

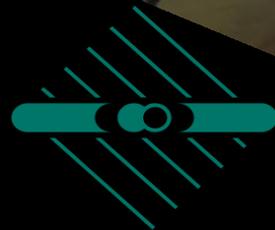
Neutrinos

selected topics



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MAX-PLANCK-INSTITUT
FÜR KERNPHYSIK

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Landscape of the field

Solar neutrinos

Atmospheric neutrinos

Cosmic neutrinos

Supernova neutrinos

Collective oscillations

Models of mass and mixing

Flavor symmetries

Neutrinos and unification

Future detectors and experiments

Mass hierarchy

CP-violation

2-3 mixing

Absolute mass scale

Nature of neutrinos

$\beta\beta_{\text{ov}}$ - decays

Sterile neutrinos

Non-standard interaction

Violation of fundamental symmetries

Neutrinos & LHC

Higgs physics

Dark matter

Grav. waves

Leptogenesis

Inflation

3 ν - paradigm

All well established/confirmed results fit well a framework

- three neutrinos
 - with interactions described by the standard model
 - with masses and mixing



New physics BSM behind

ν_R

Mechanism of neutrino mass generation has negligible effect (feedback) on Standard model structures and interaction

Higgs properties?

Content:

1. Masses, mixing and oscillations
2. Flavor transformations in matter
3. Phenomenology of neutrinos from various sources
4. Neutrino Mass hierarchy and CP-violation
5. Beyond the 3 neutrino paradigm
6. Towards the Underlying physics

The Nobel Prize in Physics 2015



Takaaki Kajita

Super-Kamiokande Collaboration
University of Tokyo, Kashiwa, Japan



Arthur B. McDonald

SNO Collaboration
Queen's University, Kingston, Canada

*" for the discovery of neutrino oscillations,
which shows that neutrinos have mass"*

Challenging
Nobel citation

Oscillations or no-oscillations?

A.Y.S. 1609.02386 [hep-ph]

While Super-Kamiokande (SK), indeed, has discovered oscillations, SNO observed effect of the adiabatic (almost non-oscillatory) flavor conversion of neutrinos in the matter of the Sun.

Oscillations are irrelevant for solar neutrinos apart from small regeneration inside the Earth.

Both oscillations and adiabatic conversion do not imply masses uniquely and further studies were required to show that non-zero neutrino masses are behind the SNO results.

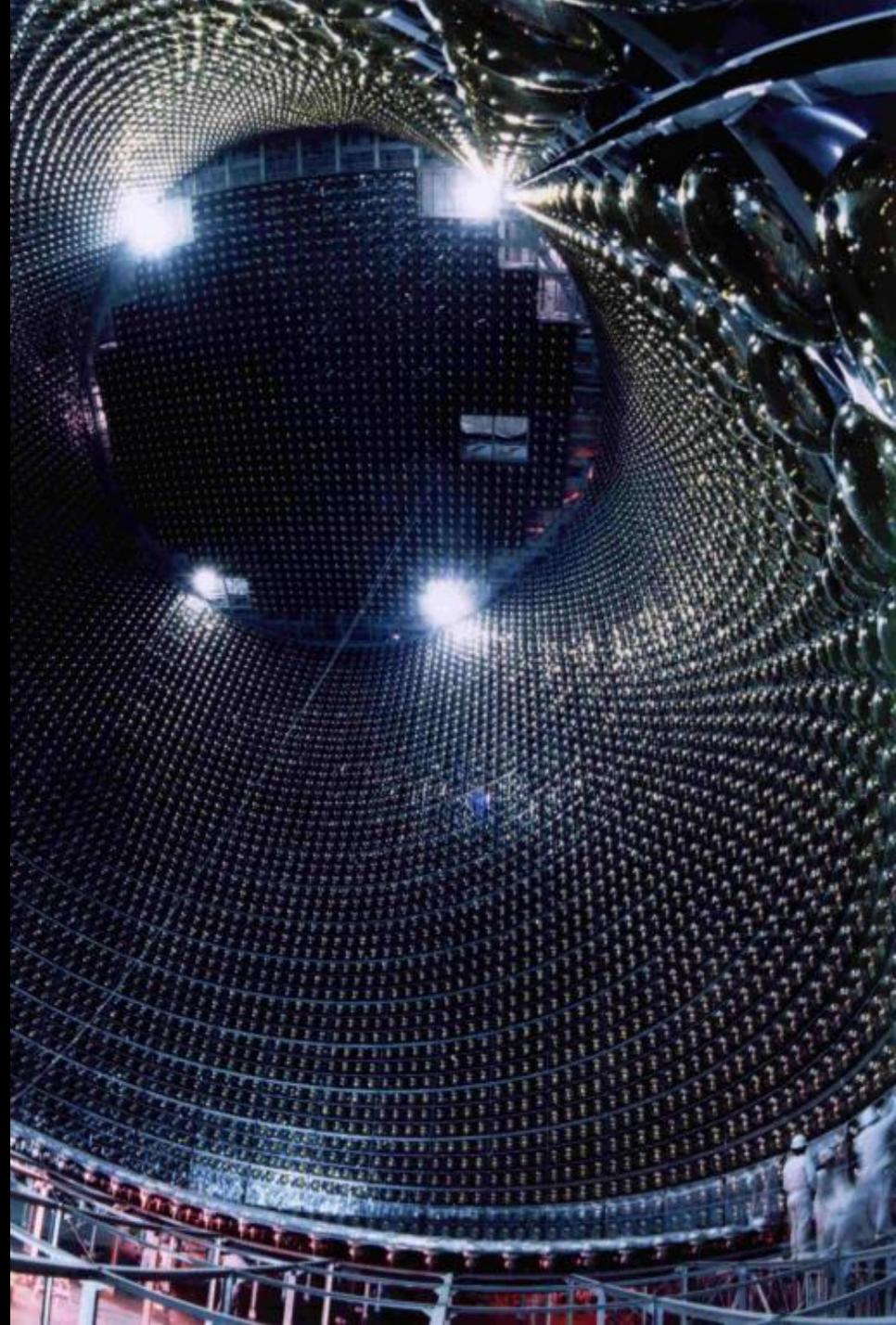
Science magazine,
Dec. 2016, Adrian Cho:

"Did the Nobel committee get the physics wrong?"

Repubblica,
Dec. 14 2016, E. Dusi:

"C'è un errore nel Nobel della fisica del 2015"

Masses, Mixing and Oscillations



Neutrinos in SM

SM definition of flavor states may differ from "physical" one if e.g. ...

