Topological defects and cosmological phase transitions

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Outline

Introduction: Symmetry, symmetry-breaking, and phase transitions

Topological defects

Phase transitions in weakly coupled gauge theories

Formation of topological defects at phase transitions

Introduction: Symmetry, symmetry-breaking, and phase transitions

Relativistic gauge field theories and the Standard Model

- Result of all particle physics experiments are well described by a relativistic gauge field theory, the Standard Model.
- Ingredients:
  - Poincaré - (3+1)D Lorentz symmetry, spacetime translations
  - Gauge symmetry SU(3)$\times$SU(2)$\times$U(1)
  - Quantum mechanics (many body)
- Formulation: Lagrangian quantum field theory
- At low energies: only U(1)$_{\text{em}}$ $\subset$ SU(2)$\times$U(1).
  - Some symmetry is spontaneously broken (SU(2)$\times$U(1))
  - Some symmetry is hidden by confinement (SU(3))

Beyond the Standard Model?

- Reasons for physics Beyond the Standard Model ("BSM")
  - Neutrino masses
  - Dark matter
  - Matter-antimatter asymmetry
  - Inflation
- Many explanations invoke extra symmetries
- Many invoke spontaneous symmetry-breaking
- Symmetry-breaking often means topological defects

BSM physics, cosmology, and topological defects

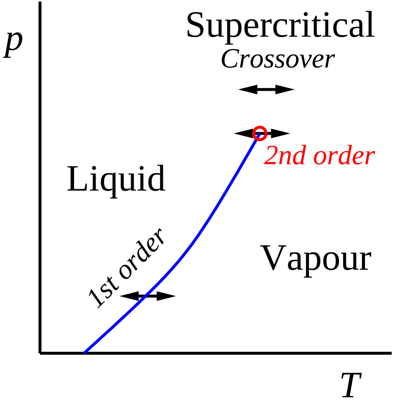
- Early universe: spontaneously broken symmetries are restored[1]
- Symmetry-breaking happens in real time at phase transitions
- At phase transitions topological defects (if allowed) are created[2]
- Search for topological defects in the universe is a search for BSM physics ...
- ... at scales much higher than those accessible by LHC

(1) Kirzhnitz & Linde (1974)
(2) Kibble, *Topology of cosmic domains and strings* (J. Phys. A 1976)

Phase transitions

Classification of phase transitions

- **1st order:** metastable states, latent heat, mixed phases
- **2nd order:** critical slowing down, diverging correlation length
- **Cross-over:** negligible departure from equilibrium



Water phase diagram (sketch)

Phase transitions & cosmology

Phase transitions happened in real time in early Universe:

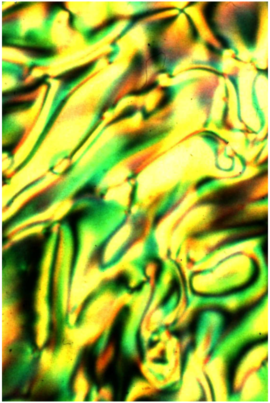
Thermal Changing $T(t)$ | Vacuum Changing field $\sigma(t)$

- **QCD phase transition**
  - Thermal, cross-over.
- **Electroweak phase transition**
  - Thermal, Cross-over (SM), 1st order (BSM): electroweak baryogenesis[3]
  - Vacuum, continuous: cold electroweak baryogenesis[4]
- **Grand Unified Theory & other high-scale phase transitions**
  - Thermal: topological defects[5]
  - Vacuum: hybrid inflation, topological defects, ... [6]

[3] Kuzmin, Rubakov, Shaposhnikov 1988
[4] Smit and Tranberg 2002-6; Smit, Tranberg & Hindmarsh 2007
[5] Kibble 1976; Zurek 1985, 1996; Hindmarsh & Rajantie 2000
[6] Copeland et al 1994; Kofman, Linde, Starobinsky 1996

Topological defects in the laboratory

- Symmetry-breaking is common in condensed matter physics
- Topological defects exist in the laboratory:
  - Vortices in superfluid Helium
  - Flux tubes in superconductors
  - Line disclinations in nematic liquid crystals (right)
  - ...
- $\rightarrow$ Ludwig Mathey's talk



Danger! Natural Units

$$\boxed{\hbar = c = k_B = 1}$$

[Mass]	GeV	10^{-27} kg	proton mass
[Length]	GeV^{-1}	10^{-15} m	proton size
[Time]	GeV^{-1}	10^{-24} s	proton light crossing time
[Temperature]	GeV	10^{13} K	proton pair creation temperature

