Topological defects in cosmology: simulations and constraints

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Outline

Topological defects and cosmology

- Made in the early universe?[a]

Physics	Temperature	Time	Length
Inflation	10^{16} GeV	10^{-36} s	10^{-31} m
Grand Unification	10^{16} GeV	10^{-36} s	10^{-31} m
Neutrino masses	10^{3} - 10^{16} GeV	10^{-36} - 10^{-13} s	10^{-31} - 10^{-18}m
just "Beyond SM"	10^{3} GeV	10^{-13} s	10^{-18}m
...			

- If formed, still here: scaling
  - Correlation length (distance between defects) $\propto$ time
  - Density $\propto$ total density
- Messengers from the very early universe and very high energies

[a] Kibble (1976); Zurek (1996); Yokoyama (1989); Kofman, Linde, Starobinski (1996); Rajantie (2002);

Cosmic strings

- Cosmic strings[1] are linear distributions of mass-energy in the universe.
- Mass per unit length μ, tension T. Normally $\mu = T/c^2$
- Dynamics: acceleration $\propto$ curvature: wave equation
- In theories of high energy physics they may be
  - Elementary (string theory): zero width
  - Solitonic (field theory): non-zero width
- Generic in Grand Unified field theories (GUTs)[2]

[1] Hindmarsh & Kibble (1994); Vilenkin & Shellard (1994); Kibble (2004)
[2] Jeannerot, Rocher & Sakellaridou 2003

Other defects: global monopoles, textures and semilocal strings

Global monopoles and textures[3]

- Self-ordering scalar fields (Goldstone modes) from global symmetry-breaking.
- Global monopoles: point-like, with attractive force proportional to distance.
- Symmetry-breaking scale $v \sim 10^{16}$ GeV: observable perturbations.

Semilocal strings[4]

- Self-ordering scalar and vector fields from “semilocal" symmetry-breaking
- Semilocal: non-trivial combination of local and global symmetries
- Symmetry-breaking scale $v \sim 10^{16}$ GeV: observable perturbations.

[3] Turok 1989; Spergel et al 1991; Pen, Spergel, Turok 1995; Durrer, Kunz, Melchiorri 1999,2002.
[4] Vachaspati, Achucarro 1991; Hindmarsh 1992,1993; Urrestilla et al. 2008.

Signals from cosmic defects

- Gravitational waves [all defects[5]]
  - scale-invariant spectrum
  - amplitude $\Omega_{\rm gw}(\omega) \sim (G\mu)^2$,
  - Cosmic string loops (Nambu-Goto scenario) $\Omega_{\rm gw}(\omega) \sim (G\mu)$ (Vilenkin 1981)
- Cosmic rays[6] [strings]
  - GeV-scale γ-rays (EGRET, FERMI/LAT)
  - UHECRs (Auger)
  - Neutrinos (Ice Cube)
- Decaying strings are sources of
  - dark matter[7]
  - baryon number[8]
- Cosmic Microwave Background perturbations [all defects[9]]

(5) Krauss 1992, Fenu et al 2009,Figueroa, Hindmarsh, Urrestilla 2012.

(6) Bhattacharjee, Sigl 1999

(7) Jeannerot, Zhang, Brandenberger 1999; MH, Kirk, West (2014)

(8) Bhattacharjee, Kibble, Turok 1984

(9) Pen, Seljak, Turok (1997); Durrer, Kunz, Melchiorri (1999); Bevis et al (2006-11)

Danger! Natural Units

$$\boxed{\hbar = c = k_B = 1}$$

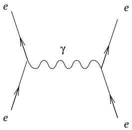
[Mass]	GeV	10^{-27} kg	proton mass
[Length]	GeV^{-1}	10^{-15} m	proton size
[Time]	GeV^{-1}	10^{-24} s	proton light crossing time
[Temperature]	GeV	10^{13} K	proton pair creation temperature

Planck mass:	$M_{\rm P} = 1/\sqrt{G}$	$\sim 10^{19}$ GeV
Reduced Planck mass:	$m_{\rm P} = 1/\sqrt{8\pi G}$	$\sim 2 \times 10^{18}$ GeV
Grand Unification (GUT) scale :	$M_{\rm GUT}$	$\sim 10^{16}$ GeV
Large Hadron Collider (LHC) energy :	$E_{\rm LHC}$	$\sim 10^{4}$ GeV

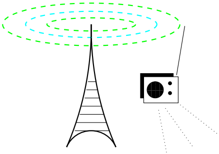
Formation & evolution of topological defects: methods

Classical field theory is good
(high occupation number)

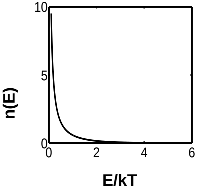
- Thermal equilibrium
  ($k \ll T$, but not $k \gtrsim T$)
- When field is “large":
  $\langle \phi \rangle^2 \gg \langle \delta\phi^2 \rangle$
  - Inflation,
  - symmetry-breaking,
  - topological defects
- Beyond classical:
  - Hartree
  - Inhomogeneous Hartree
  - 2PI
  - Stochastic quantisation
  - “Cheap" fermions



Quantum
Low occupation number



Classical
High occupation number



Breaking a global Z_2 symmetry: forming domain walls

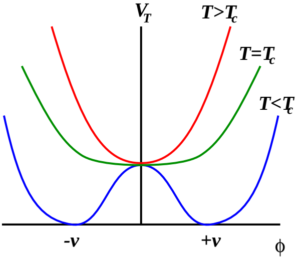
Real scalar field $\phi(\mathbf{x}, t)$, symmetry $\phi \to -\phi$. Lagrangian density:

$$\mathcal{L} = \frac{1}{2}\partial\phi^2 - V(\phi), \qquad V(\phi) = V_0 - \frac{1}{2}\mu^2\phi^2 + \frac{1}{4!}\lambda\phi^4.$$

At high temperature T, can coarse-grain for wavenumbers $k < T$. $V \to V_T$, with

$$V_T(\phi) \simeq V_0 + (\frac{1}{24}\lambda T^2 - \frac{1}{2}\mu^2)\phi^2 + \frac{1}{4!}\lambda\phi^4$$

Phase transition at $T_c \simeq \mu\sqrt{24/\lambda}$.