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \quad \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L \quad \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L$$

$$I_W = 1/2$$

$$I_{3W} = 1/2$$

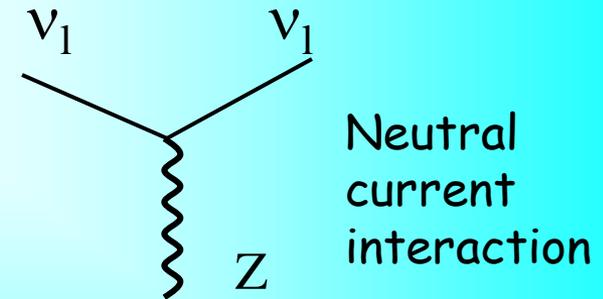
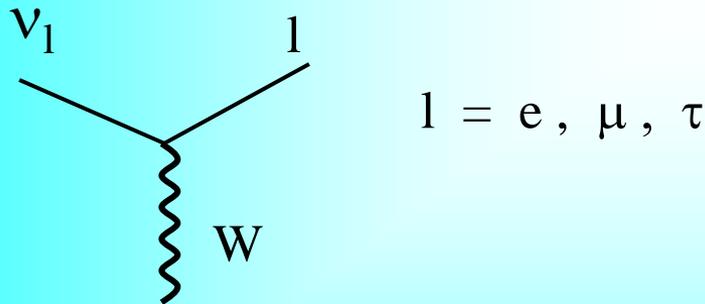
Chiral components

$$\nu_L = \frac{1}{2}(1 - \gamma_5) \nu$$

$$\nu_R = \frac{1}{2}(1 + \gamma_5) \nu$$

?

$\nu_e \quad \nu_\mu \quad \nu_\tau$ neutrino flavor states, form doublets (charged currents) with definite charged leptons,



$$\frac{g}{2\sqrt{2}} \bar{l} \gamma^\mu (1 - \gamma_5) \nu_l W_\mu^+ + h.c.$$

$$\frac{g}{4} \bar{\nu}_l \gamma^\mu (1 - \gamma_5) \nu_l Z_\mu$$

Conservation of lepton numbers L_e, L_μ, L_τ

... New heavy neutral leptons mix with neutrinos

Dirac and Majorana neutrinos

Dirac mass term

$$- m_D \bar{\nu}_R \nu_L + h.c.$$

Instead of independent
RH component

$$\nu_R \rightarrow \nu_L^C$$

$$\nu_L^C = C (\bar{\nu}_L)^T \quad C = i\gamma_0 \gamma_2 \quad \text{charge conjugate}$$



$$- \frac{1}{2} m_L \nu_L^T C \nu_L + h.c.$$

→ two component massive neutrino

corresponds to Majorana neutrino:

$$\nu_M^C = e^{i\alpha} \nu_M$$

$$\nu_M = \nu_L + e^{-i\alpha} \nu_L^C$$

α is the Majorana phase

$$- \frac{1}{2} m_M \bar{\nu}_M \nu_M = - \frac{1}{2} m_M e^{i\alpha} \nu_L^T C \nu_L + h.c.$$

No invariance under $\nu_L \rightarrow e^{i\alpha} \nu_L$

Lepton number of the mass operator: $L = 2$ and -2 (for h.c.)

mass term violates lepton number by $|\Delta L| = 2$



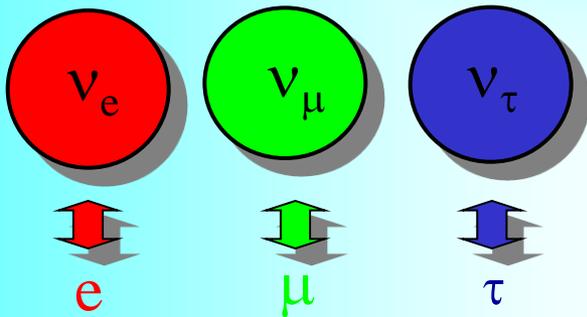
Processes with lepton number violation
by $|\Delta L| = 2$ with probabilities

$\beta\beta_{ov}$

$$\Gamma \sim m_L^2$$

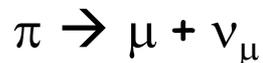
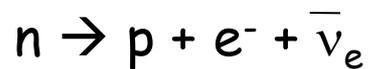
Flavors and mixing

Flavor neutrino states:

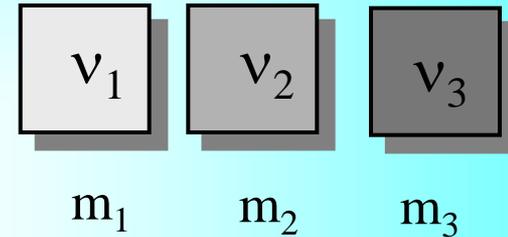


- correspond to certain charged leptons
- interact in pairs

flavor is characteristic of interactions



Mass eigenstates



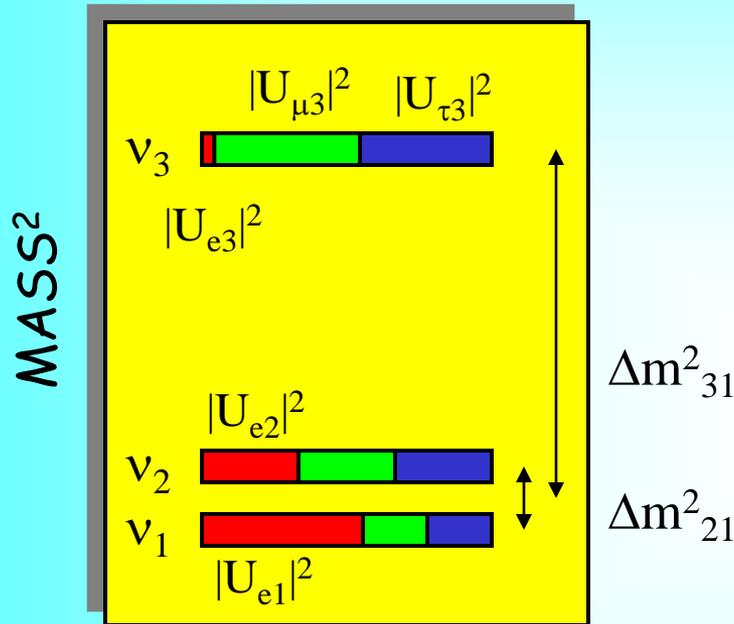
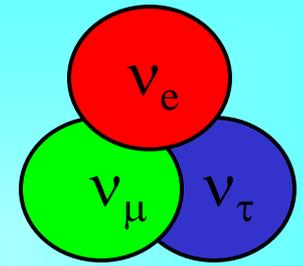
Mixing

