Light cone dynamics and Kibble-Zurek mechanism in ultra-cold atom systems

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Overview

Phase transitions in equilibrium

- Thermal phase transitions. Critical exponents. Condensation in d>2, d=2
- Quantum phase transitions. Transverse Ising model

Dynamic phase transitions

- Light cone dynamics
  - In 1D and 2D condensates
  - In the transverse Ising model
- Dynamic scaling near a phase transition
  - Kibble-Zurek mechanism

Reverse Kibble-Zurek mechanism and light cone dynamics in 2D Bose gases

Conclusions

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Reverse Kibble-Zurek mechanism and light cone dynamics in 2D Bose gases

Conclusions

Phase transitions

Qualitative change of the collective, equilibrium behavior

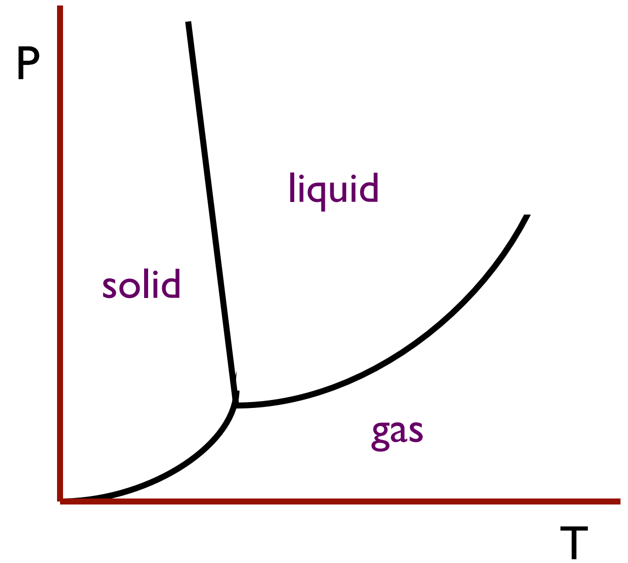
Bose-Einstein condensation

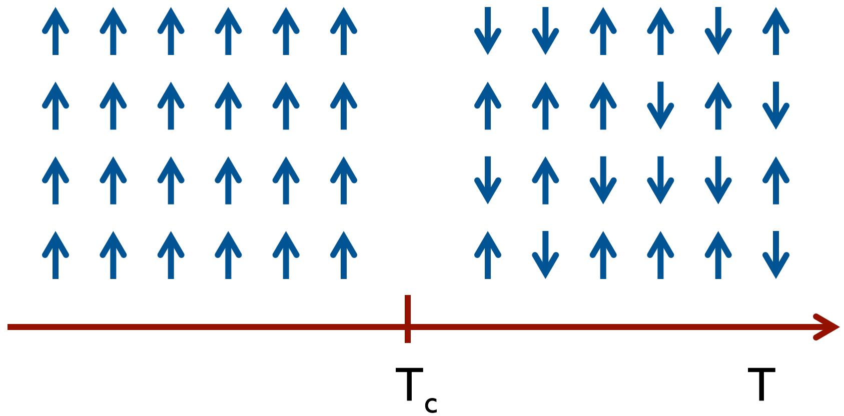
Gas-liquid transition

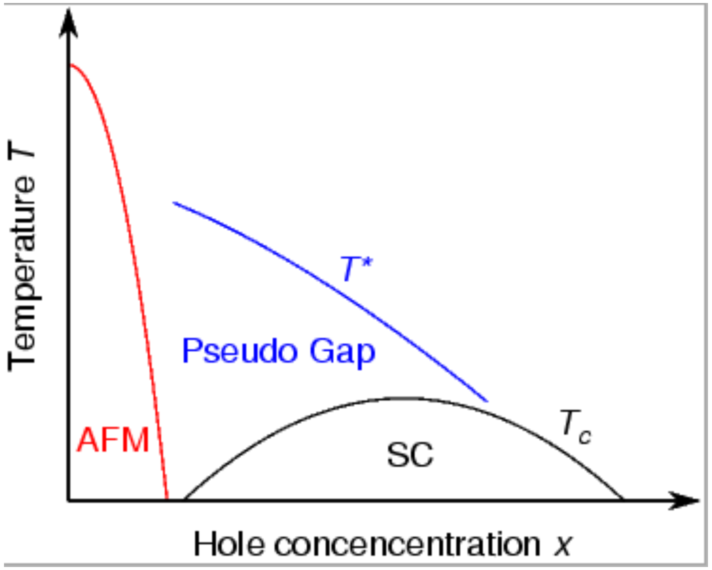
Crystallization

Metal-superconductor transition

Ferromagnet-paramagnet transition







Phase transitions

*with long-range order

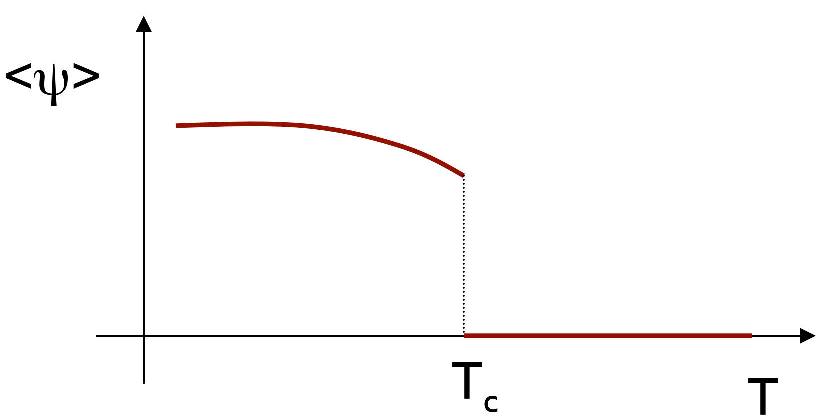
Characterized by an order parameter ψ

Transition	BEC	Magnetic	Gas-liquid	BCS	Crystal
ψ	$b \sim \exp(i\phi)$	S^z	ρ	$c_\uparrow c_\downarrow$	$\rho(\vec{k}_l)$

Ordered phase: $\langle \psi^+(0)\psi(r)\rangle \xrightarrow[r\to\infty]{} const. = \text{"}\langle \psi^+(0)\rangle\langle \psi(r)\rangle\text{"} \neq 0$

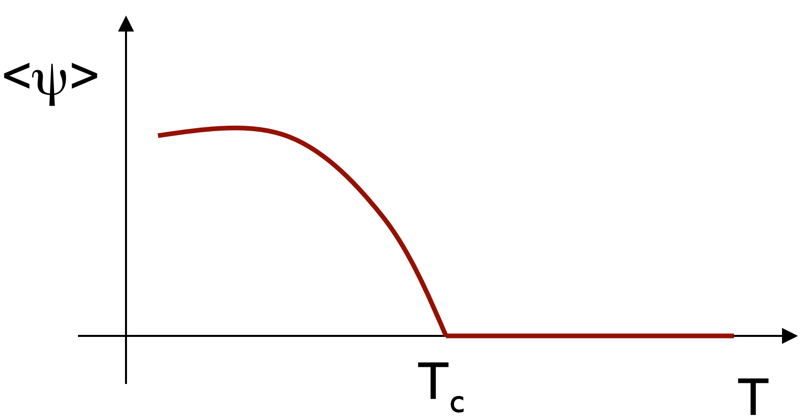
Disordered phase: $\langle \psi^+(0)\psi(r)\rangle \xrightarrow[r\to\infty]{} 0$

First order



Latent heat

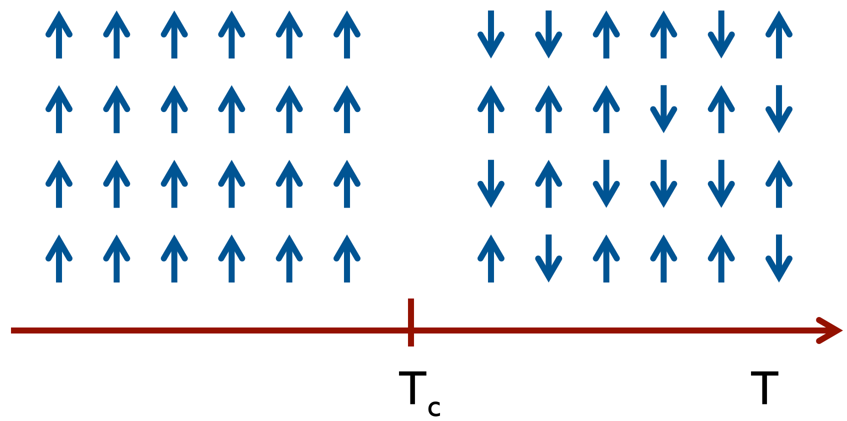
Second order



No latent heat

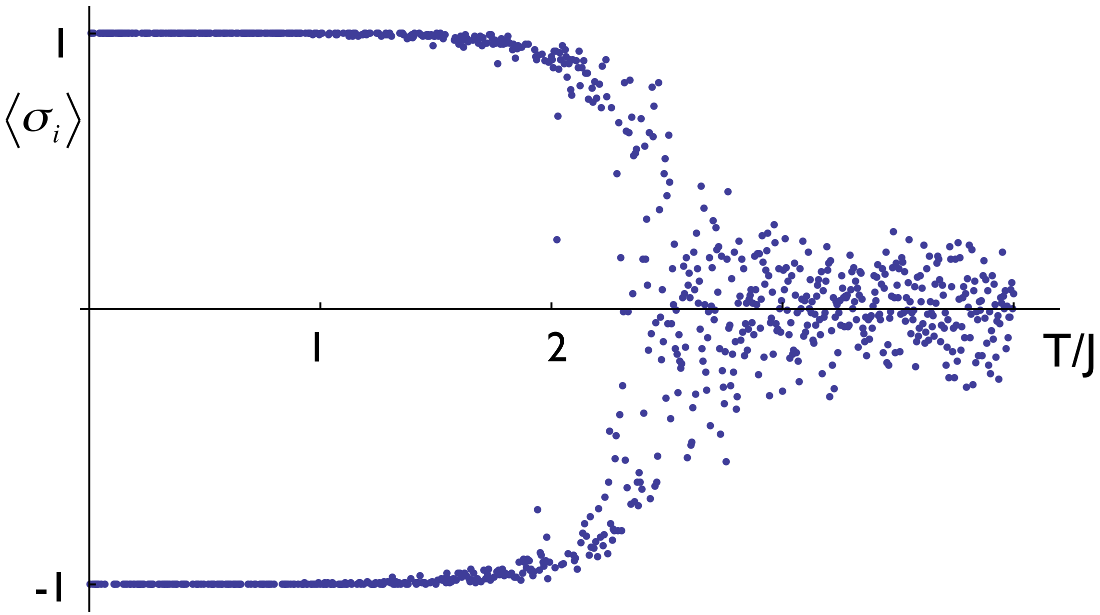
Ising transition

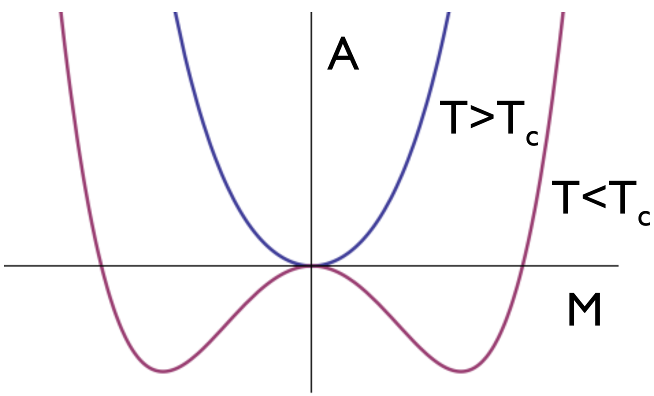
Second order transition



Model $H = -J \sum_{<ij>} \sigma_i \sigma_j$

Monte Carlo scatterplot



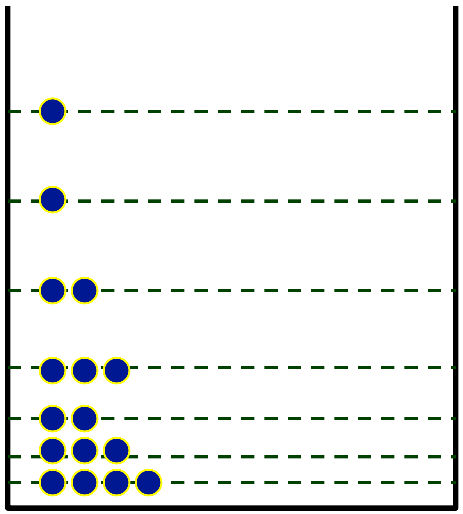


Effective, real-valued ϕ^4 theory!

Bose Einstein condensation

- Second order transition
- Complex-valued ϕ^4 theory!

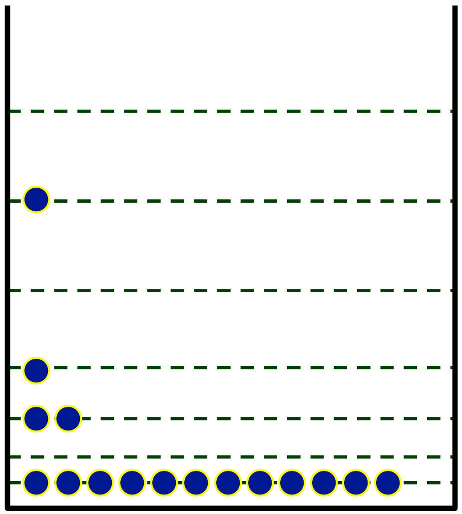
$T > T_c$



Thermal gas

$$\langle n_0 \rangle / n = 0$$

$T < T_c$



Condensate

$$\langle n_0 \rangle / n \neq 0$$

n: total density

n_0: condensate density

Critical exponents for thermal phase transitions of 2nd order

Near the critical point, the order parameter vanishes as

$$\langle \psi \rangle \sim (T_c - T)^{\beta} \qquad \text{for} \qquad T < T_c$$

The correlation function approaches

$$G(\vec{R}) = \langle \psi^{+}(0)\psi(\vec{R}) \rangle \sim \frac{\exp(-R/\xi)}{R^{d-2+\eta}}$$

η parametrizes the power-law scaling at criticality

ξ diverges as

$$\xi \sim |T_c - T|^{-\nu}$$

The critical exponents β, η, ν (and others) have the same combination of values for physically entirely different systems. Systems with the same critical exponents are a universality class.

