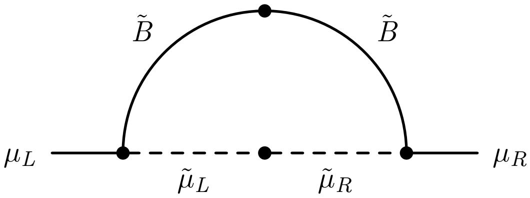
$\tilde{B}, \tilde{\mu}_L$–$\tilde{\mu}_R$ :



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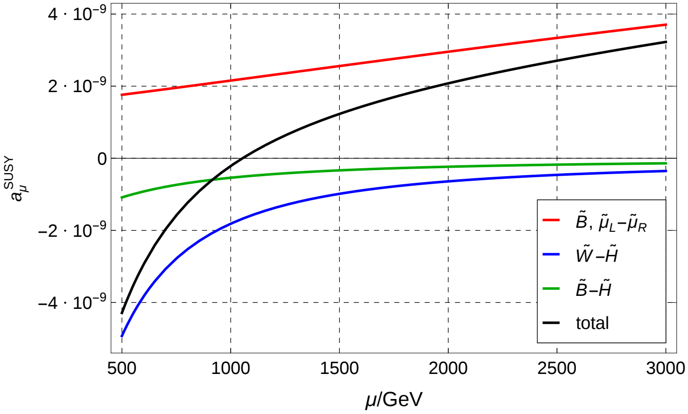
$$\begin{aligned}a_\mu^{\mathsf{SUSY}} \approx y_\mu \Bigg[ &\operatorname{sgn}(M_2)\, 2.483 \cdot 10^{-8}\ \frac{(1000\ \mathrm{GeV})^2}{\mu\,|M_2|}\ \hat{K}_W\left(\frac{M_2^2}{m_L^2}, \frac{\mu^2}{m_L^2}\right) \\ &+ 0.712 \cdot 10^{-8}\ \frac{(1000\ \mathrm{GeV})^2}{\mu\, M_1}\ \hat{K}_N\left(\frac{M_1^2}{m_L^2}, \frac{\mu^2}{m_L^2}\right) \\ &- 1.425 \cdot 10^{-8}\ \frac{(1000\ \mathrm{GeV})^2}{\mu\, M_1}\ \hat{K}_N\left(\frac{M_1^2}{m_R^2}, \frac{\mu^2}{m_R^2}\right) \\ &+ 1.425 \cdot 10^{-8}\ \frac{\mu\, M_1\, (1000\ \mathrm{GeV})^2}{m_L^2\, m_R^2}\ \hat{K}_N\left(\frac{m_L^2}{M_1^2}, \frac{m_R^2}{M_1^2}\right) \Bigg]\end{aligned}$$

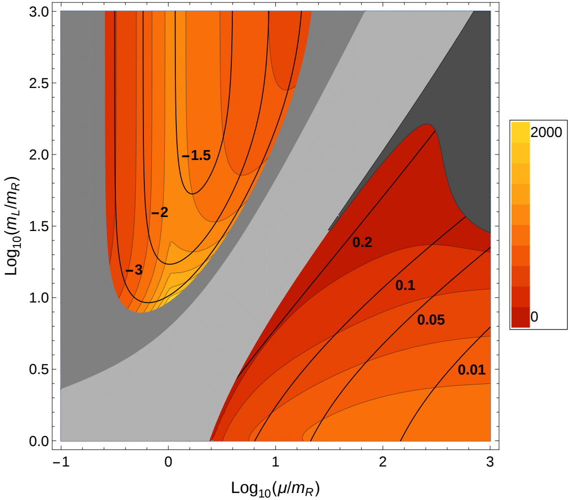
# Dependence of $a_\mu^{\text{SUSY}}$ on the Higgs parameter $\mu$

$M_2 = -3000\,\text{GeV}$ and $M_1 = m_L = m_R = 600\,\text{GeV}$ fixed



# Minimal SUSY Particle Mass for $a_\mu^{\text{SUSY}} = 3 \cdot 10^{-9}$

use $M_2 = -\mu$ and $M_1 = m_R$



# Conclusion and Outlook

- MSSM at $\tan\beta = \infty$ is capable of explaining the $a_\mu$ discrepancy
- mass splitting required; large masses compared to the normal MSSM
- huge $y_\mu$ values $\Rightarrow$ interesting consequences
- further analyses: Higgs mass, tau sector, vacuum stability
- paper on this topic will be published soon

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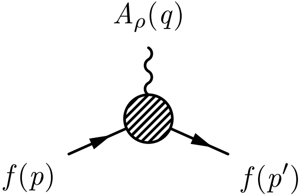
## Thanks for your attention!