Planck mass:	$M_\mathrm{P} = 1/\sqrt{G}$	$\sim 10^{19}\ \mathrm{GeV}$
Reduced Planck mass:	$m_\mathrm{P} = 1/\sqrt{8\pi G}$	$\sim 2 \times 10^{18}\ \mathrm{GeV}$
Grand Unification (GUT) scale :	M_GUT	$\sim 10^{16}\ \mathrm{GeV}$
Large Hadron Collider (LHC) energy :	E_LHC	$\sim 10^{4}\ \mathrm{GeV}$

Topological defects

Kinks: (1+1)D model

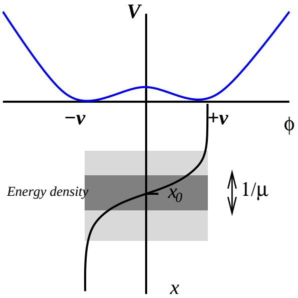
Real scalar field $\phi(x,t)$, symmetry $\phi \to -\phi$. Lagrangian density:

$$\mathcal{L} = \frac{1}{2}\partial\phi\cdot\partial\phi - V(\phi), \qquad V(\phi) = \frac{1}{4}(\lambda\phi^2 - v^2)^2.$$

Field eqn.

$$\frac{\partial^2\phi}{\partial t^2} - \frac{\partial^2\phi}{\partial x^2} + \lambda(\phi^2 - v^2)\phi = 0$$

- "Kink" solutions $\phi = v\tanh(\mu x)$ (where $\mu^2 = \lambda v^2/2$)
- Boosted: $\phi = v\tanh(\gamma\mu(x - vt))$
- Strongly localised energy density
- Energy: $E_K = \frac{2}{3}\sqrt{\frac{2}{\lambda}}v^3$
- "classical particle"



Global vortices in (2+1)D

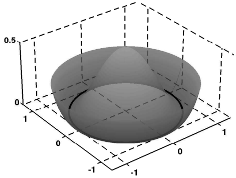
Complex scalar field $\phi(\mathbf{x}, t)$, symmetry $\phi \to e^{i\alpha}\phi$. Lagrangian:

$$\mathcal{L} = \partial\phi^* \cdot \partial\phi - V(|\phi|)$$

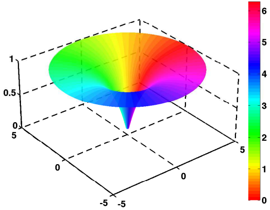
$V(\phi) = \frac{1}{2}\lambda(|\phi|^2 - v^2)^2$.

Field eqn.

$$\frac{\partial^2\phi}{\partial t^2} - \nabla^2\phi + \lambda(|\phi|^2 - v^2)\phi = 0$$



- "Vortex" solution: $\phi = vf(r)e^{i\theta}$,
  $f(r) \to \begin{cases} 0, & r \to 0, \\ 1, & r \to \infty. \end{cases}$
- Energy density: $\rho = |\nabla\phi|^2 + V(\phi)$
- ρ peaked in region $r < r_s = 1/\sqrt{\lambda}v$
- $\rho \to v^2/r^2$ as $r \to \infty$
- Global vortex energy in disk radius R:
  $E_V(R) = 2\pi v^2 \ln(R/r_s)$



Gauge vortices in (2+1)D

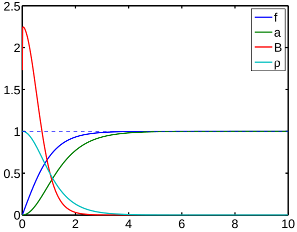
Complex scalar $\phi(x)$, vector $A_\mu(x)$, symmetry $\begin{cases} \phi \to e^{i\alpha(x)}\phi \\ A_\mu \to A_\mu - \frac{1}{e}\partial_\mu \alpha \end{cases}$

$\mathcal{L} = (D\phi)^\dagger \cdot (D\phi) - V(|\phi|) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$ where $D_\mu\phi = (\partial_\mu + ieA_\mu)\phi$

Field eqn.

$$D^2\phi + \lambda(|\phi|^2 - v^2)\phi = 0$$
$$\partial_\mu F^{\mu\nu} - ie(\phi^* D^\mu \phi - D^\mu \phi^* \phi) = 0$$

- Vortex solution:
  $\phi = vf(r)e^{i\theta}, \quad A_i = \frac{1}{er}a(r)\hat{\theta}_i$
- Magnetic field: $B = a'(r)/er$
- Energy density: $\rho = |D_i\phi|^2 + V(\phi) + \frac{1}{2}B^2$
- ρ confined to region $r < \max(1/\sqrt{\lambda}v, 1/ev)$
- Gauge vortex energy: $E_v = 2\pi v^2 G(\lambda/2e^2)$
  [G - slow function, $G(1) = 1$]