Bose Einstein condensation of an ideal gas

dimension d=3

Condensate fraction

$$\langle n_0 \rangle = \langle \psi_0^+ \psi_0 \rangle \sim 1 - \left(\frac{T}{T_c} \right)^{d/2} \sim |t|$$

$$t = \frac{T - T_c}{T_c}$$

The order parameter scales as

$$\langle \psi_0 \rangle \sim |t|^{1/2}$$

So

$$\beta = \frac{1}{2}$$

Correlation function

$$G(\vec{R}) = \langle \psi^+(0) \psi(\vec{R}) \rangle \sim \frac{\exp(-R/\xi)}{R}$$

$$T > T_c$$

with

$$\xi \sim t^{-1/(d-2)}$$

So

$$\nu = \frac{1}{d-2} \qquad \eta = 0$$

n=∞ exponents, i.e. mean-field exponents

A transition without long-range order: Kosterlitz-Thouless

dimension d=2

Instead of long-range order, the system shows quasi-long range order:

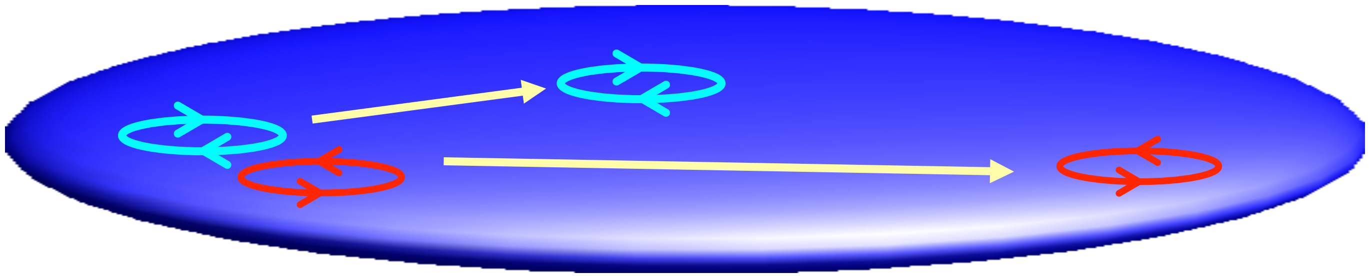
$$G(\vec{R}) = \left\langle \psi^+(0)\psi(\vec{R}) \right\rangle \sim R^{-\frac{\tau}{4}}$$

$$\tau \approx \frac{T}{T_c}$$ BdG estimate

Prokof'ev, Ruebenacker, Svistunov, '00, '01, '02

Kosterlitz-Thouless transition *driven by vortex unbinding*

Disordered phase: $$G(\vec{R}) = \left\langle \psi^+(0)\psi(\vec{R}) \right\rangle \sim \exp(-R/\xi)$$



Berezinskii 1972, Kosterlitz and Thouless 1973

Critical scaling of the Kosterlitz-Thouless transition

dimension d=2

Disordered phase: $$G(\vec{R}) = \left\langle \psi^{+}(0)\psi(\vec{R}) \right\rangle \sim \exp(-R/\xi)$$

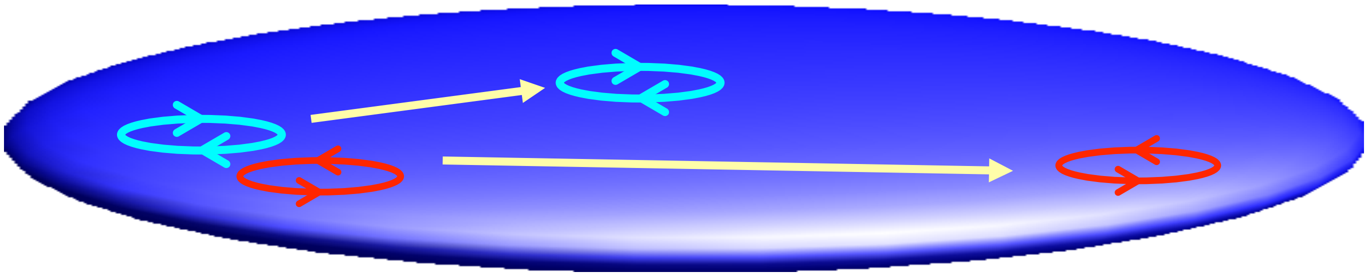
Transition of *infinite* order, i.e. $\nu = \infty$

$\xi \sim |T_c - T|^{-\nu}$ is replaced by

$$\xi \sim \exp\left(\frac{b}{\sqrt{1 - T/T_c}}\right)$$

$$\xi \sim \exp\left(\frac{b'}{T - T_c}\right)$$

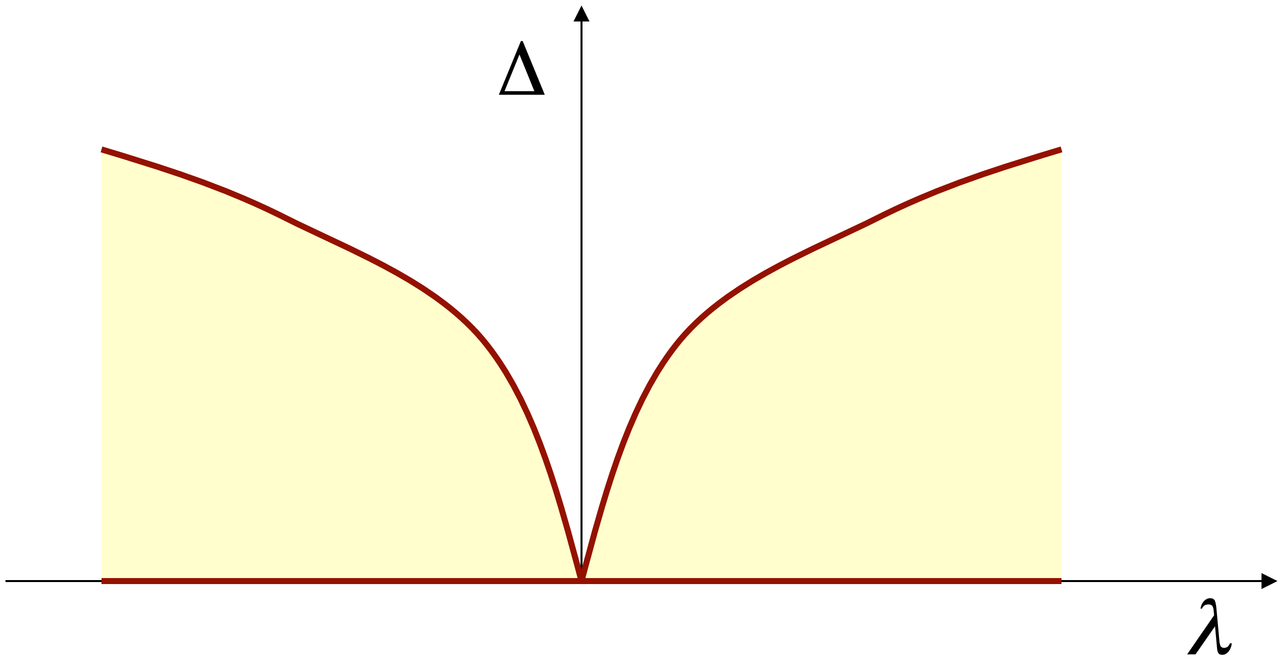
near criticality



Berezinskii 1972, Kosterlitz and Thouless 1973

Quantum phase transitions

Transition at *zero* temperature, as a function of a parameter λ



The energy gap Δ vanishes at λ=0, and scales as $\Delta \sim |\lambda|^{z\nu}$

The correlation length ξ diverges at λ=0, and scales as $\xi \sim |\lambda|^{-\nu}$

The parameter z describes the underlying dispersion, $\omega \sim k^z$

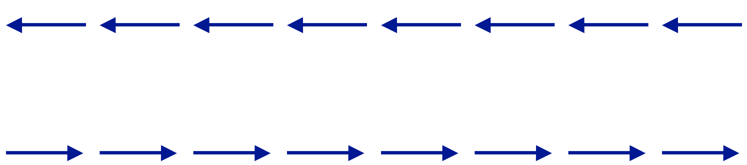
See e.g. Sachdev 'Quantum phase transitions'

Example: Transverse quantum Ising model

Ising spins σ_i with nearest-neighbor coupling, and an external z-field

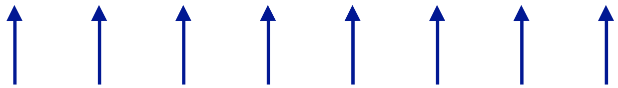
$$H = -J\sum_i \left(\sigma_i^x \sigma_{i+1}^x + g\sigma_i^z\right)$$

$g << 1$



double degenerate ground state

$g >> 1$



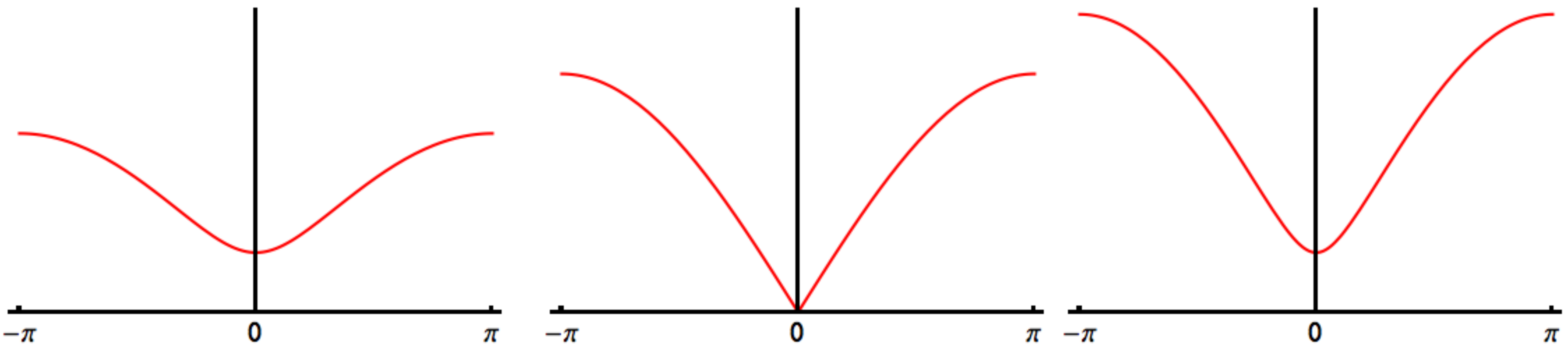
single ground state

Quantum phase transition at g=1

Example: Transverse quantum Ising model

Exactly solvable via Jordan-Wigner transformation. Dispersion:

$$\varepsilon_k = 2J\sqrt{g^2 - 2g\cos k + 1}$$



$g < 1$ $\quad$ $g = 1$ $\quad$ $g > 1$

We choose λ = g - 1.

The energy gap scales as $\Delta = \varepsilon_{k=0} = 2J\left|g-1\right| \sim \left|\lambda\right|^{z\nu}$

so: z ν = 1.

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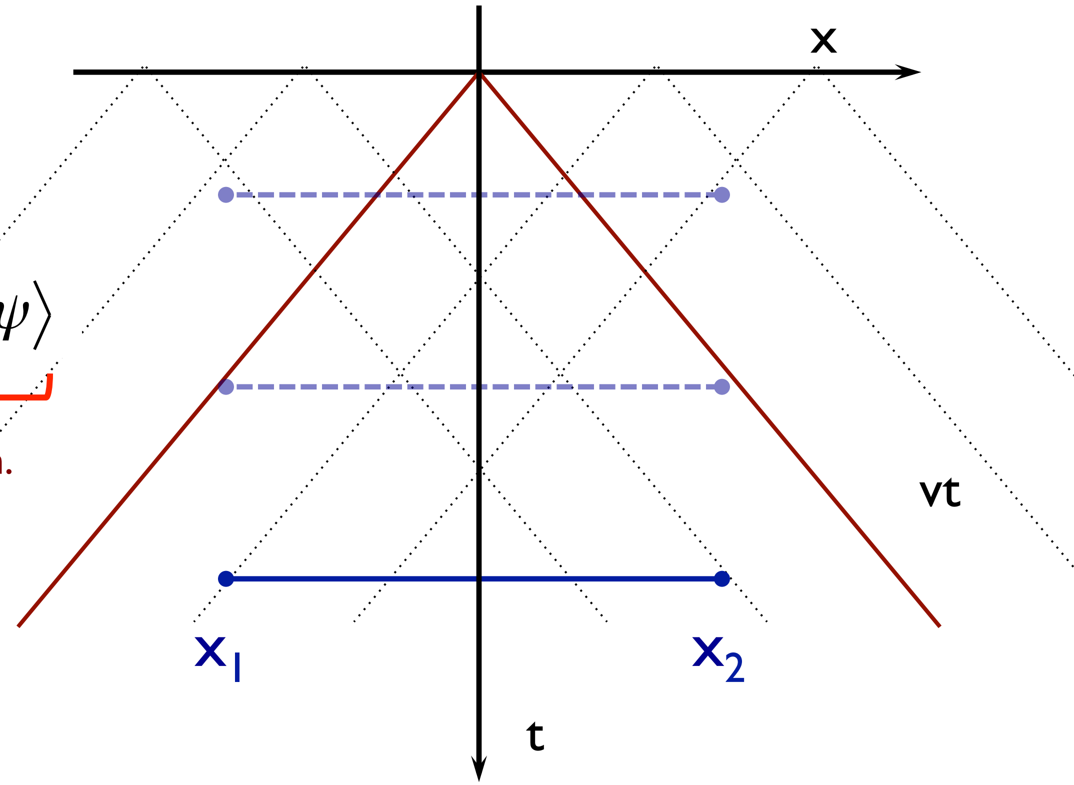
Reverse Kibble-Zurek mechanism and light cone dynamics in 2D Bose gases

Conclusions

Light cone dynamics

Dynamics separates G(x,t) into connected and disconnected part:

$$G(x,t) = \underbrace{\langle \psi^+ \psi \rangle - \langle \psi^+ \rangle \langle \psi \rangle}_{\text{con.}} + \underbrace{\langle \psi^+ \rangle \langle \psi \rangle}_{\text{discon.}}$$

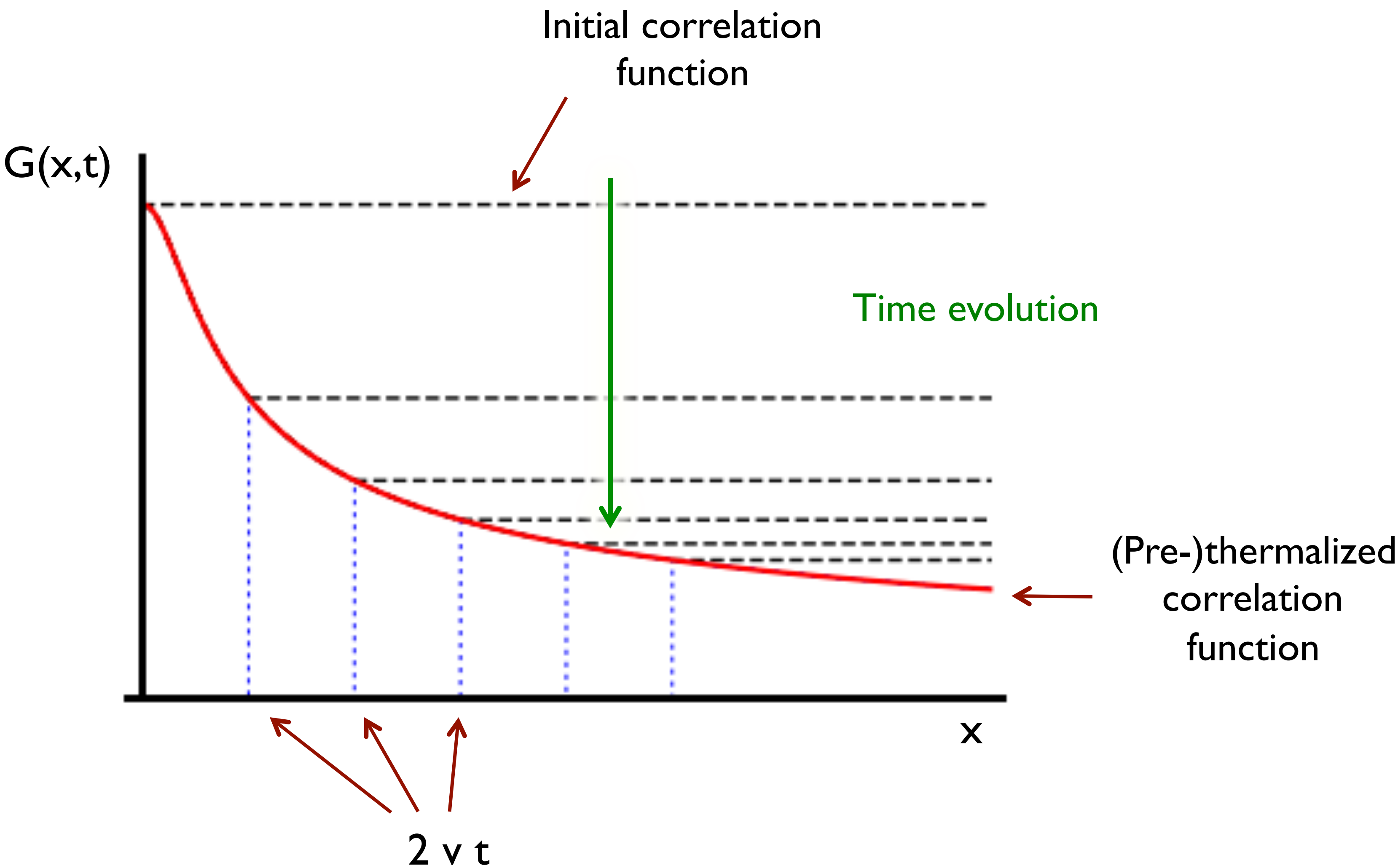


The disconnected part is independent of $x = x_1 - x_2$, and only depends on t.