Flavor states

\neq

Mass eigenstates

Mixing angles



FLAVOR

Normal mass hierarchy

$$\Delta m_{31}^2 = m_3^2 - m_1^2$$

$$\Delta m_{21}^2 = m_2^2 - m_1^2$$

Mixing determines the flavor composition of mass states

Mixing parameters

$$\tan^2 \theta_{12} = |U_{e2}|^2 / |U_{e1}|^2$$

$$\sin^2 \theta_{13} = |U_{e3}|^2$$

$$\tan^2 \theta_{23} = |U_{\mu 3}|^2 / |U_{\tau 3}|^2$$

Mixing matrix:

$$\nu_f = U_{\text{PMNS}} \nu_{\text{mass}}$$

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U_{\text{PMNS}} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

$$U_{\text{PMNS}} = U_{23} I_\delta U_{13} I_{-\delta} U_{12}$$

Standard parametrization

$$U_{\text{PMNS}} = U_{23} I_\delta U_{13} I_{-\delta} U_{12}$$

$$I_\delta = \text{diag}(1, 1, e^{i\delta})$$

$$U_{\text{PMNS}} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ s_{12}c_{23} + c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & -s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & c_{12}s_{23} + s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

$c_{12} = \cos \theta_{12}$, etc.

δ is the Dirac CP violating phase

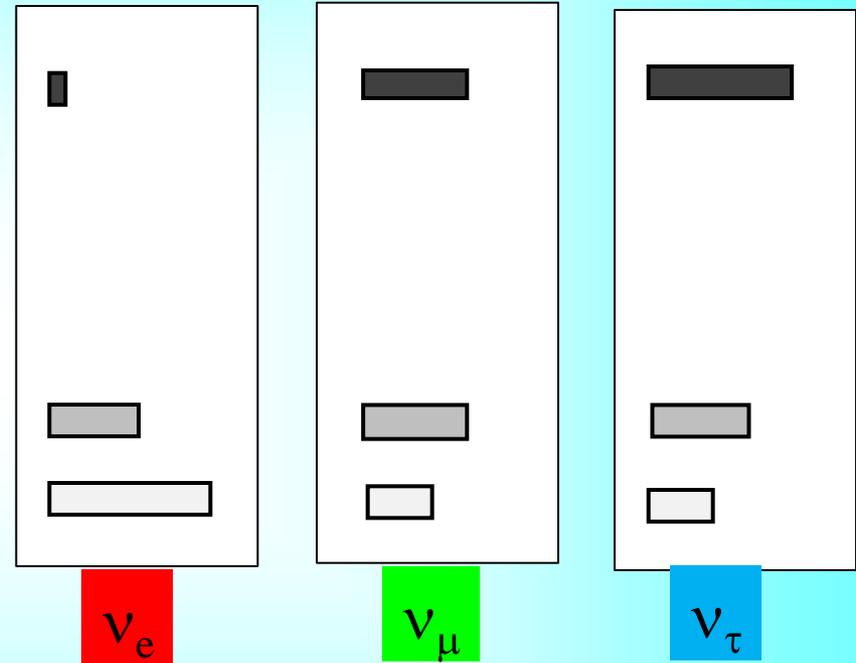
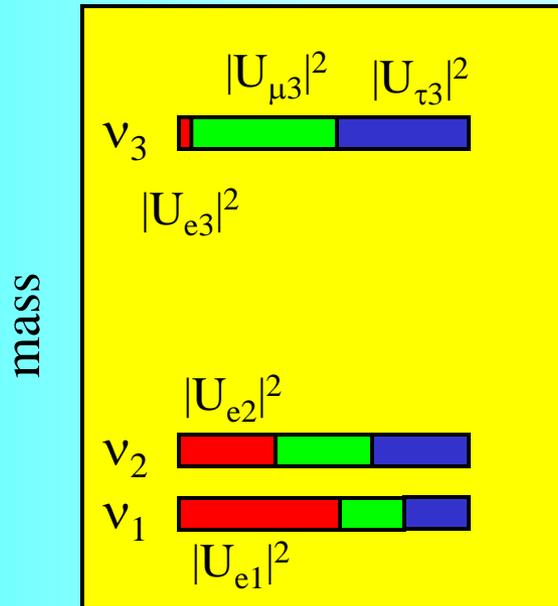
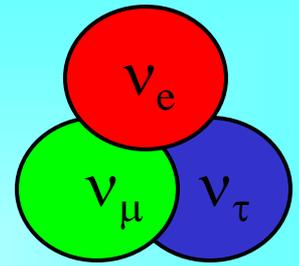
θ_{12} is the ``solar'' mixing angle

θ_{23} is the ``atmospheric'' mixing angle

θ_{13} is the mixing angle determined by T2K, Daya Bay, CHOOZ, DC...

Mixing

Dual
role



Flavor content of
mass states

$$v_{\text{mass}} = U_{\text{PMNS}}^\dagger v_f$$

Mass content of flavor states

$$v_f = U_{\text{PMNS}} v_{\text{mass}}$$

Who mixes neutrinos?

= creates certain (coherent) combinations of mass states

Mixing in CC \rightarrow mixing in produced states

Non-trivial
interplay
of

Charged current
weak interactions

Kinematics
of specific
reactions

Difference
of the charged
lepton masses

ν_e



β^- decays,
energy conservation

ν_μ



π^- decays,
chirality suppression

ν_τ



Beam dump,
D - decay

Energy interval selection,
loss of coherence

What about neutral currents?