# Backup Slides

# Definition of the Anomalous Magnetic Moment

- magn. moment of a fermion: $\vec{\mu} = g \, \frac{Q \, e \, \hbar}{2 \, m \, c} \, \vec{s}$ ($g$ - gyromagn. ratio)
- interaction with an external photon:




$$= -\mathrm{i} \, Q \, e \, \bar{u}(p') \, \Lambda_\rho \, u(p)$$

- decomposition:

$$\bar{u}(p') \, \Lambda_\rho \, u(p) = \bar{u}(p') \left[ \gamma_\rho \, \left( F_E(q^2) - 2 \, m \, F_M(q^2) \right) + (p + p')_\rho \, F_M(q^2) + \ldots \right] u(p)$$

- classical limit: $g = 2 \left( 1 - 2 \, m \, F_M(0) \right)$
- anomalous magnetic moment: $a := \frac{g - 2}{2} = -2 \, m \, F_M(0)$

# Experimental Determination of $a_\mu$

- muons from pion decay in a storage ring
- spin precession in magnetic field $\vec{B}$ relative to the momentum with

$$\vec{\omega} = \frac{e}{m_\mu}\left[a_\mu\,\vec{B} - \left(a_\mu - \frac{1}{\gamma^2 - 1}\right)\frac{\vec{\beta}\times\vec{E}}{c}\right]$$

vanishes for “magic” momentum $p = 3.094\,\mathrm{GeV}$

- detection of electrons from muon decay
- result from E821 experiment at Brookhaven National Laboratory:

$$a_\mu^{\mathsf{exp}} = (11\,659\,208.9 \pm 6.3)\cdot 10^{-10}$$

# Standard Model Prediction for $a_\mu$

- different kinds of contributions
  - QED: massless class, massive class
  - hadronic: vacuum polarization, light-by-light scattering
  - electroweak
- combined result [Davier et al., arXiv:1010.4180]:

$$a_\mu^{\mathsf{SM}} = (11\,659\,180.2 \pm 4.9) \cdot 10^{-10}$$

- discrepancy between experiment and theory:

$$\Delta a_\mu = a_\mu^{\mathsf{exp}} - a_\mu^{\mathsf{SM}} = (28.7 \pm 8.0) \cdot 10^{-10}$$

new physics?

# Muon Sneutrino and Smuon Masses

- muon sneutrino mass: $m^2_{\tilde{\nu}_\mu} = m^2_L + \frac{1}{2}\, M^2_Z \cos 2\beta$
- smuon mass matrix:

$$M^2_{\tilde{\mu}} = \begin{pmatrix} (m^0_\mu)^2 + m^2_L + M^2_Z\,(s^2_W - \frac{1}{2}) \cos 2\beta & m^0_\mu\,(A^*_\mu - \mu \tan\beta) \\ m^0_\mu\,(A_\mu - \mu^* \tan\beta) & (m^0_\mu)^2 + m^2_R - M^2_Z\, s^2_W \cos 2\beta \end{pmatrix}$$

- diagonalization: $U^{\tilde{\mu}}\, M^2_{\tilde{\mu}}\, U^{\tilde{\mu}\dagger} = \mathrm{diag}(m^2_{\tilde{\mu}_1}, m^2_{\tilde{\mu}_2})$

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$$M_{\tilde{\mu}}^2 = \begin{pmatrix} m_L^2 - M_Z^2\,(s_W^2 - \frac{1}{2}) & -\sqrt{2}\, y_\mu\, \mu\, M_W/g_2 \\ -\sqrt{2}\, y_\mu\, \mu^*\, M_W/g_2 & m_R^2 + M_Z^2\, s_W^2 \end{pmatrix}$$

- diagonalization: $U^{\tilde{\mu}} M_{\tilde{\mu}}^2\, U^{\tilde{\mu}\dagger} = \mathrm{diag}(m_{\tilde{\mu}_1}^2, m_{\tilde{\mu}_2}^2)$
- smuon masses:

$$m_{\tilde{\mu}_{1/2}}^2 = \frac{1}{2}\left( m_L^2 + m_R^2 + \frac{1}{2}\, M_Z^2 \pm \sqrt{\left[m_L^2 - m_R^2 + M_Z^2\left(\frac{1}{2} - 2\, s_W^2\right)\right]^2 + 8\, y_\mu^2\, |\mu|^2\, \frac{M_W^2}{g_2^2}} \right)$$