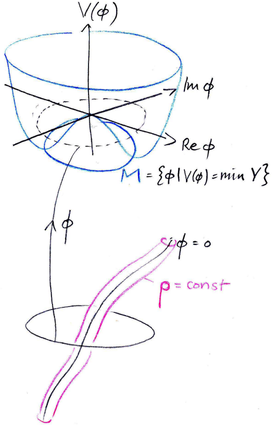


Global strings (3+1)D

- Can construct solution from (2+1)D, independent of z coordinate
- Straight static infinite string
- Energy per unit length $\mu \simeq 2\pi v^2 \ln(Rm_h)$
- Non-static solutions:
  ∞ string with waves, oscillating loops.
- Propagating modes:
  scalar ("Higgs"): $h = |\phi| - v$
  Goldstone boson: $a = v \arg(\phi)$

$$(\Box - m_h^2)h = 0, \quad \Box a = 0$$

$$m_h = \sqrt{\lambda/2}v, \; m_a = 0.$$



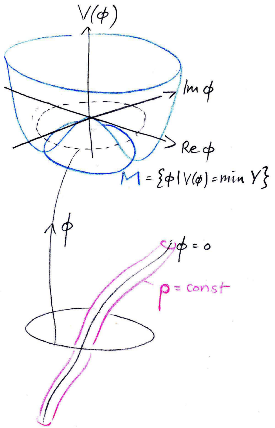
Gauge strings (3+1)D

- Can construct solution from (2+1)D, independent of z coordinate
- Straight static infinite string
- Energy per unit length $\mu \simeq 2\pi v^2$
- Non-static:
  ∞ string with waves, oscillating loops.
- Propagating modes:[a]
  scalar: $h = |\phi| - v$
  gauge: $a_\mu = A_\mu + \frac{1}{e}\partial_\mu \arg(\phi)$

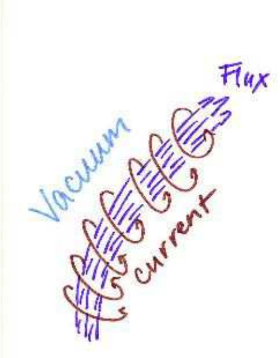
$$(\Box - m_h^2)h = 0, \quad (\Box - m_V^2)a_\mu = 0$$

$$m_h = \sqrt{\lambda}v,\ m_v = \sqrt{2}ev.$$

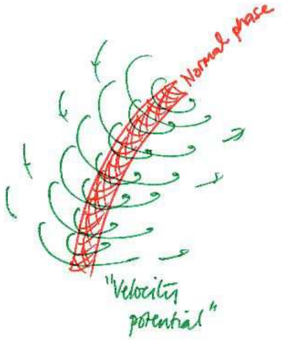
[a] Unitary gauge, $\partial \cdot a = 0$



Comparison: global and local strings from field theory



Gauge/local string
Nearest living relative:
Type II superconductor flux tube



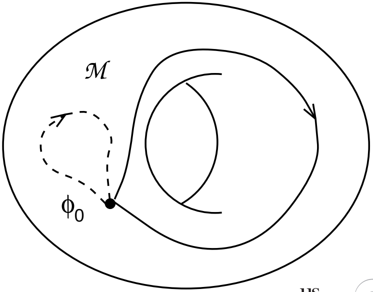
Global string
Nearest living relative:
superfluid vortex

Gauge field theory: Classification of string solutions

$$\mathcal{L} = D_\mu\phi^\dagger D^\mu\phi - V(\phi) - \frac{1}{2e^2}\,\mathrm{tr}\,F_{\mu\nu}F^{\mu\nu}$$

Symmetry group G: $\phi \to g\phi$, $A_\mu \to gA_\mu g^{-1} + ig\partial_\mu g^{-1}$: $S \to S$
Vacuum manifold $\mathcal{M} = \{\phi | V(\phi) = \min V\}$

- Let $\phi_0 \in \mathcal{M}$: unbroken symmetry group $H = \{h \in G | h\phi_0 = \phi_0\}$
- Note $\mathcal{M} \simeq G/H$[a]
- $\exists$ strings if non-contractible loops in $\mathcal{M}$ i.e. $\pi_1(\mathcal{M}) \neq 0$
- Exact sequence: $\pi_1(G/H) \simeq \pi_0(H)$[b]
- GUT example: $Spin(10) \overset{126}{\to} SU(5) \times Z_2$



[a] $\mathcal{M} \supset G/H$ if there are accidental global symmetries
[b] Provided $\pi_1(G) = \pi_0(H) = 0$

