Toward a unified interpretation of quark and lepton mixing from flavor and CP symmetries

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Based on arXiv:1704.xxxxx, in collaboration with Cai-Chang Li

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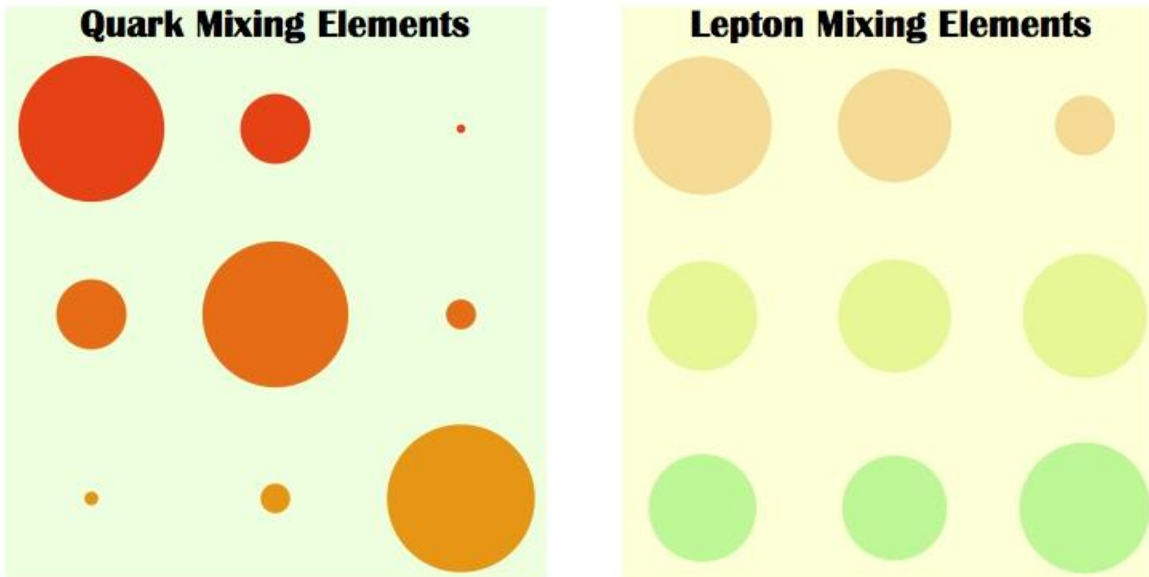
CKM vs. PMNS



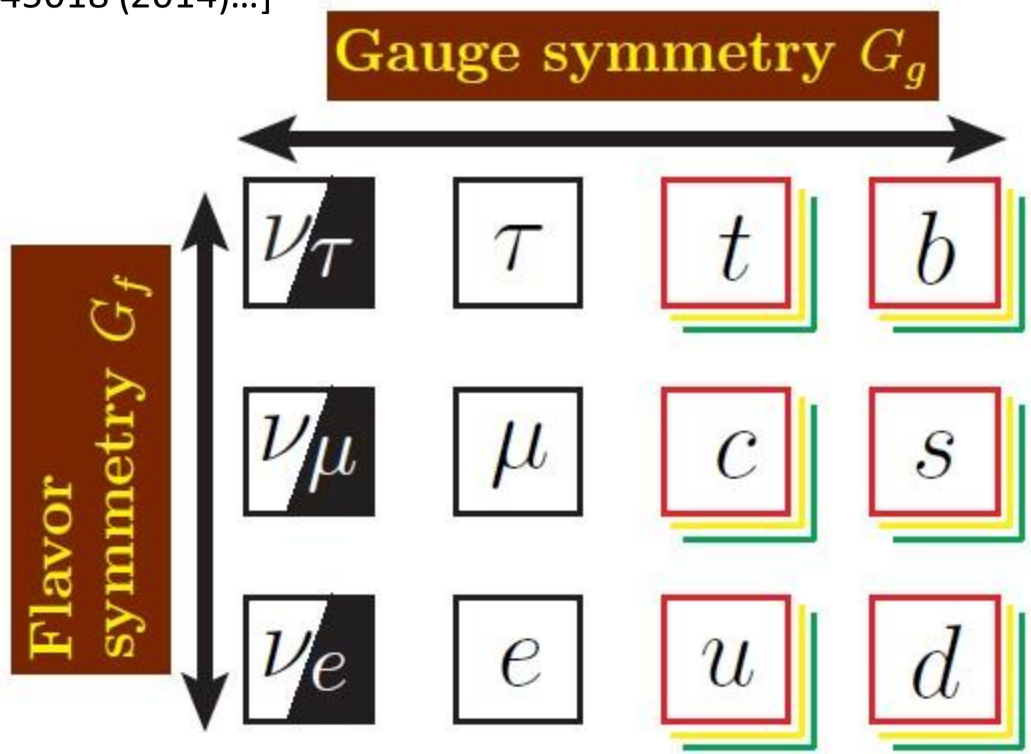
- Why these values?
- Are the two related?
- Are they related to masses?

Who ordered that?

Flavor mixing from flavor symmetry

➢The paradigm of flavor symmetry

[Froggatt, Nielsen, Nucl.Phys. B147 (1979) 277-298; Altarelli and Feruglio, Rev. Mod. Phys. 82, 2701 (2010); King and Luhn, Rept. Prog. Phys. 76, 056201 (2013); King, Merle, Morisi, Shimizu and Tanimoto, New J. Phys. 16, 045018 (2014)…]



	Continuous	Discrete
Abelian	U(1)	Z_n
Non-Abelian	U(2), SU(3),SO(3) …	A_4, S_4, A_5, $\Delta(6n^2)$, …

Classification of U_{PMNS} from finite flavor symmetries

The PMNS matrix can take **17 discrete patterns or the trimaximal form**

➢ Only **trimaximal** mixing can be compatible with data

$$U_{PMNS}=\frac{1}{\sqrt{3}}\begin{pmatrix}\sqrt{2}\cos\theta & 1 & -\sqrt{2}\sin\theta \\ -\sqrt{2}\cos(\theta-\pi/3) & 1 & \sqrt{2}\sin(\theta-\pi/3) \\ -\sqrt{2}\cos(\theta+\pi/3) & 1 & \sqrt{2}\sin(\theta+\pi/3)\end{pmatrix}$$

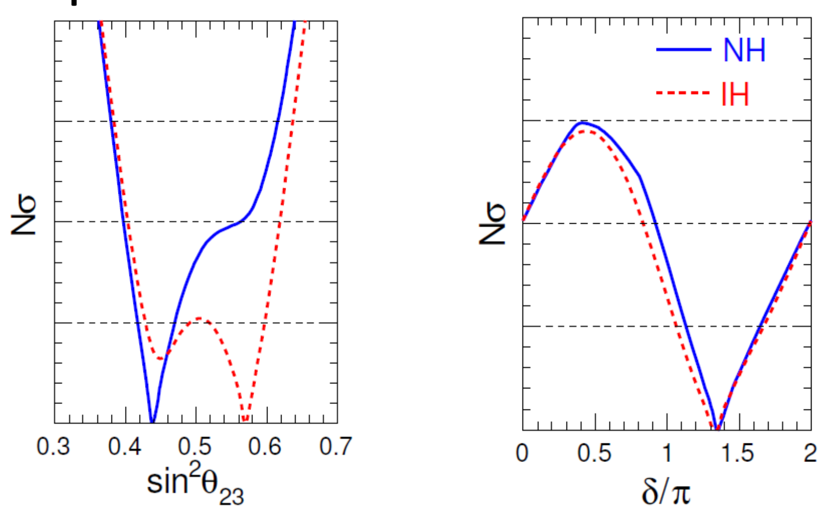
[Fonseca and Grimus, JHEP 1409, 033 (2014);
Holthausen, Lim, Lindner, Phys.Lett. B721 (2013) 61-67 ;
Yao, Ding, Phys.Rev. D92 (2015) no.9, 096010]

- testable sum rules:

$$3\sin^2\theta_{12}\cos^2\theta_{13}=1,\quad \sin^2\theta_{23}=\frac{1}{2}\pm\frac{1}{2}\tan\theta_{13}\sqrt{2-\tan^2\theta_{13}}$$

$$(\sin^2\theta_{13})^{\rm bf}\simeq 0.0218\Rightarrow \sin^2\theta_{12}\simeq 0.341,\quad \sin^2\theta_{23}\simeq 0.391\ \text{ or }\ 0.609$$

- Dirac CP phase is conserved : $\sin\delta=0$



[Capozzi, Lisi et al., Nucl.Phys. B908 (2016) 218-234]

- Underlying flavor symmetry

$$G_f = \Delta(6n^2) \cong (Z_n \times Z_n) \rtimes S_3$$

$$\text{or} \quad G_f = D^{(1)}_{9n,3n} \cong (Z_{9n} \times Z_{3n}) \rtimes S_3$$

Examples:

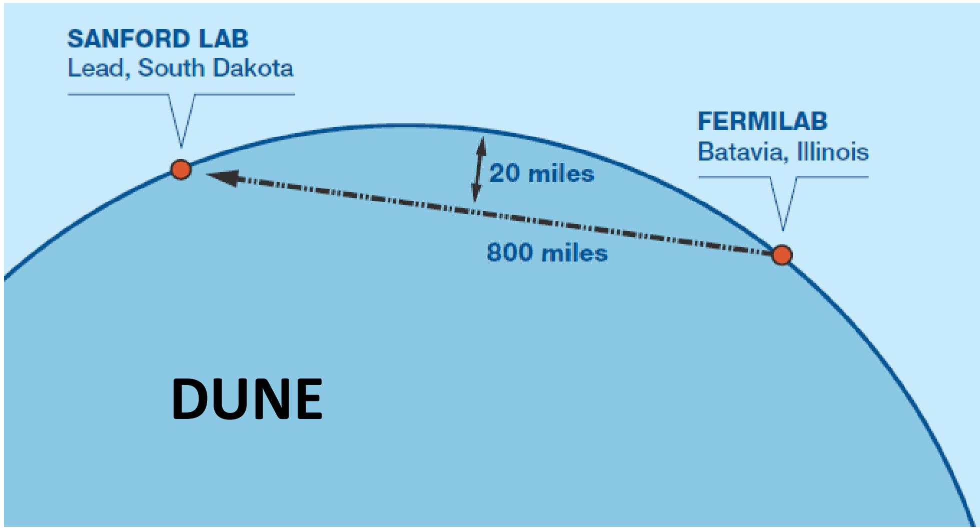
G_f	**Δ(600)**	$D^{(1)}_{18,6}$	**Δ(1536)**
θ	$\pm\pi/15$	$\pm\pi/18$	$\pm\pi/16$
$\sin^2\theta_{13}$	0.0288	0.0201	0.0254