Neutrino oscillations

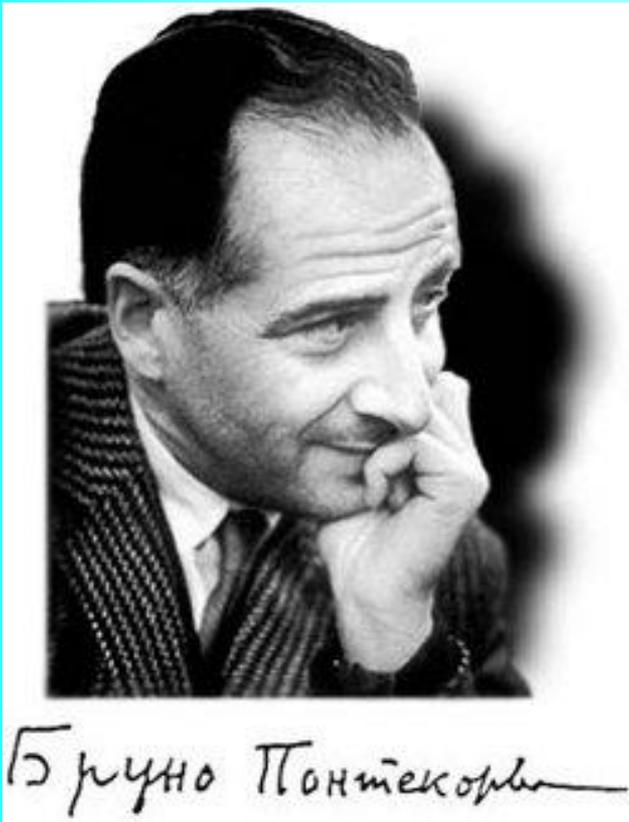
periodic transformation of one neutrino species (flavor) into another

- Consequence of mixing: production of mixed states

- effect of propagation of mixed states

- interference effect

- effect of the relative phase increase with time / distance :



B. Pontecorvo, 1957

"Mesonium and antimesonium"

Zh. Eksp. Teor. Fiz. 33, 549 (1957)

[Sov. Phys. JETP 6, 429 (1957)]

Neutrino oscillations

THEORY:

Simple but not as simple as in most of textbooks (pointlike or plane wave)

BB

Conceptually wrong but gives correct final results for physically relevant situations

Wave packet picture (or QFT) should be used to avoid misconception. Complications related to WP disappear in final result in physically relevant cases

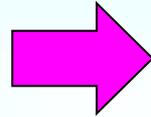
...disappear in normalization

In principle:

Lagrangian

$$\begin{aligned} & \frac{g}{2\sqrt{2}} \bar{l} \gamma^\mu (1 - \gamma_5) \nu_l W_\mu^+ \\ & - \frac{1}{2} m_L \nu_L^T C \nu_L \\ & - \bar{l}_L m_l l_R + \text{h.c.} \end{aligned}$$

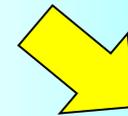
Starting from
the first principles



QFT

QM

Amplitudes,
probabilities
of processes



Observables,
number of
events, etc..

Actually not very simple

Quantum mechanics at macroscopic distances

What is the problem?

Set-up

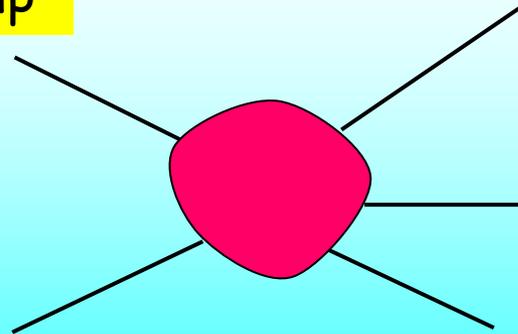
Formalism should be adjusted to specific physics situation

Initial conditions

Recall, the usual set-up

asymptotic states described by plane waves

- enormous simplification



single interaction region

Approximations

Approximations, if one does not want to consider whole history of the Universe to compute e.g. signal in Daya Bay

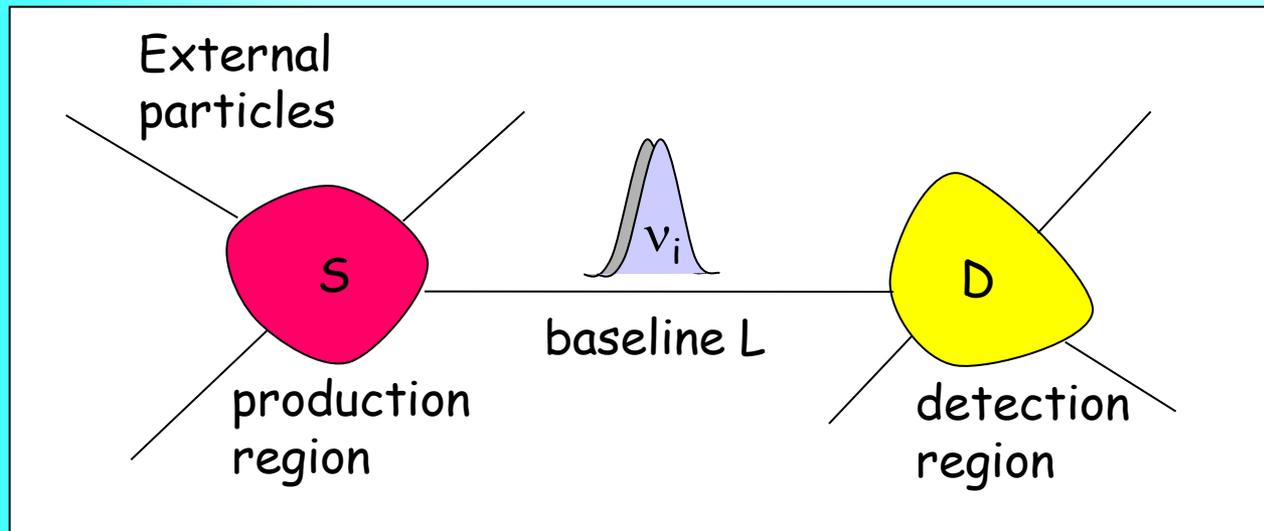
Truncating the process

Localizations

Of some initial and final particles

Oscillation set-up: localization

E. Akhmedov, A.S.



Finite space and time phenomenon

BB2

Two interaction regions in contrast to usual scattering problem : wave packets (wave functions) of external particles (e.g. nuclei) which determine localization

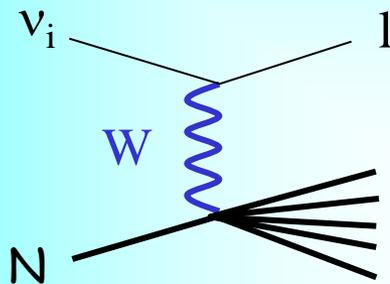
Neutrinos: propagators for mass states or as real particles on mass shell

