Neutrinos
selected topics

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Oscillation probability

$|\nu(x,t)> = \cos\theta\, g_2(x - v_2 t)\, |\nu_2> + \sin\theta\, g_3(x - v_3 t) e^{i\phi} |\nu_3>$

$|\nu_\mu> = \cos\theta\, |\nu_2> + \sin\theta\, |\nu_3>$

Mass states are orthogonal and normalized: $<\nu_i | \nu_k> = \delta_{ik}$

$A(\nu_\mu) = <\nu_\mu | \nu(x,t)> = \cos^2\theta\, g_2(x - v_2 t) + \sin^2\theta\, g_3(x - v_3 t)\, e^{i\phi}$

Oscillation probability

Amplitude of (survival) probability

$$A(\nu_\mu) = \langle \nu_\mu | \nu(x,t) \rangle = \cos^2\theta\, g_2(x - v_2 t) + \sin^2\theta\, g_3(x - v_3 t)\, e^{i\phi}$$

Probability in the moment of time t

interference

$$P(\nu_\mu) = \int dx\, |\langle \nu_\mu | \nu(x,t) \rangle|^2 =$$

$$= \cos^4\theta + \sin^4\theta + 2\sin^2\theta \cos^2\theta \cos\phi \int dx\; g_2(x - v_2 t)\, g_3(x - v_3 t)$$

since $\int dx\, |g|^2 = 1$

Similarly one can integrate over time for fixed x

If $g_3 = g_2$

$$P(\nu_\mu) = 1 - 2\sin^2\theta \cos^2\theta\, (1 - \cos\phi) = 1 - \sin^2 2\theta \sin^2 \tfrac{1}{2}\phi$$

depth of oscillations

$$\phi = \frac{\Delta m^2\, x}{2E} = \frac{2\pi x}{l_\nu}$$

$$\boxed{l_\nu = \frac{4\pi E}{\Delta m^2}}$$

Oscillation length

Coherence in propagation

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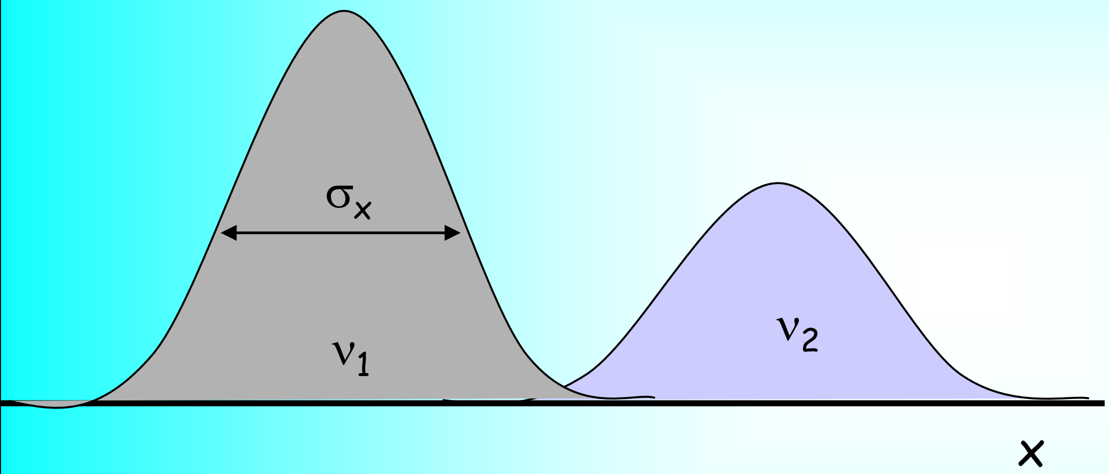
In the configuration space: separation of the wave packets due to difference of group velocities



$\Delta v_{gr} = \Delta m^2 / 2E^2$

separation: $\Delta v_{gr} L = \Delta m^2 L / 2E^2$

no overlap: $\Delta v_{gr} L > \sigma_x$

coherence length:

$L_{coh} = \sigma_x E^2 / \Delta m^2$

In the energy space: averaging over oscillations

BB3

Oscillatory period in the energy space $E^T = 4\pi E^2 / (\Delta m^2 L)$

Averaging (loss of coherence) if energy resolution σ_E is $E^T < \sigma_E$

→ leads to the same coherence length

If $E^T > \sigma_E$ - restoration of coherence even if the wave packets separated

Equivalence of considerations in p- and x- spaces

**

Averaging over energy – loss of coherence due to separation of WP

on BB

L Stodolsky theorem

in stationary state approximation

or if there is no time tagging

→

Wave packets are unnecessary for computation of observable effects

The only what is needed
is to make correct integration
over

Production energy spectrum

Energy resolution of a detector

Spread of wave packets

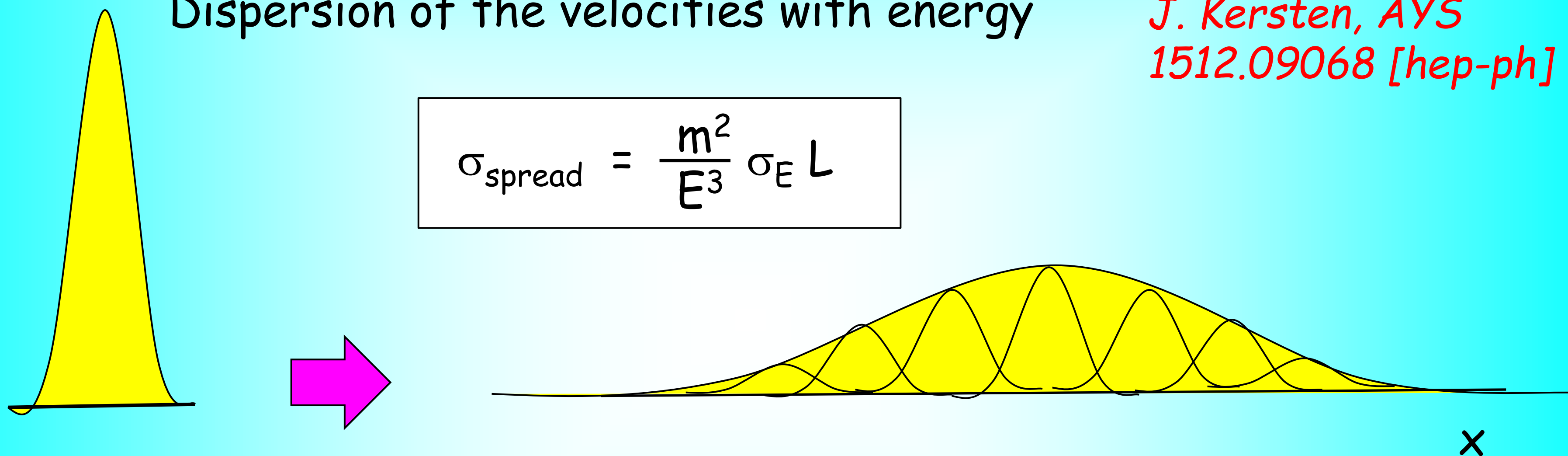
BB4

Due to presence of waves with different energies in the packet

Dispersion of the velocities with energy

J. Kersten, AYS
1512.09068 [hep-ph]

$$\sigma_{spread} = \frac{m^2}{E^3} \sigma_E L$$



Beryllium neutrino line:

$\sigma_x = 2\pi / \Gamma_{Be} = 6 \; 10^{-8}$ cm → $\sigma_{spread} \sim 4 \; 10^{-6}$ cm

$\Delta x_{sep}{}^S \sim 2 \; 10^{-3}$ cm → $\Delta x_{sep}{}^S \gg \sigma_{spread}$

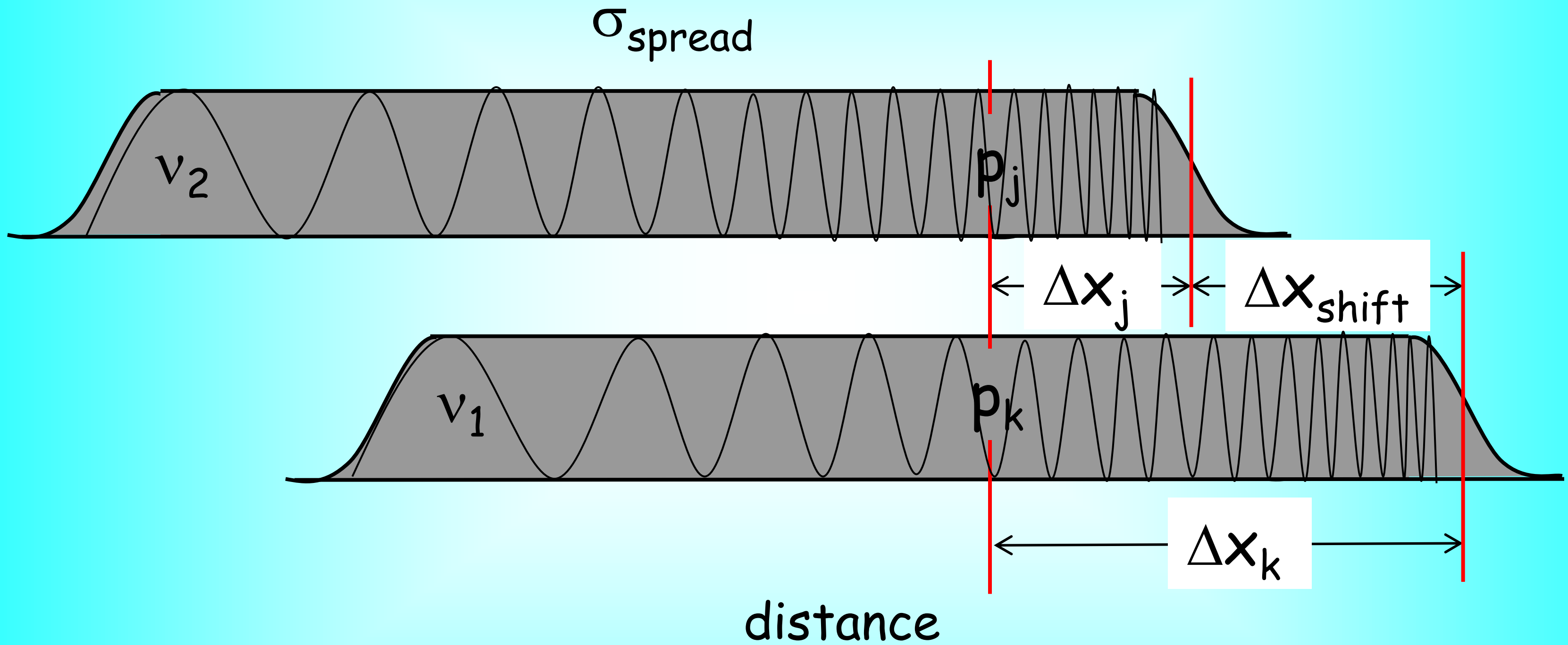
Oscillations of mass states in the Earth

$\Delta x_{sep}{}^E \sim 5 \; 10^{-8}$ cm → $\sigma_x \sim \Delta x_{sep}{}^E \ll \sigma_{spread}$

Loss of coherence: YES NO?