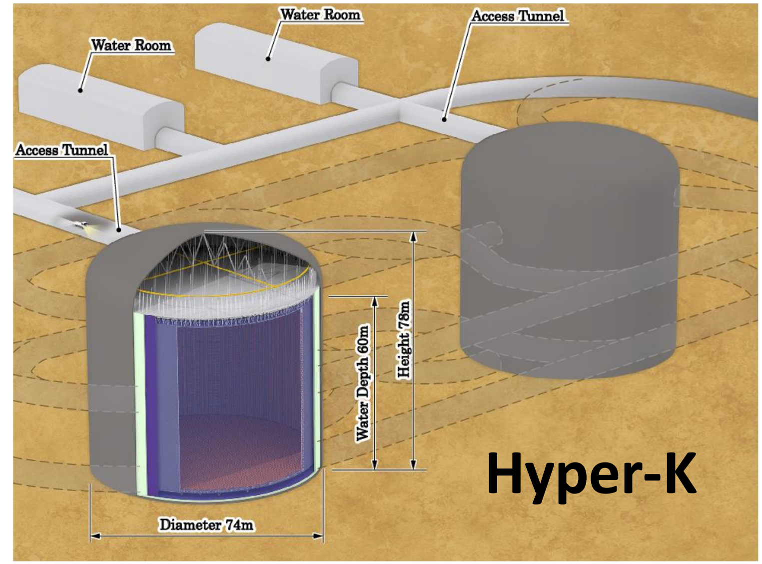
- Quark sector: only the Cabibbo mixing angle can be reproduced for both the triplet and doublet+singlet assignment of left-handed quarks

$$V_{CKM} = \begin{pmatrix} \cos\theta_C & \sin\theta_C & 0 \\ -\sin\theta_C & \cos\theta_C & 0 \\ 0 & 0 & 1 \end{pmatrix}, \qquad \theta_C \approx \frac{\pi}{14}$$

[C.S. Lam, Phys.Lett.B656:193-198,2007;
Blum, Hagedorn and Lindner, Phys.Rev. D77 (2008) 076004 ;
Ishimori,King,Okada,Tanimoto,Phys.Lett. B743 (2015) 172-179 ;
Yao, Ding, Phys.Rev. D92 (2015) no.9, 096010]

Next goal: measure leptonic CP violation





➢Theoretical idea: flavor symmetry → flavor+CP symmetries

Flavor symmetry

↕

CP symmetry

→ **Mixing angles+CP phases**

See talks by Morimitsu, Walter, Claudia

[Grimus et al.,J. Phys. A 20 (1987) L807; Harrison and Scott, Phys. Lett. B 535 (2002) 163; Lindner et al., JHEP 1304, 122; Feruglio et al., JHEP 1307, 027 (2013); Ding,King,Luhn,Stuart,JHEP1305,084(2013); Chen, Fallbacher,Mahanthappa,Ratz and Trautner, Nucl. Phys. B883, 267 (2014)…]

Semi-direct approach to lepton mixing

See talk by Claudia

$$G_f \rtimes H_{CP}$$

charged lepton

neutrino

$$Z_n^{g_l},\ n \geq 3$$

$$Z_2^{g_\nu} \times X_\nu$$

$$U_l^\dagger \rho_3(g_l) U_l = \rho_3(g_l)_{diag}$$

$$X_\nu = \Sigma_\nu \Sigma_\nu^T,$$

$$\Sigma_\nu^\dagger \rho_3(g_\nu) \Sigma_\nu = \pm \mathrm{diag}(1,-1,-1)$$

$$U_l$$

$$U_\nu = \Sigma_\nu R_{23}(\theta) Q_\nu$$

$$U_{PMNS} = U_l^\dagger \Sigma_\nu R_{23}(\theta) Q_\nu$$

➢The mixing angles and CP violating phases are predicted in terms of a single real parameter $0 \leq \theta \leq \pi$

[Feruglio, Hagedorn, Ziegler, JHEP 1307, 027 (2013)]

Possible mixing patterns from finite flavor and CP symmetries

Only eight kinds of mixing matrices consistent with experimental data can be obtained up to row and column permutations.

$$U^{I}=\frac{1}{\sqrt{3}}\begin{pmatrix}\sqrt{2}\sin\varphi_1 & e^{i\varphi_2} & \sqrt{2}\cos\varphi_1\\ \sqrt{2}\cos\left(\varphi_1-\frac{\pi}{6}\right) & -e^{i\varphi_2} & -\sqrt{2}\sin\left(\varphi_1-\frac{\pi}{6}\right)\\ \sqrt{2}\cos\left(\varphi_1+\frac{\pi}{6}\right) & e^{i\varphi_2} & -\sqrt{2}\sin\left(\varphi_1+\frac{\pi}{6}\right)\end{pmatrix}R_{23}(\theta)Q_\nu$$

$$U^{II}=\frac{1}{\sqrt{3}}\begin{pmatrix}e^{i\varphi_1} & 1 & e^{i\varphi_2}\\ \omega e^{i\varphi_1} & 1 & \omega^2 e^{i\varphi_2}\\ \omega^2 e^{i\varphi_1} & 1 & \omega e^{i\varphi_2}\end{pmatrix}R_{13}(\theta)Q_\nu$$

[Yao, Ding, Phys.Rev. D94 (2016) no.7, 073006]

$R_{ij}(\theta)$ is the rotation matrix in the ij plane

$$U^{III}=\frac{1}{\sqrt{3}}\begin{pmatrix}\sqrt{2}e^{i\varphi_1}\sin\varphi_2 & 1 & \sqrt{2}e^{i\varphi_1}\cos\varphi_2\\ \sqrt{2}e^{i\varphi_1}\cos\left(\varphi_2+\frac{\pi}{6}\right) & 1 & -\sqrt{2}e^{i\varphi_1}\sin\left(\varphi_2+\frac{\pi}{6}\right)\\ -\sqrt{2}e^{i\varphi_1}\cos\left(\varphi_2-\frac{\pi}{6}\right) & 1 & \sqrt{2}e^{i\varphi_1}\sin\left(\varphi_2-\frac{\pi}{6}\right)\end{pmatrix}R_{13}(\theta)Q_\nu$$

$$U^{IV(a)} = \begin{pmatrix} -\sqrt{\frac{\phi_g}{\sqrt{5}}} & \sqrt{\frac{1}{\sqrt{5}\phi_g}} & 0 \\ \sqrt{\frac{1}{2\sqrt{5}\phi_g}} & \sqrt{\frac{\phi_g}{2\sqrt{5}}} & -\frac{1}{\sqrt{2}} \\ \sqrt{\frac{1}{2\sqrt{5}\phi_g}} & \sqrt{\frac{\phi_g}{2\sqrt{5}}} & \frac{1}{\sqrt{2}} \end{pmatrix} R_{13}(\theta)Q_\nu, \qquad U^{IV(b)} = \begin{pmatrix} -i\sqrt{\frac{\phi_g}{\sqrt{5}}} & \sqrt{\frac{1}{\sqrt{5}\phi_g}} & 0 \\ i\sqrt{\frac{1}{2\sqrt{5}\phi_g}} & \sqrt{\frac{\phi_g}{2\sqrt{5}}} & -\frac{1}{\sqrt{2}} \\ i\sqrt{\frac{1}{2\sqrt{5}\phi_g}} & \sqrt{\frac{\phi_g}{2\sqrt{5}}} & \frac{1}{\sqrt{2}} \end{pmatrix} R_{13}(\theta)Q_\nu$$

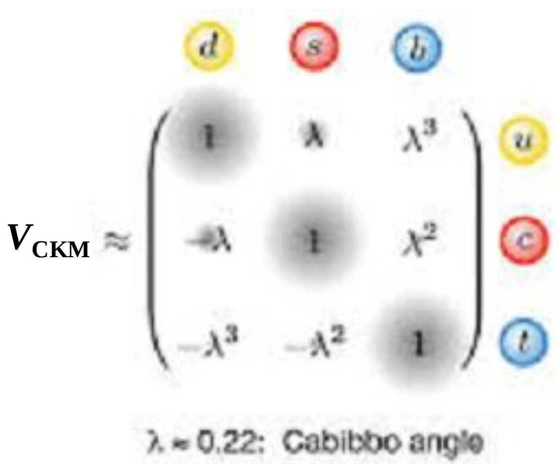
$$U^{V} = \frac{1}{2}\begin{pmatrix} \phi_g & 1 & \phi_g - 1 \\ \phi_g - 1 & -\phi_g & 1 \\ 1 & 1-\phi_g & -\phi_g \end{pmatrix} R_{23}(\theta)Q_\nu, \quad U^{VI} = \frac{1}{2\sqrt{3}}\begin{pmatrix} (\sqrt{3}-1)e^{i\varphi} & 2 & -(\sqrt{3}+1)e^{i\left(\varphi+\frac{3\pi}{4}\right)} \\ -(\sqrt{3}+1)e^{i\varphi} & 2 & (\sqrt{3}-1)e^{i\left(\varphi+\frac{3\pi}{4}\right)} \\ 2e^{i\varphi} & 2 & 2e^{i\left(\varphi+\frac{3\pi}{4}\right)} \end{pmatrix} R_{13}(\theta)Q_\nu$$

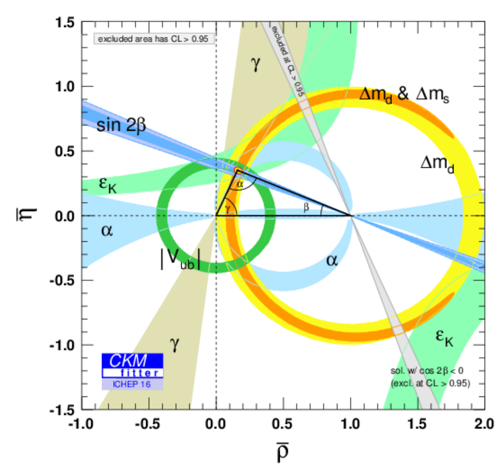
$$U^{VII} = \frac{1}{2\sqrt{6}}\begin{pmatrix} -\frac{\sqrt{3}}{s_3} & 2\sqrt{2} & \frac{s_2 - s_1}{s_1 s_2} \\ \frac{\sqrt{3}}{s_2} & 2\sqrt{2} & -\frac{s_1 + s_3}{s_1 s_3} \\ \frac{\sqrt{3}}{s_1} & 2\sqrt{2} & \frac{s_2 + s_3}{s_2 s_3} \end{pmatrix} R_{23}(\theta)Q_\nu, \qquad U^{VIII} = \frac{1}{2} R_{13}^T(\theta)\begin{pmatrix} \sqrt{2}e^{i\varphi_1} & -\sqrt{2}e^{i\varphi_1} & 0 \\ 1 & 1 & -\sqrt{2}e^{i\varphi_2} \\ 1 & 1 & \sqrt{2}e^{i\varphi_2} \end{pmatrix} Q_\nu$$

➢All these viable mixing patterns can be obtained from the SU(3) finite subgroups $\Delta(6n^2)$, $D^{(1)}_{9n,3n}$, A_5 and $\Sigma(168)$ combined with CP symmetry.