Interference of two amplitudes with exchange of ν_1 and ν_2

In terms of mass eigenstates

Without flavor states

Scattering



$$\frac{g}{2\sqrt{2}} U_{PMNS} \bar{l}_i \gamma^\mu (1 - \gamma_5) \nu_i W_\mu^+ + \text{h.c.}$$



interaction constant

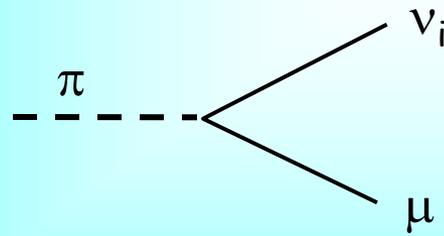


Eigenstates of the Hamiltonian in vacuum

Lagrangian of interactions

wave functions of accompanying particles

$\pi \rightarrow \mu \nu_i$

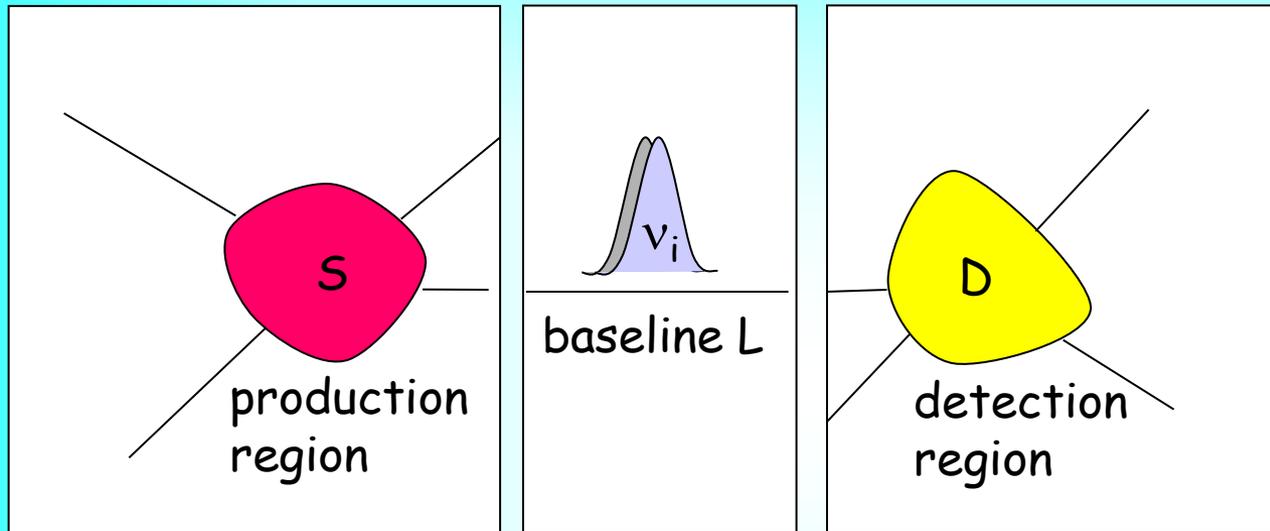


compute the wave functions of neutrino mass eigenstates

Wave packets



Factorization



If oscillation effect in production/detection regions can be neglected

$$r_D, r_S \ll l_\nu$$



factorization

Production, propagation and detection can be considered as three independent processes

Wave packets and oscillations

Suppose v_α is produced in the source centered at $x = 0, t = 0$

After formation of the wave packet (outside the production region)

$$|v_\alpha(x, t)\rangle = \sum_k U_{\alpha k}^* \Psi_k(x, t) |v_k\rangle$$

$$\Psi_k \sim \int dp f_k(p - p_k) e^{ipx - iE_k(p)t} \quad - \text{WF of } k\text{-mass state}$$

$$E_k(p) = \sqrt{p^2 + m_k^2} \quad - \text{dispersion relation}$$

$f_k(p - p_k)$ - the momentum distribution function peaked at
 p_k - the mean momentum

Expanding around mean momentum

describes spread of
the wave packets

$$E_k(p) = E_k(p_k) + \left. \frac{dE_k}{dp} \right|_{p_k} (p - p_k) + \left. \frac{d^2E_k}{dp^2} \right|_{p_k} (p - p_k)^2 + \dots$$



$$v_k = \left. \frac{dE_k}{dp} \right|_{p_k} = \left. \frac{p}{E_k} \right|_{p_k} \quad - \text{group velocity of } v_k$$



Shape factor and phase factor

$$E_k(p) = E_k(p_k) + v_k(p - p_k)$$

(neglecting spread of the wave packets)

Inserting into $\Psi_k \sim \int dp f_k(p - p_k) e^{ipx - iE_k(p)t}$

$$\Psi_k \sim e^{ip_k x - iE_k(p_k)t} g_k(x - v_k t)$$

Phase factor

$$e^{i\phi_k}$$

$$\phi_k = p_k x - E_k t$$

Depends on mean characteristics p_k and corresponding energy:

$$E_k(p_k) = \sqrt{p_k^2 + m_k^2}$$

Shape factor

$$g_k(x - v_k t) = \int dp f_k(p) e^{ip(x - v_k t)}$$

Depends on x and t only in combination $(x - v_k t)$ and therefore describes propagation of the wave packet with group velocity v_k without change of the shape

Mixing & mixed states **

One needs to compute the state which is produced
i.e. compute

the shape factors

$$g_k(x - v_k t)$$

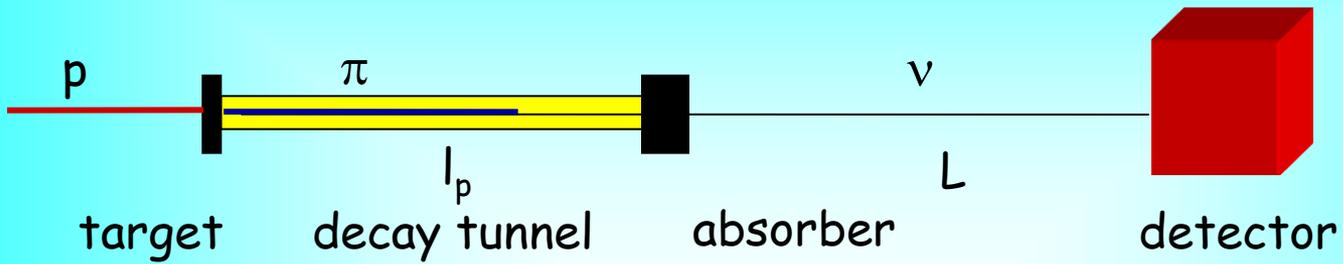
mean momenta p_k

- Fundamental interactions
- Kinematics
- characteristics of parent and accompanying particles

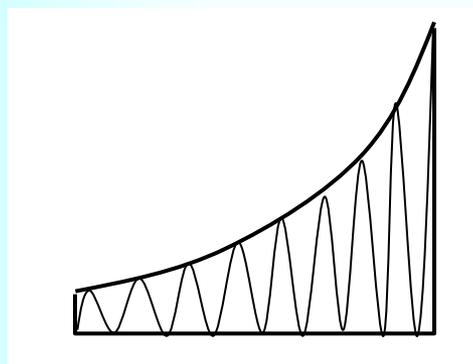
Process dependent

If heavy neutrinos are present but can not be produced for kinematical reasons, flavor states in Lagrangian differ from the produced states, etc..