Global monopoles

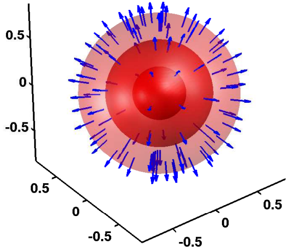
N scalar fields $\phi^A(\mathbf{x}, t)$, $N = 3$, O(3) symmetry,

$$\mathcal{L} = \frac{1}{2}\partial\phi^A \cdot \partial\phi^A - V(|\vec{\phi}|), \qquad V(|\vec{\phi}|) = \frac{1}{4}\lambda(\phi^A\phi^A - v^2)^2$$

Field eqn.

$$\frac{\partial^2\phi^A}{\partial t^2} - \nabla^2\phi^A + \lambda(\vec{\phi}^2 - v^2)\phi^A = 0$$

- Only static stable solution: $|\vec{\phi}| = v$
- Global symmetry is broken to O(2).
- Vacuum manifold $M \simeq O(3)/O(2) \simeq S^2$
- Monopole solution $\phi^A(\mathbf{x}) = \hat{x}^A v f(r)$
  $f(r) \to \begin{cases} 0, & r \to 0, \\ 1, & r \to \infty. \end{cases}$
- Energy density $\rho \to v^2/r^2$ as $r \to \infty$
- Monopole energy in ball radius R:
  $E_{gm}(R) = 4\pi v^2 R$
- Unstable: $M - \bar{M}$ has linear potential.

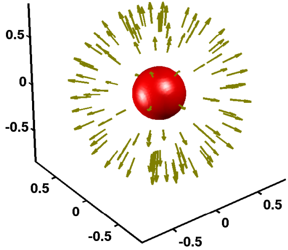


Gauge ('t Hooft-Polyakov) monopoles

3 scalar fields ϕ^A, SO(3) gauge fields A^A_μ. Let $D_\mu \phi^A = \partial_\mu \phi^A + g\epsilon^{ABC} A^B_\mu \phi^C$

$\mathcal{L} = \frac{1}{2} D\phi^A \cdot D\phi^A - V(|\vec{\phi}|) - V(|\vec{\phi}|) - \frac{1}{4} F^A_{\mu\nu} F^{A\mu\nu}$

- Symmetry broken to O(2) $\simeq$ U(1).
- Monopole solution
  $\phi^A(\mathbf{x}) = \hat{x}^A v f(r), \quad A^A_i = \frac{P^A_i}{gr}(1 - a(r))$
  where $P^A_i = \delta^A_i - \hat{x}_i \hat{x}^A$.
- Magnetic field: $B_i = \frac{1}{2}\epsilon_{ijk} F^A_{jk} \hat{\phi}^A = \frac{a'(r)}{gr^2}$
- Monopole energy: $E_m = \frac{4\pi}{g} v^2 H(\lambda/g^2)$
  $H(0) = 1$ (Bogomol'nyi-Prasad-Somerfeld)
- $\exists$ monopoles if non-contractible spheres in $\mathcal{M}$ i.e. $\pi_2(\mathcal{M}) \neq 0$
- Exact sequence: $\pi_2(G/H) \simeq \pi_1(H)$[a]
- Grand Unification $\implies$ monopoles
  (H = SU(3)×SU(2)×U(1))



[a] Provided $\pi_2(G) = \pi_1(H) = 0$

Textures

N scalar fields $\phi^A(\mathbf{x}, t)$, O(N) symmetry, $N > 3$

$$\mathcal{L} = \frac{1}{2}\partial\phi^A \cdot \partial\phi^A - V(|\vec{\phi}|), \qquad V(|\vec{\phi}|) = \frac{1}{4}\lambda(\phi^A\phi^A - v^2)^2$$

Field eqn.

$$\frac{\partial^2\phi^A}{\partial t^2} - \nabla^2\phi^A + \lambda(\vec{\phi}^2 - v^2)\phi^A = 0$$

- Only static stable solution: $|\vec{\phi}| = v$
- Global symmetry is broken to O($N - 1$).
- Low-energy dynamics: non-linear σ-model
  $\left(\Box - \partial\hat{\phi} \cdot \partial\hat{\phi}\right) \phi^A = 0, \quad \hat{\phi} = \vec{\phi}/|\vec{\phi}|.$
- Vacuum manifold $M \simeq O(N)/O(N-1) \simeq S^{N-1}$
- $N = 4$: non-static solutions with $|\vec{\phi}|$ vanishing at one space-time point
- Textures exist in any kind of non-Abelian global symmetry-breaking.