Cases	Group series	Minimal flavor group
I, II, III	$\Delta(6n^2)$, $D^{(1)}_{9n,3n}$	S_4 (A_4 for III)
IV, V	A_5, [120, 35], [180, 19] $\cdots$	A_5
VI, VII	$\Sigma(168)$, [336, 209], [504,157], $\cdots$	$\Sigma(168)$
VIII	$\Delta(6n^2)$, $D^{(1)}_{9n,3n}$	S_4

➢The quark mixing angles and CP phase still **can not** be explained in this approach.





Results collected on the website

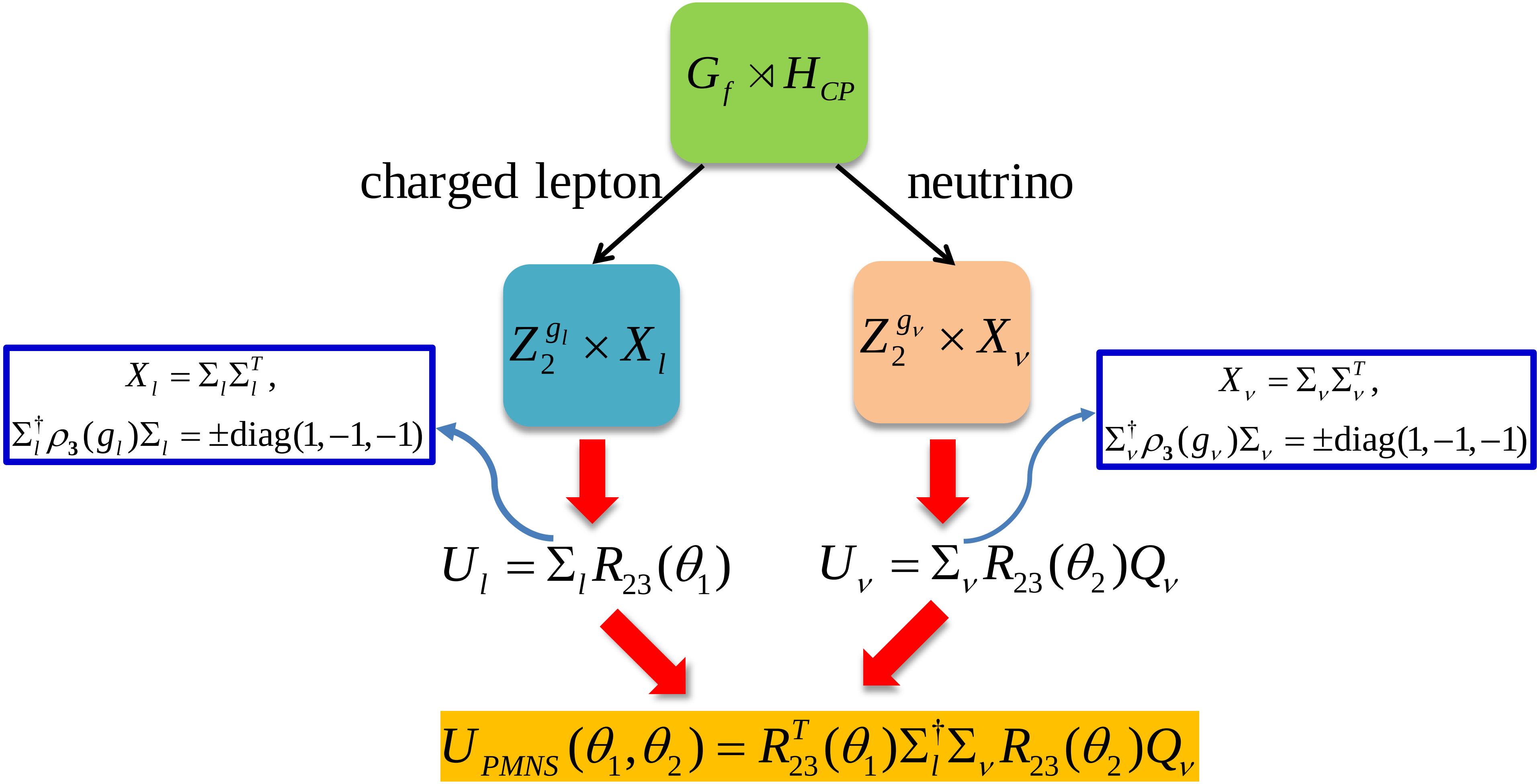
I(a)	I(b)	II	III	IV	V	VI	VII	VIII

$$U_{\rm PMNS}^{I(b)} = \frac{1}{\sqrt{3}}\begin{pmatrix} \sqrt{2}\cos\varphi_1 & e^{i\varphi_2} & \sqrt{2}\sin\varphi_1 \\ -\sqrt{2}\sin\left(\varphi_1-\frac{\pi}{6}\right) & -e^{i\varphi_2} & \sqrt{2}\cos\left(\varphi_1-\frac{\pi}{6}\right) \\ -\sqrt{2}\sin\left(\varphi_1+\frac{\pi}{6}\right) & e^{i\varphi_2} & \sqrt{2}\cos\left(\varphi_1+\frac{\pi}{6}\right) \end{pmatrix} S_{12}(\theta)$$

Group ID	(φ_1, φ_2)
[648,259]	$\left(\frac{\pi}{18},-\frac{\pi}{6}\right), \left(\frac{\pi}{18},0\right), \left(\frac{\pi}{18},\frac{\pi}{3}\right), \left(\frac{\pi}{18},\frac{\pi}{2}\right), \left(\frac{17\pi}{18},-\frac{\pi}{6}\right), \left(\frac{17\pi}{18},0\right), \left(\frac{17\pi}{18},\frac{\pi}{3}\right), \left(\frac{17\pi}{18},\frac{\pi}{2}\right)$
[726,5]	$\left(\frac{2\pi}{33},-\frac{2\pi}{11}\right), \left(\frac{2\pi}{33},0\right), \left(\frac{2\pi}{33},\frac{\pi}{11}\right), \left(\frac{2\pi}{33},\frac{3\pi}{11}\right), \left(\frac{2\pi}{33},\frac{4\pi}{11}\right), \left(\frac{2\pi}{33},\frac{5\pi}{11}\right), \left(\frac{31\pi}{33},-\frac{2\pi}{11}\right), \left(\frac{31\pi}{33},0\right), \left(\frac{31\pi}{33},\frac{\pi}{11}\right), \left(\frac{31\pi}{33},\frac{3\pi}{11}\right), \left(\frac{31\pi}{33},\frac{4\pi}{11}\right), \left(\frac{31\pi}{33},\frac{5\pi}{11}\right)$
[1734,5]	$\left(\frac{\pi}{17},-\frac{8\pi}{17}\right), \left(\frac{\pi}{17},-\frac{6\pi}{17}\right), \left(\frac{\pi}{17},0\right), \left(\frac{\pi}{17},\frac{\pi}{17}\right), \left(\frac{\pi}{17},\frac{2\pi}{17}\right), \left(\frac{\pi}{17},\frac{3\pi}{17}\right), \left(\frac{\pi}{17},\frac{4\pi}{17}\right), \left(\frac{\pi}{17},\frac{5\pi}{17}\right), \left(\frac{\pi}{17},\frac{7\pi}{17}\right), \left(\frac{16\pi}{17},-\frac{8\pi}{17}\right), \left(\frac{16\pi}{17},-\frac{6\pi}{17}\right), \left(\frac{16\pi}{17},0\right), \left(\frac{16\pi}{17},\frac{\pi}{17}\right), \left(\frac{16\pi}{17},\frac{2\pi}{17}\right), \left(\frac{16\pi}{17},\frac{3\pi}{17}\right), \left(\frac{16\pi}{17},\frac{4\pi}{17}\right), \left(\frac{16\pi}{17},\frac{5\pi}{17}\right), \left(\frac{16\pi}{17},\frac{7\pi}{17}\right)$

http://staff.ustc.edu.cn/~dinggj/cp_scan.html

Another scheme to predict lepton mixing from flavor and CP



$$U_{PMNS}(\theta_1,\theta_2)=R_{23}^T(\theta_1)\Sigma_l^\dagger\Sigma_\nu R_{23}(\theta_2)Q_\nu$$

- Since the lepton masses are not constrained, permutations of rows and columns of U_{PMNS} are possible

$$P_{12}=\begin{pmatrix}0&1&0\\1&0&0\\0&0&1\end{pmatrix},\ P_{13}=\begin{pmatrix}0&0&1\\0&1&0\\1&0&0\end{pmatrix},\ P_{23}=\begin{pmatrix}1&0&0\\0&0&1\\0&1&0\end{pmatrix}$$

- One element is completely fixed by the residual symmetry

- All mixing angles and CP phases are expressed in terms of two free parameters $\theta_{1,2}\epsilon[0,\pi)$

Δ(6n²) flavor group and CP symmetry

- Δ(6n²) is a non-abelian finite subgroup of SU(3), it is isomorphic to $(Z_n \times Z_n) \rtimes S_3$. Its four generators satisfy:

$$a^3 = b^2 = (ab)^2 = 1,$$

$$c^n = d^n = 1, \qquad cd = dc,$$

$$aca^{-1} = c^{-1}d^{-1}, \quad ada^{-1} = c, \quad bcb^{-1} = d^{-1}, \quad bdb^{-1} = c^{-1}$$

- Familiar examples: $\Delta(6\times 1^2) \cong S_3, \quad \Delta(6\times 2^2) \cong S_4$
- Irreducible representations : 1-dim, 2-dim, 3-dim, 6-dim $\qquad \eta = e^{2\pi i/n}$

$$a = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, \quad b = -\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad c = \begin{pmatrix} \eta & 0 & 0 \\ 0 & \eta^{-1} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad d = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \eta & 0 \\ 0 & 0 & \eta^{-1} \end{pmatrix}$$

- Physical CP transformations are of the same form as the flavor symmetry transformations in the chosen basis.

$$X\rho^*(g)X^{-1} = \rho(g')$$

[Hagedorn, Meroni and Molinaro, Nucl. Phys. B 891, 499 (2015); Ding, King and Neder, JHEP 1412, 007 (2014)]

Predictions for lepton mixing from $\Delta(6n^2)$ and CP

We find **four** independent viable lepton mixing patterns

Case (I): $Z_2^{g_l} = Z_2^{bc^x d^x},\ X_l = \left\{c^\gamma d^{-2x-\gamma}, bc^{x+\gamma} d^{-x-\gamma}\right\},$

$Z_2^{g_\nu} = Z_2^{bc^y d^y}, X_\nu = \left\{c^\delta d^{-2y-\delta}, bc^{y+\delta} d^{-y-\delta}\right\}$

- The lepton mixing matrix is determined to be

$$U_I = \begin{pmatrix} \cos\varphi_1 & s_2 \sin\varphi_1 & -c_2 \sin\varphi_1 \\ -s_1 \sin\varphi_1 & c_1 c_2 e^{i\varphi_2} + s_1 s_2 \cos\varphi_1 & c_1 s_2 e^{i\varphi_2} - c_2 s_1 \cos\varphi_1 \\ c_1 \sin\varphi_1 & c_2 s_1 e^{i\varphi_2} - c_1 s_2 \cos\varphi_1 & s_1 s_2 e^{i\varphi_2} + c_1 c_2 \cos\varphi_1 \end{pmatrix} Q_\nu$$

with

$$\varphi_1 = \frac{x-y}{n}\pi, \quad \varphi_2 = \frac{3(x-y+\gamma-\delta)}{n}\pi$$

depend on residual symmetry

$$c_1 \equiv \cos\theta_1,\ c_2 \equiv \cos\theta_2,\ s_1 \equiv \sin\theta_1,\ s_2 \equiv \sin\theta_2,\ 0 \le \theta_{1,2} \le \pi$$