Neutrino wave packets ^{**}



ν wave packet



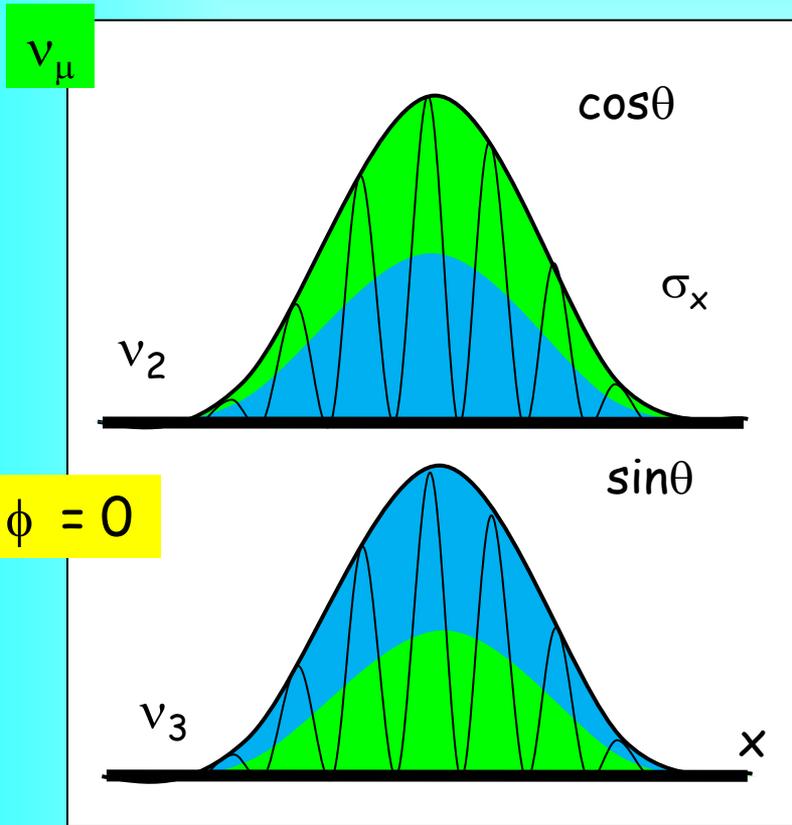
*D. Hernandez, AS
E. Kh Akhmedov,
D. Hernandez, AS
arXiv:1110.5453*

The length of the ν wave packet emitted in the forward direction

$$\sigma = l_p \frac{v - v_\pi}{v_\pi}$$

Doppler effect

Mixing and wave packets



$$\begin{aligned}
 v_\mu &= \cos\theta v_2 + \sin\theta v_3 & \phi &= 0 \\
 v_\tau &= -\sin\theta v_2 + \cos\theta v_3 & \phi &= \pi
 \end{aligned}$$

differ by

- mass content
- phase difference

Inverting

$$\begin{aligned}
 v_2 &= \cos\theta v_\mu - \sin\theta v_\tau \\
 v_3 &= \cos\theta v_\tau + \sin\theta v_\mu
 \end{aligned}$$

Interference of the same flavor parts

$k = 2, 3$

$$\Psi_k = g_k(x - v_k t) e^{i\phi_k}$$

Shape factor v_k - group velocity

Phase factor $\phi_k = p_k x - E_k t$

E_k and p_k - mean energy and momentum in packet

... for maximal mixing

$$\theta = \pi/4$$

$$v_{\mu} = (v_2 + v_3) / 2^{1/2}$$

$$v_{\tau} = (-v_2 + v_3) / 2^{1/2}$$

The difference in phase only, mass composition is the same!
Interaction of neutrino state depends on the phase difference between mass eigenstates

$$v(\phi) = (e^{i\phi} v_2 + v_3) / 2^{1/2}$$

$$v(\phi) = \begin{cases} v_{\mu} & \phi = 0 \\ v_{\tau} & \phi = \pi \\ v_{\mu}, v_{\tau} & 0 < \phi < \pi \end{cases}$$

Flavor composition
(interaction properties)
depends on ϕ

Propagation

Additional phase difference due to difference of masses

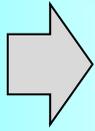
$$|v(x,t)\rangle = \cos \theta g_2(x - v_2 t) |v_2\rangle + \sin \theta g_3(x - v_3 t) e^{i\phi} |v_3\rangle$$

$\phi = \phi_3 - \phi_2$ - oscillation phase changes with (x,t)

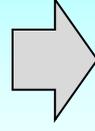
for $\phi \neq 0$ components v_τ will not cancel \rightarrow appearance of v_τ