Equation of motion of the (coarse-grained) field:

$$\frac{\partial^2\phi}{\partial t^2} - \nabla^2\phi + \frac{1}{12}\lambda(T^2 - T_c^2)\phi + \frac{1}{3!}\lambda\phi^3 = 0$$

Evolution of domain walls/strings in 2D

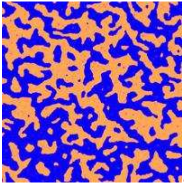
Solve classical field equation:

$$\ddot{\phi} + \eta(t)\dot{\phi} - \nabla^2\phi + (\phi^2 - \mu^2(t))\phi = 0$$

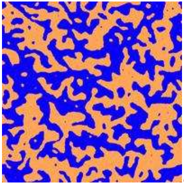
- $\eta(t) = \theta(t_{\rm damp} - t)$ models cooling, expansion:[10]
- $\mu^2(t) = \theta(t) - \theta(-t)$ models rapid transition
- Initial conditions: $\phi(\mathbf{x})$ Gaussian random variable on each lattice site (both thermal and quantum correlators vanish as $|\mathbf{x} - \mathbf{y}| \to \infty$)
- Interested only in $t \to \infty$ behaviour of topological defects

[10] Garagounis and Hindmarsh, arXiv:hep-ph/0212359 (2002)
http://www.sussex.ac.uk/tpp/arXiv/hep-ph/0212359/

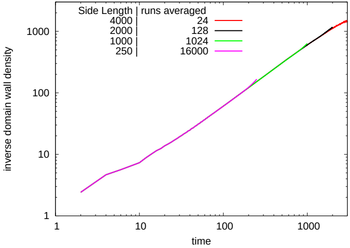
Domain walls/strings in 2D: scaling



$t = 50$



$t = 100$, $\times\frac{1}{2}$



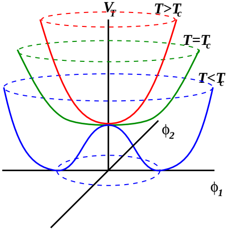
- Define network scale length: $\xi = \text{Area}/(\text{Wall length})$
- Scaling: $\xi \propto t$
- Scaling lasts $\xi M > 10^3$ [a]

[a] Borsanyi & Hindmarsh (2007)

Abelian Higgs model

$$S = -\int d^4x \left(D_\mu \phi^* D^\mu \phi + V(\phi) + \frac{1}{4e^2} F_{\mu\nu} F^{\mu\nu} \right),$$

- Complex scalar field $\phi(\mathbf{x}, t)$,
- vector field $A_\mu(\mathbf{x}, t)$
- Covariant derivative $D_\mu = \partial_\mu - iA_\mu$.
- Potential $V(\phi) = \frac{1}{2}\lambda(|\phi|^2 - v^2)^2$.

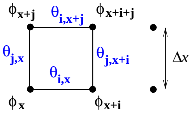


Temporal gauge ($A_0 = 0$) field equations:

$$\begin{aligned} \ddot{\phi} - \mathbf{D}^2\phi + \lambda(|\phi|^2 - v^2)\phi &= 0, \\ \dot{E}_i + \epsilon_{ijk}\partial_j B_k - ie^2(\phi^* D_i\phi - D_i\phi^*\phi) &= 0. \end{aligned}$$

Abelian Higgs model on the lattice (Minkowski space)

- $\phi(\mathbf{x}, t) \to \phi_{\mathbf{x}}(t)$, , defined on sites
- Canonical momentum $\pi_{\mathbf{x}}(t) = \dot{\phi}_{\mathbf{x}}(t)$
- $\mathbf{A}(\mathbf{x}, t) \to \theta_{i,\mathbf{x}} = -e\Delta x A_{i,\mathbf{x}}(t)$ on links
- Electric field $\epsilon_{i,\mathbf{x}}(t) = \dot{\theta}_{i,\mathbf{x}}$



Discretisation: covariant derivative $\mathbf{D}\phi(\mathbf{x})$, B-field energy density $\frac{1}{2}\mathbf{B}^2$

$$|\mathbf{D}\phi(\mathbf{x})|^2 \to \frac{1}{\Delta x^2}\sum_i |e^{-i\theta_{i,\mathbf{x}}}\phi_{\mathbf{x+i}} - \phi_{\mathbf{x}}|^2$$

$$\frac{1}{2}\mathbf{B}^2 \to \frac{1}{2\Delta x^4 e^2}\sum_{\langle i,j\rangle}[1 - \cos(\theta_{i,\mathbf{x}} + \theta_{j,\mathbf{x+i}} - \theta_{i,\mathbf{x+j}} - \theta_{j,\mathbf{x}})]$$

Time evolution: Leapfrog. $O(\Delta x^2)$ accurate, conserves (pseudo-)energy.

$$\phi^n_{\mathbf{x}} = \phi^{n-1}_{\mathbf{x}} + \pi^{n-\frac{1}{2}}_{\mathbf{x}} \cdot \Delta t, \qquad \pi^{n+\frac{1}{2}}_{\mathbf{x}} = \pi^{n-\frac{1}{2}}_{\mathbf{x}} + F^n_{\mathbf{x}} \cdot \Delta t,$$

$$\theta^n_{i,\mathbf{x}} = \theta^{n-1}_{i,\mathbf{x}} + \epsilon^{n-\frac{1}{2}}_{i,\mathbf{x}} \cdot \Delta t \qquad \epsilon^{n+\frac{1}{2}}_{i,\mathbf{x}} = \epsilon^{n-\frac{1}{2}}_{i,\mathbf{x}} + G^n_{i,\mathbf{x}} \cdot \Delta t.$$

Preserves discrete version of Gauss's Law $\nabla \cdot E = \rho$.