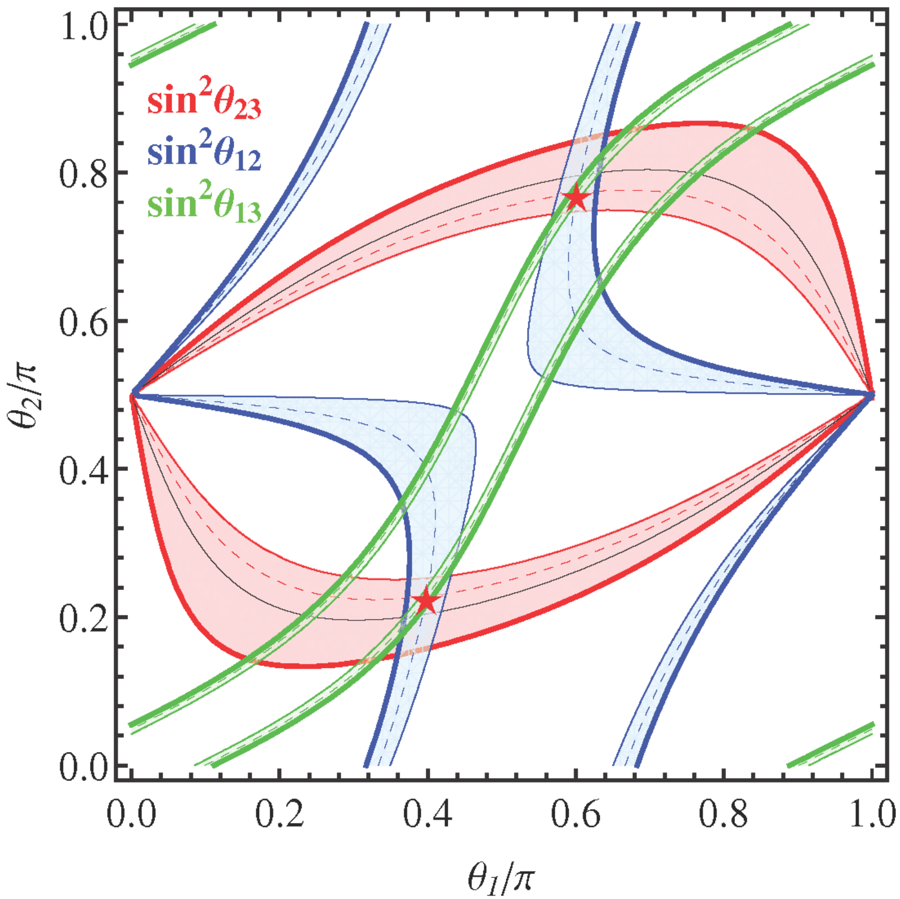
- Nine independent permutations

$$U_{I,1}=U_I,\qquad U_{I,2}=U_IP_{12},\qquad U_{I,3}=U_IP_{13},$$
$$U_{I,4}=P_{12}U_I,\quad U_{I,5}=P_{12}U_IP_{12},\quad U_{I,6}=P_{12}U_IP_{13},$$
$$U_{I,7}=P_{13}U_I,\quad U_{I,8}=P_{13}U_IP_{12},\quad U_{I,9}=P_{13}U_IP_{13},$$

- Sum rules between Dirac CP phase and mixing angles

$$U_{I,1}:\cos^2\theta_{12}\cos^2\theta_{13}=\cos^2\varphi_1,\qquad U_{I,2}:\sin^2\theta_{12}\cos^2\theta_{13}=\cos^2\varphi_1,$$
$$U_{I,6}:\sin^2\theta_{23}\cos^2\theta_{13}=\cos^2\varphi_1,\qquad U_{I,9}:\cos^2\theta_{23}\cos^2\theta_{13}=\cos^2\varphi_1,$$
$$U_{I,4}:\cos\delta_{CP}=\frac{2(\cos^2\varphi_1-\sin^2\theta_{12}\cos^2\theta_{23}-\sin^2\theta_{13}\cos^2\theta_{12}\sin^2\theta_{23})}{\sin 2\theta_{12}\sin\theta_{13}\sin 2\theta_{23}},$$
$$U_{I,5}:\cos\delta_{CP}=-\frac{2(\cos^2\varphi_1-\cos^2\theta_{12}\cos^2\theta_{23}-\sin^2\theta_{13}\sin^2\theta_{12}\sin^2\theta_{23})}{\sin 2\theta_{12}\sin\theta_{13}\sin 2\theta_{23}},$$
$$U_{I,7}:\cos\delta_{CP}=-\frac{2(\cos^2\varphi_1-\sin^2\theta_{12}\sin^2\theta_{23}-\sin^2\theta_{13}\cos^2\theta_{12}\cos^2\theta_{23})}{\sin 2\theta_{12}\sin\theta_{13}\sin 2\theta_{23}},$$
$$U_{I,8}:\cos\delta_{CP}=\frac{2(\cos^2\varphi_1-\cos^2\theta_{12}\sin^2\theta_{23}-\sin^2\theta_{13}\sin^2\theta_{12}\cos^2\theta_{23})}{\sin 2\theta_{12}\sin\theta_{13}\sin 2\theta_{23}},$$

Simple example: $U_{l,4}$ with $\varphi_1 = \frac{\pi}{3}, \varphi_2 = 0$ in n=3



θ_l^{bf}/π	θ_ν^{bf}/π	χ^2_{min}	$\sin^2\theta_{13}$	$\sin^2\theta_{12}$	$\sin^2\theta_{23}$	$\sin\delta$	$\sin\alpha_{21}$	$\sin\alpha_{31}$
0.398	0.228	0.00143	0.0234	0.308	0.438	0	0	0

Nontrivial CP phases can be achieved from groups with n≥4

Case (II): $Z_2^{g_l} = Z_2^{bc^x d^x},\ X_l = \left\{c^{\gamma} d^{-2x-\gamma}, bc^{x+\gamma} d^{-x-\gamma}\right\},$

$$Z_2^{g_\nu} = Z_2^{abc^y}, X_\nu = \left\{c^{\delta} d^{2y+2\delta}, abc^{y+\delta} d^{2y+2\delta}\right\}$$

- The PMNS mixing matrix is

$$U_{II} = \frac{1}{2}\begin{pmatrix} 1 & c_2 + \sqrt{2}e^{i\varphi_4}s_2 & s_2 - \sqrt{2}e^{i\varphi_4}c_2 \\ s_1 + \sqrt{2}e^{i\varphi_3}c_1 & s_1c_2 - \sqrt{2}(e^{i\varphi_3}c_1c_2 + e^{i\varphi_4}s_1s_2) & s_1s_2 - \sqrt{2}(e^{i\varphi_3}c_1s_2 - e^{i\varphi_4}c_2s_1) \\ c_1 - \sqrt{2}e^{i\varphi_3}s_1 & c_1c_2 + \sqrt{2}(e^{i\varphi_3}c_2s_1 - e^{i\varphi_4}c_1s_2) & c_1s_2 + \sqrt{2}(e^{i\varphi_3}s_1s_2 + e^{i\varphi_4}c_1c_2) \end{pmatrix} Q_\nu$$

with $$\varphi_3 = \frac{3\gamma + 2(x+y)}{n}\pi,\ \varphi_4 = -\frac{3\delta + 2(x+y)}{n}\pi$$

- Four viable row and column permutations

$$U_{II,1} = P_{12}U_{II}, \quad U_{II,2} = P_{12}U_{II}P_{12},$$
$$U_{II,3} = P_{13}U_{II}, \quad U_{II,4} = P_{13}U_{II}P_{12}$$

- Sum rules

$$U_{II,1}:\ \cos\delta_{CP}=\frac{1-4\sin^2\theta_{12}\cos^2\theta_{23}-4\sin^2\theta_{13}\cos^2\theta_{12}\sin^2\theta_{23}}{2\sin 2\theta_{12}\sin\theta_{13}\sin 2\theta_{23}},$$

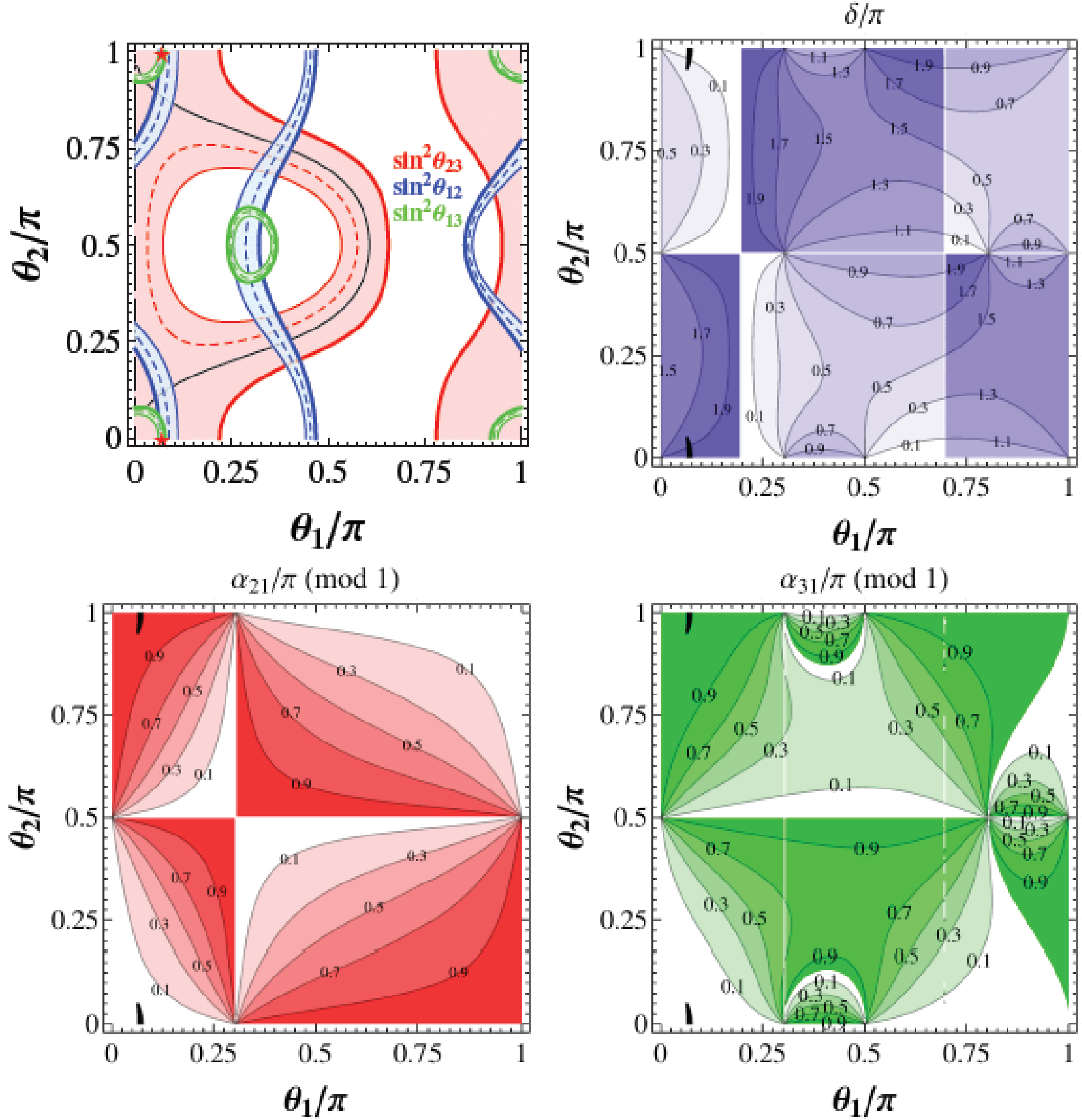
$$U_{II,2}:\ \cos\delta_{CP}=-\frac{1-4\cos^2\theta_{12}\cos^2\theta_{23}-4\sin^2\theta_{13}\sin^2\theta_{12}\sin^2\theta_{23}}{2\sin 2\theta_{12}\sin\theta_{13}\sin 2\theta_{23}},$$