Spread of the wave packets and coherence**

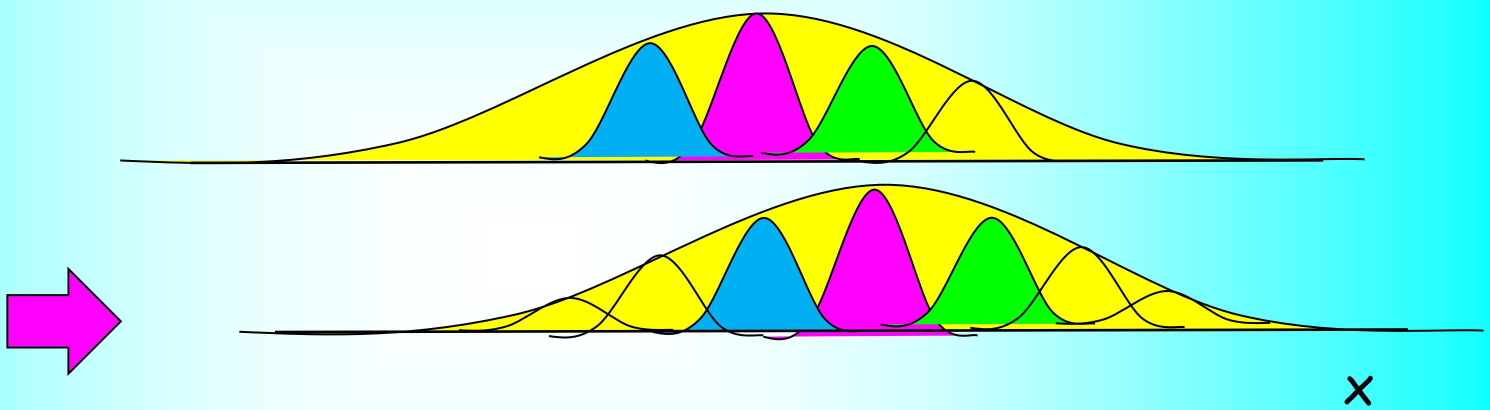
J. Kersten, AYS
1512.09068 [hep-ph]



Averaged (effective) momenta in overlapping parts are different → averaging

Spread of wave packets **

Coherent parts (same color)
– stop to overlap



Loss of coherence between overlapping parts of the WP

WP becomes "classical": describing that the highest energy neutrinos arrive first

No effect on coherence if considered in the p-space

Propagation of wave packets

What happens?

Phase difference change

Due to different masses (dispersion relations) → phase velocities

Oscillations

Separation of wave packets

Due to different group velocities

Loss of coherence

Spread of individual wave packets

Due to presence of waves with different momenta and energy in the packet

Loss of coherence within within WP

Physics summary

Appearance probability

$$P(\nu_\mu) = \sin^2 2\theta \sin^2 \frac{\pi x}{l_\nu}$$

All complications are “absorbed” in normalization
or reduced to partial averaging of oscillations
or lead to negligible corrections of order $m/E \ll 1$

Oscillations - effect of the phase difference increase between mass eigenstates

Admixtures of the mass eigenstates ν_i in a given neutrino state do not change during propagation

Flavors (flavor composition) of the eigenstates are fixed by the vacuum mixing angle

Evolution equation

If loss of coherence and other complications related to WP picture are irrelevant –
`` point-like" picture

$$i \frac{d\Psi}{dt} = H\Psi$$

$$\Psi = \begin{pmatrix} \Psi_e \\ \Psi_\mu \\ \Psi_\tau \end{pmatrix}$$

$$H = \frac{M M^+}{2E}$$

generalization for ultra relativistic ν of

$$E \sim p + \frac{m^2}{2E}$$

p is omitted

$m^2 \to M M^+$

M is the mass matrix

mixing matrix

$$M M^+ = U M_{diag}^2 U^+$$
$$M_{diag}^2 = \mathrm{diag}\,(m_1^2, m_2^2, m_3^2)$$

For 2 flavors on BB5

Neutrino polarization vectors

$\psi = \begin{pmatrix} \nu_e \\ \nu_\tau \end{pmatrix}$ ⇒ Polarization vector:

$$\boxed{\mathbf{P} = \psi^+ \sigma/2\, \psi} \quad (*)$$

Pauli matrices

$$\mathbf{P} = \begin{pmatrix} \mathrm{Re}\, \nu_e^+ \nu_\tau, \\ \mathrm{Im}\, \nu_e^+ \nu_\tau, \\ \nu_e^+ \nu_e - 1/2 \end{pmatrix}$$

Elements of density matrix

Evolution equation:

$$i \frac{d\Psi}{dt} = H\Psi \Rightarrow \boxed{i \frac{d\Psi}{dt} = -(\mathbf{B}\, \sigma/2)\,\Psi}$$

where $\mathbf{B} = \frac{2\pi}{l_\nu}(\sin 2\theta,\ 0,\ \cos 2\theta)$

Differentiating **P** from (*) and using equation of motion for Ψ

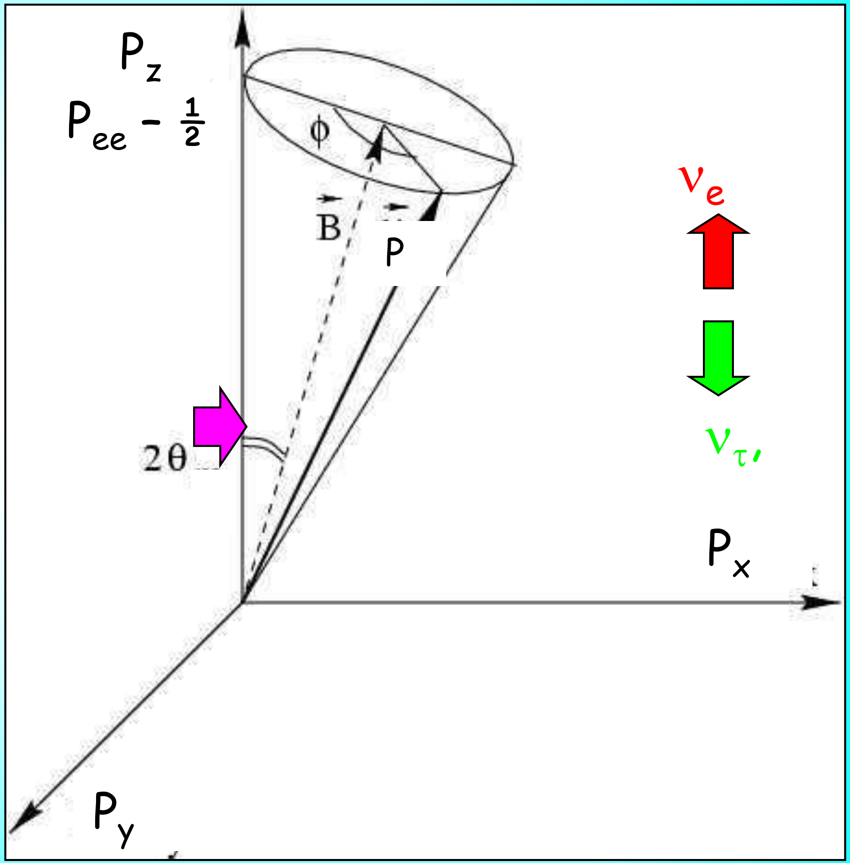
$$\boxed{\frac{d\mathbf{P}}{dt} = -(\mathbf{B} \times \mathbf{P})}$$

Coincides with equation for the electron spin precession in the magnetic field

Graphic representation

$\vec{\nu} = \mathbf{P} =$
$(\mathrm{Re}\ \nu_e^+ \nu_\tau, \quad \mathrm{Im}\ \nu_e^+ \nu_\tau, \quad \nu_e^+ \nu_e - 1/2)$