Parallel simulations of field theories: LATfield

- Public C++ library of objects for parallel classical lattice fields[11]
- Rewrite of MDP/FermiQCD[12] to optimise memory efficiency
- Objects:

  Lattice: Domain decomposition, (toroidal) boundary conditions
  Field: Template - can have real, complex, user-defined object.
  Site: Accesses elements of field

- Parallelisation by compiler switch

[11] Bevis & Hindmarsh http://www.latfield.org/
[12] Massimo di Pierro et al., http://www.fermiqcd.net/

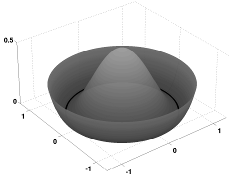
Direct simulation: standing wave on a smooth string

D. Daverio

Abelian Higgs in expanding universe

$$S = -\int d^4x \sqrt{-g} \left(g^{\mu\nu} D_\mu \phi^* D_\nu \phi + V(\phi) + \frac{1}{4e^2} g^{\mu\rho} g^{\nu\sigma} F_{\mu\nu} F_{\rho\sigma} \right),$$

- $ds^2 = g_{\mu\nu} dx^\mu dx^\nu = a^2(\tau)(-d\tau^2 + d\mathbf{x}^2)$
- $a(\tau)$: scale factor.
- τ: conformal time, related to physical time t.
- "Radiation" universe: $a \propto \tau$, $\tau \propto t^{\frac{1}{2}}$.
- "Matter" universe: $a \propto \tau^2$, $\tau \propto t^{\frac{1}{3}}$.



Temporal gauge ($A_0 = 0$) field equations:

$$\begin{aligned} \ddot{\phi} + 2\frac{\dot{a}}{a}\dot{\phi} - \mathbf{D}^2\phi + \lambda a^2(|\phi|^2 - v^2)\phi &= 0, \\ \dot{E}_i + \epsilon_{ijk}\partial_j B_k - ie^2a^2(\phi^* D_i\phi - D_i\phi^*\phi) &= 0. \end{aligned}$$

Abelian Higgs model: shrinking string problem

- Most convenient to solve equations in comoving coordinates ... BUT
- Comoving width of string shrinks as a^{-1} ($a \sim \tau, \tau^2$ in rad, mat era)
- Modify field equations[13]

$$
\begin{aligned}
\ddot{\phi} + 2\frac{\dot{a}}{a}\dot{\phi} - D^2\phi + \lambda a^{2s}(|\phi|^2 - \phi_0^2)\phi &= 0, \\
\dot{E}_i + 2(1-s)\frac{\dot{a}}{a}E_i + \epsilon_{ijk}\partial_j B_k - ie^2 a^{2s}(\phi^* D_i\phi - D_i\phi^*\phi) &= 0.
\end{aligned}
$$

- Physical width of string “fattens” if $s < 1$
- Preserves Gauss’s Law, violates EM conservation
- 2014: now running with $s = 1$ on 4096^3 lattices

[13] Press, Ryden, Spergel (1989); Bevis et al astro-ph/0605018.

Abelian Higgs model simulations: Field isosurfaces, electric fields

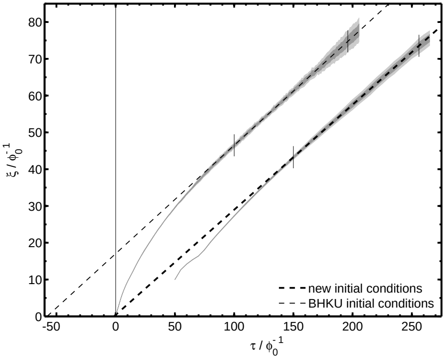
D. Daverio

Abelian Higgs model simulations: string length scale

Total length of string: L
Network scale: $\xi = \sqrt{(V/L)}$
Scaling: $\xi \propto \tau$

Couplings:	$\lambda = 2, e = 1$
Masses:	$m_h = m_A = 1$
Lattice link:	$\Delta x = 0.4295, 0.5$
Time step:	$\Delta t = 0.1$
Volume:	768^3,1024^3
s parameter	0.3, 0

Matter era, different initial conditions
Average over 3 runs



Bevis et al (2011)

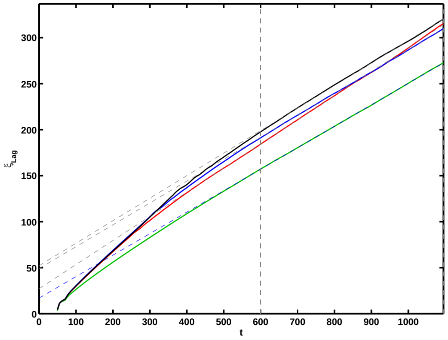
Abelian Higgs model simulations: dependence on s

Fits ($600 < t$):

s	$d\xi/dt$
1.0	0.246
0.8	0.275
0.6	0.252
0.0	0.245

Mean slopes:

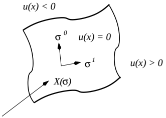
s	$d\xi/dt$	$N_{\rm runs}$
1.0	0.247 ± 0.005	6
0.0	0.234 ± 0.011	7



- Black: $s = 1.0$
- Blue: $s = 0.8$
- Red: $s = 0.6$
- Green: $s = 0.0$

Approximations: Nambu-Goto strings

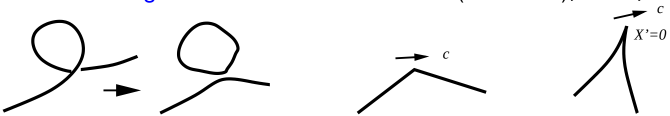
Limit $w/R \to 0$ (w – width, R – curvature radius)[a]
$\dot{\mathbf{X}} = \partial \mathbf{X}/\partial\sigma^0$ (local velocity)
$\mathbf{X}' = \partial \mathbf{X}/\partial\sigma^1$ (local tangent vector)



[a]Förster (1974); Carter & Gregory (1994); Arodz (1997)

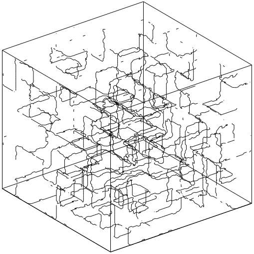
$$\ddot{\mathbf{X}} - \mathbf{X}'' = 0 \qquad \text{(Mink. space equations of motion)}$$
$$\dot{\mathbf{X}}^2 + \mathbf{X}'^2 = 1, \qquad \dot{\mathbf{X}} \cdot \mathbf{X}' = 0. \qquad \text{(Mink. space constraints)}$$

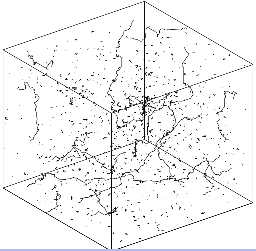
Huom! generally no Magnus force on cosmic strings
Solitonic strings: NG fails at reconnections (Prob $= 1$), kinks, and cusps



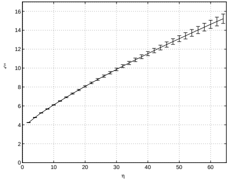
Elementary strings: Prob(reconnection) < 1

Nambu-Goto numerical simulations





Total string length (energy) conserved
"Long" string length scale: $\xi = \sqrt{V/L}$
Minkowski space:[a] Long string $\xi \propto t$



FLRW simulations:[b] Long string $\xi \propto t$ also.