$$U_{II,3}:\ \cos\delta_{CP}=-\frac{1-4\sin^2\theta_{12}\sin^2\theta_{23}-4\sin^2\theta_{13}\cos^2\theta_{12}\cos^2\theta_{23}}{2\sin 2\theta_{12}\sin\theta_{13}\sin 2\theta_{23}},$$

$$U_{II,4}:\ \cos\delta_{CP}=\frac{1-4\cos^2\theta_{12}\sin^2\theta_{23}-4\sin^2\theta_{13}\sin^2\theta_{12}\cos^2\theta_{23}}{2\sin 2\theta_{12}\sin\theta_{13}\sin 2\theta_{23}}$$

test the model at future facilities

Simple example: $U_{II,1}$ with $\varphi_3 = 0, \varphi_4 = \frac{\pi}{2}$ in n=2



$\theta_1^{\rm bf}/\pi$	$\theta_2^{\rm bf}/\pi$	$\chi^2_{\rm min}$	$\sin^2\theta_{13}$	$\sin^2\theta_{12}$	$\sin^2\theta_{23}$	$\sin\delta$	$\sin\alpha_{21}$	$\sin\alpha_{31}$
0.224	0	9.890	0.0248	0.343	0.513	0	0	0

Case (III): $Z_2^{g_l}=Z_2^{bc^x d^x}, X_l=\left\{c^\gamma d^{-2x-\gamma}, bc^{x+\gamma}d^{-x-\gamma}\right\}, Z_2^{g_\nu}=Z_2^{c^{n/2}}, X_\nu=c^\alpha d^\delta$

- The PMNS mixing matrix is

$$U_{III}=\frac{1}{\sqrt{2}}\begin{pmatrix} c_2 & s_2 & -e^{i\varphi_6} \\ c_2 s_1+\sqrt{2}e^{i\varphi_5}c_1 s_2 & s_1 s_2-\sqrt{2}e^{i\varphi_5}c_1 c_2 & e^{i\varphi_6}s_1 \\ c_1 c_2-\sqrt{2}e^{i\varphi_5}s_1 s_2 & \sqrt{2}e^{i\varphi_5}c_2 s_1+c_1 s_2 & e^{i\varphi_6}c_1 \end{pmatrix}Q_\nu$$

with

$$\varphi_5=\frac{2x-2\alpha+3\gamma+\delta}{n}\pi, \varphi_6=-\frac{2x+\alpha+\delta}{n}\pi$$

- Four independent viable permutations

$$U_{III,1} = P_{12} U_{III} P_{23}, \qquad U_{III,2} = P_{12} U_{III},$$
$$U_{III,3} = P_{12} P_{13} U_{III} P_{23}, \quad U_{III,4} = P_{12} P_{13} U_{III}$$

- Sum rules

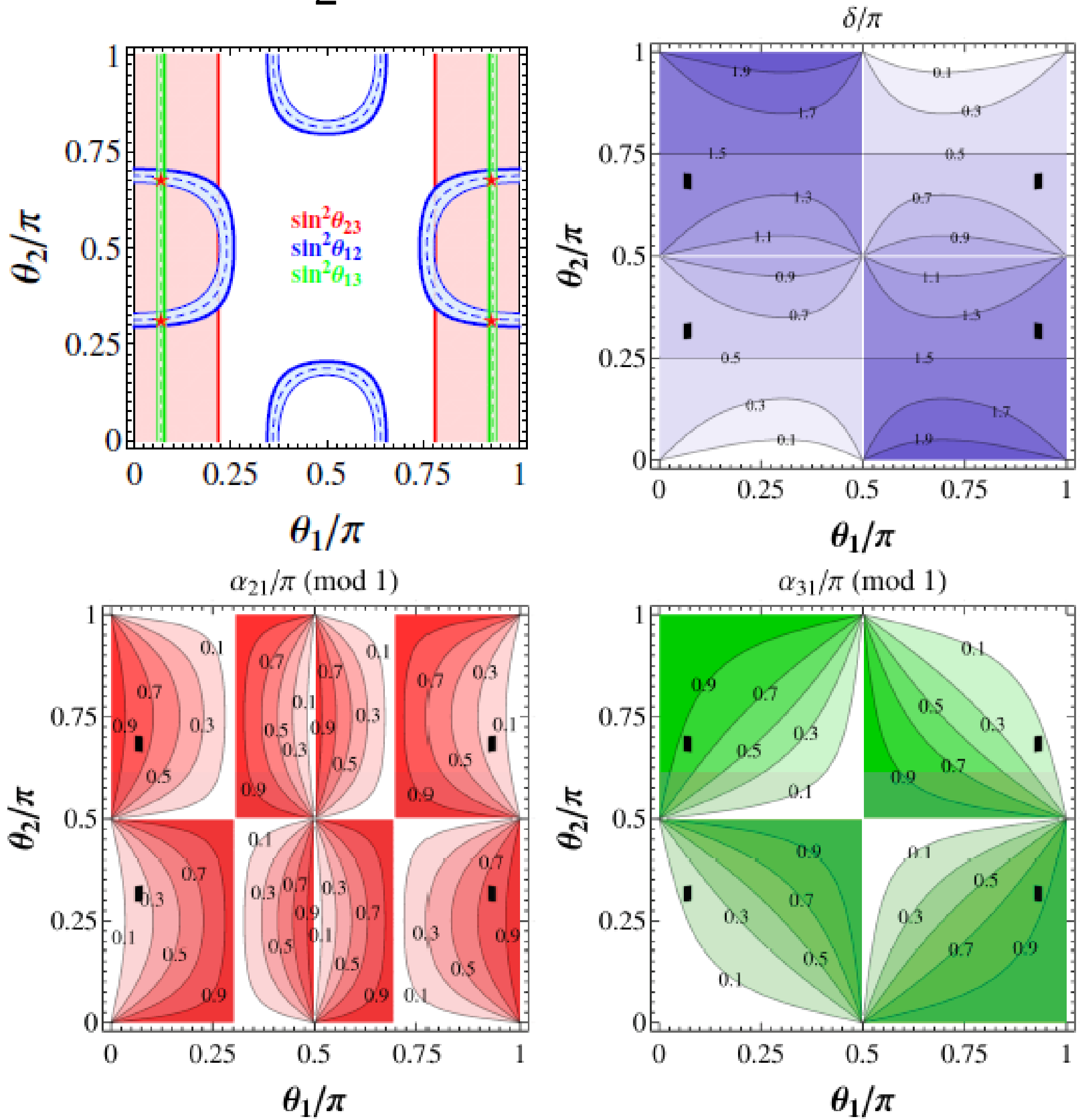
$$U_{III,2}: \ \cos^2\theta_{13}\sin^2\theta_{23} = \frac{1}{2}, \qquad U_{III,4}: \ \cos^2\theta_{13}\cos^2\theta_{23} = \frac{1}{2},$$

$$U_{III,1}: \ \cos\delta_{CP} = -\frac{1-2\cos^2\theta_{12}\cos^2\theta_{23} - 2\sin^2\theta_{13}\sin^2\theta_{12}\sin^2\theta_{23}}{\sin 2\theta_{12}\sin\theta_{13}\sin 2\theta_{23}},$$

$$U_{III,3}: \ \cos\delta_{CP} = \frac{1-2\cos^2\theta_{12}\sin^2\theta_{23} - 2\sin^2\theta_{13}\sin^2\theta_{12}\cos^2\theta_{23}}{\sin 2\theta_{12}\sin\theta_{13}\sin 2\theta_{23}}$$