$$\mathbf{B} = \frac{2\pi}{l_\nu}(\sin 2\theta,\ 0,\ \cos 2\theta)$$



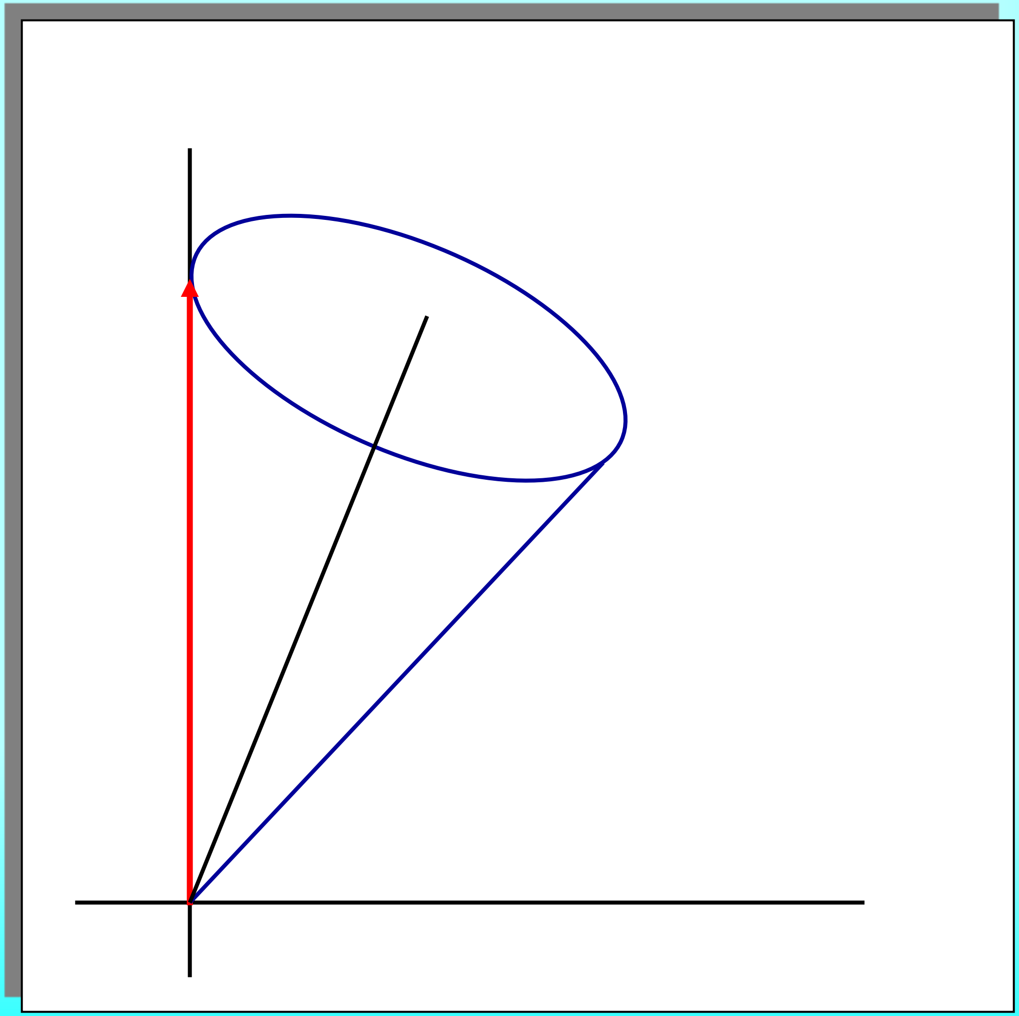
Evolution equation

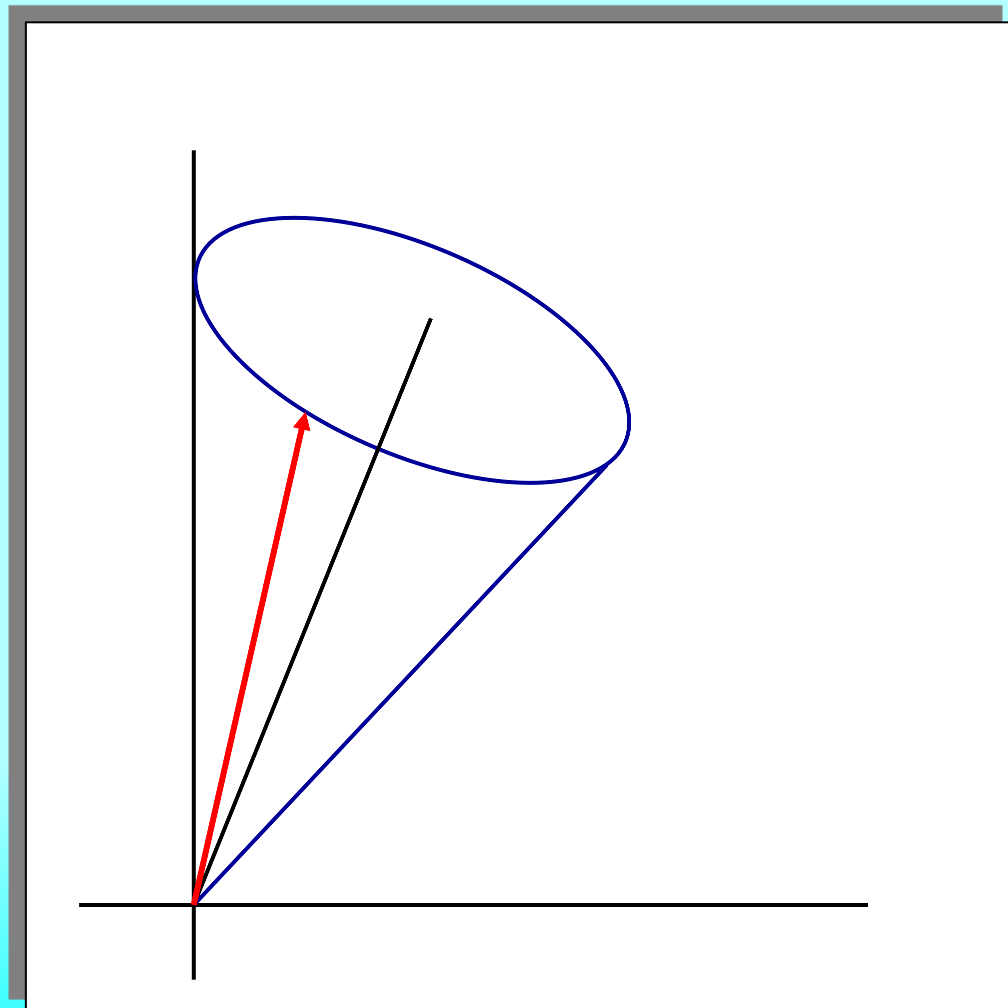
$$\frac{d\vec{P}}{dt} = -(\vec{B} \times \vec{P})$$

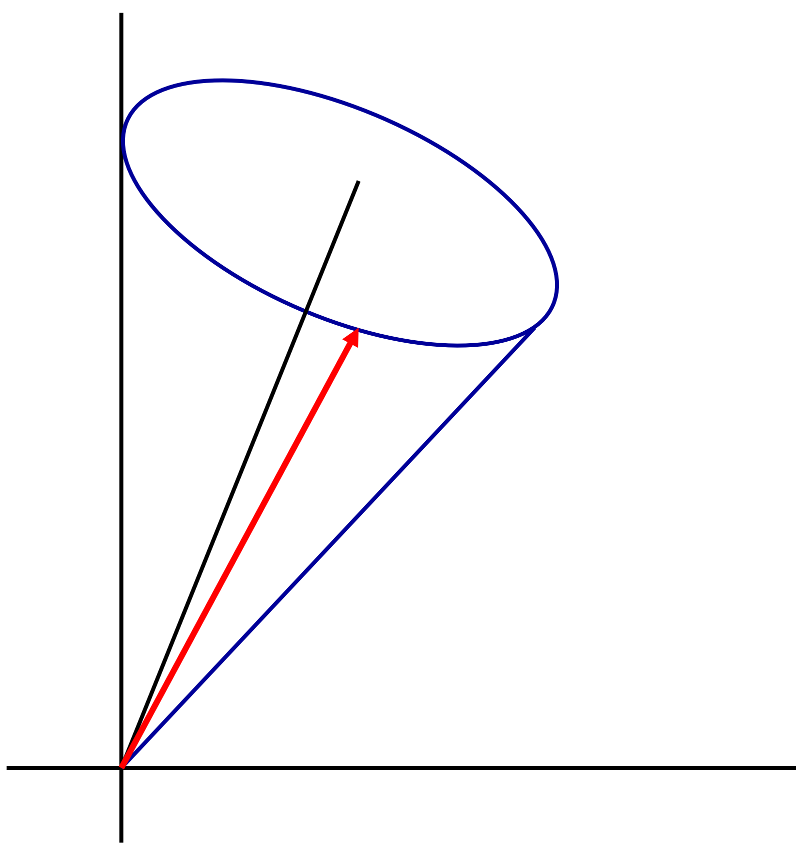
$\phi = 2\pi t / l_\nu$ - phase of oscillations

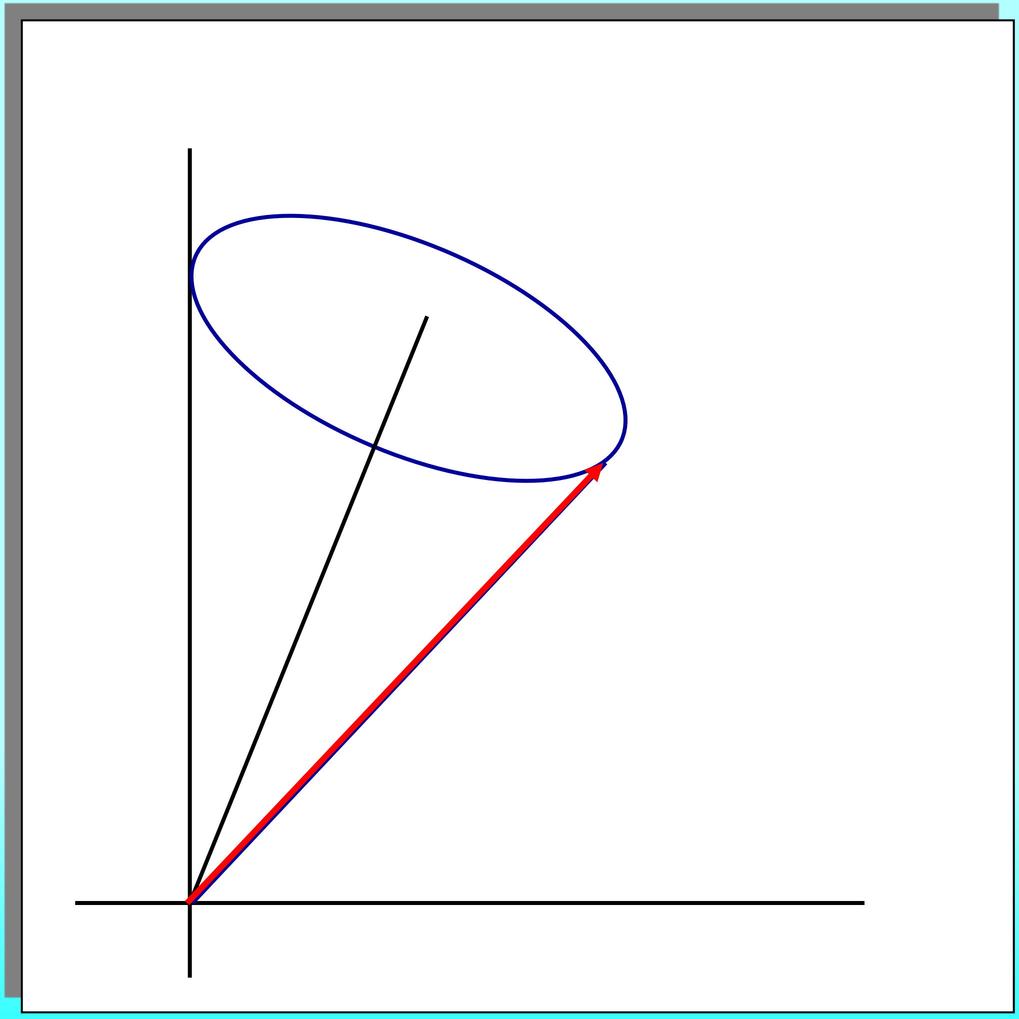
$P_{ee} = \nu_e^+ \nu_e = P_Z + 1/2 = \cos^2\theta_Z/2$ probability to find ν_e