[a] Smith, Vilenkin (1987); Sakellariadou, Vilenkin (1988); Vincent, Hindmarsh, Sakellariadou (1996); Olum, Vanchurin, Vilenkin (2005)

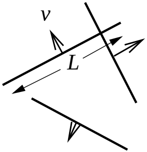
[b] Albrecht & Turok (1985); Bennet & Bouchet (1988); Allen & Shellard (1990); Ringeval, Sakellariadou, Bouchet (2005); Olum & Vanchurin (2006); Blanco-Pillado, Olum, Shlaer (2011-13)

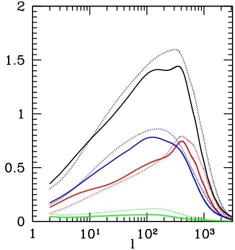
Approximations: Unconnected Segment Model (USM)

Moving sticks of energy μ & tension T:[a]

- Straight segments length $L(t)$
- Random positions, velocities
- Random decay: density $\propto t^{-2}$
- Parameters:
  - String mass/length μ
  - Segment length $\xi = L/t$
  - RMS segment velocity v
  - Segment eqn of state $\beta = \mu/T$

[a] Vincent, Hindmarsh, Sakellariadou (1997); Albrecht, Battye, Robinson (1998); Pogosian, Wyman, Wasserman (2004)





Comparison with Abelian Higgs[a]

[a] Battye, Moss (2010) [Bevis et al (2006)]

Approximations: the one-scale model(s)

- 1-scale model[14] "Infinite" string density ρ_∞

$$\dot{\rho}_\infty = -2H(1+v^2)\rho_\infty - c(\rho_\infty/\xi)$$

$\xi = \sqrt{\mu/\rho_\infty}$, c: "loop chopping efficiency"

- Velocity-dependent 1-scale (VOS) model [15]

$$\begin{aligned} \dot{\rho}_\infty &= -2H(1+v^2)\rho_\infty - \tilde{c}v(\rho_\infty/\xi) \\ \dot{v} &= -(1-v^2)\left(2H - (k/\xi)\right) \end{aligned}$$

k: velocity-curvature correlation parameter

- Reasonable fits (thin-string and field theory) for infinite string evolution

[14] Kibble, Nucl. Phys. B **252**, 227 (1985); Albrecht & Turok, (1988)
[15] Martins, Shellard, Phys. Rev. D **54**, 2535 (1996).

String network scaling hypothesis

- String network characteristic scale ξ ($= \sqrt{V/L}$, i.e. average string separation)
- Network scaling hypothesis: $\boxed{\xi = x_* t}$ (x_* constant O(1))
- String energy density: $\rho_s \simeq \mu/\xi^2$
- Total energy density: $\rho_t \sim 1/Gt^2$:
- String density fraction: $\boxed{\Omega_s \sim G\mu/x_*^2}$
- Grand Unification: $G\mu \sim 10^{-6}$

Scaling: extrapolate from $t_i \sim 10^{-36}$ s to $t_0 \sim 3 \times 10^{17}$ s today

Loops

$\boxed{\text{Abelian Higgs model}}$

- Strings lose energy primarily into Higgs and gauge radiation
- O(1) horizon-size loop per horizon volume, lifetime $O(1)t$

$\boxed{\text{Traditional scenario (based on Nambu-Goto)}}$

- Long strings lose energy primarily into loops, distribution of sizes $f(\ell, t)$
- Gravitational radiation reaction (not included) controls formation size
- $\langle \ell_{form} \rangle \simeq (G\mu)^p t$
- Power p subject to debate:
  - Siemens, Olum, Vilenkin $p = 3/2$ (radiation), $p = 5/2$ (matter)
  - Polchinski & Rocha $p = 1 + 2\chi$ [Small scale structure scaling exponent 2χ]
- Parametrise uncertainty: $\langle \ell_{form} \rangle \simeq \epsilon (G\mu) t$

Modelling cosmic strings

- Parameters of the string model:
  - $G\mu$ string (effective) mass per unit length (in Planck units)
  - GT string (effective) tension (in Planck units)
  - Ω_s (long) string density parameter
  - $\bar{x}_*$ long string correlation length (relative to horizon)
  - ℓ_{form} average loop size at formation (...)
  - f_{SM} fraction of energy density radiated into Standard Model particles
- Derived parameters:
  - $x_* = \sqrt{G\mu/\Omega_s}$ inter-string distance (relative to horizon)
  - $\beta = \sqrt{\frac{\mu}{T}}$ wiggliness parameter

Stochastic background of gravitational radiation

$\boxed{\text{Assume } \ell_{form} \simeq \alpha t}$

Abelian Higgs: effectively $\alpha = 0$

$\Omega_{gw} \sim (G\mu)/x_*^2$

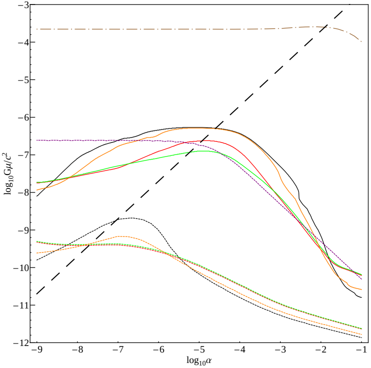
Limits & predictions:[a]