testable in future experiments

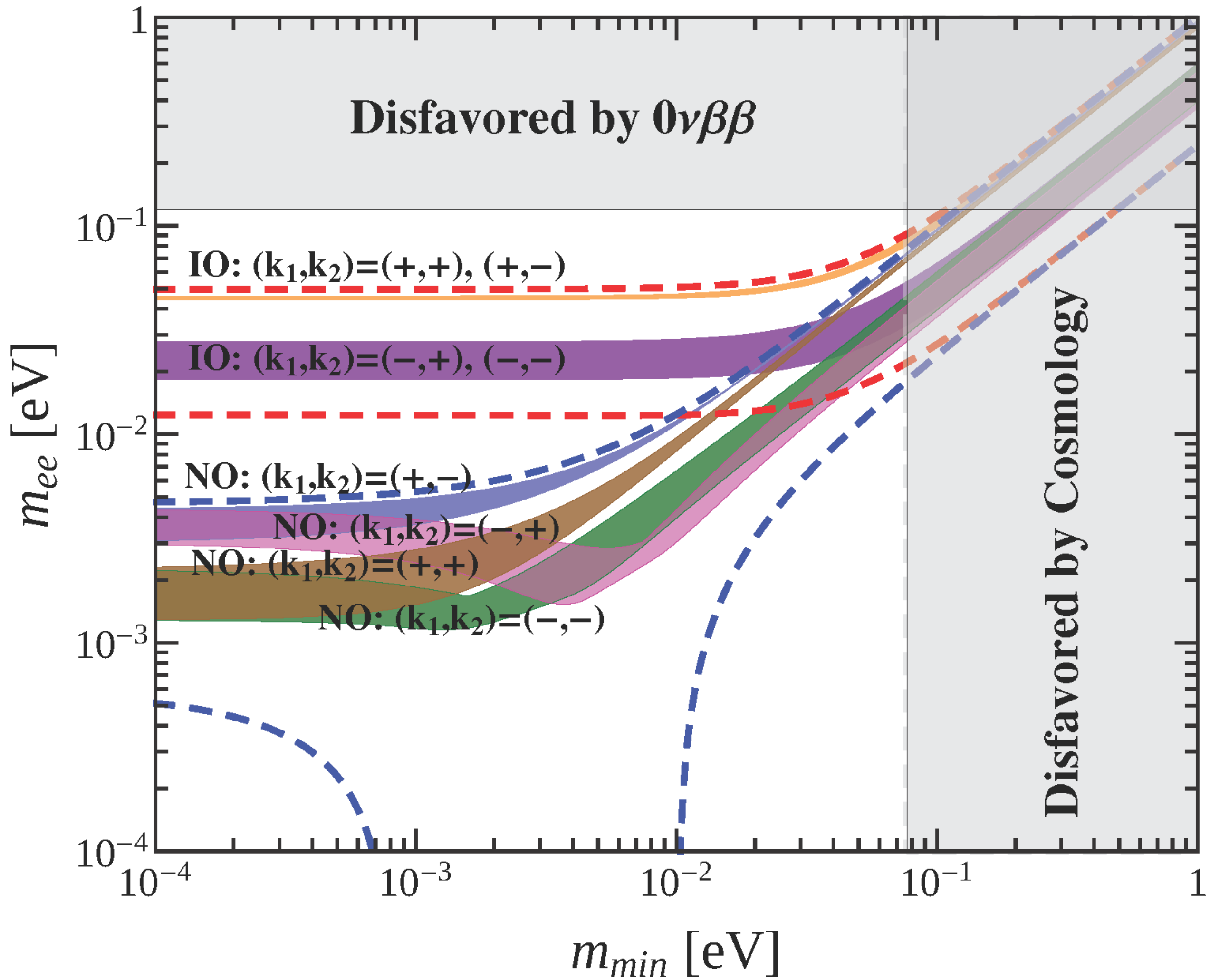
Example: $U_{III,2}$ with $\varphi_5 = \frac{\pi}{2}, \varphi_6 = 0$ in n=2



$\theta_1^{\rm bf}/\pi$	$\theta_2^{\rm bf}/\pi$	$\chi^2_{\rm min}$	$\sin^2\theta_{13}$	$\sin^2\theta_{12}$	$\sin^2\theta_{23}$	$\sin\delta$	$\sin\alpha_{21}$	$\sin\alpha_{31}$
0.0692	0.684	5.158	0.0233	0.308	0.512	−0.991	−0.624	−0.453

Predictions for neutrinoless double decay

$$m_{ee} \equiv \left| m_1 \cos^2\theta_{12}\cos^2\theta_{13} + m_2 \sin^2\theta_{12}\cos^2\theta_{13} e^{i\alpha_{21}} + m_3 \sin^2\theta_{13} e^{i(\alpha_{31}-2\delta_{CP})} \right|$$



The effective mass $m_{ee} \geq 1.3\text{x}10^{-3}\text{eV}$ for IO mass spectrum

Case (IV): $Z_2^{g_l}=Z_2^{bc^x d^x}, X_l=\left\{c^\gamma d^{-2x-\gamma}, bc^{x+\gamma}d^{-x-\gamma}\right\},$

$$Z_2^{g_\nu}=Z_2^{c^{n/2}},\ X_\nu=abc^\delta d^{2\delta},$$

- The PMNS matrix is

$$U_{IV}=\frac{1}{2}\begin{pmatrix} 1 & 1 & -\sqrt{2}e^{i\theta_2} \\ s_1+\sqrt{2}e^{i(2\theta_2+\varphi_7)}c_1 & s_1-\sqrt{2}e^{i(2\theta_2+\varphi_7)}c_1 & \sqrt{2}e^{i\theta_2}s_1 \\ c_1-\sqrt{2}e^{i(2\theta_2+\varphi_7)}s_1 & c_1+\sqrt{2}e^{i(2\theta_2+\varphi_7)}s_1 & \sqrt{2}e^{i\theta_2}c_1 \end{pmatrix}Q_\nu$$

with $\varphi_7=\dfrac{6x+3(\gamma+2\delta)}{n}\pi$

- Two independent permutations

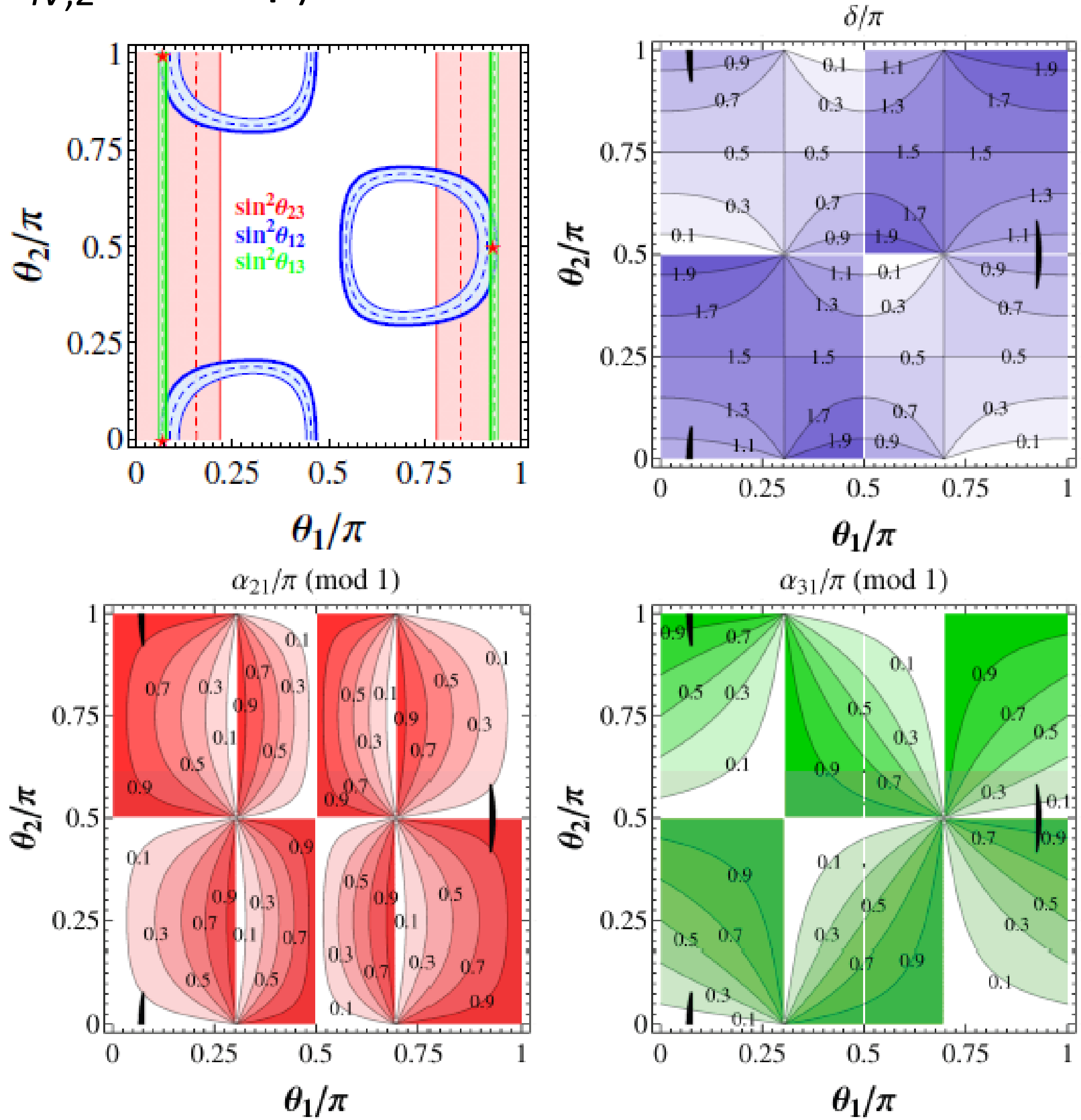
$$U_{IV,1}=P_{12}U_{IV},\quad U_{IV,2}=P_{12}P_{13}U_{IV}$$

- Sum rules

$$U_{IV,1}:\ \cos^2\theta_{13}\sin^2\theta_{23}=\frac{1}{2},\quad \cos\delta_{CP}=\frac{1-4\sin^2\theta_{12}\cos^2\theta_{23}-4\sin^2\theta_{13}\cos^2\theta_{12}\sin^2\theta_{23}}{2\sin2\theta_{12}\sin\theta_{13}\sin2\theta_{23}},$$

$$U_{IV,2}:\ \cos^2\theta_{13}\cos^2\theta_{23}=\frac{1}{2},\quad \cos\delta_{CP}=-\frac{1-4\sin^2\theta_{12}\sin^2\theta_{23}-4\sin^2\theta_{13}\cos^2\theta_{12}\cos^2\theta_{23}}{2\sin2\theta_{12}\sin\theta_{13}\sin2\theta_{23}}.$$