Semilocal strings

- N complex scalar fields $\phi^A(\mathbf{x}, t)$
- U(N) symmetry: $\mathcal{L} = (D\phi^A)^* \cdot (D\phi^A) - V(|\phi|) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$
- $\mathrm{U}(N) = \mathrm{SU}(N)_{\mathrm{global}} \times \mathrm{U}(1)_{\mathrm{local}}$.
- Potential: $V(\phi^A\phi^A) = \frac{1}{2}\lambda(\phi^{A*}\phi^A - v^2)^2$
- At low energy density, $|\phi| \simeq v$
- Symmetry is broken to $\mathrm{SU}(N-1)_{\mathrm{global}}$
- Vortex/string solutions: $\phi = v^A f(r) e^{i\theta}, \quad A_i = \frac{1}{er} a(r)\hat{\theta}_i$
- For gauge coupling $\gg$ scalar coupling: vortices stable
- For gauge coupling $\ll$ scalar coupling: vortices unstable
- Low-energy dynamics: non-linear σ-model, $\partial_\mu(G_{AB}(\phi)\partial^\mu\phi^A) = 0$.
- Vacuum manifold: $M \simeq \mathrm{SU}(N)/\mathrm{SU}(N-1) \simeq \mathrm{CP}^{N-1}$, metric $G_{AB}(\phi)$.

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- Salomaa and Volovik, *Quantized vortices in superfluid* ^{3}*He* (Rev. Mod. Phys. 1987)
- Volovik, *The universe in a Helium droplet* (Clarendon, Oxford, 2003)

Phase transitions in weakly coupled gauge theories

Thermodynamic relations for cosmology

Particle reaction rates large compared with expansion rate $H \propto 1/t$

$$n\langle \sigma v \rangle \ll H \qquad \begin{cases} \sigma & \text{Scattering cross-section} \\ n & \text{Number density of scatterers} \\ v & \text{Relative speed} \\ \langle \ldots \rangle & \text{Thermal average} \end{cases}$$

Early Universe very close to thermal equilibrium: expansion isentropic.

$$S = sa^3 = \text{const.} \qquad \text{Entropy density } s.$$

Thermodynamic relations:

$$s = \frac{dp}{dT}, \quad sT = \rho + p \qquad \left(\to \rho = T^2 \frac{d}{dT}\left(\frac{p}{T}\right)\right)$$

NB Need to calculate only pressure (easiest in QFT)
NB Eqm fails for neutrinos at $T \simeq 1$ MeV, WIMPs at $T \simeq 1 - 10$ GeV.

Free energy of an ideal gas

- Free energy density $f = \rho - Ts$ (also $f = -p$)
- To find equilibrium state we minimise free energy
- Dimensions: $f = T^4\phi(m/T)$ with $\phi(0) = -g\pi^2/90$.

Pressure due to particles of mass m in equilibrium (zero chemical potential) $\eta = \pm 1$ (FD/BE)):

$$p = \int \frac{\bar{d}^3 k}{2E_k} \frac{1}{e^{E_k/T} + \eta} \frac{2k^2}{3}, \qquad E_k = (k^2 + m^2)^{\frac{1}{2}}$$

Free energy density ($f = -k_B T \ln Z/V$):

$$f = -\eta T \int \bar{d}^3 k \ln(1 + \eta e^{-E_k/T})$$

Note $f = -p$ by partial integration.

Free energy: exact formulae in high T expansion

Bosons:

$$f_B = -\frac{\pi^2}{90}T^4 + \frac{m^2T^2}{24} - \frac{(m^2)^{\frac{3}{2}}T}{12\pi} - \frac{m^4}{64\pi^2}\ln\left(\frac{m^2}{a_bT^2}\right) - \frac{m^4}{16\pi^{\frac{5}{2}}}\sum_\ell(-1)^\ell\frac{\zeta(2\ell+1)}{(\ell+1)!}\left(\frac{m^2}{4\pi^2T^2}\right)^\ell$$

Fermions:

$$f_F = -\frac{\pi^2}{90}\frac{7}{8}T^4 + \frac{m^2T^2}{48} + \frac{m^4}{64\pi^2}\ln\left(\frac{m^2}{a_fT^2}\right) + \frac{m^4}{16\pi^{\frac{5}{2}}}\sum_\ell(-1)^\ell\frac{\zeta(2\ell+1)}{(\ell+1)!}(1-2^{-2\ell-1})\Gamma(\ell+\tfrac{1}{2})\left(\frac{m^2}{4\pi^2T^2}\right)^\ell$$

$a_b = 16\pi^2\ln(\frac{3}{2} - 2\gamma_E)$, $a_f = a_b/16$, $\gamma_E = 0.5772\ldots$ (Euler's constant)