If $x = x_1 - x_2 < 2\,v\,t$, the correlation function approaches a new steady state, either thermalized or pre-thermalized.

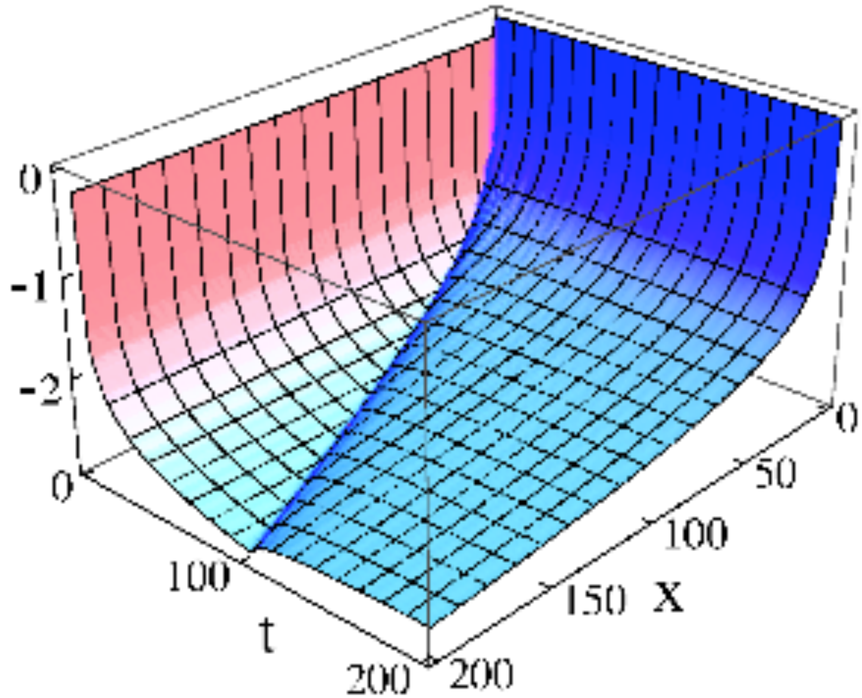
Lieb & Robinson, Cardy & Calabrese, …

Light cone dynamics



Light cone dynamics in 2d condensates

Splitting a 2d condensate:



Outside the light cone: $x >> 2vt \quad : \quad G \sim |t|^{-T^*/4T_c}$

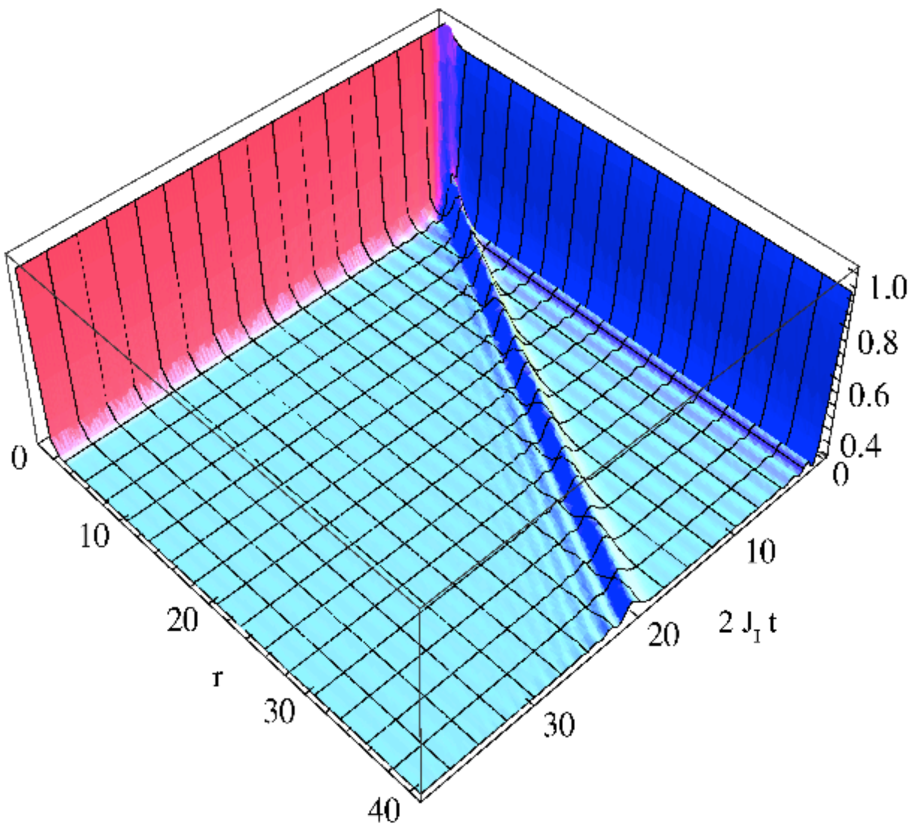
Inside the light cone: $x << 2vt \quad : \quad G \sim |x|^{-T^*/4T_c}$

LM + Polkovnikov, '10

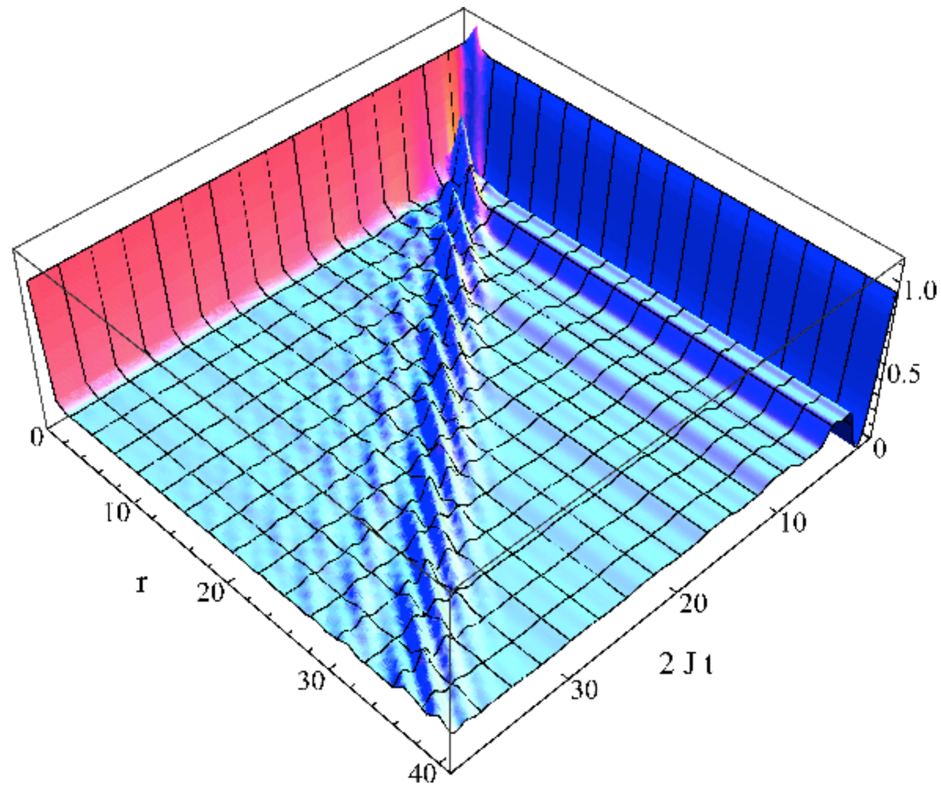
Light cone dynamics in the transverse Ising model

Quench from high magnetic field

$g = 3.0 \rightarrow g' = 1.0$



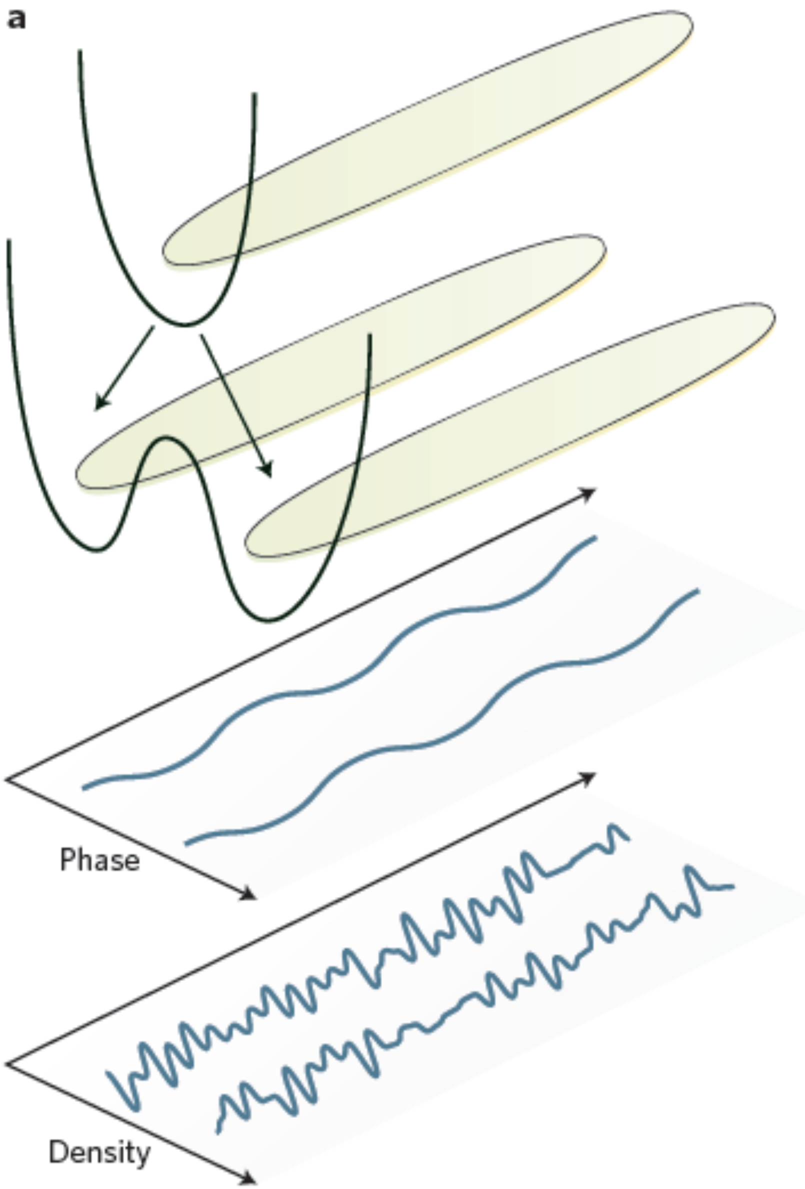
$g = 3.0 \rightarrow g' = 0.5$



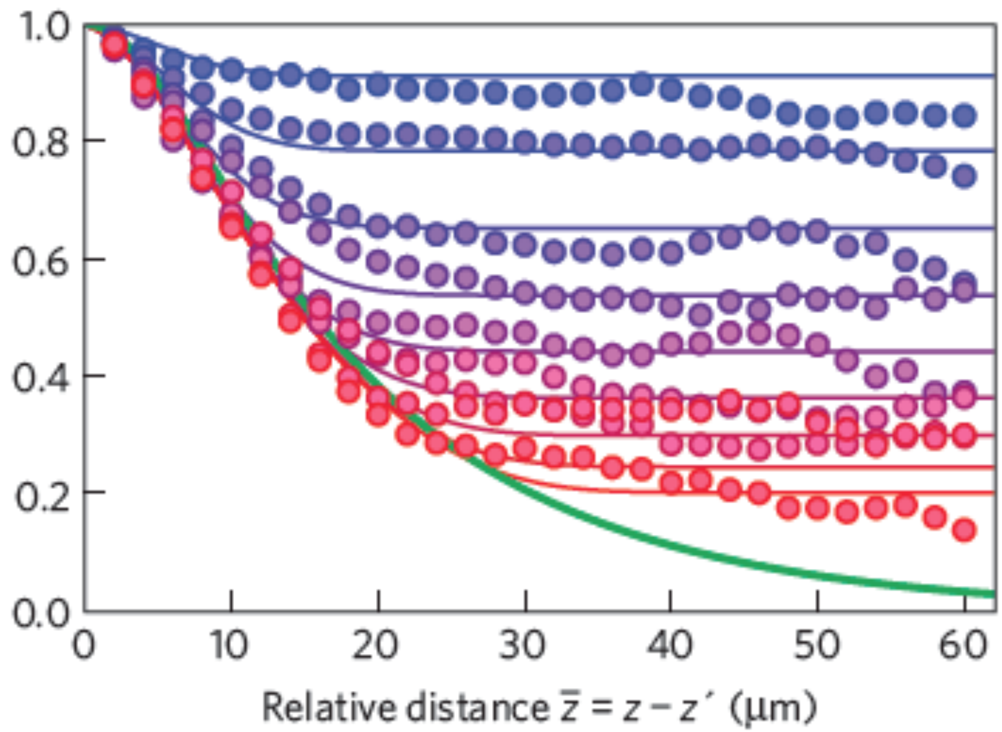
$$\left\langle \sigma_i^z(t)\sigma_j^z(t) \right\rangle$$