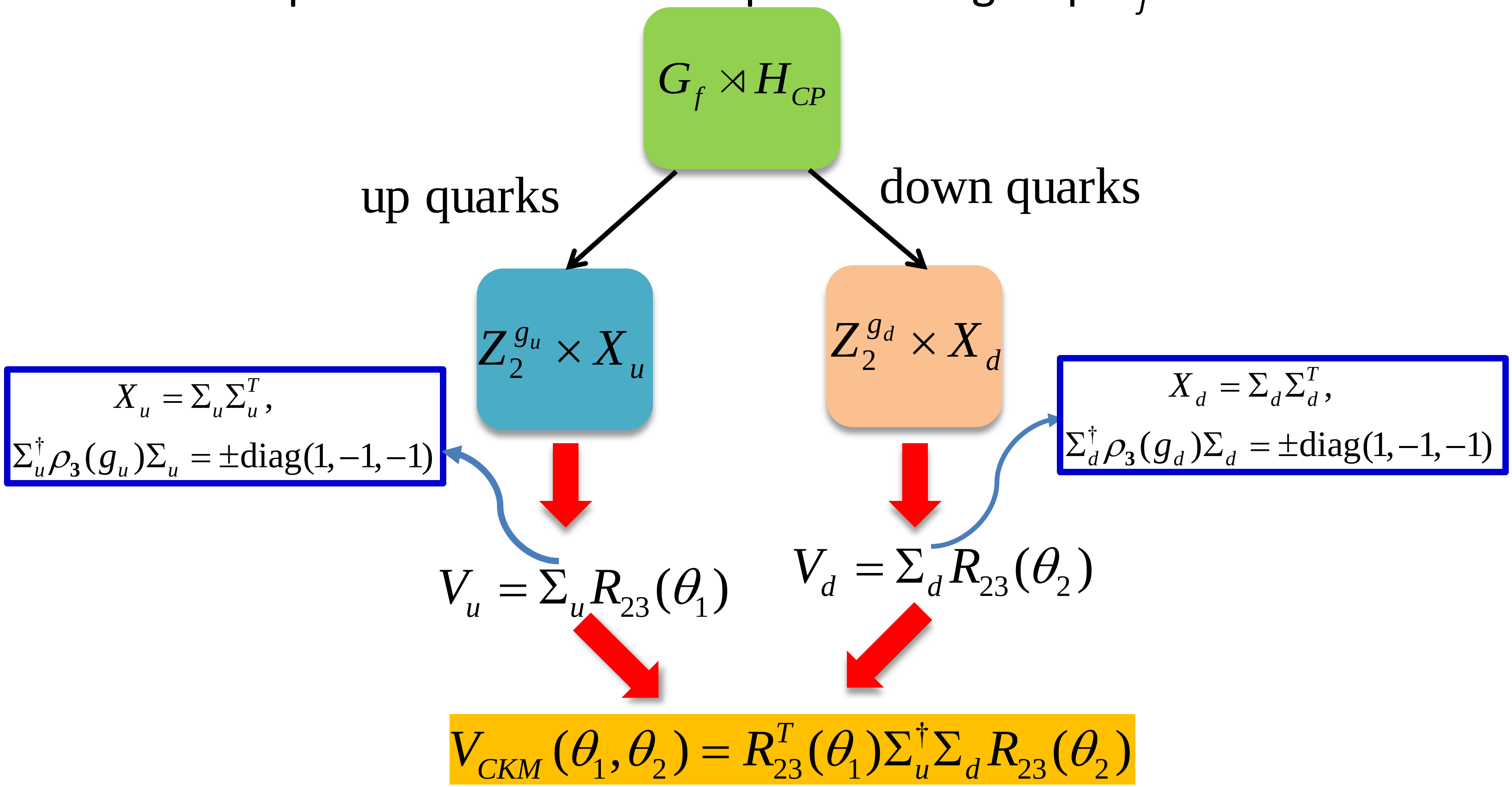
Example of $U_{IV,2}$ with φ_7=0 in n=2



$\theta_1^{\rm bf}/\pi$	$\theta_2^{\rm bf}/\pi$	$\chi^2_{\rm min}$	$\sin^2\theta_{13}$	$\sin^2\theta_{12}$	$\sin^2\theta_{23}$	$\sin\delta$	$\sin\alpha_{21}$	$\sin\alpha_{31}$
0.0718	0	6.938	0.0250	0.342	0.487	0	0	0

Extending to the quark sector

Left-handed quarks transform as triplet of the group G_f



- Four observables: three quark mixing angles+one CP phase are predicted in terms of two parameters $\theta_{1,2}$

Predictions for quark mixing from $\Delta(6n^2)$ and CP

One viable quark mixing pattern is found up to row and column permutations

Residual symmetry:

$$Z_2^{g_u} = Z_2^{bc^x d^x},\ X_u = \left\{c^\gamma d^{-2x-\gamma}, bc^{x+\gamma} d^{-x-\gamma}\right\},$$

$$Z_2^{g_d} = Z_2^{bc^y d^y}, X_d = \left\{c^\delta d^{-2y-\delta}, bc^{y+\delta} d^{-y-\delta}\right\}$$

- The CKM matrix is determined to be

$$V_{CKM} = \begin{pmatrix} s_2 \sin\varphi_1 & \cos\varphi_1 & -c_2 \sin\varphi_1 \\ c_1 c_2 e^{i\varphi_2} + s_1 s_2 \cos\varphi_1 & -s_1 \sin\varphi_1 & c_1 s_2 e^{i\varphi_2} - c_2 s_1 \cos\varphi_1 \\ c_2 s_1 e^{i\varphi_2} - c_1 s_2 \cos\varphi_1 & c_1 \sin\varphi_1 & s_1 s_2 e^{i\varphi_2} + c_1 c_2 \cos\varphi_1 \end{pmatrix}$$

with

$$\varphi_1 = \frac{x-y}{n}\pi, \quad \varphi_2 = \frac{3(x-y+\gamma-\delta)}{n}\pi$$

← fixed by residual symmetry

- Expressions of mixing parameters

$$\sin^2\theta_{13} = \sin^2\varphi_1\cos^2\theta_2, \quad \sin^2\theta_{12} = \frac{\cos^2\varphi_1}{1-\sin^2\varphi_1\cos^2\theta_2},$$

$$\sin^2\theta_{23} = \frac{2\cos^2\theta_1\sin^2\theta_2 + 2\sin^2\theta_1\cos^2\theta_2\cos^2\varphi_1 - \cos\varphi_1\cos\varphi_2\sin 2\theta_1\sin 2\theta_2}{2-2\cos^2\theta_2\sin^2\varphi_1},$$

$$J_{CP} = \frac{1}{8}\sin\varphi_1\sin 2\varphi_1\sin\varphi_2\sin 2\theta_1\sin 2\theta_2$$

- Two correlations

$$\cos^2\theta_{13}\sin^2\theta_{12} = \cos^2\varphi_1,$$

$$J_{CP} \approx \pm\frac{1}{2}\sin\varphi_2\sin 2\varphi_1\sin\theta_{13}\sin\theta_{23}.$$

Experimental data can be accommodated for certain choices of residual symmetry parameters $\varphi_{1,2}$

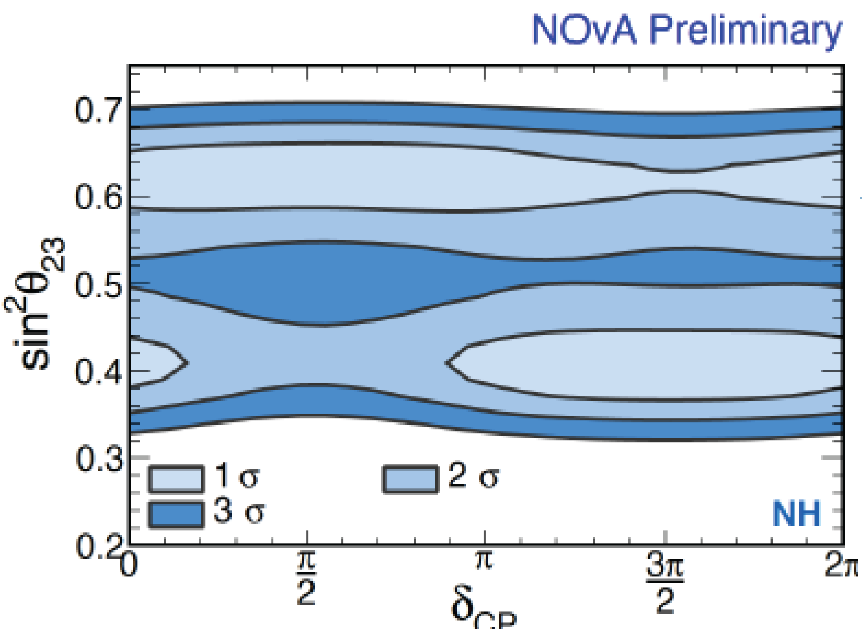
Some viable choices of parameters in n=7

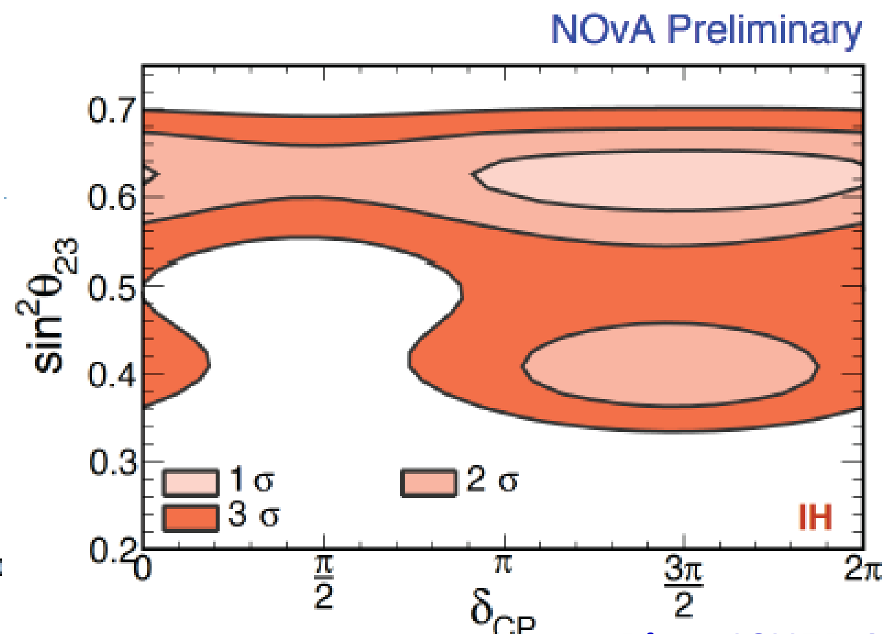
- Quark sector : $\varphi_1 = \varphi_2 = 3\pi/7$

	$\theta_1^{\rm bf}/\pi$	$\theta_2^{\rm bf}/\pi$	$\sin\theta_{12}$	$\sin\theta_{23}$	$\sin\theta_{13}$	J_{CP}
Our	0.4867	0.4988	0.22252	0.04158	0.00357	3.14615×10^{-5}
Data	—	—	0.22523 ± 0.00065	0.0417 ± 0.00057	0.00360 ± 0.00012	$(3.109 \pm 0.086) \times 10^{-5}$

- Lepton sector : $\varphi_3 = 2\pi/7$, $\varphi_4 = \pi/7$ for $U_{II,4}$

	$\theta_1^{\rm bf}/\pi$	$\theta_2^{\rm bf}/\pi$	$\chi^2_{\rm min}$	$\sin^2\theta_{13}$	$\sin^2\theta_{12}$	$\sin^2\theta_{23}$	$\sin\delta$	$\sin\alpha_{21}$	$\sin\alpha_{31}$
Our	0.381	0.956	3.006	0.0238	0.325	0.404	-0.992	0.784	0.995
Data	—	—	—	$0.0176 \to 0.0295$	$0.259 \to 0.359$	$0.374 \to 0.626$	$-1 \to 1$	$-1 \to 1$	$-1 \to 1$





Bian, ICHEP 16

Summary

- Flavor and CP symmetries broken to Z2xCP in both neutrino and charged lepton sectors can predict mixing angles and CP phases in terms of two parameters.

- The quark mixing angles and CP violation phase can also be accommodated in this approach.

- A unified description of quark and lepton mixing can be achieved, $\Delta(6\cdot 7^2) = \Delta(294)$ is a good candidate for flavor symmetry.

- It is interesting to consider model construction and strong CP problem.

Thank you for your attention!