# Chargino Masses

- chargino mass matrix: $X = \begin{pmatrix} M_2 & \sqrt{2}\, M_W \sin\beta \\ \sqrt{2}\, M_W \cos\beta & \mu \end{pmatrix}$
- diagonalization: $U^* X\ V^{-1} = \mathrm{diag}(m_{\chi_1^-}, m_{\chi_2^-})$
- Hermitian form: $V\ X^\dagger X\ V^{-1} = \mathrm{diag}(m^2_{\chi_1^-}, m^2_{\chi_2^-})$

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- chargino mass matrix: $X = \begin{pmatrix} M_2 & \sqrt{2}\, M_W \\ 0 & \mu \end{pmatrix}$
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$$m^2_{\chi^-_{1/2}} = \frac{1}{2}\Big(|\mu|^2 + |M_2|^2 + 2\, M_W^2 \pm \sqrt{\left(|\mu|^2 + |M_2|^2 + 2\, M_W^2\right)^2 - 4\, |\mu|^2\, |M_2|^2}\,\Big)$$

# Neutralino Masses

- neutralino mass matrix:

$$Y = \begin{pmatrix} M_1 & 0 & -M_Z\, s_W \cos\beta & M_Z\, s_W \sin\beta \\ 0 & M_2 & M_Z\, c_W \cos\beta & -M_Z\, c_W \sin\beta \\ -M_Z\, s_W \cos\beta & M_Z\, c_W \cos\beta & 0 & -\mu \\ M_Z\, s_W \sin\beta & -M_Z\, c_W \sin\beta & -\mu & 0 \end{pmatrix}$$

- diagonalization: $N^*\ Y\, N^{-1} = \mathrm{diag}(m_{\chi_1^0}, m_{\chi_2^0}, m_{\chi_3^0}, m_{\chi_4^0})$
- Hermitian form: $N\ Y^\dagger\, Y\, N^{-1} = \mathrm{diag}(m^2_{\chi_1^0}, m^2_{\chi_2^0}, m^2_{\chi_3^0}, m^2_{\chi_4^0})$

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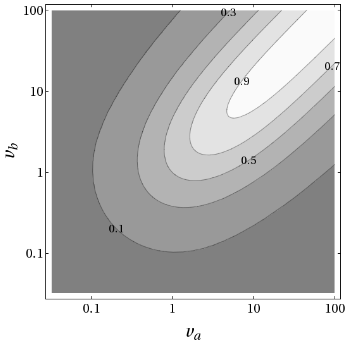
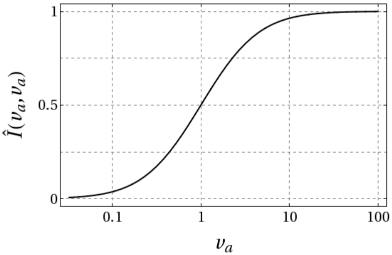
# Loop Functions I

- dimensionless loop function:

$$\hat{I}\left(\frac{m_a}{m_c}, \frac{m_b}{m_c}\right) := m_a\, m_b\, I(m_a^2, m_b^2, m_c^2)$$

- $\forall\, v_a, v_b : \quad 0 < \hat{I}(v_a, v_b) < 1$



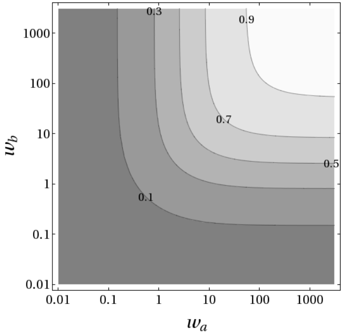



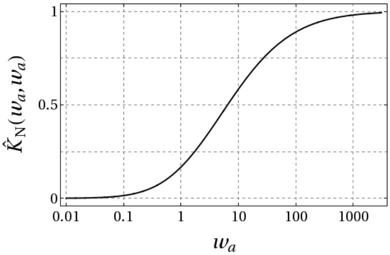
# Loop Functions II

- dimensionless loop function:

$$\hat{K}_{\mathrm{N}}\left(\frac{q_a}{q_c}, \frac{q_b}{q_c}\right) := -q_a\, q_b\, K_{\mathrm{N}}(q_c, q_a, q_b)$$

- $\forall\, w_a, w_b : \quad 0 < \hat{K}_{\mathrm{N}}(w_a, w_b) < 1$



# Loop Functions II

- dimensionless loop function:

$$\hat{K}_{\mathrm{W}}\left(\frac{q_a}{q_c}, \frac{q_b}{q_c}\right) := q_a\, q_b \left[\, 2\, K_{\mathrm{C}}(q_c, q_a, q_b) + K_{\mathrm{N}}(q_c, q_a, q_b) \,\right]$$

- $\forall\, w_a, w_b : \quad 0 < \hat{K}_{\mathrm{W}}(w_a, w_b) < 1.155$