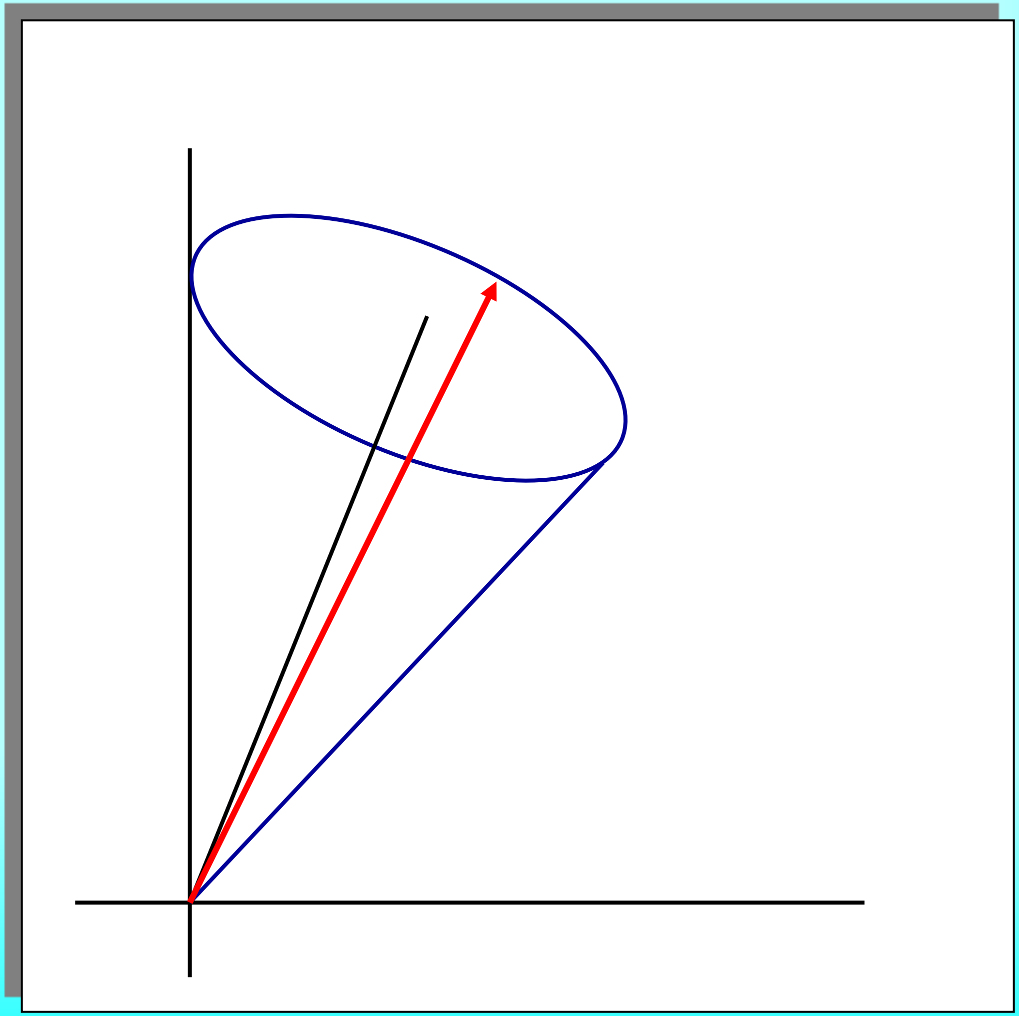
Oscillations

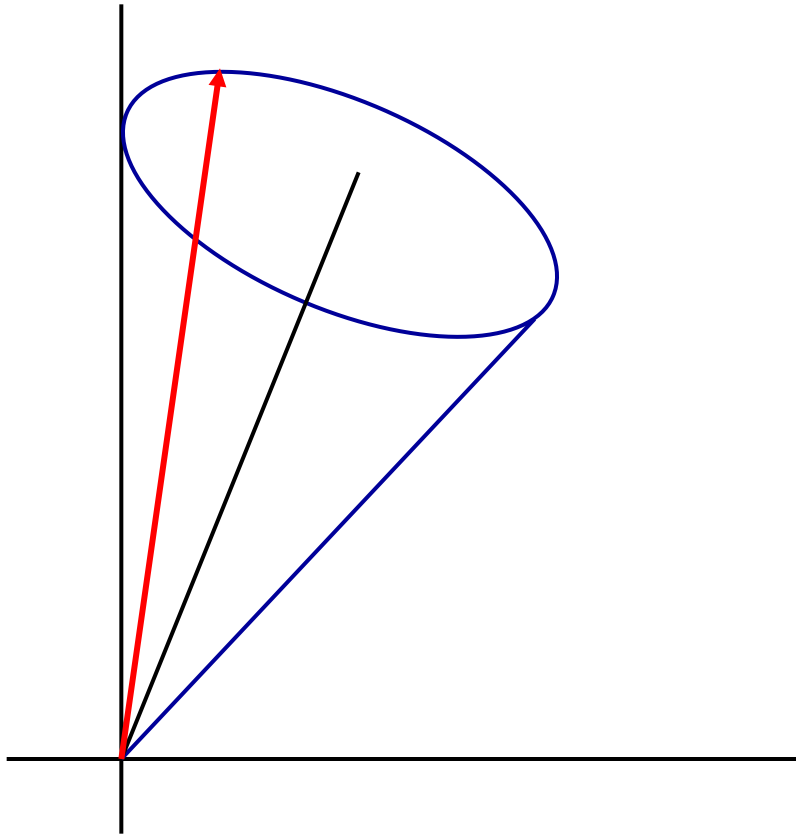


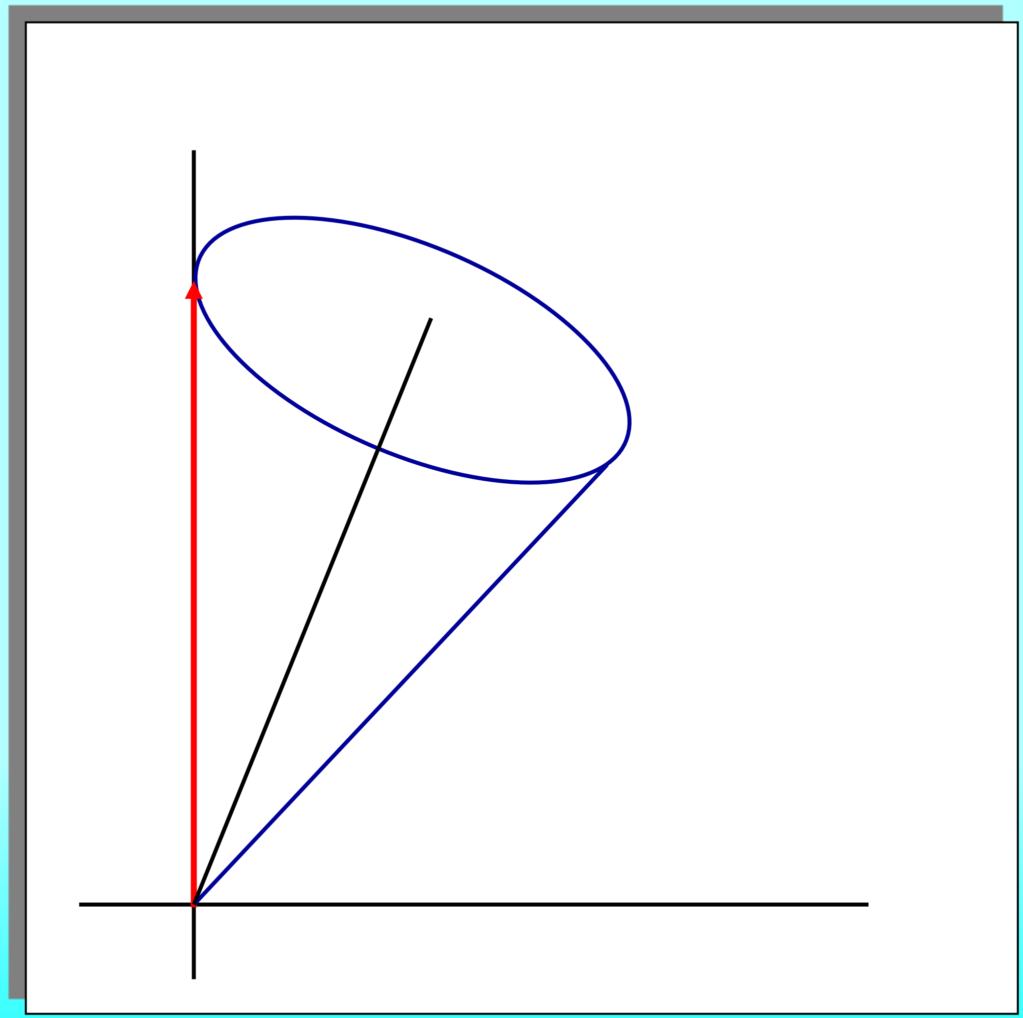




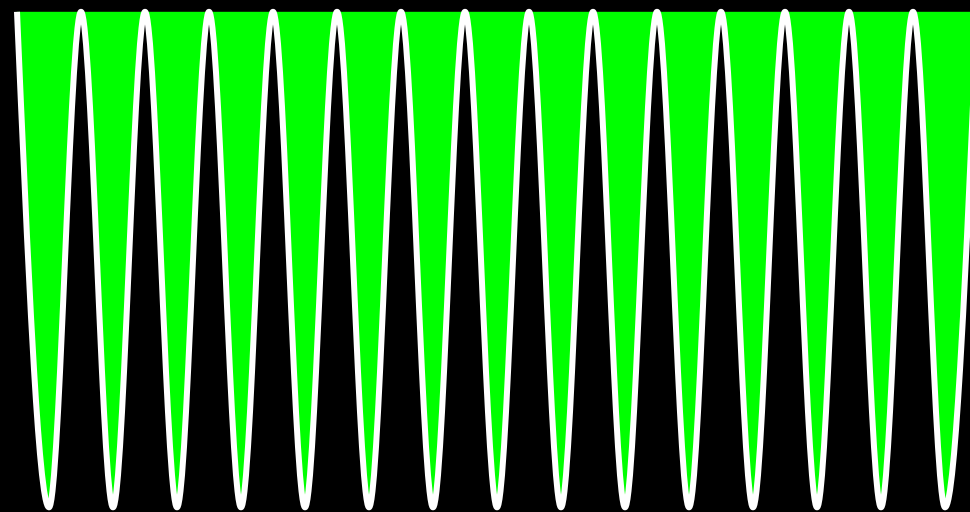




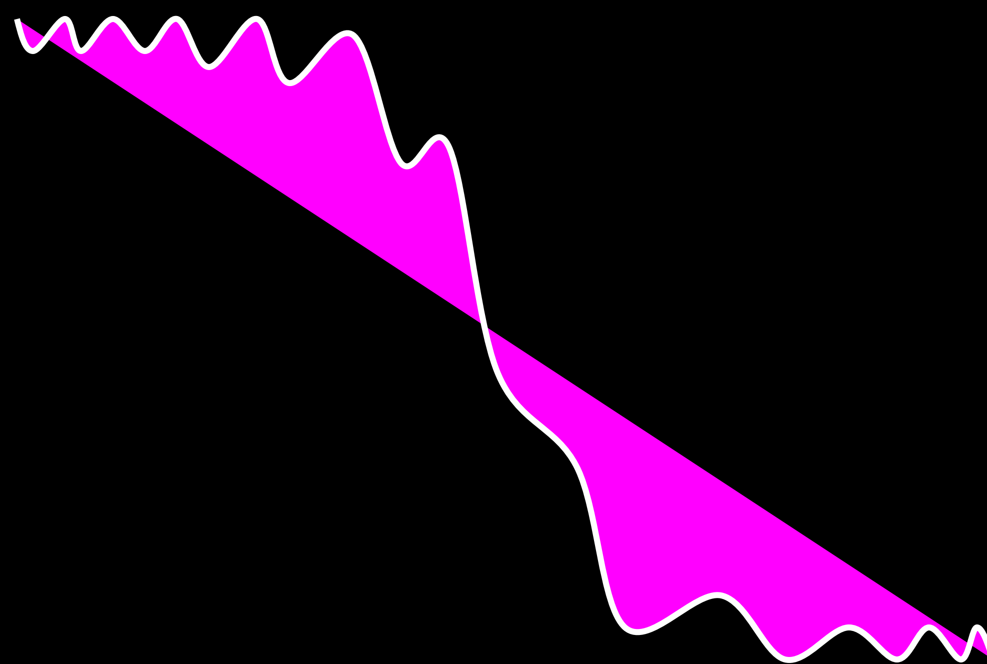




Matter effects oscillations



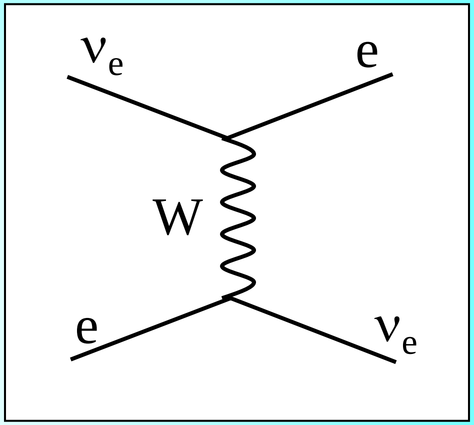
adiabatic conversion



Matter potential

L. Wolfenstein, 1978

for $\nu_e \ \nu_\mu$



at low energies Re A >> Im A
inelastic interactions can be neglected

Elastic forward scattering

$V_e, \ V_\mu$

potentials

Refraction index:

$$n - 1 = V / p$$

for E = 10 MeV

$$n - 1 = \begin{cases} \sim 10^{-20} & \text{inside the Earth} \\ < 10^{-18} & \text{inside the Sun} \end{cases}$$

difference of potentials

$$V = V_e - V_\mu = \sqrt{2}\, G_F\, n_e$$

$V \sim 10^{-13}$ eV inside the Earth

Refraction length:

$$l_0 = \frac{2\pi}{V}$$

Matter potential

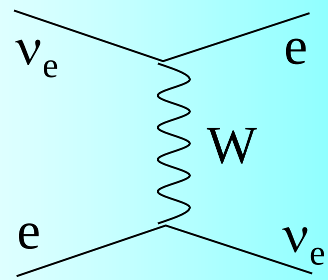
At low energies: neglect the inelastic scattering and absorption
effect is reduced to the elastic forward scattering (refraction)
described by the potential V (mean field approximation):

$$H_{int}(\nu) = \langle \psi \,|\, H_{int} \,|\, \psi \rangle = V \, \bar{\nu} \, \nu$$

ψ is the wave function of the medium

CC interactions with electrons

$$H_{int} = \frac{G_F}{\sqrt{2}} \, \bar{\nu} \, \gamma^{\mu}(1 - \gamma_5) \, \nu \; \bar{e} \, \gamma_{\mu}(1 - \gamma_5) \, e$$



$\langle \bar{e} \, \gamma_0 \, (1 - \gamma_5) \, e \rangle = n_e$ - electron number density

$\langle \bar{e} \, \vec{\gamma} \, e \rangle = n_e \vec{v}$ - velocity of electrons

$\langle \bar{e} \, \vec{\gamma} \, \gamma_5 \, e \rangle = n_e \, \vec{\lambda}_e$ - averaged polarization vector of electrons

For unpolarized medium at rest:

$$\boxed{V = \sqrt{2} \, G_F n_e}$$

Mixing in matter

Mixing is determined with respect to the eigenstates of propagation

$\nu_e \quad \nu_\mu$

ν_f

in vacuum:

H_0

θ

ν_{mass} $\quad \nu_1 \quad \nu_2$

Eigenstates in vacuum H_0

in matter:

$H(n_e, E) = H + V$

θ_m

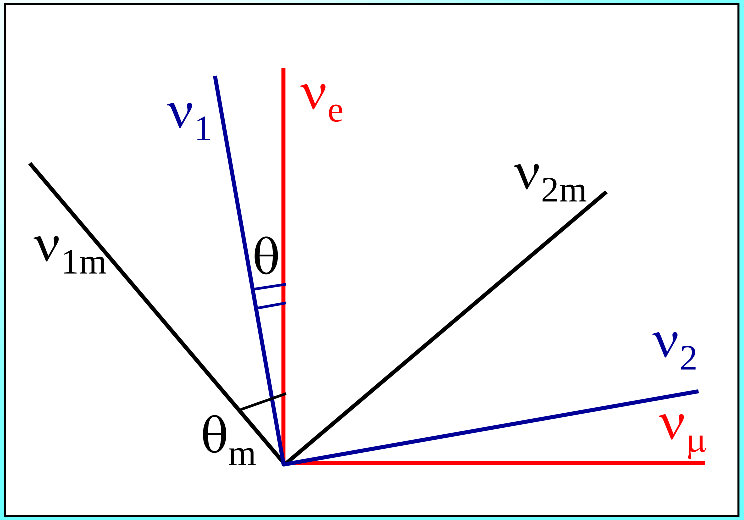
ν_H $\quad \nu_{1m} \quad \nu_{2m}$

Eigenstates in medium

Mixing angle determines flavors (flavor content) of eigenstates of propagation

θ_m depends on n_e, E

Flavor basis is the same, Eigenstates basis changes



Hamiltonian, eigenstates and eigenvalues

$H(n_e, E) = H + V$

$V = \mathrm{diag}\,(V_e\ ,\ 0\ )$

In the flavor basis $(\nu_e\, , \nu_\mu)^T$

$$H_{tot} = \frac{\Delta m^2}{4E}\begin{pmatrix} -\cos 2\theta\ +\xi & \sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{pmatrix}$$

$$\xi = \frac{4V_e E}{\Delta m^2} \qquad V_e = 2\sqrt{G}_F n_e$$

Mixing in matter

Eigenstates and eigenvalues

Diagonalization of the Hamiltonian:

$$\sin^2 2\theta_m = \frac{\sin^2 2\theta}{(\cos 2\theta - 2EV/\Delta m^2)^2 + \sin^2 2\theta}$$

$V = \sqrt{2}\, G_F\, n_e$

Mixing is maximal if

$$V = \frac{\Delta m^2}{2E} \cos 2\theta$$

Resonance condition

$H_e = H_\mu$

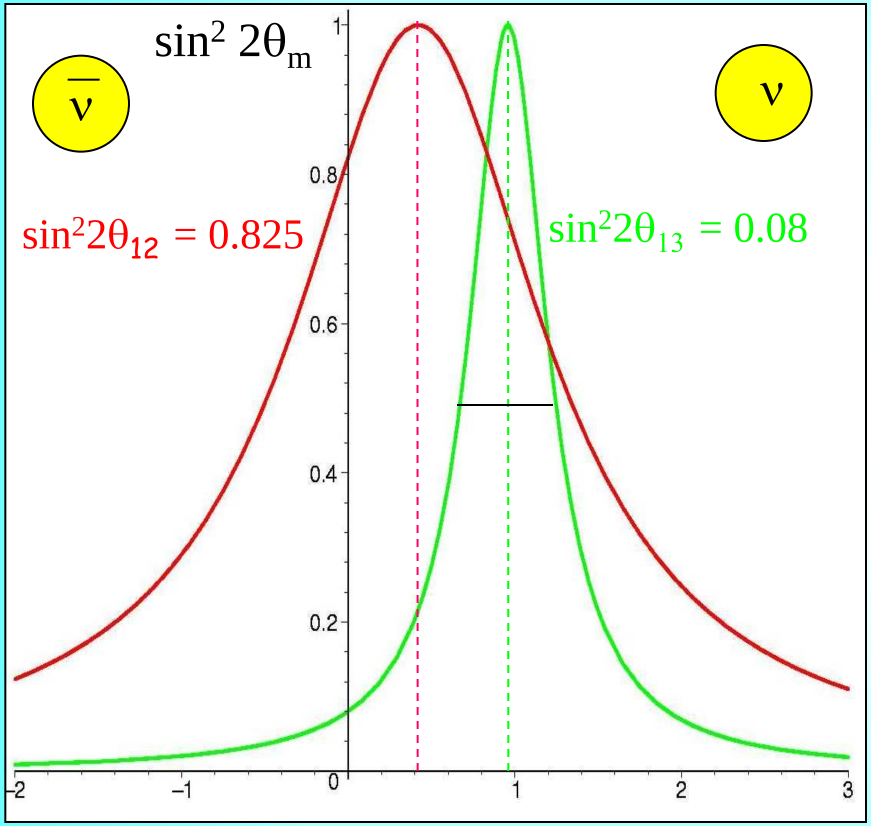
$$\sin^2 2\theta_m = 1$$

Difference of the eigenvalues

$$H_{2m} - H_{1m} = \frac{\Delta m^2}{2E} \sqrt{(\cos 2\theta - 2EV/\Delta m^2)^2 + \sin^2 2\theta}$$

Resonance

Dependence of mixing on density, energy has a resonance character



In resonance: $\sin^2 2\theta_m = 1$

Flavor mixing is maximal

$l_\nu = l_0 \cos 2\theta$

Vacuum oscillation length $\approx$ Refraction length

Resonance width: $\Delta n_R = 2 n_R \tan 2\theta$

density $l_\nu / l_0 \sim n E$

$\theta_m \rightarrow 0$ $\theta_m = \theta$ $\theta_m = \pi/4$ $\theta_m \rightarrow \pi/2$

Mixing is suppressed at high densities

$|V| \gg \frac{\Delta m^2}{2E}$ ⇒ Flavor states coincide with eigenstates and vice versa

Mixing in matter: dynamical variable

θ_m depends on n_e, E

Changes when propagates in matter with varying density

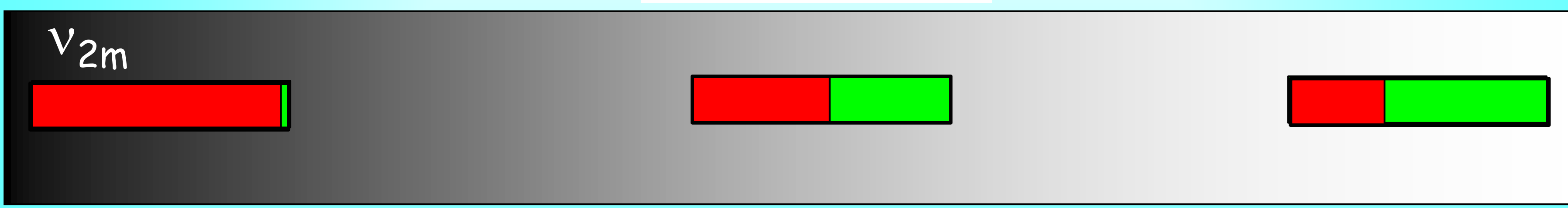
Mixing angle determines flavor content of eigenstates of propagation