LM + Polkovnikov, '10

Splitting a 1d condensate



Correlation function of the relative phase



Schmiedmayer group, '13

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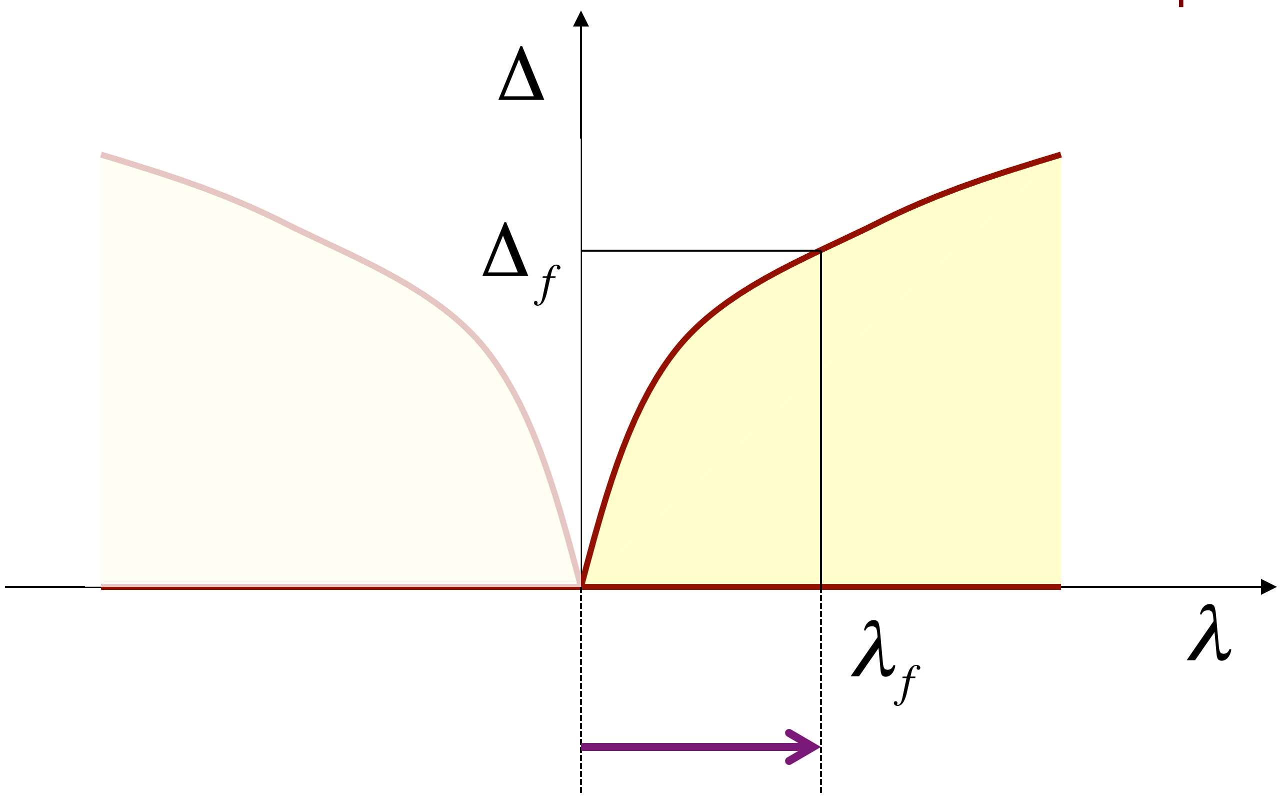
Universal scaling of a quench near criticality

Quench from critical point

$$H(\lambda) = H_0 + \lambda V$$

- H_0: Hamiltonian at criticality
- λ: external parameter
- V: Relevant operator



Dynamics

$$\lambda = \lambda(t)$$

Polkovnikov, '03. Dorner, Zurek, Zoller, '05

Slow quench near a quantum phase transition

Sweep: $\lambda(t) = v_q t$

Below energy Δ^*, states are excited, above they are not.

Landau-Zener criterium $\partial_t \Delta^* \sim (\Delta^*)^2$

diabatic-adiabatic condition

From $\Delta^* \sim (v_q t^*)^{z\nu}$ we have

$$t^* \sim (v_q)^{-z\nu/(z\nu+1)}$$

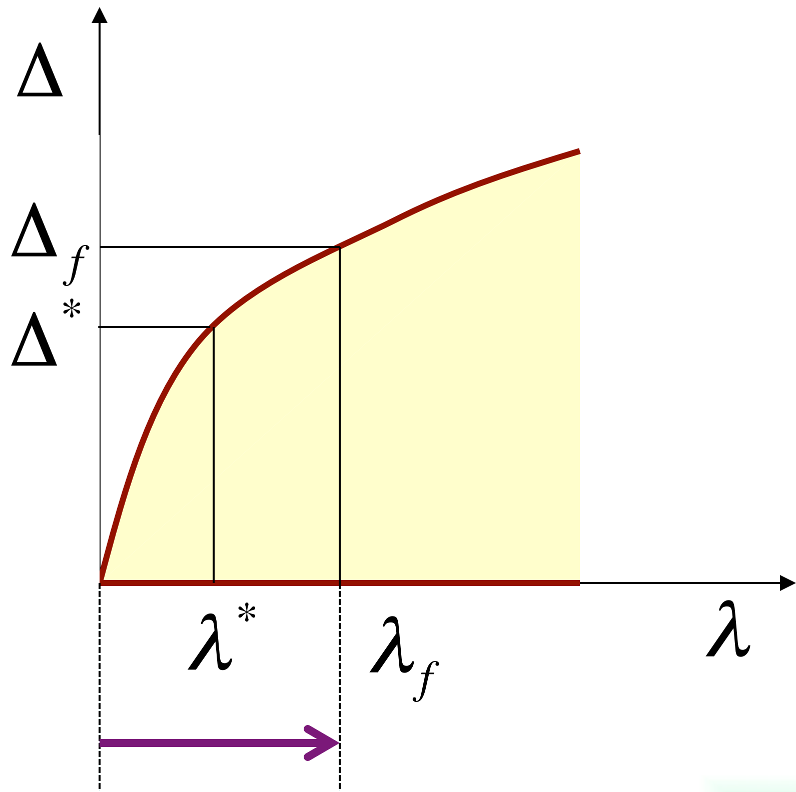
$$\xi^* \sim (v_q)^{-\nu/(z\nu+1)}$$

$$\Delta^* \sim (v_q)^{z\nu/(z\nu+1)}$$

$$n_{ex} \sim 1/(\xi^*)^d \sim (v_q)^{d\nu/(z\nu+1)}$$

$$Q \sim \Delta^* n_{ex} \sim (v_q)^{(d+z)\nu/(z\nu+1)}$$

Comparable to latent heat



Kibble-Zurek mechanism

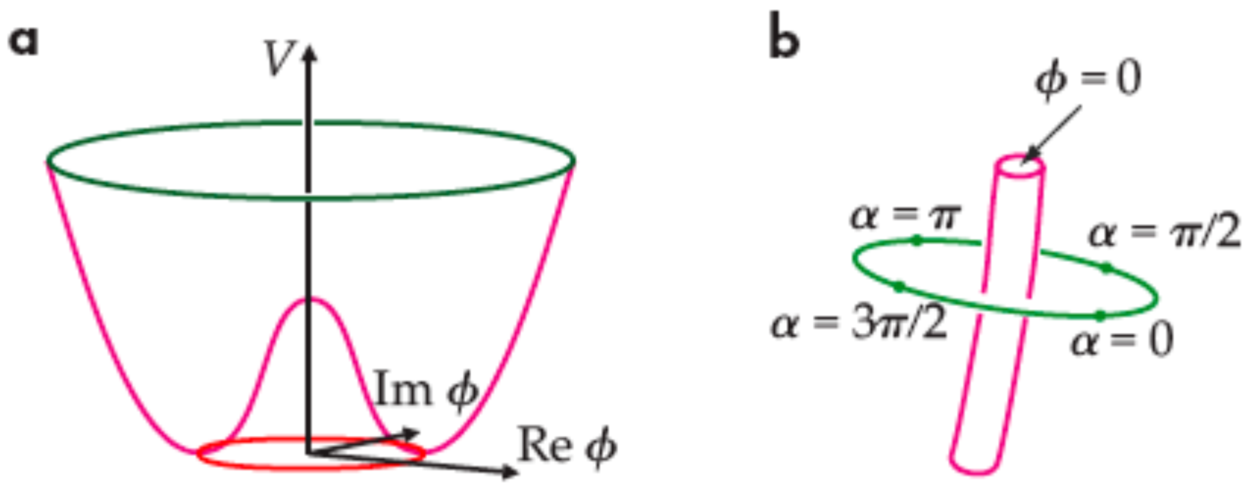
Short review: T.W.B. Kibble, Physics Today, 60, 47 (2007)

Originally a cosmological idea regarding the rapid cooling of the universe. Defects in the Higgs field.

General feature:

Ramp across a phase transition, from the disordered to the ordered side

Domains form → generation of topological defects



T.W.B. Kibble, J. Phys. A 9, 1387 (1976); W.H. Zurek, Nature 317, 505 (1985)

Kibble-Zurek scaling

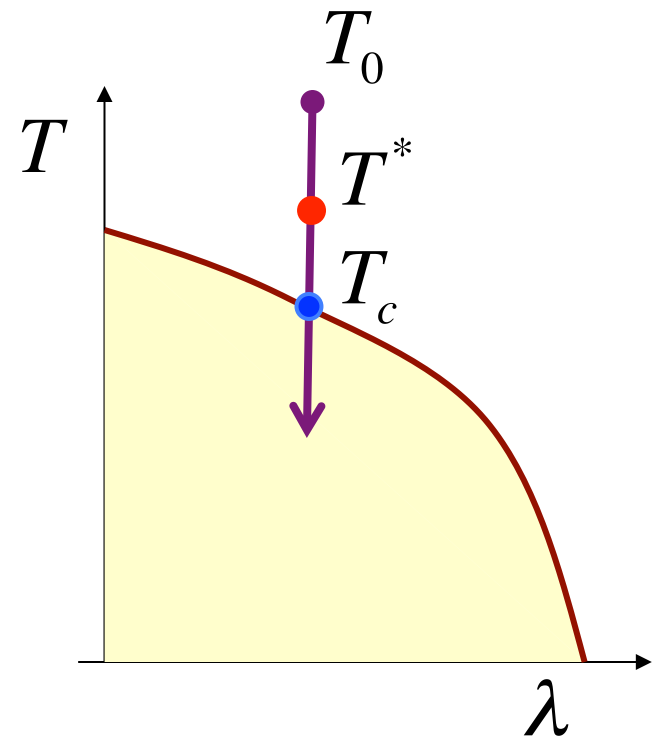
Temperature quench $T(t) = T_0 - v_q t$

Energy gap ~ relaxation time

Correlation length

$$\tau \sim |T - T_c|^{-z\nu}$$

$$\xi \sim |T - T_c|^{-\nu}$$



Adiabatic to diabatic:

$$t^* \sim (T^* - T_c)/v_q \sim \tau \sim |T^* - T_c|^{-z\nu}$$

$$T^* - T_c \sim v_q^{1/(1+z\nu)}$$

Correlation length: $\xi^* \sim v_q^{-\nu/(1+z\nu)}$

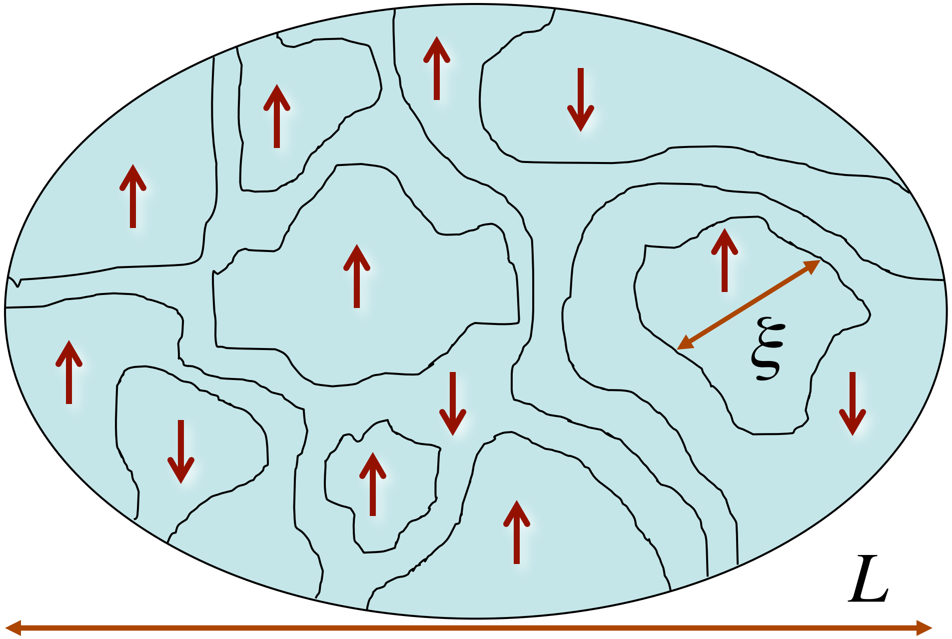
Density of excitations: $n_{ex} \sim (\xi^*)^{-d} \sim v_q^{d\nu/(1+z\nu)}$