Oscillations

Difference of masses of ν_2 and ν_3



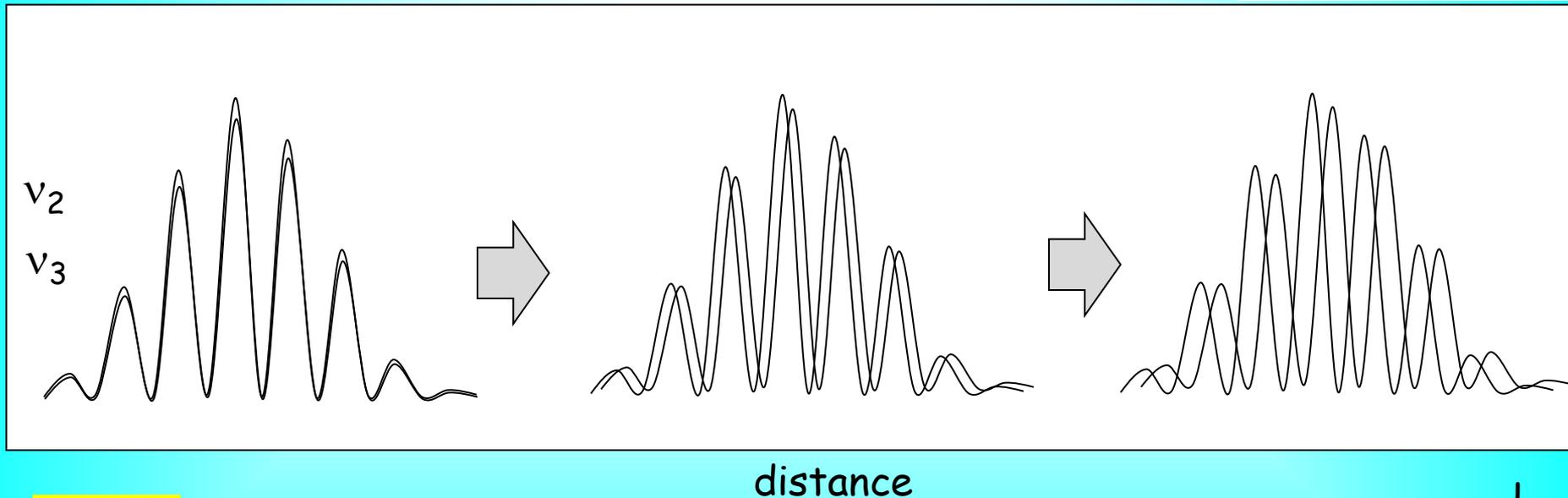
Difference of phase velocities



Phase difference increase with distance

$$\phi = \frac{\Delta m^2 L}{2E}$$

$$\Delta m^2 = m_3^2 - m_2^2$$



$$\phi = 0$$

shift of oscillatory patterns

ν_μ

L

Oscillation phase

$$\phi = \phi_2 - \phi_1$$

$$\phi_i = -E_i t + p_i x$$

$$p_i = \sqrt{E_i^2 - m_i^2}$$

Dispersion relation

$$\phi = \Delta E t - \Delta p x$$

These are averaged characteristics of WP

where

$$\Delta p = (dp/dE)\Delta E + (dp/dm^2)\Delta m^2 = 1/v_g \Delta E + (1/2p) \Delta m^2$$

insert

group velocity

$$\phi = (\Delta E/v_g) (v_g t - x) + \frac{\Delta m^2}{2E} x$$

standard
oscillation
phase

$$\Delta E \sim \Delta m^2/2E$$

$$< \sigma_x$$

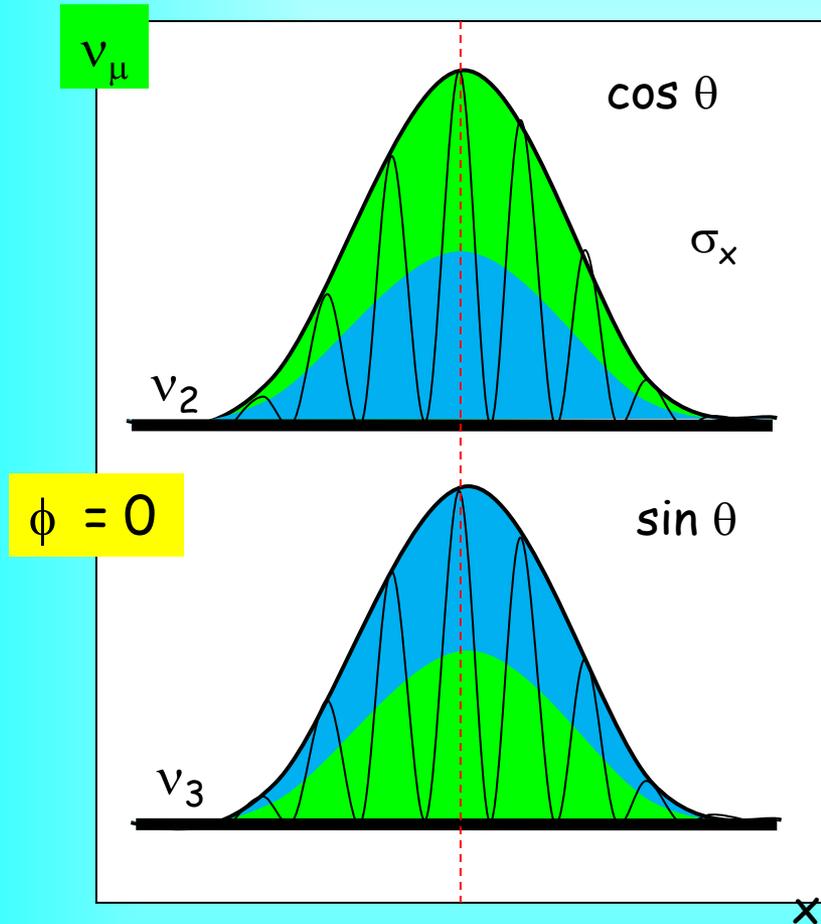
Averaged
energies

$$\sigma_x \Delta m^2/2E$$

Oscillation effect
over the size of WP
usually- small

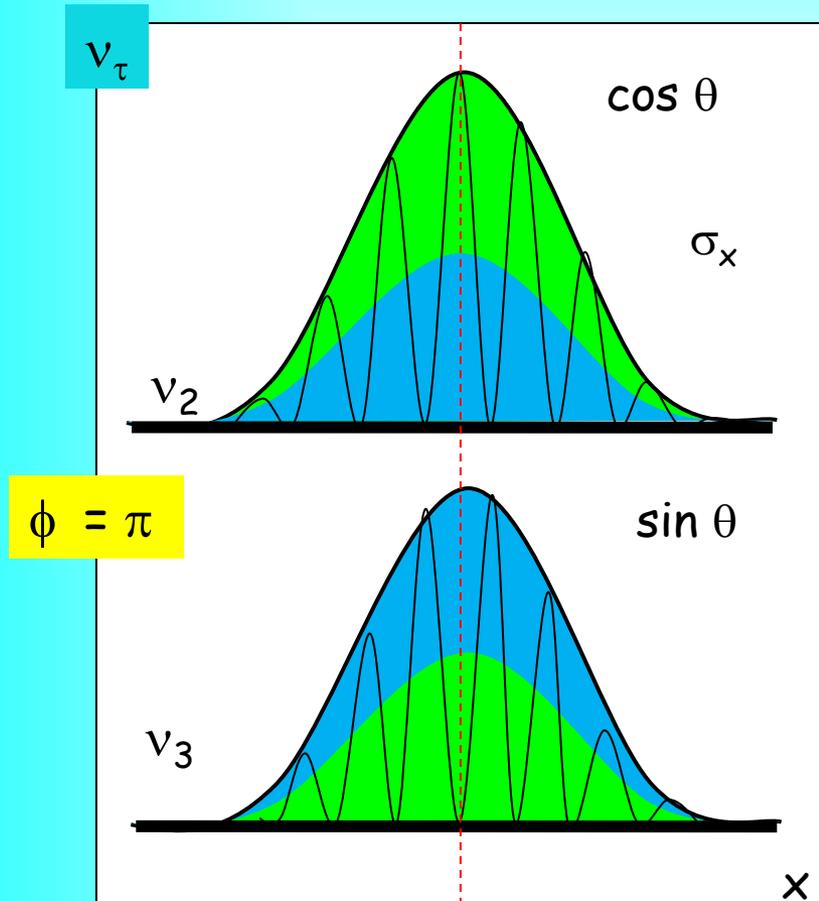
Phase difference
along the wave
packets is the same