High density
Mixing suppressed

Resonance:
Maximal mixing

$l_\nu = l_0 \cos 2\theta$

Low density
Vacuum mixing



Level crossing

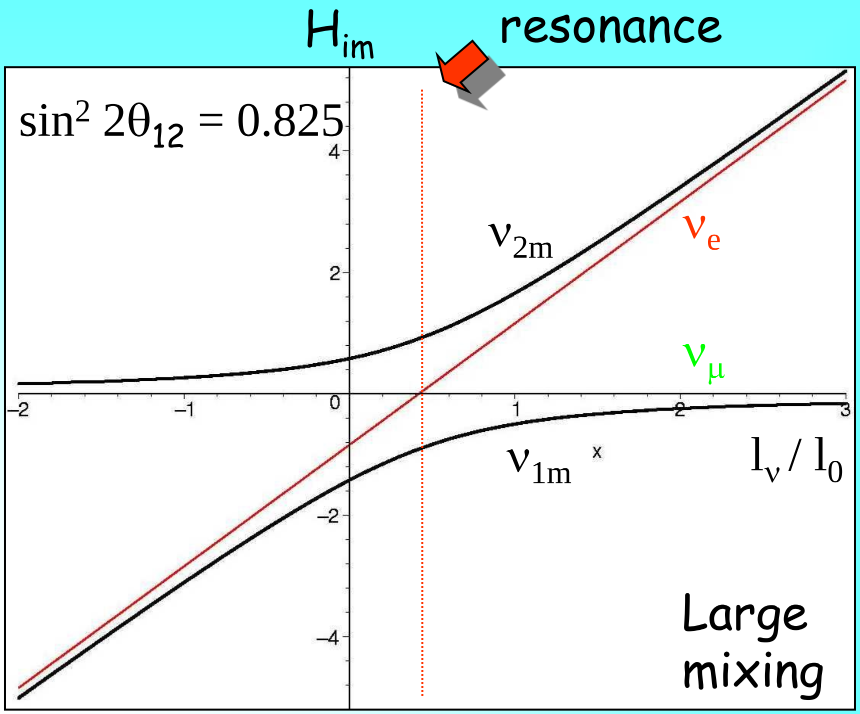
V. Rubakov, private comm.
N. Cabibbo, Savonlinna 1985
H. Bethe, PRL 57 (1986) 1271

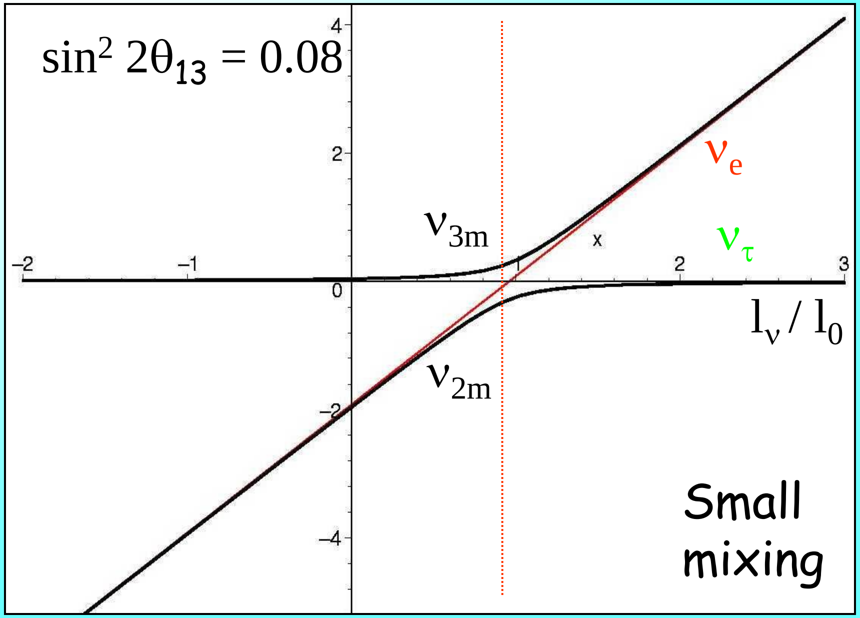
Dependence of the neutrino eigenvalues on the matter potential (density):

$$\frac{l_\nu}{l_0} = \frac{2E\,V}{\Delta m^2}$$

$$\frac{l_\nu}{l_0} = \cos 2\theta$$

Crossing point - resonance
- the level split is minimal
- the oscillation length is maximal



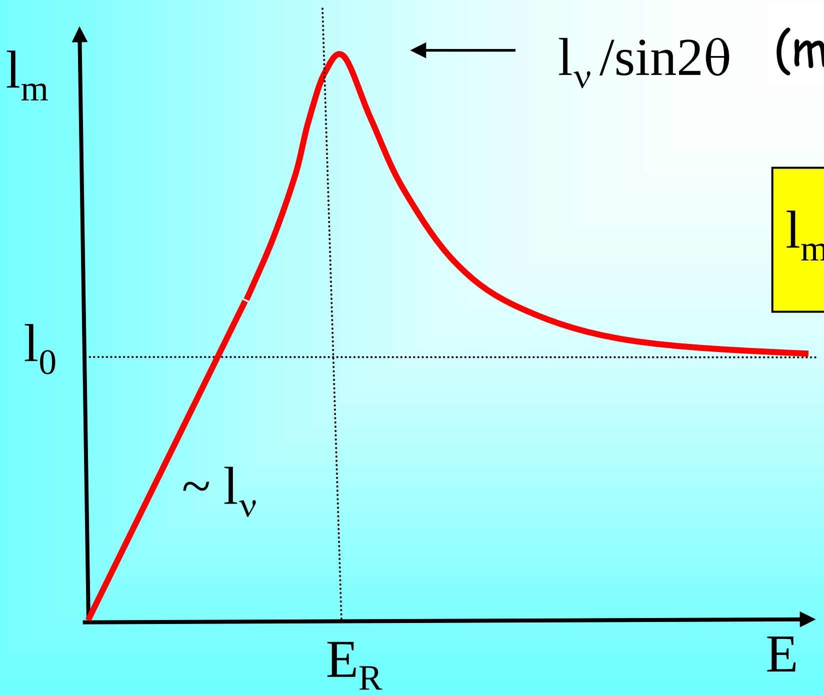


Oscillation length in matter

Oscillation length in vacuum $$l_\nu = \frac{4\pi E}{\Delta m^2}$$

Refraction length $$l_0 = \frac{2\pi}{\sqrt{2}\, G_F n_e}$$

- determines the phase produced by interaction with matter



$l_\nu/\sin 2\theta$ (maximum at $l_\nu = l_0/\cos 2\theta$)

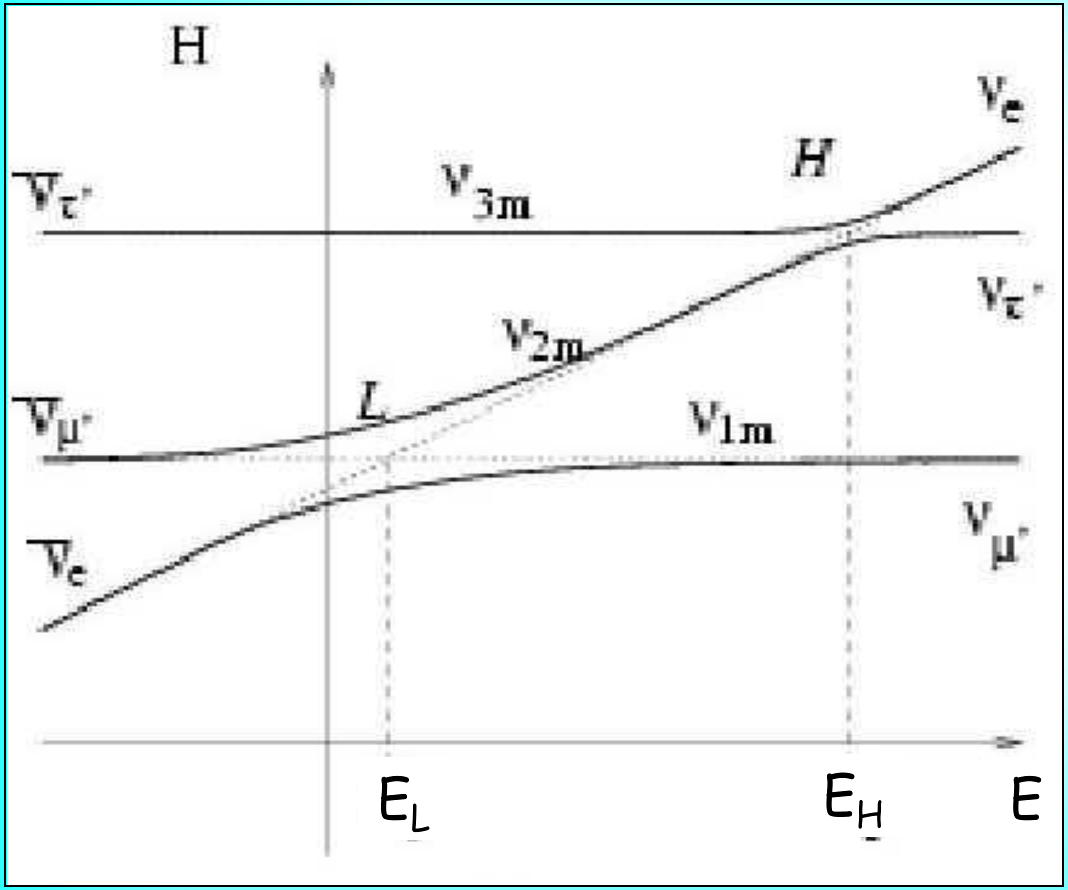
$$l_m = \frac{2\pi}{H_{2m} - H_{1m}}$$

shifts with respect resonance energy:

$$l_\nu(E_R) = l_0 \cos 2\theta$$

converges to the refraction length

Level crossings



Normal mass hierarchy

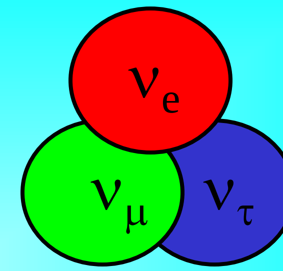
0.1 GeV

6 GeV

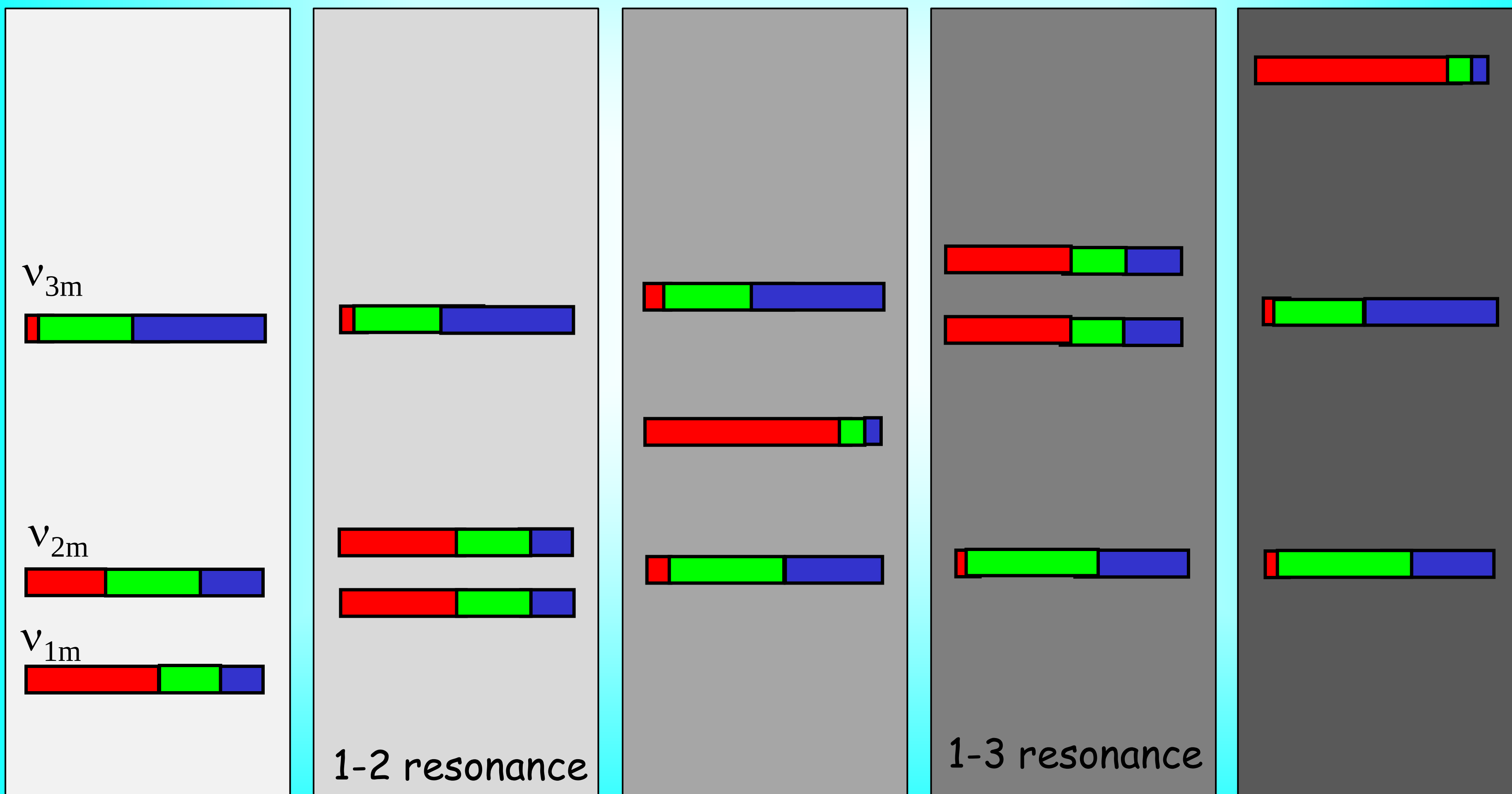
Resonance region

High energy range

Flavor in matter



Density increase → Normal mass hierarchy, neutrinos



Propagation effects

Evolution equation

$$\boxed{i\,\frac{d\nu_f}{dt} = H_{tot}\,\nu_f}$$

$$\nu_f = \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}$$

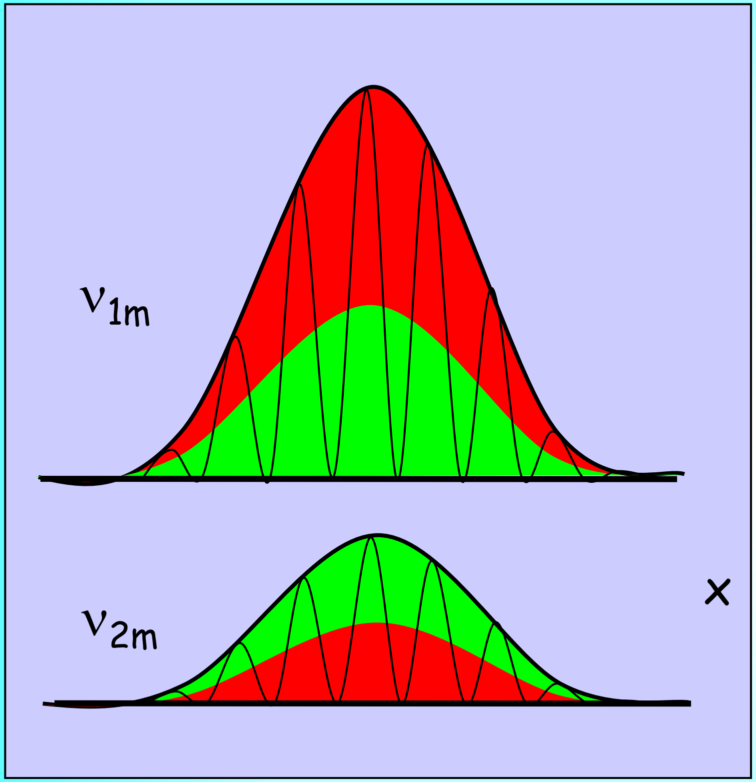
$H_{tot} = H_{vac} + V$ is the total Hamiltonian

$H_{vac} = \dfrac{M^2}{2E}$ is the vacuum (kinetic) part

$V = \begin{pmatrix} V_e & 0 \\ 0 & 0 \end{pmatrix}$ matter part $V_e = \sqrt{2}\,G_F n_e$

$$i\,\frac{d}{dt}\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \underbrace{\begin{pmatrix} -\dfrac{\Delta m^2}{2E}\cos 2\theta + V_e & \dfrac{\Delta m^2}{4E}\sin 2\theta \\ \dfrac{\Delta m^2}{4E}\sin 2\theta & 0 \end{pmatrix}}_{H_{tot}} \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}$$

Oscillations in matter



$H_0 \rightarrow H = H_0 + V$

$\nu_k \rightarrow \nu_{mk}$

eigenstates of H_0 — eigenstates of H

$\theta \rightarrow \theta_m\,(n)$

Resonance - maximal mixing in matter – oscillations with maximal depth

$\theta_m = \pi/4$

Resonance condition:

$$V = \cos 2\theta \frac{\Delta m^2}{2E}$$

Constant density medium: the same dynamics

Mixing changed
phase difference changed

Oscillations in matter

Probability in constant density

$$P(\nu_e \rightarrow \nu_a) = \sin^2 2\theta_m \sin^2\left(\frac{\pi L}{l_m}\right)$$

depth of oscillations

oscillatory factor

half-phase ϕ

$\theta_m(E, n)$ - mixing angle in matter

$l_m(E, n)$ – oscillation length in matter

$$l_m = 2\pi/(H_{2m} - H_{1m})$$

In vacuum: $\theta_m \rightarrow \theta$, $l_m \rightarrow l_\nu$

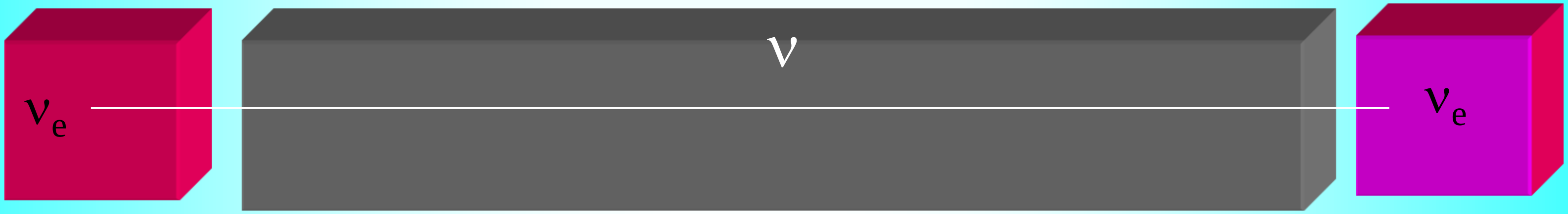
Maximal effect:

$\sin^2 2\theta_m = 1$ ⇒ MSW resonance condition

$\phi = \pi/2 + \pi k$

Resonance enhancement

Constant density



Source

$F_0(E)$

layer of length L

Detector

$F(E)$

Depth of oscillations determined by $\sin^2 2\theta_m$
the oscillation length, l_m
depends on neutrino energy