- pulsar timing
- Big Bang Nucleosynthesis
- Cosmic Microwave Background
- LIGO, AdvLIGO, LISA

[a] Vilenkin (1981); Hogan & Rees (1984); Caldwell & Allen (1992); Damour & Vilenkin (2004); Siemens, Mandic, Creighton (2006); de Pies & Hogan (2007)

Pulsars (EPTA):

$G\mu < 5.3 \times 10^{-7}$[a]



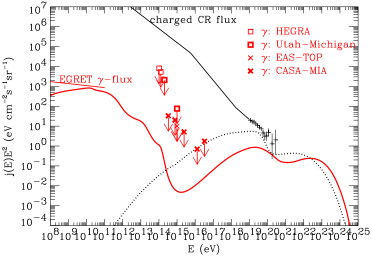
[a] Sanidas, Battye, Stappers (2012)

Cosmic rays from cosmic strings

- "Top-down" scenario: X particles mass m_X
- $\dot{n}_X = \frac{Q_0}{m_X}\left(\frac{t}{t_0}\right)^{-4+p}$
- Cosmic strings: $p = 1$ (scaling)
- No bound from Ultra-High Energy CRs:[a] range 20 Mpc.
- Diffuse γ-ray background bound: $Q_0 < 2.2 \times 10^{-23} h(3p - 1)$ eV cm^{-3} s^{-1}
- Translates to: $G\mu < 10^{-9} f_{SM}^{-1}$ (fraction f_{SM} into SM particles)

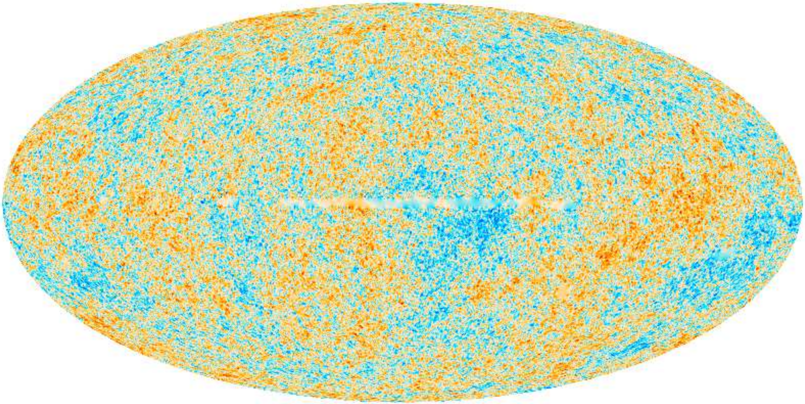
[a] Error in Vincent, Hindmarsh, Antunes (1998)

Example spectrum:[a]
$m_X = 10^{16}$ GeV
$X \to q, \bar{q}$ only
Normalised to UHECR flux



[a] Sigl et al (1998); Bhattacharjee, Sigl (2000)

Cosmic Microwave Background constraints



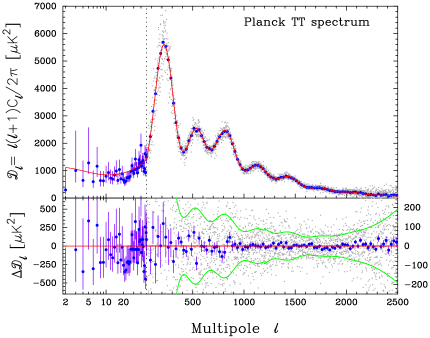
The angular power spectrum of the temperature anisotropy

Multipole moments:
$a_{lm} = \int d\Omega \Delta T(\mathbf{n}) Y^*_{lm}(\mathbf{n})$

Angular power spectrum:

$$C_l = \sum_{m=-l}^{l} |a_{lm}|^2$$

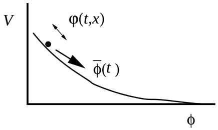
Anisotropy power:
$D_l = l(l+1)C_l/(2\pi)$

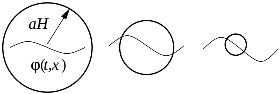


Explanation - Inflation

Energy density of Universe dominated by homogeneous scalar field $\bar{\phi}(t)$

- Scalar field equation: $\ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} + V'(\phi) = 0$
- Friedmann equation: $\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho_\phi$
- "Slow roll" $|\ddot{\phi}| \ll |\dot{\phi}|$: overdamped evolution, $\rho_\phi \simeq V(\phi)$
- Accelerated expansion: $a(t) \sim t^{\text{huge}}$,
- Quantum fluctuations in field: $\phi(x) = \bar{\phi}(t) + \varphi(t, \mathbf{x})$
  - Density perturbations
- Quantum fluctuations in metric: $g_{ij}(x) = a(t)^2(\delta_{ij} + h_{ij}(t, \mathbf{x}))$
  - Gravitational waves
- Perturbations are in phase at $t = 0$

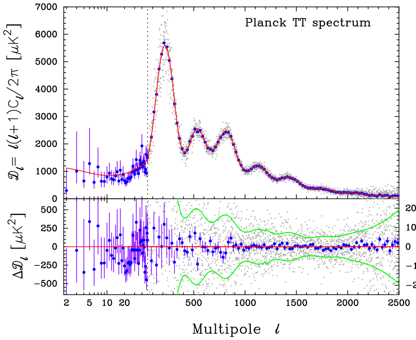




CMB anisotropy power spectrum and inflation

Anisotropy power: $D_l = l(l+1)C_l/(2\pi)$

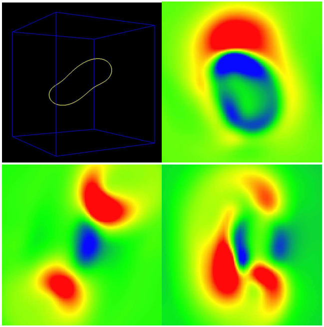
- Inflation predicts perturbation power spectrum
- Simplest model: “single field”:
  - Gravitational potential (curvature $\mathcal{R}$): $\mathcal{P}_{\mathcal{R}}(k) = A_\mathrm{s}^2 (k/k_0)^{n_\mathrm{s}-1}$
  - Gravitational wave (tensor): $\mathcal{P}_\mathrm{t}(k) = A_\mathrm{t}^2 (k/k_0)^{n_\mathrm{t}}$
- 4 “base” parameters: $A_\mathrm{s}^2, n_\mathrm{s}, r = A_\mathrm{t}^2/A_\mathrm{s}^2, n_\mathrm{t}$
  - Plus small corrections: $\alpha_s = dn_s/d\ln k$, ...
- Perturbations make cosmic fluid oscillate in phase