Oscillations



- Destructive interference of the tau parts
- Constructive interference of muon parts

Oscillations



- Destructive interference of the muon parts
- Constructive interference of tau parts

Detection:

**

As important as production
should be considered symmetrically with production

Detection effect can be included in
the generalized shape factors

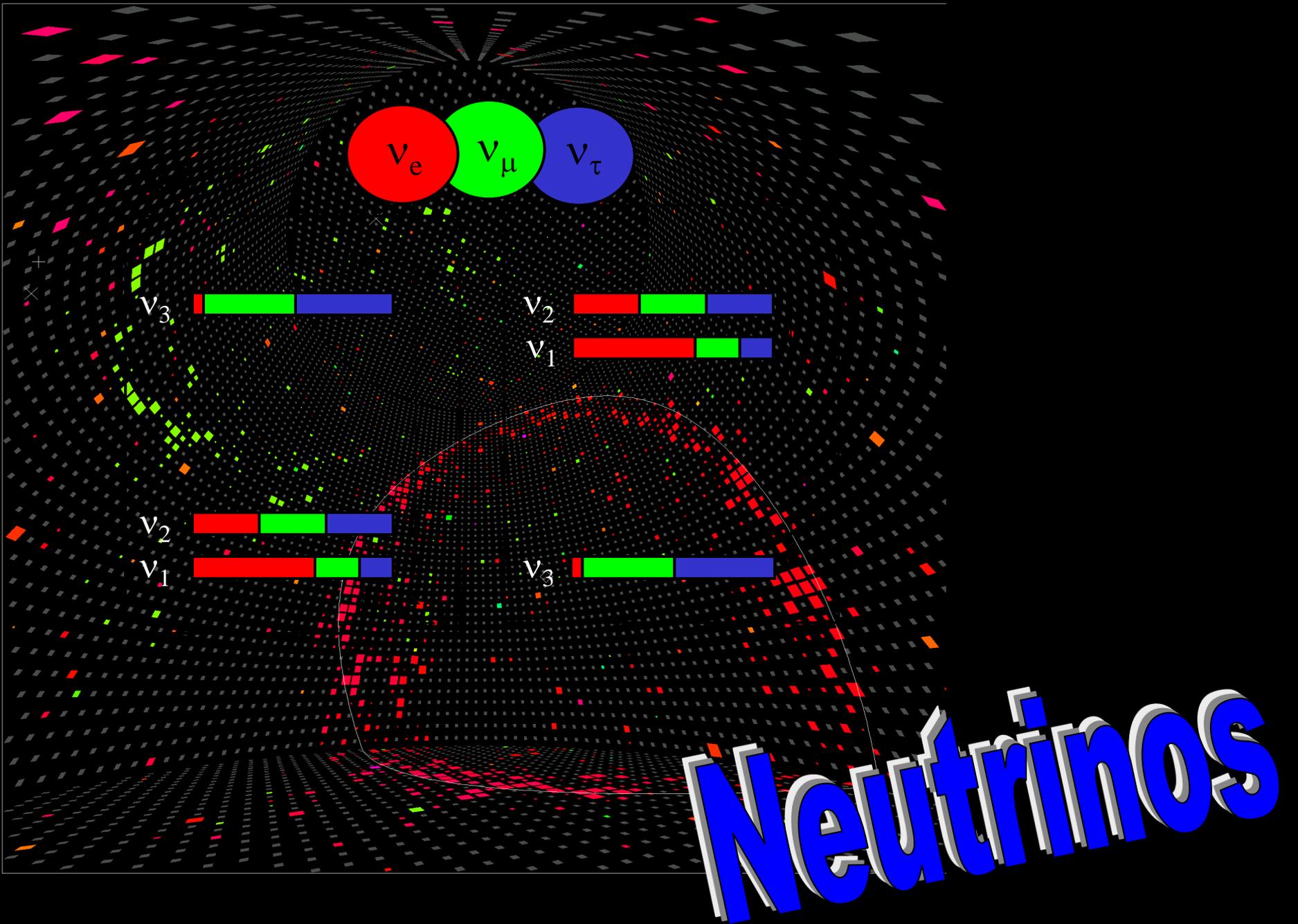
$$g_k(x - v_k t) \rightarrow G_k(L - v_k t)$$

$x \rightarrow L$ - distance between central points of the
production and detection regions

HOMEWORK...

Based on

- [1] Neutrino production coherence and oscillation experiments.
E. Akhmedov, D. Hernandez, A. Smirnov, JHEP 1204 (2012) 052,
arXiv:1201.4128 [hep-ph]
- [2] Neutrino oscillations: Entanglement, energy-momentum conservation and QFT.
E.Kh. Akhmedov, A.Yu. Smirnov, Found. Phys. 41 (2011) 1279-1306
arXiv:1008.2077 [hep-ph]
- [3] Paradoxes of neutrino oscillations.
E. Kh. Akhmedov, A. Yu. Smirnov Phys. Atom. Nucl. 72 (2009) 1363-1381
arXiv:0905.1903 [hep-ph]
- [4] Active to sterile neutrino oscillations: Coherence and MINOS results.
D. Hernandez, A.Yu. Smirnov, Phys.Lett. B706 (2012) 360-366
arXiv:1105.5946 [hep-ph]
- [5] Neutrino oscillations: Quantum mechanics vs. quantum field theory.
E. Kh. Akhmedov, J. Kopp, JHEP 1004 (2010) 008
arXiv:1001.4815 [hep-ph]



Neutrinos