Effective potential for scalar field with gauge fields and fermions

Suppose scalar field gives masses to

- scalars ($M_S^2(\bar{\phi}) \propto 3\lambda\bar{\phi}^2 + \mu^2$)
- vectors ($M_V^2(\bar{\phi}) \propto g^2\bar{\phi}^2$)
- (Dirac) fermions ($M_F^2(\bar{\phi}) \propto y^2\bar{\phi}^2$)

$$
\begin{aligned}
V_T(\bar{\phi}) &= V_T(0) + \tfrac{1}{2}\mu^2\bar{\phi}^2 + \tfrac{1}{4!}\lambda\bar{\phi}^4 \\
&+ \frac{T^2}{24}\left(\sum_S M_S^2(\bar{\phi}) + 3\sum_V M_V^2(\bar{\phi}) + 2\sum_F M_F^2(\bar{\phi})\right) \\
&- \frac{T}{12\pi}\left(\sum_S (M_S^2(\bar{\phi}))^{\frac{3}{2}} + 3\sum_V (M_V^2(\bar{\phi}))^{\frac{3}{2}}\right) + \cdots
\end{aligned}
$$

Neglect higher order terms when $M^2(\phi)/T^2 \ll 1$.

Symmetry restoration at high T

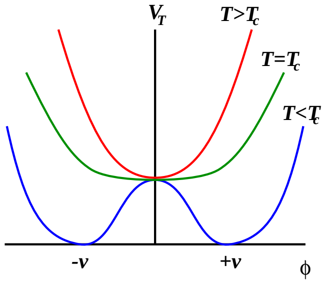
Consider 1 real scalar

Suppose $\mu^2 < 0$ and $M(\bar\phi)/T \ll 1$.

$$\Delta V_T = \frac{1}{2}(-|\mu|^2 + \frac{1}{24}\lambda T^2)\bar\phi^2 + \frac{1}{4!}\lambda\bar\phi^4$$

Equilibrium at

$$\begin{aligned} \bar\phi^2 &= 6m^2(T)/\lambda \\ m^2(T) &= (|\mu^2| - \frac{1}{24}\lambda T^2) \end{aligned} \quad (1)$$



- Critical temperature $T_c^2 = 24|\mu^2|/\lambda$
- Above T_c, equilibrium state is $\bar\phi = 0$
- $\phi \to -\phi$ symmetry is restored
- Second-order phase transition(7)
  discontinuity in specific heat, correlation length diverges $\xi = 1/|m(T)|$
- Careful: we must only consider $\bar\phi$ for which $M^2(\bar\phi) > 0$.

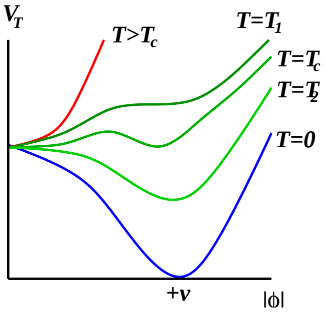
(7) Kirzhnitz & Linde (1974), Dolan & Jackiw (1974)

First order phase transition

Gauge fields and fermions: when $\lambda \ll g^2$ cubic term can be trusted

$$\Delta V_T \simeq \frac{\gamma}{2}(T^2 - T_2^2)|\bar{\phi}|^2 - \delta T|\bar{\phi}|^3 + \frac{1}{4!}\lambda|\bar{\phi}|^4$$

- γ, δ are functions of couplings g, y, λ
- Second minimum develops at T_1
- Critical temperature T_c: free energies are equal.
- System can supercool below T_c.
- First order transition discontinuity in free energy
- Note: when $\lambda \simeq g^2$, transition is a cross-over (Standard Model)



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Formation of topological defects at phase transitions

Breaking a global Z_2 symmetry: forming domain walls

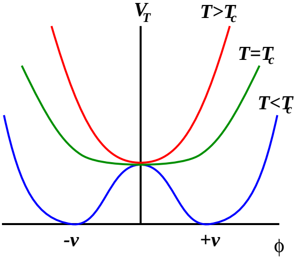
Real scalar field $\phi(\mathbf{x}, t)$, symmetry $\phi \to -\phi$. Lagrangian density:

$$\mathcal{L} = \tfrac{1}{2}\partial\phi^2 - V(\phi), \qquad V(\phi) = V_0 - \tfrac{1}{2}\mu^2\phi^2 + \tfrac{1}{4!}\lambda\phi^4.$$

At high temperature T, can coarse-grain for wavenumbers $k < T$. $V \to V_T$, with

$$V_T(\phi) \simeq V_0 + (\frac{1}{24}\lambda T^2 - \tfrac{1}{2}\mu^2)\phi^2 + \tfrac{1}{4!}\lambda\phi^4$$

Phase transition at $T_c \simeq \mu\sqrt{24/\lambda}$.