Excess energy: $Q \sim (1/t^*) n_{ex} \sim (v_q)^{(d+z)\nu/(z\nu+1)}$

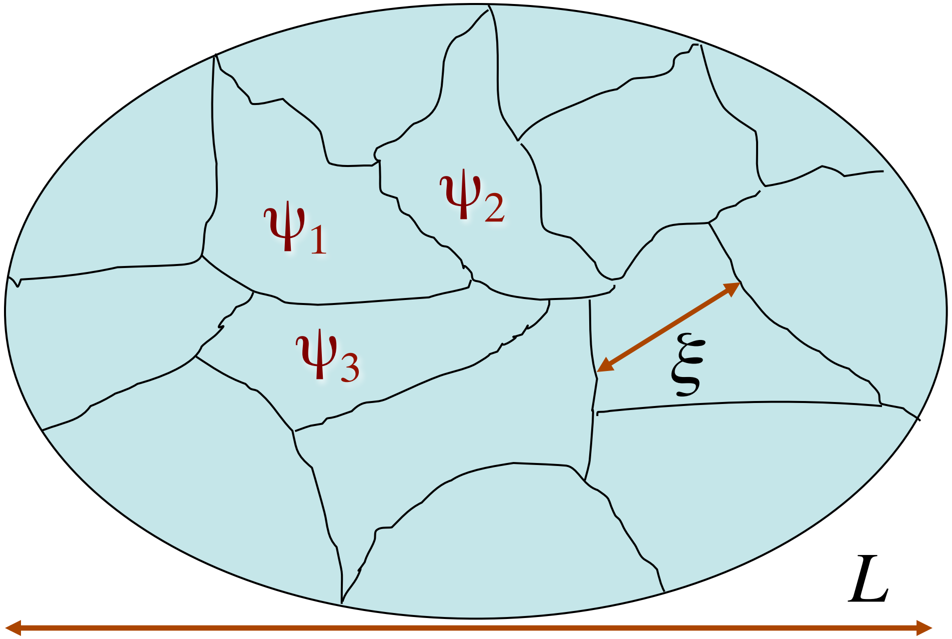
Comparable to latent heat

Domains after quench

real-valued, Ising order parameter



complex-valued order parameter

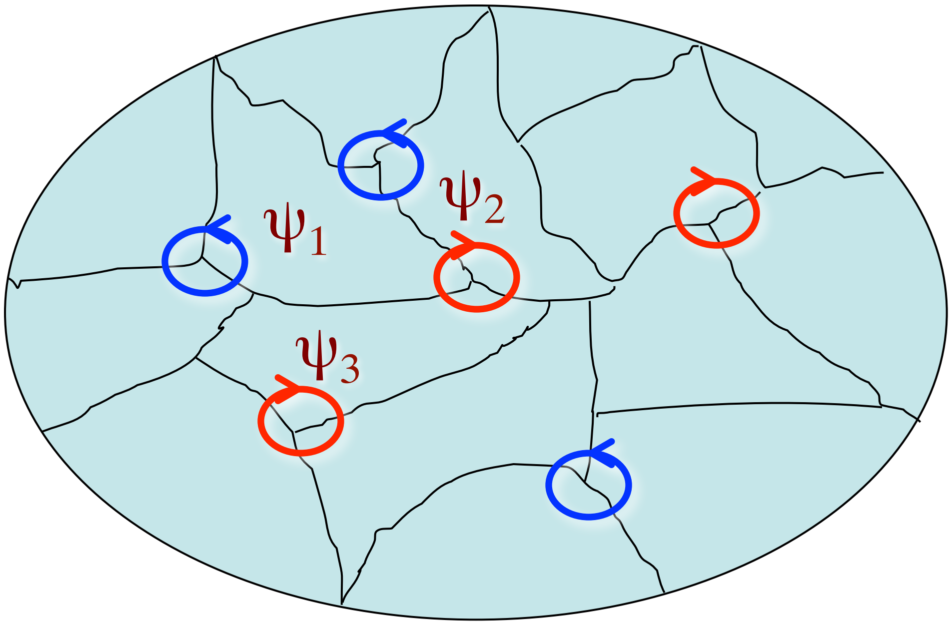


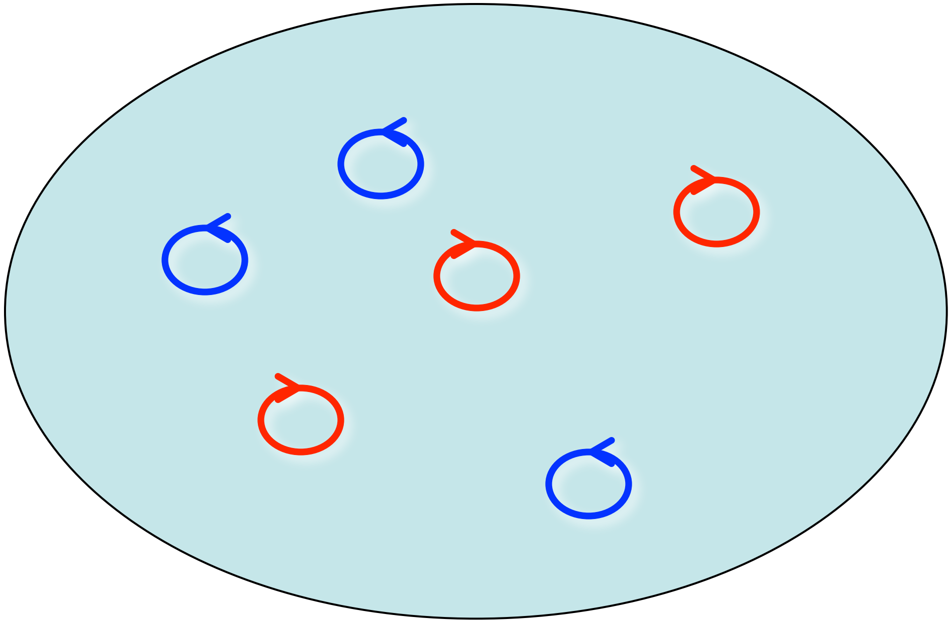
Number of domains

$$\sim N_{dom} \sim \frac{L^2}{\xi^2}$$

Topological defects

Vortices form, phonons dephase





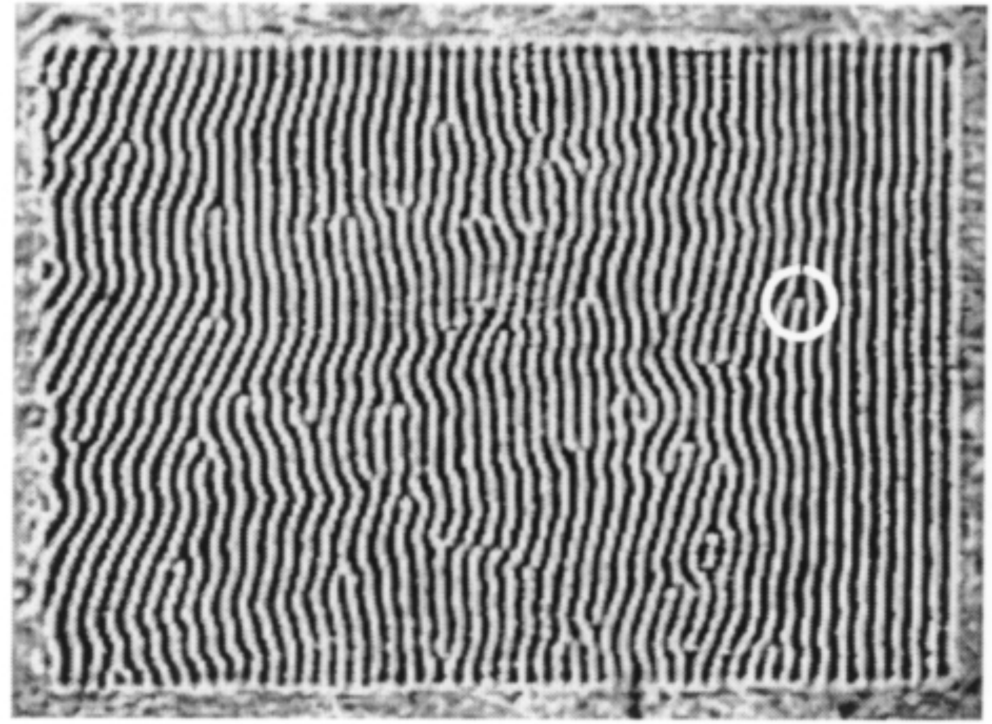
Vortices annihilate on a very long time scale

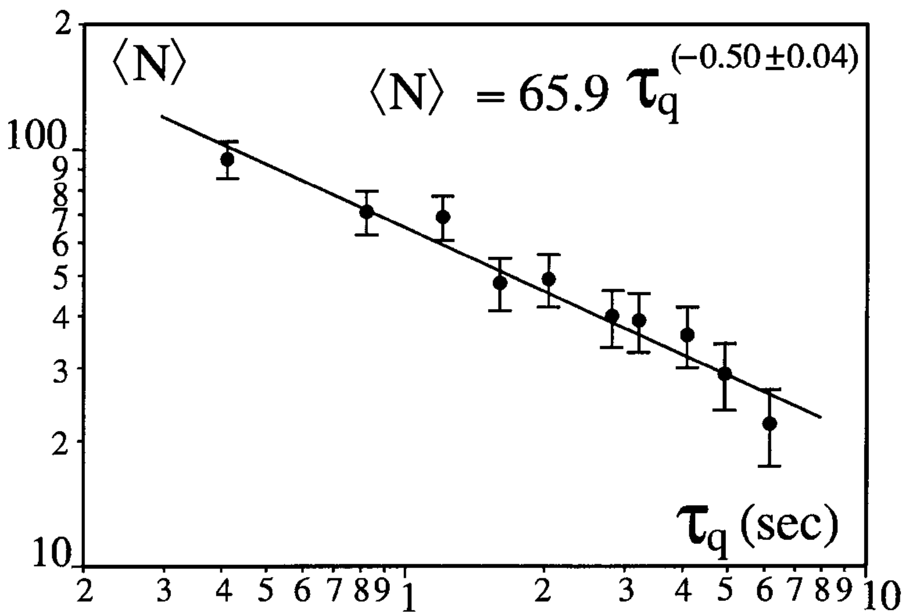
In the early universe physics: cosmological strings

Kibble-Zurek scaling

Optical Kerr medium with feedback displays pattern formation instability at a critical light intensity.

Complex Landau Ginzburg universality class.





Scaling of the defect number with the quench time $\tau_q \sim 1/v_q$

$$n_{ex} \sim \tau_q^{-d\nu/(1+z\nu)} \sim \tau_q^{-1/2}$$

with d=2, z=2, ν=1/2

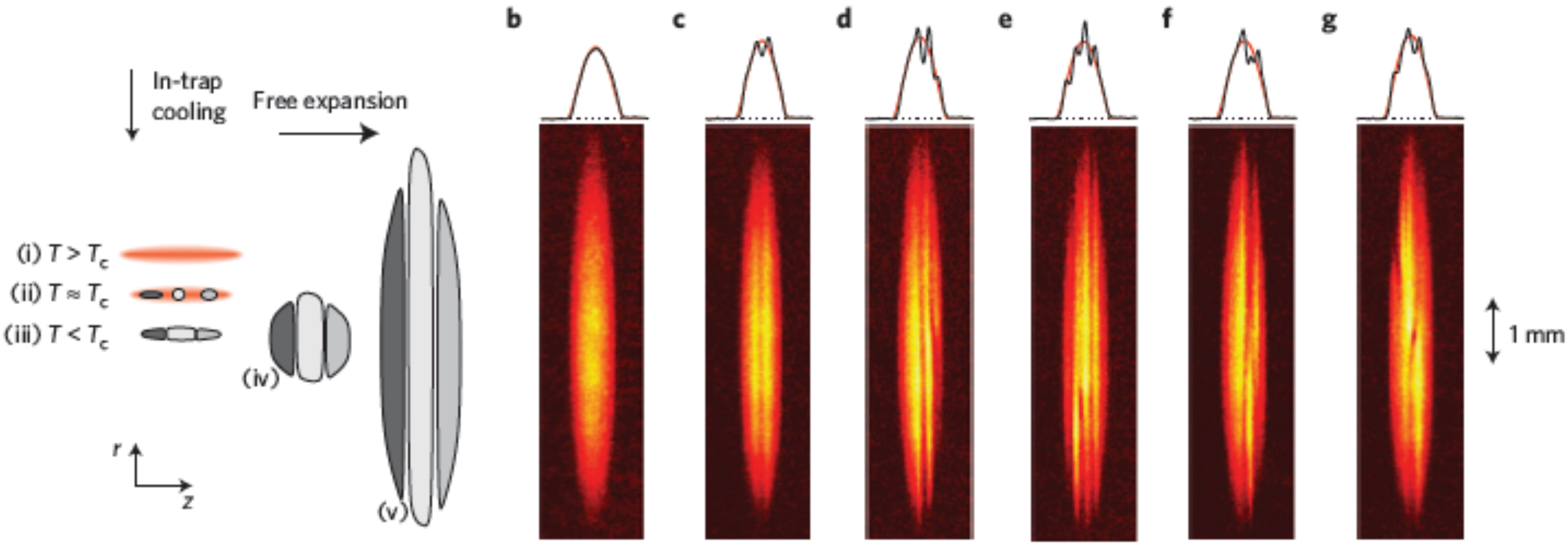
Arecchi group, PRL '99

Kibble-Zurek solitons

BEC quenched across phase transition by evaporative cooling

Build-up of phase coherent domains

Domain boundaries visible as solitons

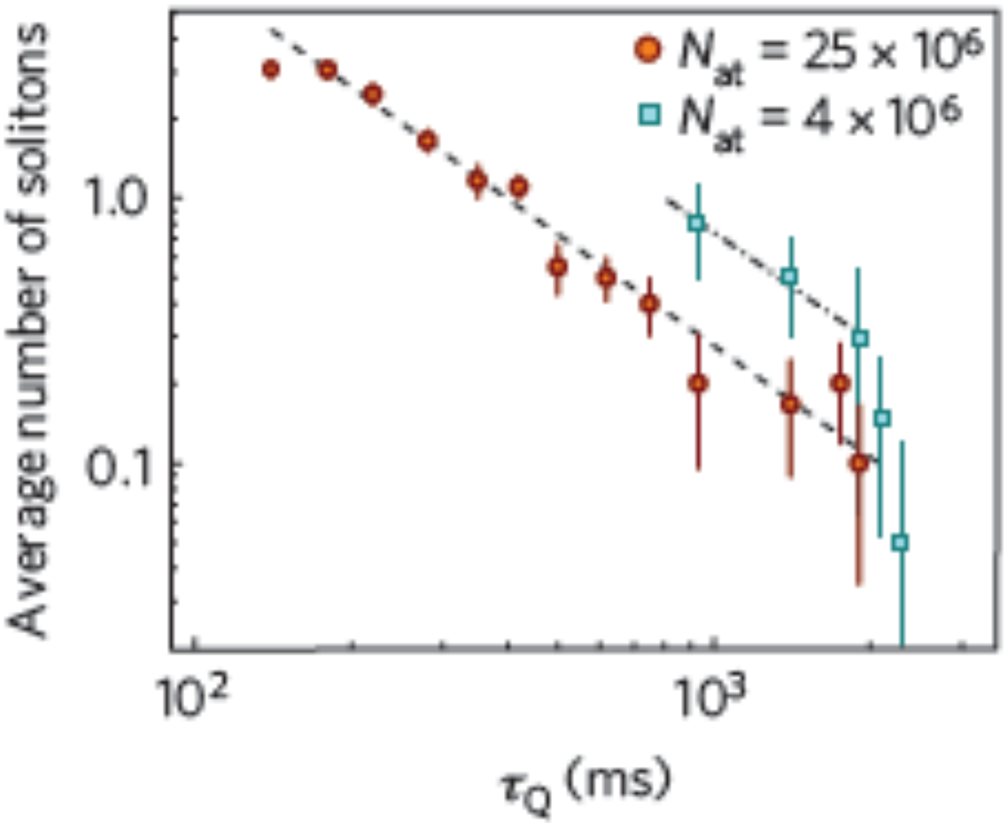


Ferrari group, NatPhys '13

Scaling

Number of solitons as a function og quench time

➡ approximately power-law!