CMB and strings: Gott-Kaiser-Stebbins (GKS) effect

Discontinuity[a] $\Delta T \simeq 8\pi(G\mu)vT_{\rm CMB}$
Need high resolution & high sensitivity

[a] Kaiser, Stebbins (1984)



Landriau and Shellard (2002)

Calculating CMB perturbations from defects: UETC method

$h_\alpha(\tau, k)$: linear perturbation (metric, matter, temperature . . .)
$S_\alpha(\tau, k)$: source (defect energy-momentum, separately conserved)
$D_{\alpha\beta}(\tau, k)$: time dependent differential operator

Perturbation equation: $\mathcal{D}_{\alpha\beta}(\tau, k)h_\beta(\tau, k) = S_\alpha(\tau.k)$

Power spectrum:[16] $\langle|h_\alpha(\tau_0, k)|^2\rangle = \int\int \mathcal{D}^{-1}\mathcal{D}^{-1}\langle S_\alpha(\tau, k)S^*_\alpha(\tau', k)\rangle$

Need unequal-time correlators (UETCs) of energy-momentum tensor

$$C_{\mu\nu\rho\lambda}(k, \tau, \tau') = \left\langle T_{\mu\nu}(k, \tau)T^*_{\rho\lambda}(k, \tau')\right\rangle$$

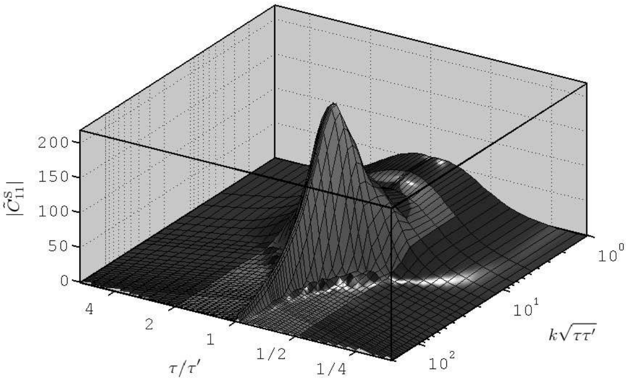
5 independent UETCS [3 scalar, 1 vector, 1 tensor] (simulations)
$\mathcal{D}^{-1}$ is e.g. CMBEASY, CAMB, applied to eigenvectors of UETCs

$\boxed{\text{Scaling: small times, lengths} \rightarrow \text{large times, lengths}}$

[16] Pen, Seljak, Turok (1997); Dürrer, Kunz, Melchiorri (1998,2002)

String scalar ($\Phi\Phi$) unequal time correlator

Scaling: function of $(k\tau, k\tau')$ or $(k\sqrt{\tau\tau'}, \tau'/\tau)$



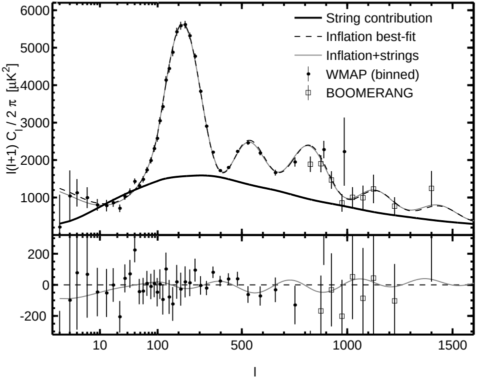
Cosmic string CMB using Abelian Higgs

Multipole moments:
$a_{lm} = \int d\Omega \Delta T(\mathbf{n}) Y^*_{lm}(\mathbf{n})$

Angular power spectrum:

$$C_l = \sum_{m=-l}^{l} |a_{lm}|^2$$

Anisotropy power:
$l(l+1)C_l/(2\pi)$



Top: Strings normalised to WMAP3 ($\ell = 10$)[a]
Bottom: Differences from best-fit ΛCDM

[a] Bevis, Hindmarsh, Kunz, Urrestilla (2006)

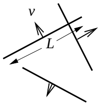
Fitting CMB with inflation & cosmic strings

- Two sources of perturbations: incoherent - add in quadrature
- Cosmological model with 1 more parameter: $G\mu$
- Use $f_{10} = C_{10}^{\text{string}} / C_{10}^{\text{total}}$. Proportional to $(G\mu)^2$.
- Perform Monte Carlos fits: maximise likelihood over model parameters: $\mathcal{L} \propto \exp\left(-(\text{data} - \text{theory(params)})^2/(\text{error})^2\right)$

Summary: CMB constraints on string tension (Planck)

Model	Data set	$10^7(G\mu)$ (95%)	f_{10} (95%)
AH	Planck + WMAP	3.2	0.038
USM-NG	Planck + WMAP	1.5	0.015
USM-NG	Planck + WMAP + ACT/SPT	1.3	0.010
Texture	Planck + WMAP	11	
Semilocal string	Planck + WMAP	11	

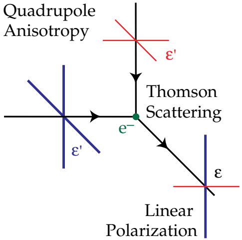
AH - Abelian Higgs model[a]
USM - Unconnected Segment Model[b]
USM-NG: USM modelling Nambu-Goto strings
USM-AH: USM modelling Abelian Higgs strings



[a] Bevis et al (2011)
[b] Albrecht, Battye, Robinson (1998); Pogosian, Vachaspati (1998)