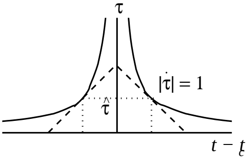


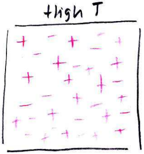
Equation of motion of the (coarse-grained) field:

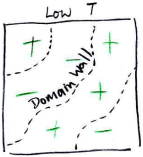
$$\frac{\partial^2\phi}{\partial t^2} - \nabla^2\phi + \frac{1}{12}\lambda(T^2 - T_c^2)\phi + \frac{1}{3!}\lambda\phi^3 = 0$$

Formation of defects: Kibble-Zurek mechanism

- "Quench" through transition in time τ_Q:
  $T_c - T(t) = T_c[(t - t_c)/\tau_Q]$
- Equilibrium correlation length:
  $\xi \simeq 1/\sqrt{\lambda(T^2 - T_c^2)} \propto [(t - t_c)/\tau_Q]^{-\frac{1}{2}}$
- Field relaxation time:
  $\tau \simeq 1/\sqrt{\lambda(T^2 - T_c^2)} \propto [(t - t_c)/\tau_Q]^{-\frac{1}{2}}$
- Out of equilibrium at t_*, when $|d\tau/dt| > 1$.
- Correlation length is "stuck" at ξ_*
- Defects are formed with initial correlation length ξ_*



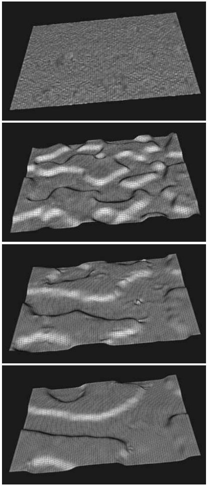




Formation of walls in 2D: numerical simulation[8]

$$\ddot{\phi} + \eta(t)\dot{\phi} - \nabla^2\phi + (\phi^2 - \mu^2(t))\phi = 0$$

- $\eta(t) = \theta(t_{\rm damp} - t)$ models cooling
- $\mu^2(t) = \theta(t) - \theta(-t)$ models rapid transition
- Initial conditions: $\phi(\mathbf{x})$ Gaussian random variable on each lattice site
- Late time behaviour ("coarsening" dynamics): $\xi(t) \propto t$
- $\xi(t) \propto t$ also called scaling

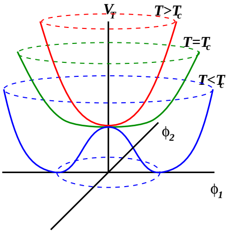


(8) Garagounis and Hindmarsh, arXiv:hep-ph/0212359 (2002)

Gauge field theories: Abelian Higgs model

$$S = -\int d^4x \left(D_\mu \phi^* D^\mu \phi + V(\phi) + \frac{1}{4e^2} F_{\mu\nu} F^{\mu\nu} \right)$$

Complex scalar field $\phi(\mathbf{x}, t)$, vector field $A_\mu(\mathbf{x}, t)$
Covariant derivative $D_\mu = \partial_\mu + iA_\mu$.
Potential $V(\phi) = \frac{1}{2}\lambda(|\phi|^2 - v^2)^2$.
"Relativistic Ginzburg-Landau"



Temporal gauge ($A_0 = 0$) field equations

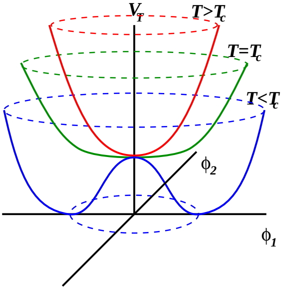
$$\begin{aligned} \ddot{\phi} - D_i^2 \phi + \lambda(|\phi|^2 - v^2)\phi &= 0, \\ \frac{\partial}{\partial t} E_i + \epsilon_{ijk}\partial_j B_k - ie(\phi^* D_i \phi - D_i \phi^* \phi) &= 0, \end{aligned}$$

Formation of gauge vortices

Abelian Higgs model: complex scalar field $\phi(x)$, vector field $A_\mu(x)$.

$$\mathcal{L}_{\rm eff} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + |D\phi|^2 - V_{\rm eff}(\phi),$$
$$V_{\rm eff}(\phi) \simeq V_0 + m_{\rm eff}^2|\phi^2| + \frac{1}{4}\lambda|\phi|^4$$