Exponent: 1.38 +/- 0.06

(Zurek, PRL ´09 : 7/6)

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Dynamic Kosterlitz Thouless transition

with K. Günter, J. Dalibard, A. Polkovnikov

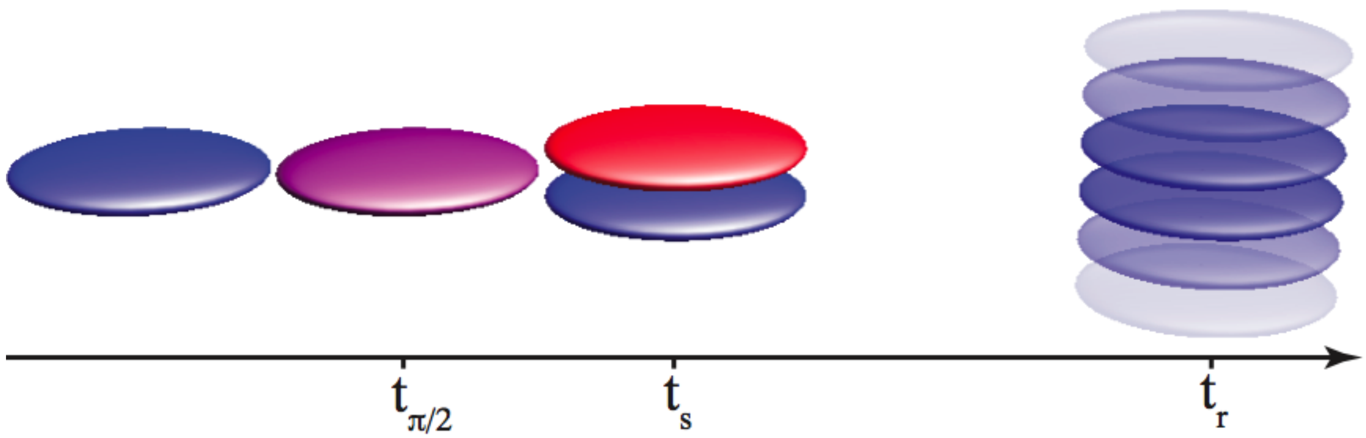
Quenching across phase transitions

Short-time behavior: Light-cone dynamics

Kibble-Zurek mechanism

Real-time renormalization group approach

Detecting a dynamic KT transition via matter wave interferometry

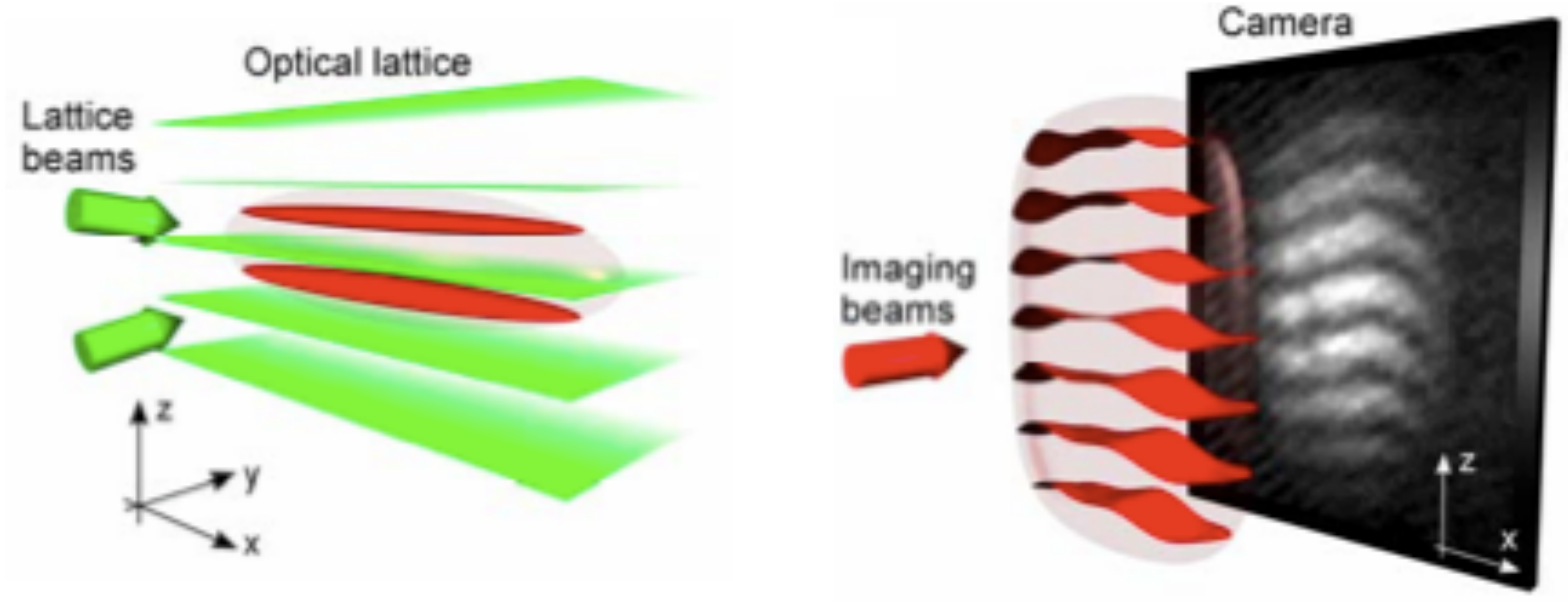


PRA 80, 041601(R) (2009); 81, 033605 (2010); c-m/1112.1204

Interfering 2D Bose gases

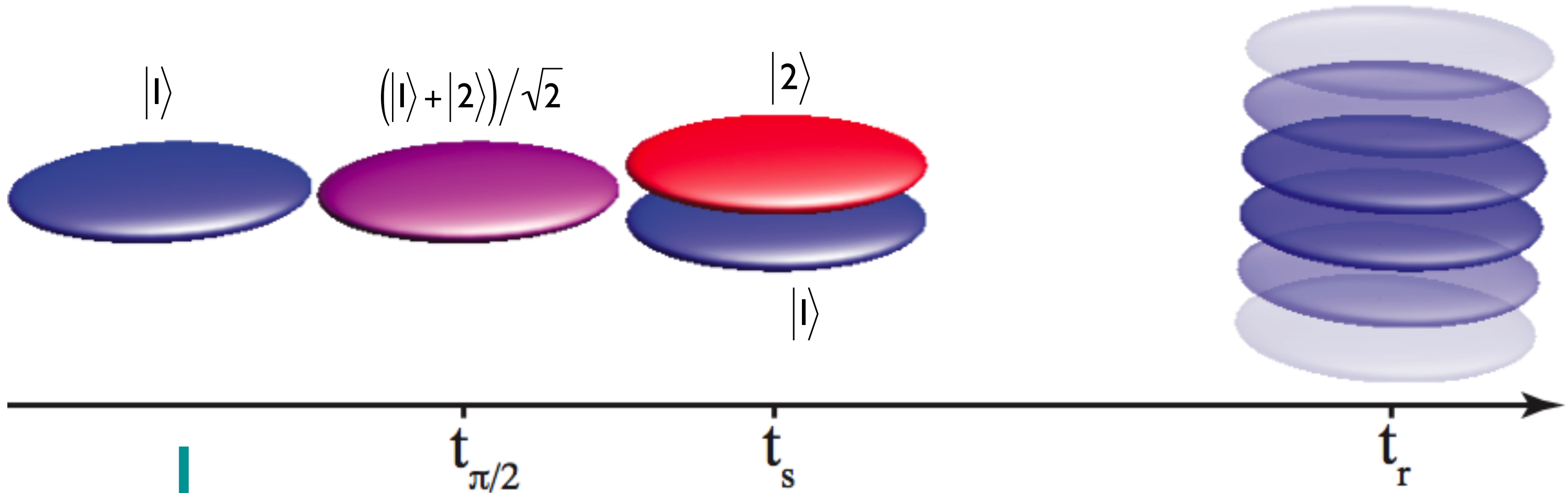
- Realization of quasi-condensates and KT transition
- algebraic → exponential scaling. Vortices

Dalibard group, Phillips/Helmerson group, Chin group, Jin group, …



Review: Z. Hadzibabic, J. Dalibard, Rivisto del Nuovo Cimento, 34, 389 (2011)

Preparation and measurement sequence



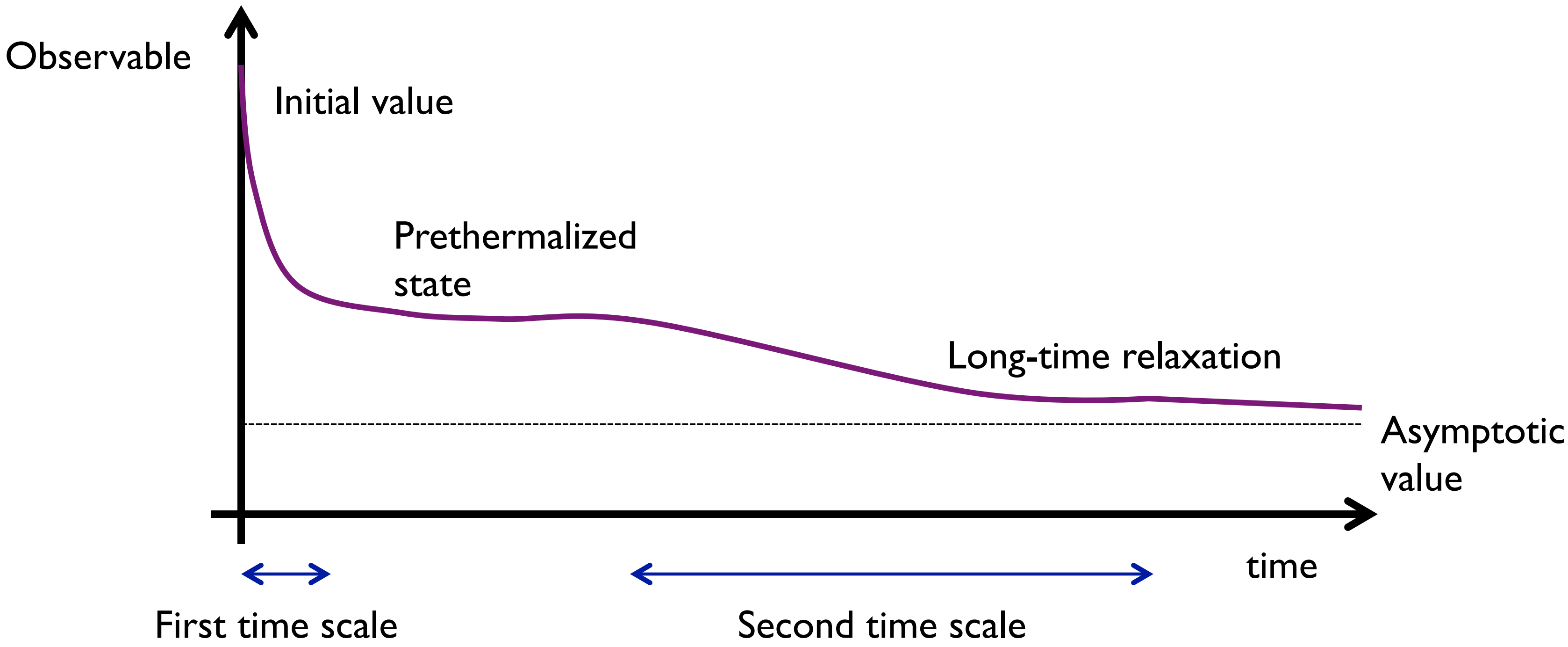
Prepare a 2D superfluid in internal state $|1\rangle$

Apply $\pi/2$ pulse to create a superfluid in $(|1\rangle + |2\rangle)/\sqrt{2}$

Apply field gradient, turn off the inter-species interaction

Measure interference to detect critical dynamics

Many-body thermalization with two time scales



Initial relaxation: Dephasing, perturbation theory, thermal ensemble of a subset of degrees of freedom

Long-time relaxation: thermal ensemble of all degrees of freedom

We propose to describe the long-time relaxation via real-time RG

Truncated Wigner approximation

Semi-classical phase space method

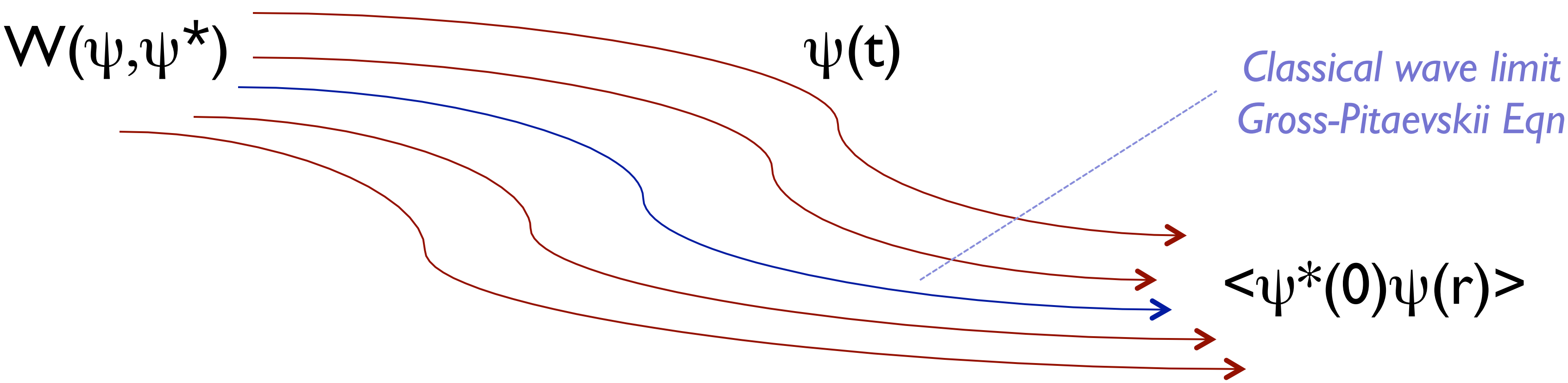
Initial, thermal state
$\rho = e^{-\beta H}$

→

Time propagation
e^{-iHt}

→

Average over realizations
<O(t)>

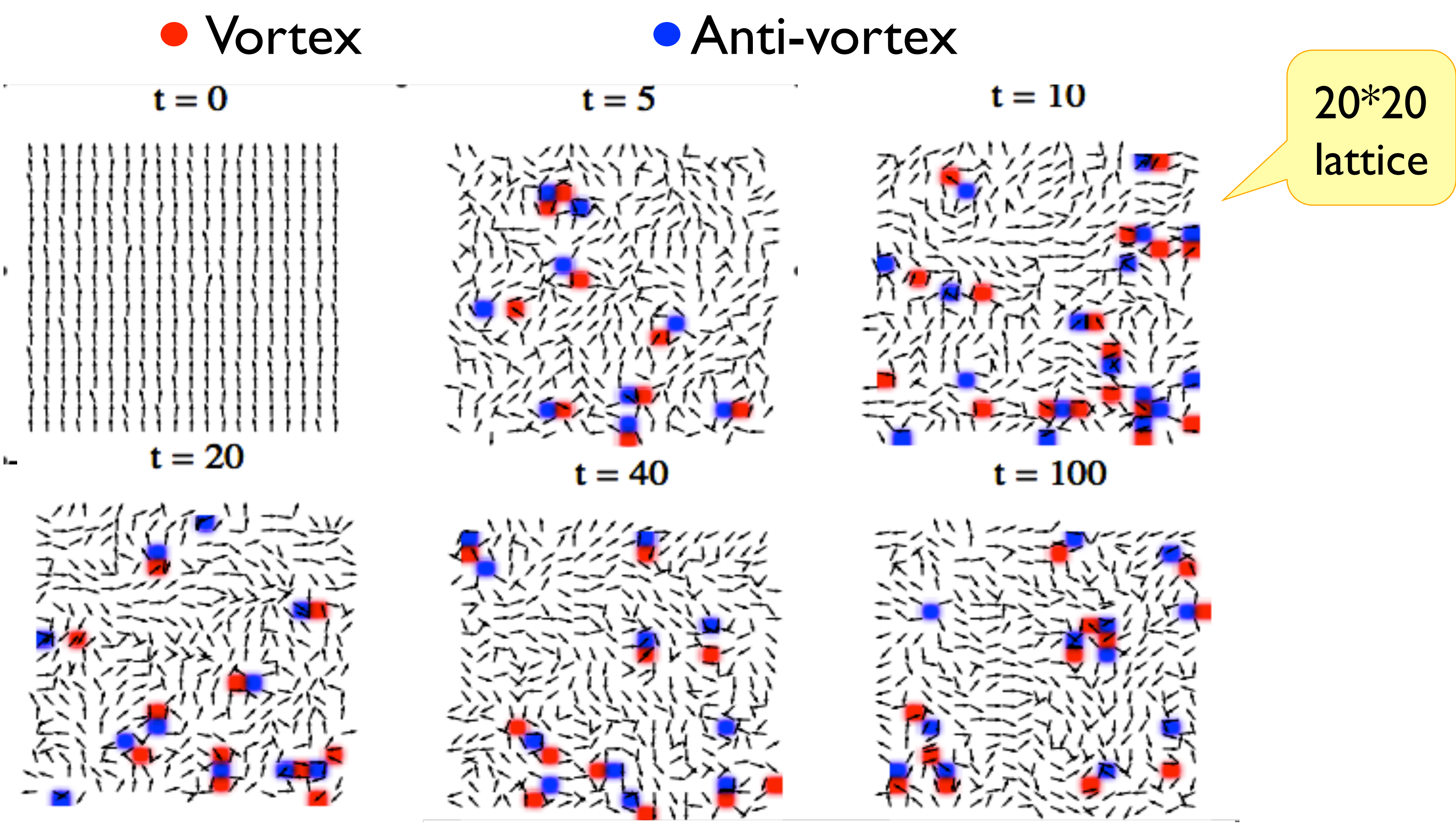


Includes the lowest order of quantum and thermal fluctuations
Becomes exact for classical systems