CMB polarisation

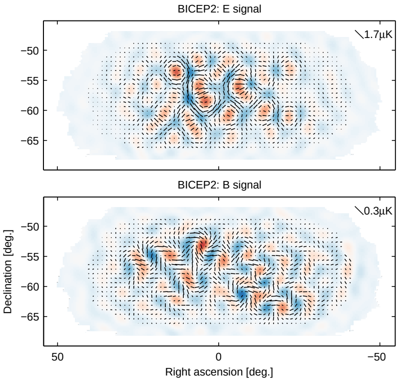
- Cosmic microwaves are polarised
- Thomson scattering is anisotropic
- Polarisation sourced by the quadrupole moment of perturbations
- Polarisation vector field $\mathbf{n}(\theta, \phi)$ decomposable:
  - **div** - E-mode
  - **curl** - B-mode
- B-mode polarisation sourced by
  - gravitational waves from inflation
  - GWs and vorticity perturbations from strings & other defects



Hu, White
astro-ph/9702270

CMB polarisation: the BICEP2 results March 2014

- Claim: detection of B-mode polarisation on angular scales $\ell \lesssim 100$
- ... with just the power spectrum to be gravitational waves from inflation
- Headline result: $r = 0.2^{+0.07}_{-0.05}$
- Paper has 153 citations since March 16 (11:20 am today)



Can topological defects mimic the BICEP2 results?

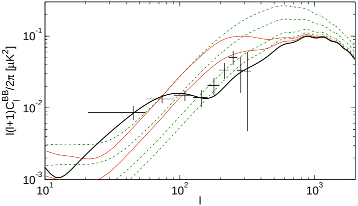
Paper 1 (Top)[a]

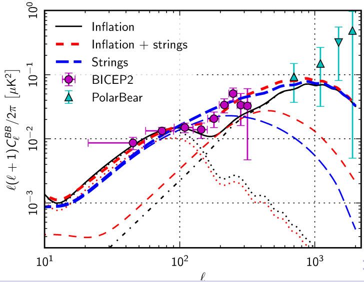
- Classical field theory simulations of Abelian Higgs strings, $N = 4$ textures, semilocal strings
- No

Paper 2 (Bottom)[b]

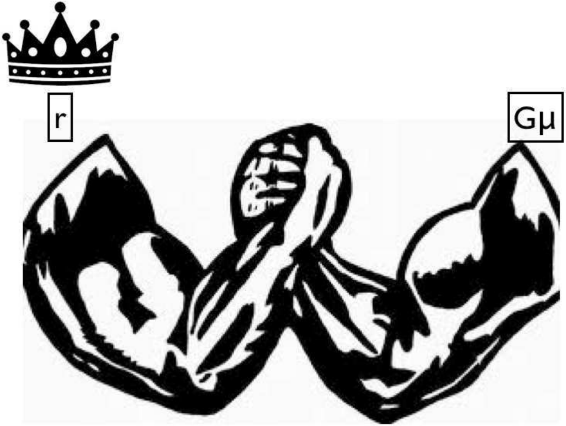
- Unconnected Segment Model
- Maybe (if segments are longer than the causal horizon at $t \sim 400000$ Year

[a] Lizarraga et al (2014)
[b] Moss Pogosian (2014)



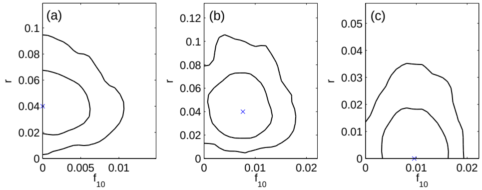


BICEP2, inflation, defects: summary



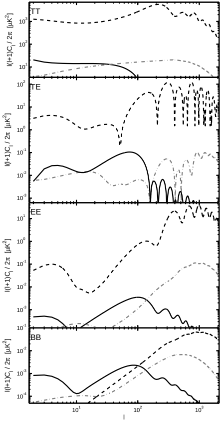
Allen Fisher, Jon Urrestilla

Future: Planck, Future B-mode polarisation satellite



- Xpol $f_{10} \gtrsim 10^{-3} - 10^{-5}$[a] (foregrounds!)

[a] Seljak, Slosar (2006); Garcia-Bellido et al (2010); Mukherjee et al (2011)



$r = 0.04$, grey dot-dash
$f_{10} = 0.01$, solid

Summary and conclusions

- Topological defects are messengers from the very early universe
- Cosmic strings: generic in Grand Unification
- Rich phenomenology in early universe cosmology
- CMB is providing strong constraints on GUT-scale defects
- Future:
  - B-modes (Keck, SPTpol, ACTpol, PRISM)
  - Gravitational waves (EPTA, eLISA, Big Bang Observer)
  - Cosmic rays (Fermi-LAT, Ice Cube)
- Defects in the lab: modelling "vacuum" phase transitions (i.e. post-inflation) with cold atom systems?

Abelian Higgs model on the lattice (FLRW spacetime)

Hamiltonian: $\mathcal{H} = \sum_{\mathbf{x}} \left[\frac{1}{2a^2} E_{i,\mathbf{x}}^2 + \frac{1}{2a^2} \mathbf{B}^2 + |\pi_{\mathbf{x}}|^2 + |D\phi|_{i,\mathbf{x}}^2 + a^2 V(\phi_{\mathbf{x}}) \right]$

$$\phi_{\mathbf{x}}^n = \phi_{\mathbf{x}}^{n-1} + \pi_{\mathbf{x}}^{n-\frac{1}{2}} \cdot \Delta t, \qquad \pi_{\mathbf{x}}^{n+\frac{1}{2}} = \frac{\pi_{\mathbf{x}}^{n-\frac{1}{2}}(1 - H\Delta t) + F_{\mathbf{x}}^n \cdot \Delta t}{1 + H\Delta t},$$

$$\theta_{i,\mathbf{x}}^n = \theta_{i,\mathbf{x}}^{n-1} + \epsilon_{i,\mathbf{x}}^{n-\frac{1}{2}} \cdot \Delta t \qquad \epsilon_{i,\mathbf{x}}^{n+\frac{1}{2}} = \frac{\epsilon_{i,\mathbf{x}}^{n-\frac{1}{2}}(1 - (1-s)H\Delta t) + G_{i,\mathbf{x}}^n \cdot \Delta t}{1 + (1-s)H\Delta t}.$$

where

$$F_{\mathbf{x}}^n = \frac{\partial \mathcal{H}}{\partial \phi_{\mathbf{x}}^n}, \quad G_{i,\mathbf{x}}^n = \frac{\partial \mathcal{H}}{\partial \theta_{i,\mathbf{x}}^n} \tag{1}$$