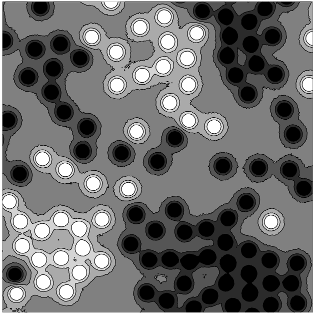
- Scalar expectation value: $v(T) = \theta(T_c - T)\sqrt{T_c^2 - T^2}$
- Gauge field mass: $m_v^2(T) = \frac{1}{2}e^2v^2(T)$
- Correlation function: $\langle B_i(\mathbf{k})B_j(\mathbf{k}')\rangle = G(k)P_{ij}(k)\delta^3(\mathbf{k}-\mathbf{k}')$
- Equilibrium: $G_{\rm eq}(k) = \frac{k^2T}{k^2+m_v^2(\phi)}$



Gauge field theories: flux trapping mechanism

- Decay rate of eqm. correlator:
  $\Gamma_k = \frac{d \ln G_{\text{eq}}(k)}{dt} = \frac{e^2 T_c^2}{2\tau_Q k^2}$ (for $k^2 < m_v^2$)
- Response time of magnetic field:
  $\tau = k^2/\sigma$ (conductivity σ)
- Wavenumbers out of equilibrium:
  $$k < \hat{k} = \left(\frac{e^2 T_c^2 \sigma}{4\tau_Q}\right)^{\frac{1}{4}}$$
- Hence non-eqm correlator
  $$G(k) \simeq \begin{cases} T & k < \hat{k} \\ 0 & k > \hat{k} \end{cases}$$
- Net vortex number in disc of radius R:
  $N_V(R) = (e/2\pi) \int^R d^2 x B(x)$.
- Prediction: $\langle N_V(R)^2 \rangle \propto T_c R$
- Prediction: vortices are correlated

Numerical simulation of vortex formation in 2D



[Stephens, Bettencourt, Zurek (2002)]

More topics ...

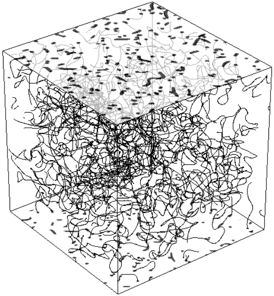
- Dynamics of first order phase transitions
- Formation of defects at first order phase transitions
- Dynamics of vacuum phase transitions
- Formation of defects at vacuum phase transitions

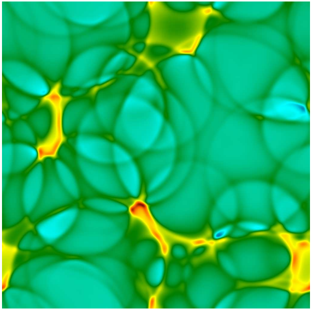
References

- de Campo & Zurek, *Universality of phase transition dynamics: Topological Defects from Symmetry Breaking* (Int. J. Mod. Phys. 2014)
- Rajantie, *'Phase transitions in the early universe' and 'Defect formation'* (hep-ph/0311262)
- Zurek, *Cosmological experiments in condensed matter systems* (Phys. Rep. 1996)
- Rajantie & Tranberg, *Counting defects with the two-point correlator* (JHEP 2010)
- Berges & Roth, *Topological defect formation from 2PI effective action techniques* (Nucl. Phys. B 2010)

Next time ...

- Dynamics of topological defects in the early universe
- Observational signals (Cosmic Microwave Background B-modes!)
- Gravitational waves from phase transitions





Connection to time-dependent Gross-Pitaevskii equation

- Let $\phi(x) = e^{-imt}\psi(x)$: results in a number density
  $$n = i(\phi^*\dot{\phi} - \dot{\phi}^*\phi) = 2m|\psi(x)|^2 + i(\psi^*\dot{\psi} - \dot{\psi}^*\psi).$$
- Slowly varying number density: $\omega \ll |\dot{\psi}/\psi|$,

$\ddot{\phi} - \nabla^2\phi + \lambda(|\phi|^2 - v^2)\phi$ becomes

$$-2im\dot{\psi} - \nabla^2\psi + \left[\lambda(|\psi|^2 - v^2) - m^2\right]\psi \quad \simeq \quad 0,$$

Equivalent to time-dependent Gross-Pitaevskii equation ($\hbar = 1$)

$$i\dot{\psi} = -\frac{1}{2m}\nabla^2\psi + \left(g|\psi|^2 + V\right)\psi,$$

Dispersion relation:

$$\omega = \sqrt{\frac{\mathbf{k}^2}{2m}\left(\frac{\mathbf{k}^2}{2m} + 2gn\right)}$$