A. Polkovnikov, Annals of Phys. 325, 1790 (2010)

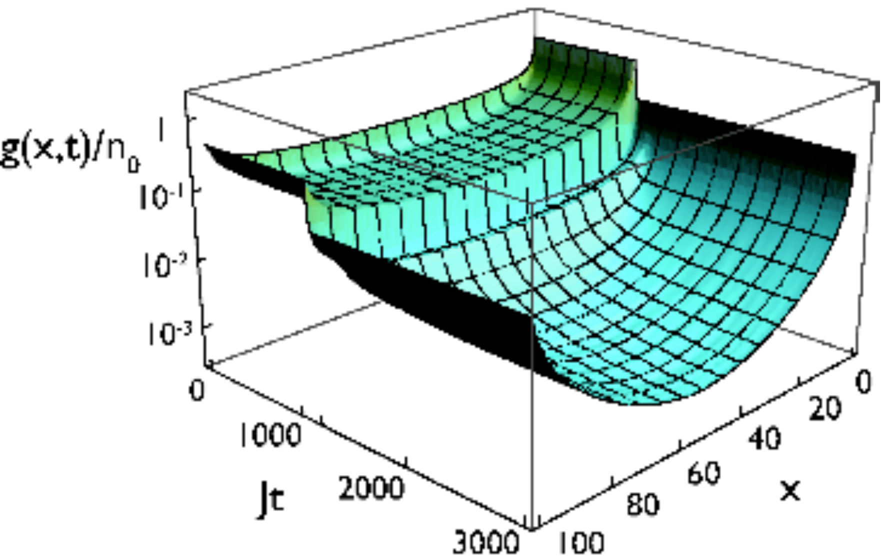
Simulating the quench

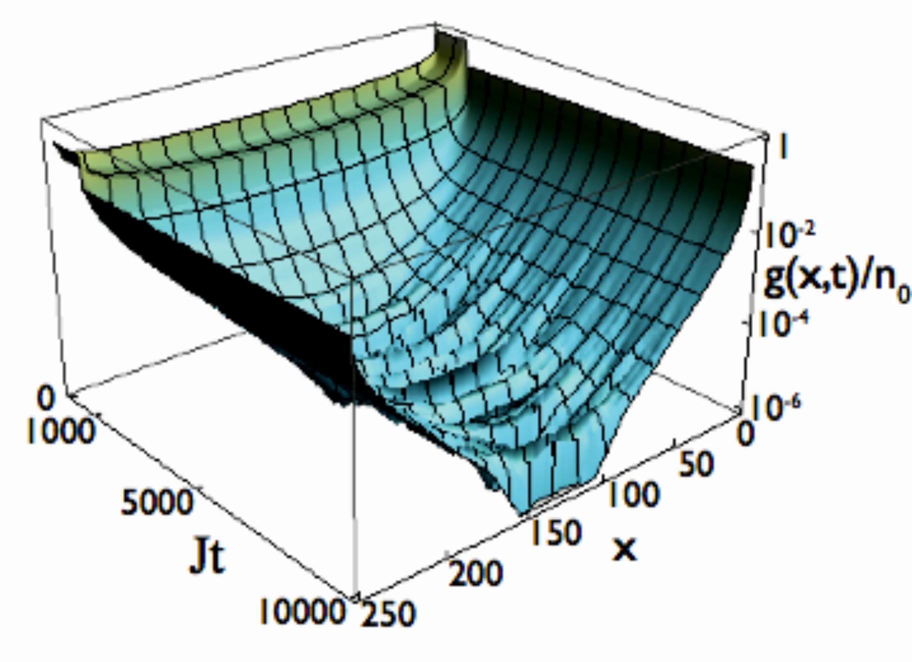
The relative phase is nearly perfectly ordered after the quench. The only fluctuations are due to vacuum fluctuations in the unoccupied |2> state



Phys. Rev. A 80, 041601(R) (2009); 81, 033605 (2010)

Dynamics of G(x,t) on different time scales





On intermediate scales, a metastable supercritical state with algebraic scaling emerges.

On a very long time scale, the scaling changes from algebraic to exponential

→ dynamic KT transition!

$$\hbar/J = 0.3ms \quad \rho_0 = 50/\mu m^2 \quad l = 0.3\mu m \quad {}^{87}Rb$$

Quantifying the change of functional form

We fit g(x,t) with an algebraic and an exponential fitting function:

Algebraic: $\sim |x|^{-\tau/4}$

$$f_a(x) = C\left(|\sin(\pi x/L)|\,L/\pi\right)^{-\tau/4}$$

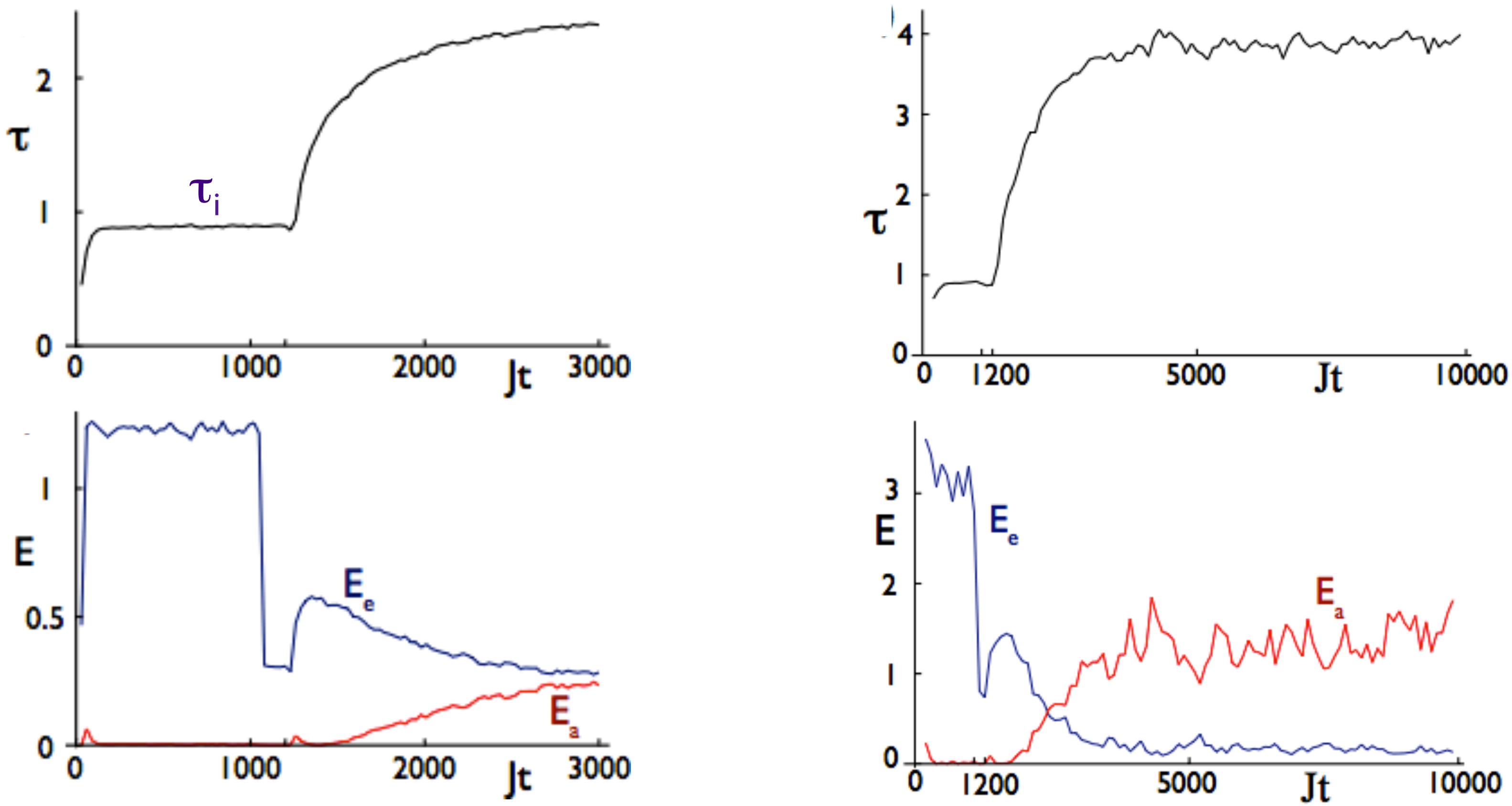
Exponential: $\sim \exp\left(-|x|/x_o\right)$

$$f_e(x) = C\exp\left(-|\sin(\pi x/L)|/x_o\right)$$

We define the two fitting errors:

$$E_{e,a}(t) = \sum\left(g(x,t) - f_{e,a}(x)\right)^2$$

Fitting results



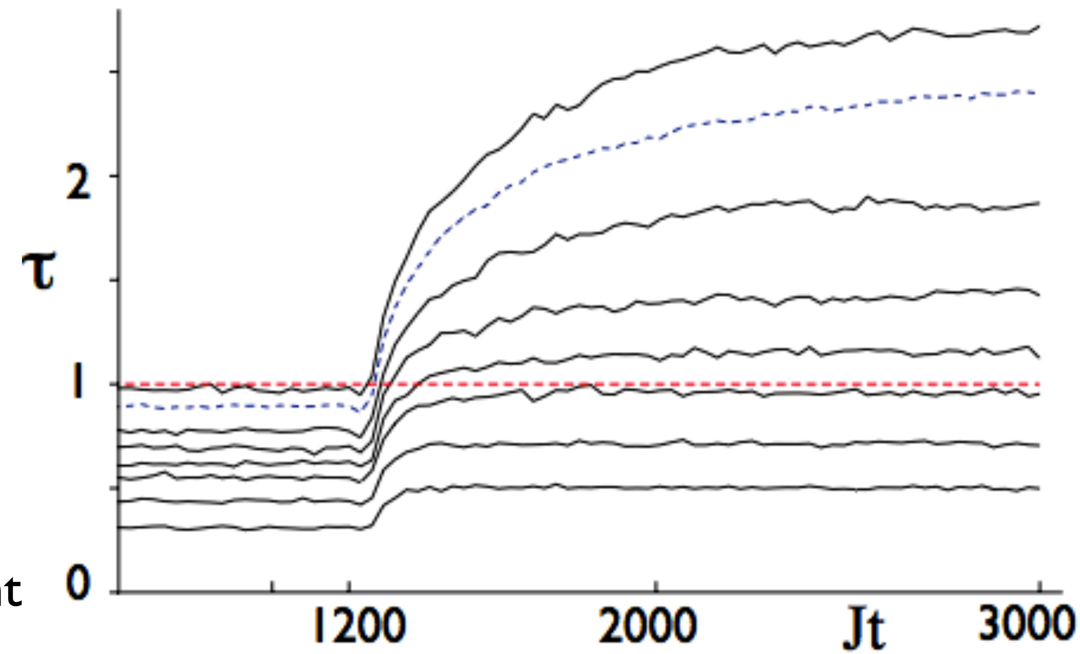
- The system relaxes to a steady state with exponent τ_i
- A supercritical superfluid state is observed
- The relaxation to the disordered groundstate is slowed down critically

Mapping out the dynamic phase transition

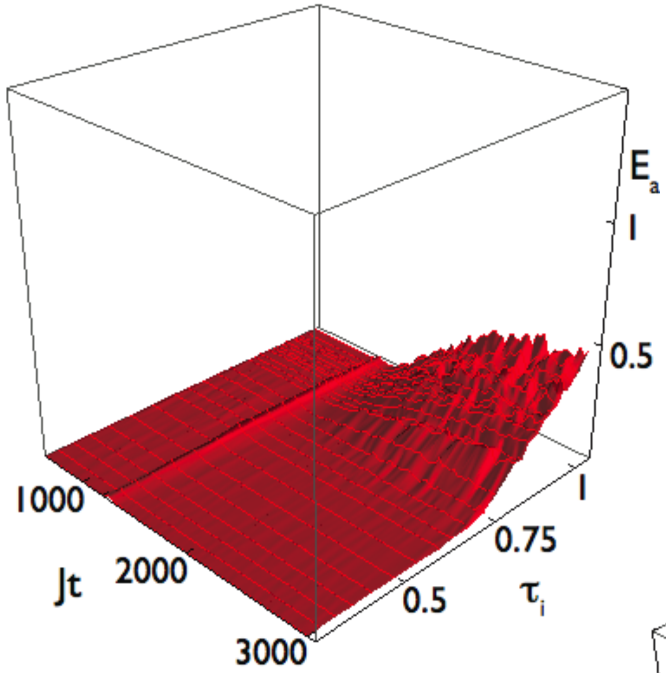
- $\tau(t)$, with τ_I as initial value

- For small τ_i, the system equilibrates at some final $\tau_f > \tau_i$

- For larger τ_i, the system continues to increase, and event unbinding

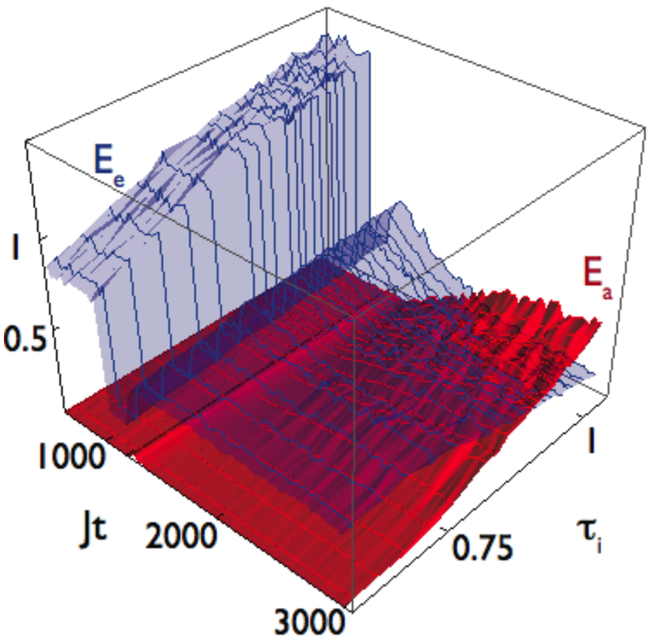


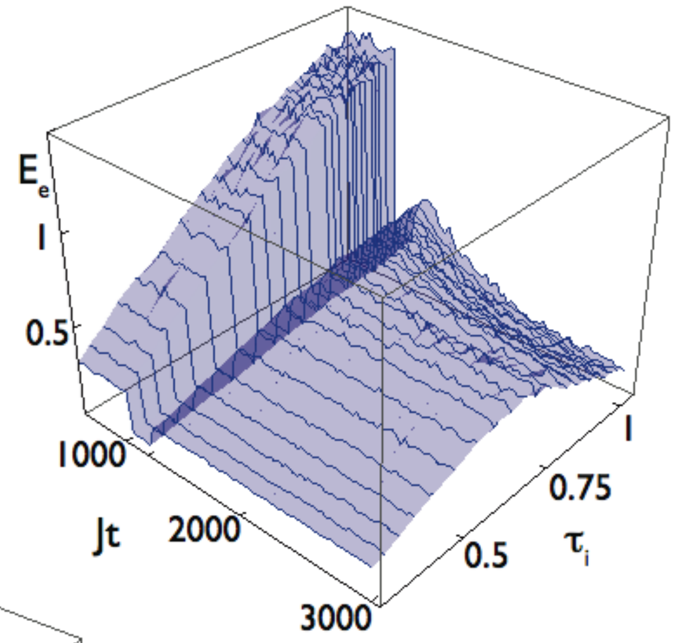
Errors indicate change of functional form



Algebraic error

Dynamic
KT transition!