For neutrinos propagating
in the mantle of the Earth

Large mixing $\sin^2 2\theta = 0.824$

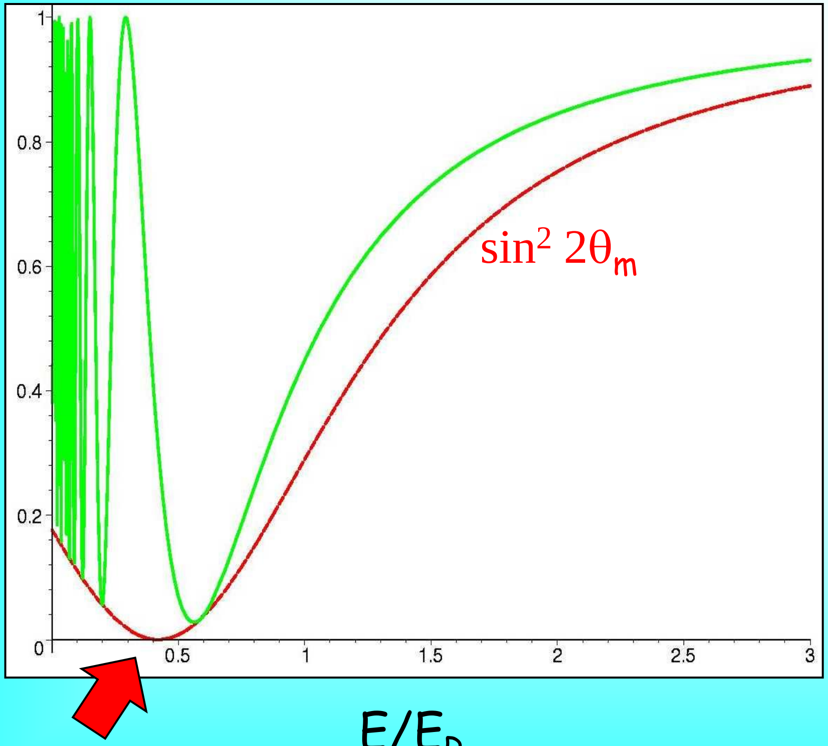
Layer of length L $k = \pi L / l_0$

ν

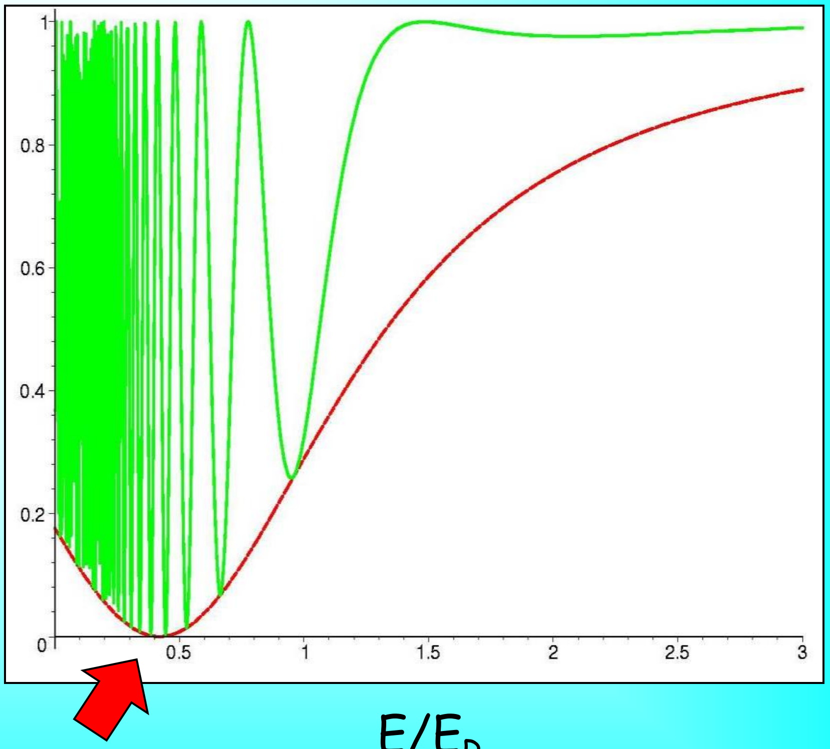
thin layer $k = 1$

thick layer $k = 10$

$\frac{F(E)}{F_0(E)}$



E/E_R



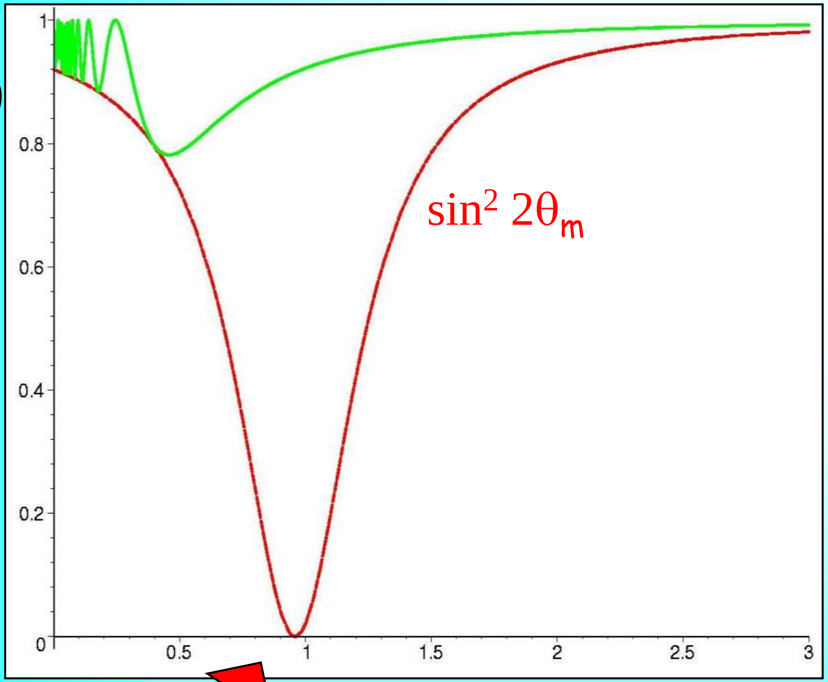
E/E_R

Oscillation in the Earth,
ORCA/PINGU

Small mixing $\sin^2 2\theta = 0.08$

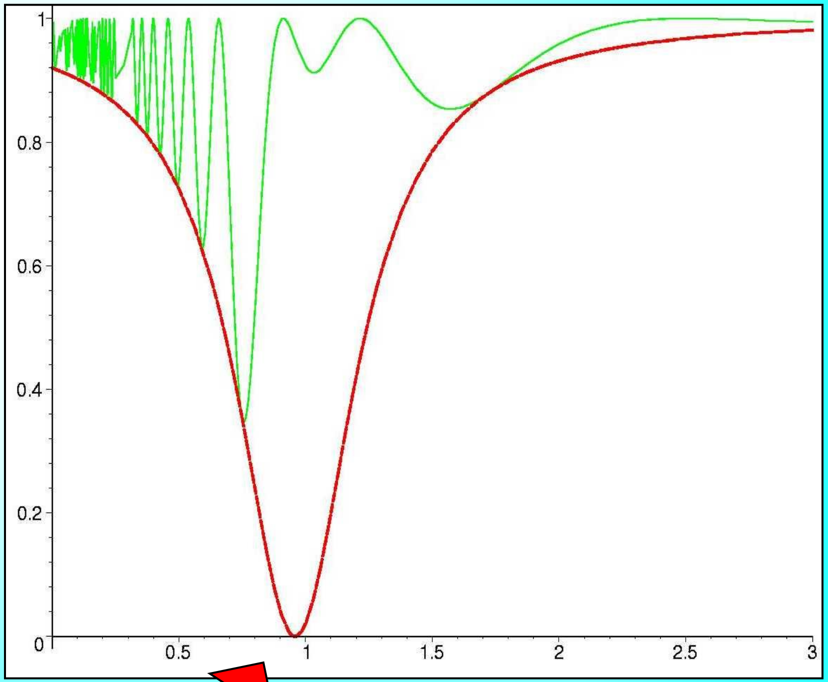
thin layer $k = 1$

$\frac{F(E)}{F_0(E)}$



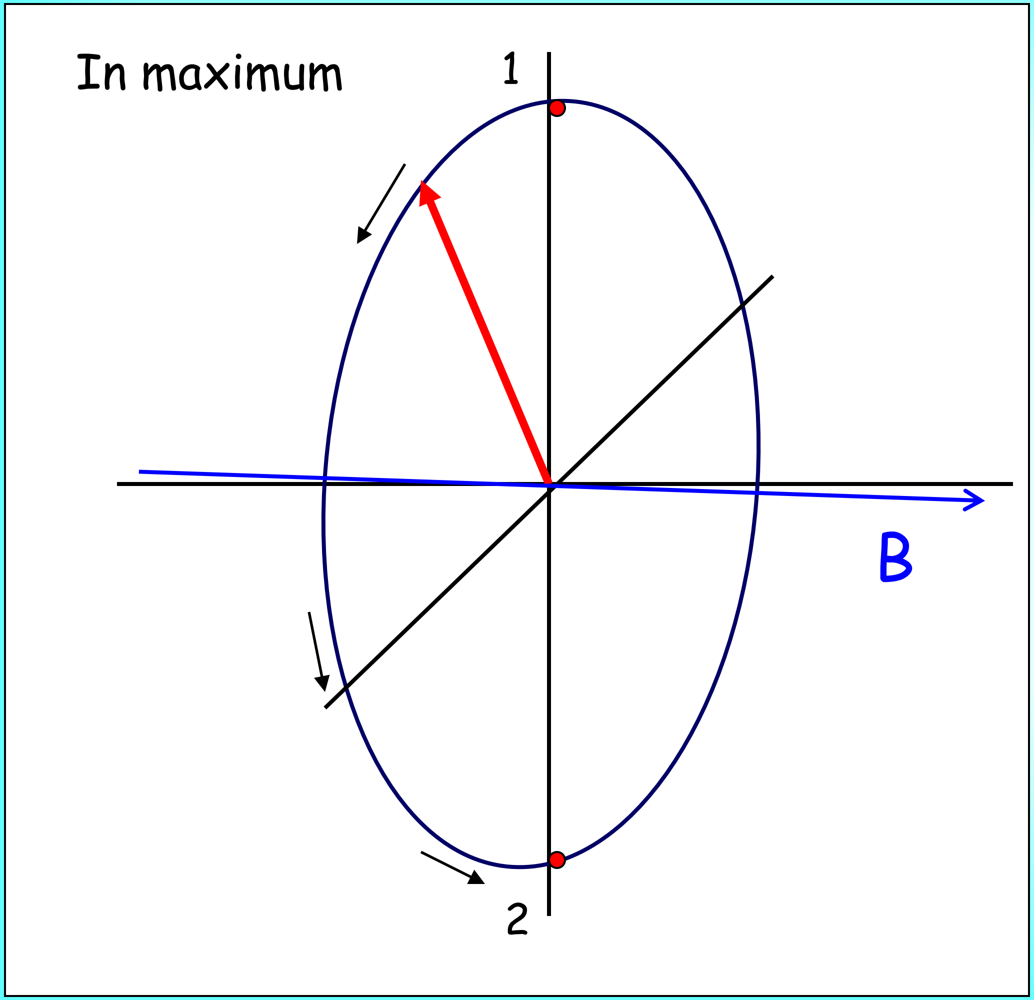
E/E_R

thick layer $k = 10$

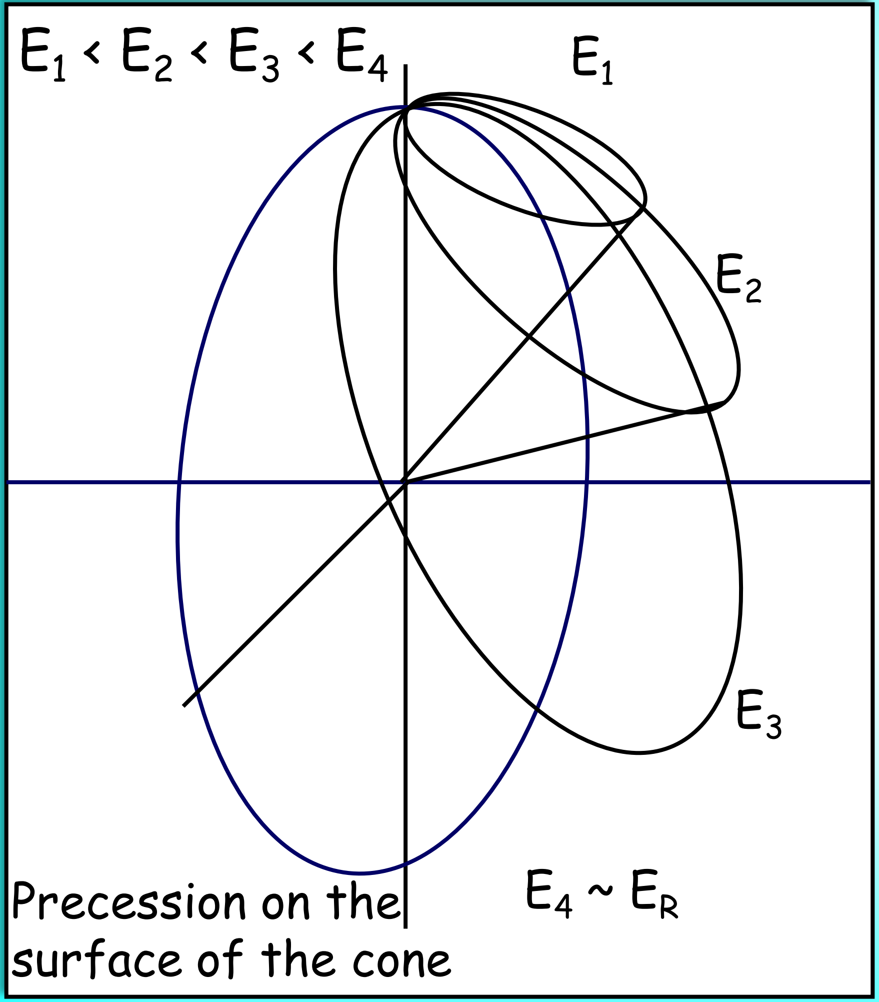


E/E_R

Resonance enhancement



Graphic representation



Resonance enhancement