Calculating perturbations from defects: UETC method

$h_\alpha(\tau, k)$: linear perturbation (metric, matter, temperature ...)
$S_\alpha(\tau, k)$: source (energy-momentum of defects)
$\mathcal{D}_{\alpha\beta}(\tau, k)$: time dependent differential operator

- Perturbation equation: $\mathcal{D}_{\alpha\beta}(\tau, k) h_\beta(\tau, k) = S_\alpha(\tau.k)$
- Power spectrum:[17] $\langle |h_\alpha(\tau_0, k)|^2 \rangle = \int \int \mathcal{D}^{-1}\mathcal{D}^{-1} \langle S_\alpha(\tau, k) S^*_\alpha(\tau', k) \rangle$
- Need unequal-time correlators (UETCs) of energy-momentum tensor

$$C_{\mu\nu\rho\lambda}(k, \tau, \tau') = \left\langle T_{\mu\nu}(k, \tau) T^*_{\rho\lambda}(k, \tau') \right\rangle$$

[17] Pen, Seljak, Turok (1997); Dürrer, Kunz, Melchiorri (1998,2002)

... UETC method

- Isotropy + EM conservation + parity: $C_{\mu\nu\rho\lambda}$: 3 scalar, 1 vector, 1 tensor
- (S,V,T) correlators can be diagonalised:
  $C(k\tau, k\tau') = \sum_n \lambda_n v_n(k\tau) v_n^*(k\tau')$
- Eigenvectors $v_n(k\tau)$ as sources: $h^n_\alpha(\tau, k) = \int_{\tau_i}^{\tau} \mathcal{D}^{-1}_{\alpha\beta}(\tau, \tau', k) v_n(\tau', k)$
- Reconstruct complete power spectrum:

$$C_\ell = \sum_n \lambda_n^{(S)} C_\ell^{(S)n} + \sum_n \lambda_n^{(V)} C_\ell^{(V)n} + \sum_n \lambda_n^{(T)} C_\ell^{(T)n}$$

- Easy to adapt numerical packages designed for inflation[18]

[18] CMBeasy, CAMB, CLASS ...

UETC method for power spectrum: summary

$$\boxed{\text{Calculate UETCs}}$$
$$\downarrow$$
$$\boxed{\text{Diagonalise UETCs}}$$
$$\downarrow$$
$$\boxed{\text{Solve perturbation equations with eigenfunctions as sources}}$$
$$\downarrow$$
$$\boxed{\text{Square (e.g.) } (\Delta T)_\ell^{(S,V,T)n} \text{ and sum}}$$

Energy momentum tensor decomposition

Scalar (S), Vector (V) and Tensor (T) under 3D rotation group[19]

$$T_{00}^{(S)} = v^2 f_\rho \qquad T_{j0}^{(S)} = iv^2 k_j f_v \qquad T_{jl}^{(S)} = v^2 \left[(f_p + \frac{1}{3}k^2 f_\pi)\delta_{jl} - k_j k_l f_\pi \right]$$

$$T_{j0}^{(V)} = v^2 w_j^{(v)} \qquad T_{jl}^{(V)} = iv^2 \frac{1}{2} \left(k_j w_l^{(\pi)} + k_l w_j^{(\pi)} \right)$$

$$T_{jl}^{(T)} = v^2 \tau_{ij}^{(\pi)}$$

v is v.e.v. of scalar field: $V(\phi) = \frac{1}{2}\lambda(|\phi|^2 - v^2)^2$

[19] Notation of Dürrer, Kunz, Melchiorri [arXiv:astro-ph/0110348]

Linearised Einstein equations for defect "seeds"

Gauge invariant formalism: metric perturbations can be broken down to:

$$ds^2 = a^2(t)\left(-dt^2(1+2\Psi^{(s)}) + 2dtdx^i\Sigma_i^{(s)} + \delta_{ij}(1+2\Phi^{(s)}) + H_{ij}^{(s)}\right)$$

Scalar: $\Phi^{(s)}$, $\Psi^{(s)}$ Vector: $\Sigma_i^{(s)}$ Tensor: $H_{ij}^{(s)}$

Linearised Einstein equations:

$$\begin{aligned} k^2\Phi^{(s)} &= \epsilon(f_\rho + 3\frac{\dot{a}}{a}f_v) \\ \Phi^{(s)} + \Psi^{(s)} &= -2\epsilon f_\pi \\ -k^2\Sigma_i^{(s)} &= 4\epsilon w_i^{(v)} \\ \ddot{H}_{ij}^{(s)} + 2\frac{\dot{a}}{a}\dot{H}_{ij}^{(s)} + k^2 H_{ij}^{(s)} &= 2\epsilon\tau_{ij}^{(\pi)}\ . \end{aligned}$$

$\epsilon = 4\pi G v^2$, where v is v.e.v. of scalar field

Unequal time correlators: scaling

Do not have to trace entire history of network

Defect networks exhibit scaling

Scaling: correlators depend only on time τ and wavenumber k

E.g. scalar:[20] ($x = k\tau$)

$$\begin{aligned} \langle \Phi_s(\mathbf{k},\tau)\Phi_s^*(\mathbf{k},\tau')\rangle &= \frac{1}{k^4\sqrt{\tau\tau'}} C_{11}(x,x') \\ \langle \Phi_s(\mathbf{k},\tau)\Psi_s^*(\mathbf{k},\tau')\rangle &= \frac{1}{k^4\sqrt{\tau\tau'}} C_{12}(x,x') \\ \langle \Psi_s(\mathbf{k},\tau)\Psi_s^*(\mathbf{k},\tau')\rangle &= \frac{1}{k^4\sqrt{\tau\tau'}} C_{22}(x,x') \end{aligned}$$

NB scaling fails during a change in expansion rate

[20] Notation of Dürrer, Kunz, Melchiorri [arXiv:astro-ph/0110348]

String scalar (ΦΦ) unequal time correlator

Plot against $k\sqrt{\tau\tau'}$, τ'/τ instead of $k\tau$, $k\tau'$.