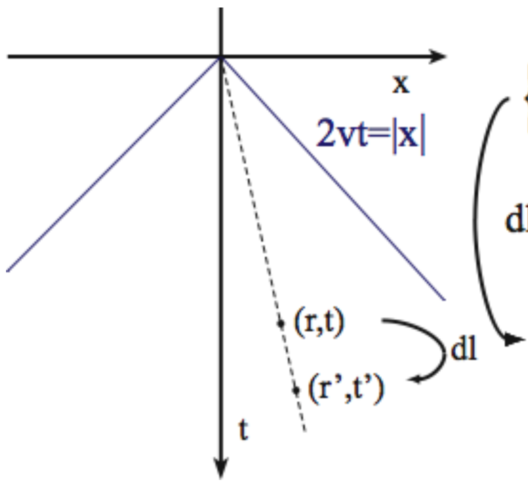


Exponential
error

Renormalization group framework for critical dynamics

LM and A. Polkovnikov, Phys. Rev. A 81, 033605 (2010)

We rescale real-time and real space, and correct for it up to 1-loop order



$$\begin{cases} \dot{\mathbf{p}}(\mathbf{r},\mathbf{t}) = -\partial_{\mathbf{q}}\mathbf{H}(\mathbf{r},\mathbf{t},\lambda,\mathbf{g}) \\ \dot{\theta}(\mathbf{r},\mathbf{t}) = \partial_{\mathbf{p}}\mathbf{H}(\mathbf{r},\mathbf{t},\lambda,\mathbf{g}) \end{cases}$$

dl = dt/t = dr/r

$$\begin{cases} \dot{\mathbf{p}}(\mathbf{r}',\mathbf{t}') = -\partial_{\mathbf{q}}\mathbf{H}(\mathbf{r}',\mathbf{t}',\lambda',\mathbf{g}') \\ \dot{\theta}(\mathbf{r}',\mathbf{t}') = \partial_{\mathbf{p}}\mathbf{H}(\mathbf{r}',\mathbf{t}',\lambda',\mathbf{g}') \end{cases}$$

$$\mathrm{I}: \quad \frac{d\tau}{dl} = \alpha g^2$$

g: vortex fugacity

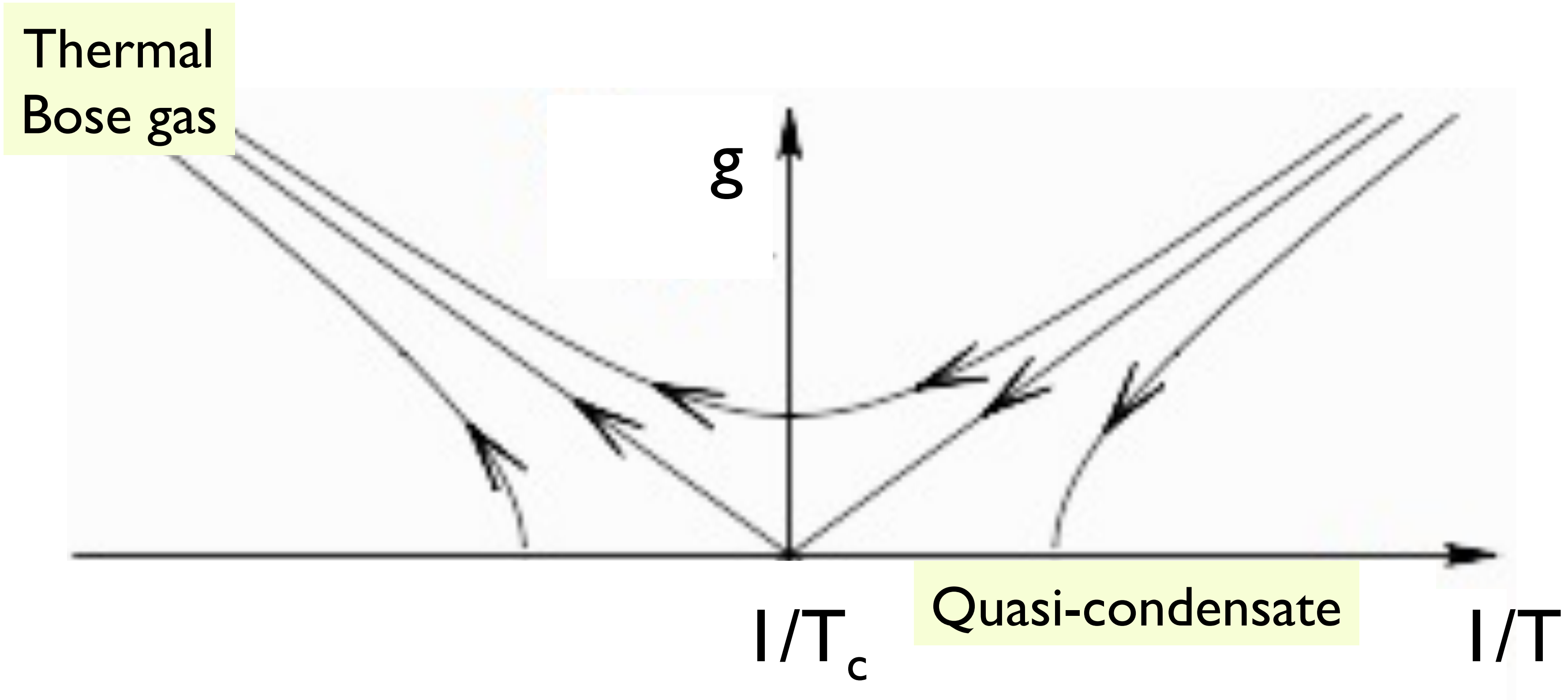
$$\mathrm{II}: \quad \frac{dg}{dl} = (2 - 2/\tau) g$$

$t = t_0 e^l$

This generates RG flow equations in real-time. To predict the dynamic behavior, we fix non-universal constants and time scales, and integrate the flow equations in time.

→ Dynamic KT transition

Renormalization group flow vs. dynamics

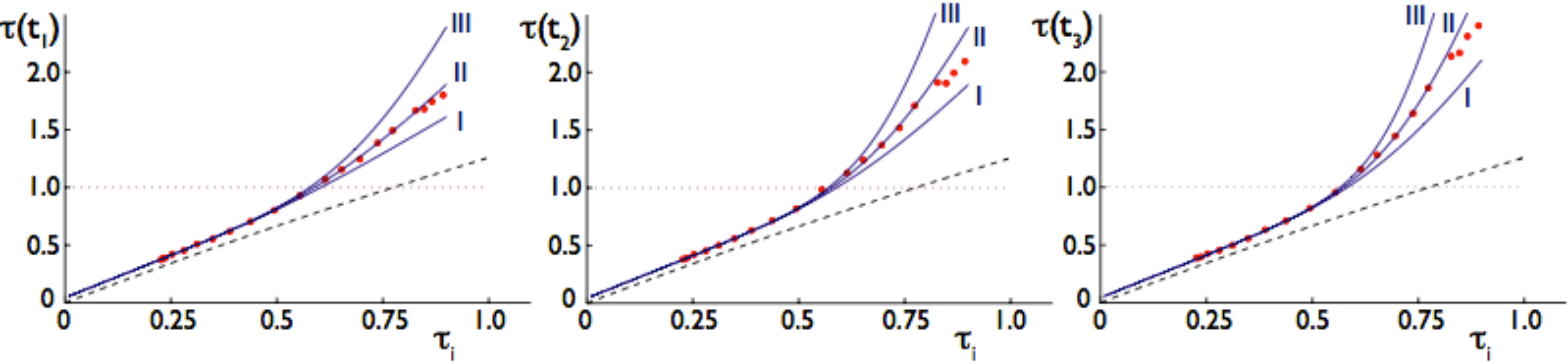


Dynamics resemble the RG flow of the equilibrium system

Phys. Rev. A 80, 041601(R) (2009); 81, 033605 (2010)

Predicting critical dynamics via real-time RG

- We integrate the flow equations and find agreement with the numerics.



Red: τ data at times t_1 = 300, t_2 = 1200, t_3 = 1800.

Blue: RG prediction; 'II' is the correct prediction, 'I' and 'III' are two near-by solutions for visible comparison.

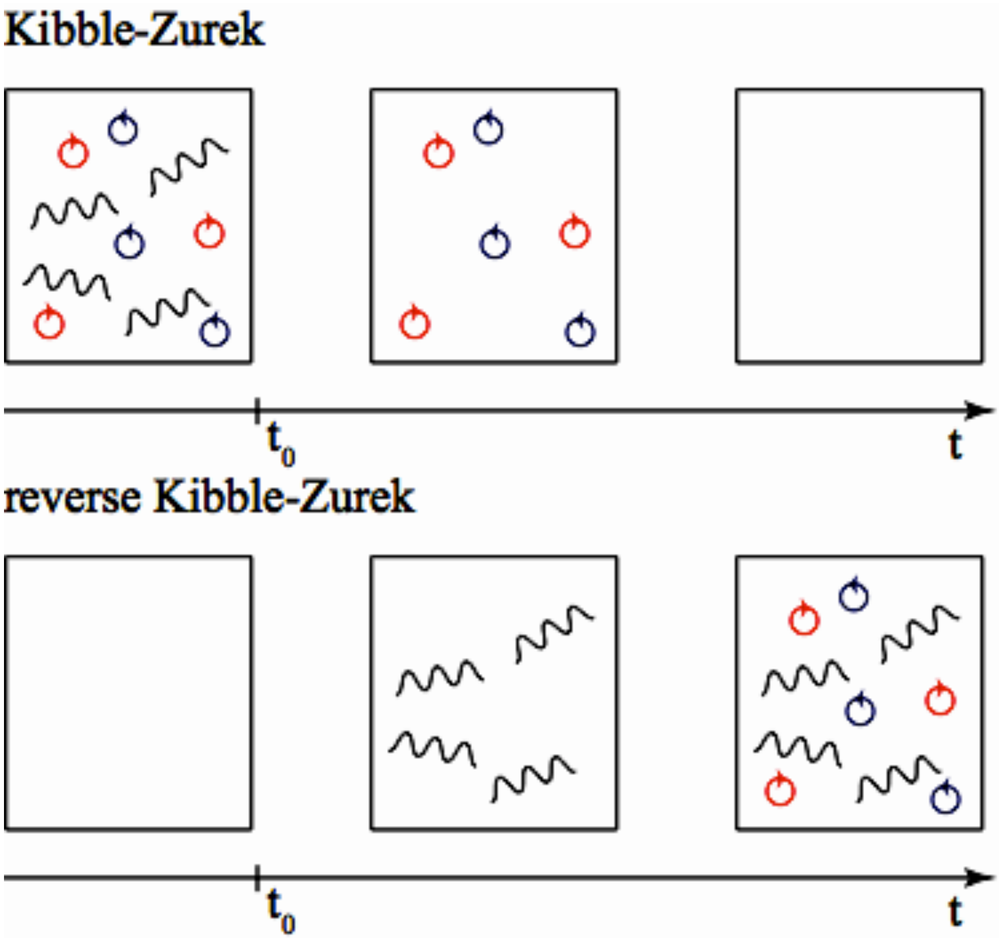
Analytical description of critical dynamics!

Reverse Kibble-Zurek

Kibble-Zurek:
Quench from disordered state

Reverse Kibble-Zurek:
Quench from ordered states

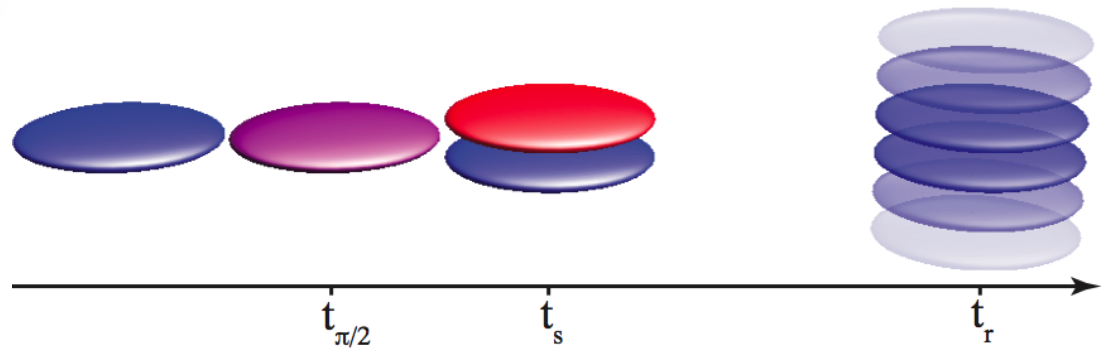
Separation of time scales
of phonon and vortex dynamics



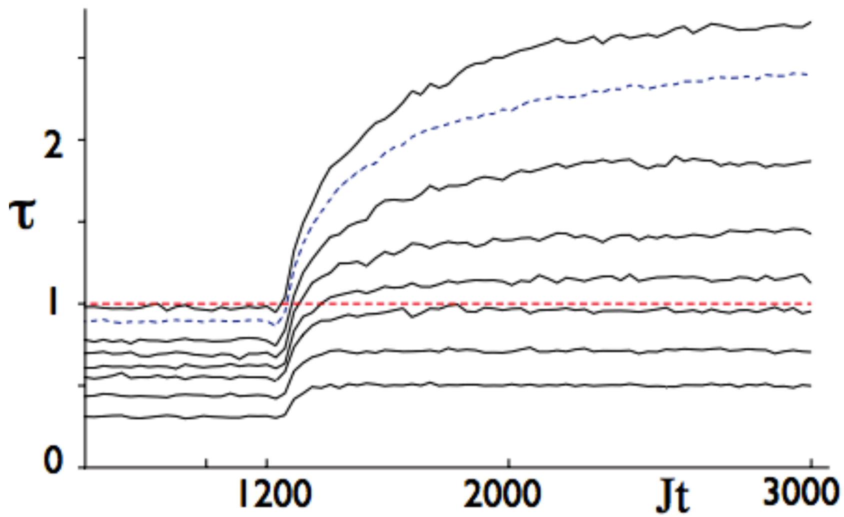
LM + Polkovnikov, '10, '11

Reverse Kibble-Zurek

Split a 2d condensate...



...and watch τ flow according to real-time RG...



...vortices unbind on the time scale

$$\begin{cases} t^* \sim \exp\left(\dfrac{\exp(-E_c/2T)}{\sqrt{1-T^*/T_c}}\right) \\ t^* \sim \exp\left(\dfrac{E_c}{T^*-T_c}\right) \end{cases}$$

Kibble Zurek scaling!

LM + Polkovnikov, '10,'11

Conclusions

- Phase transitions at zero and non-zero temperature
- Critical scaling in equilibrium
- Emergence of a (pre-)thermalized state via light-cone dynamics, following a quench across a phase transition
- Kibble-Zurek scaling following a ramp across a transition
- Domains and topological defects formed as a consequence
- Ultracold atom experiments
- Reverse Kibble-Zurek, real-time RG scaling, induced by splitting a 2D Bose gas