Backup

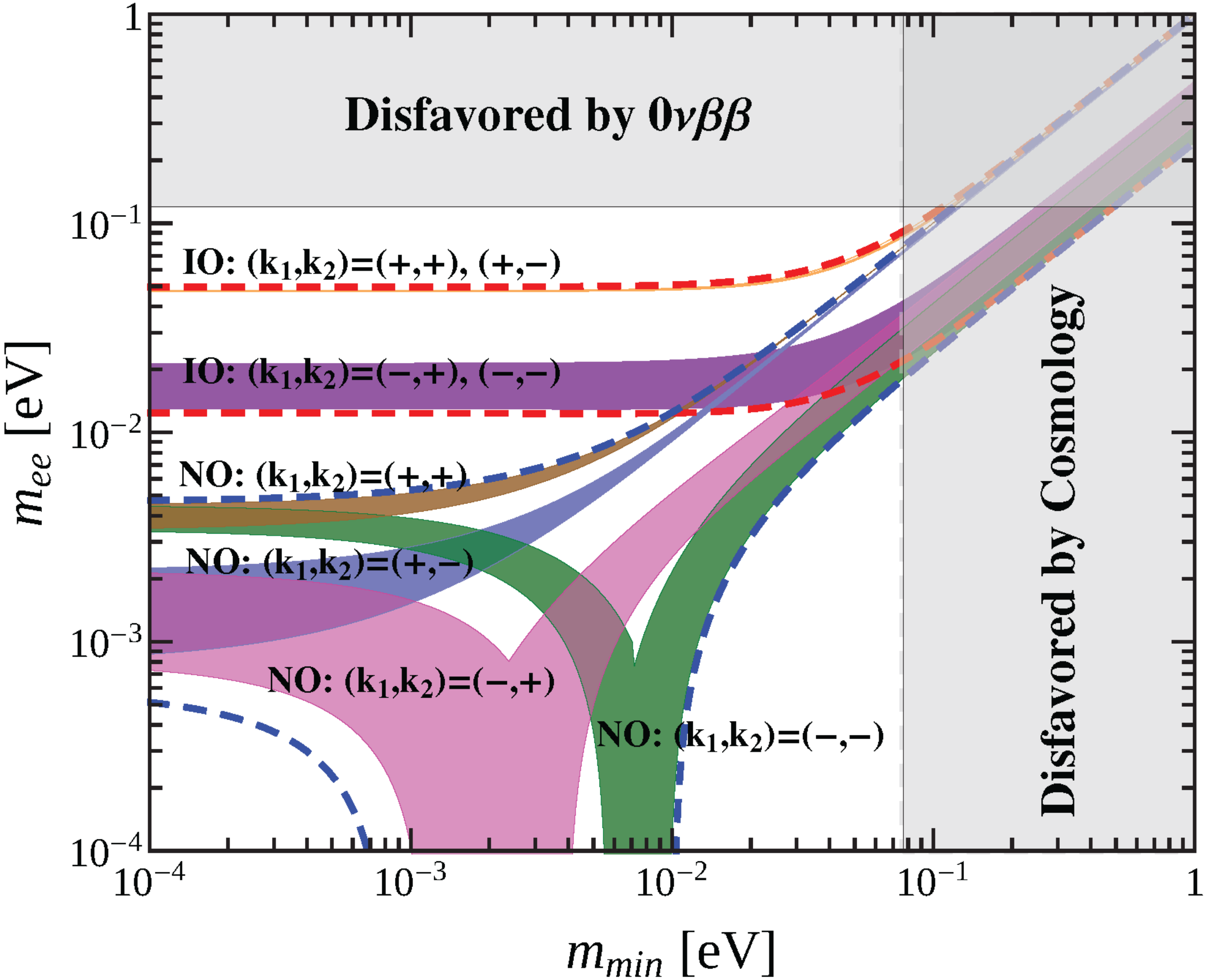
- Convention for rotation matrices

$$R_{12}(\theta) = \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad R_{13}(\theta) = \begin{pmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{pmatrix}$$

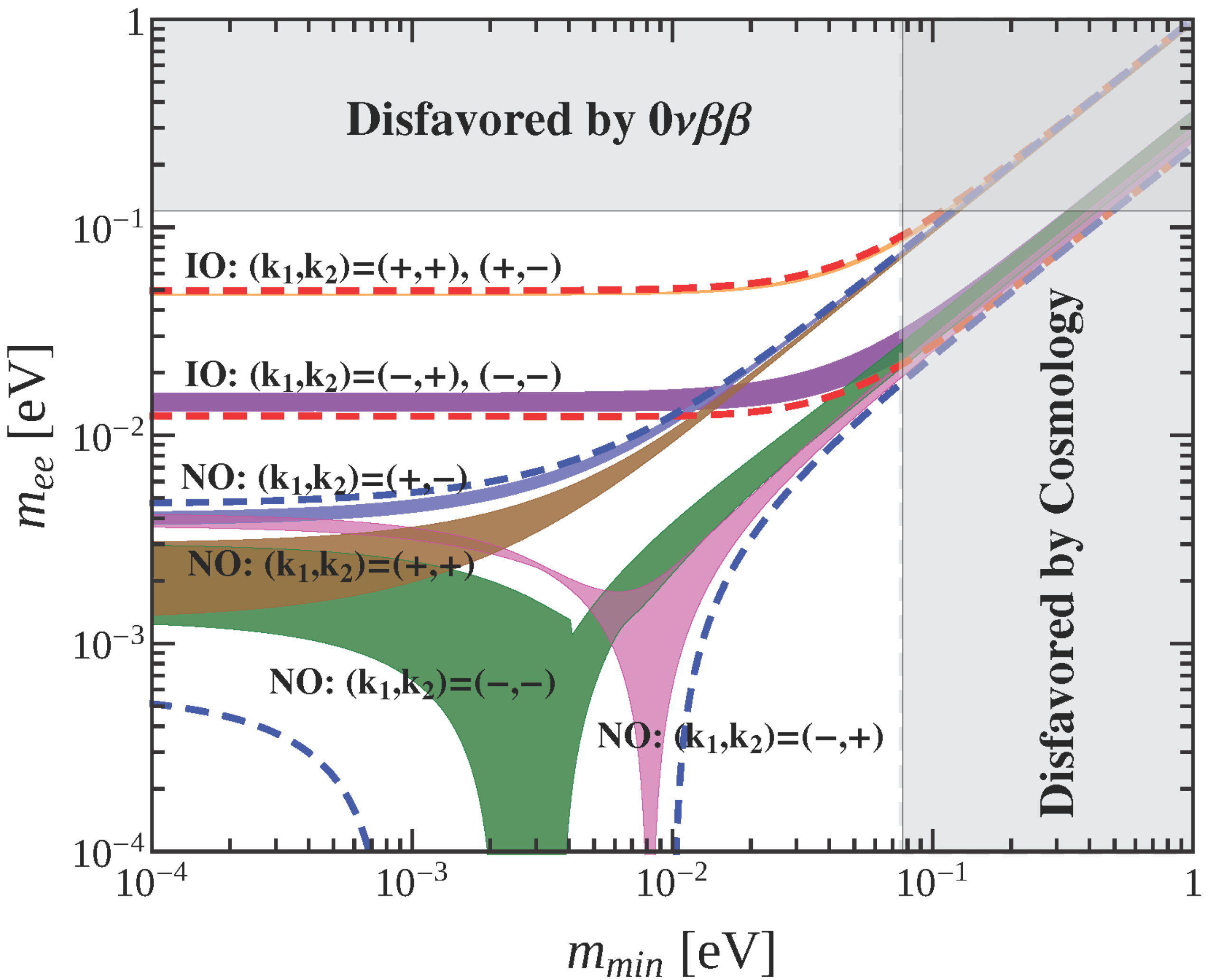
$$R_{23}(\theta) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{pmatrix}$$

predictions for neutrinoless double decay

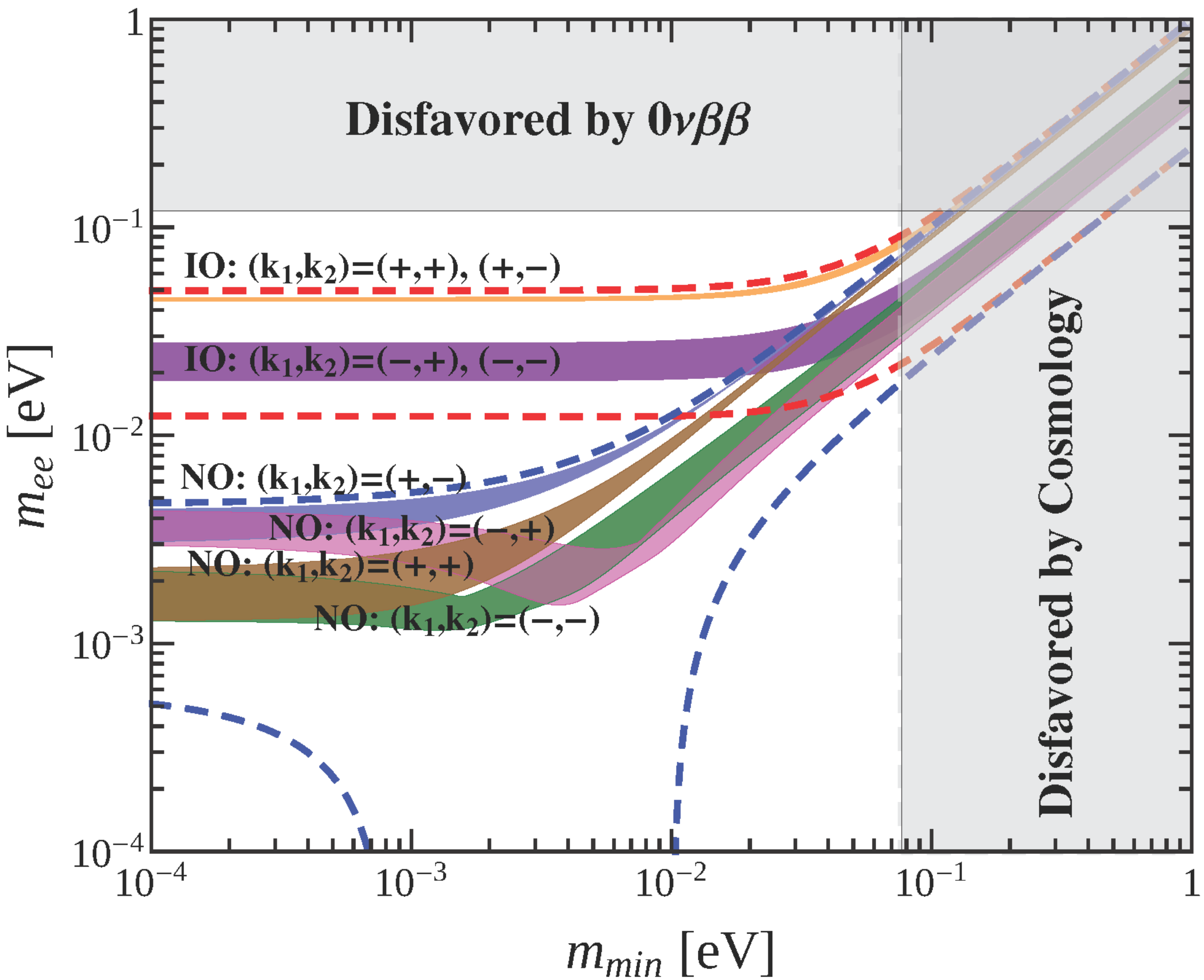
- 1^{st} example: $U_{l,4}$ with $\varphi_1 = \frac{\pi}{3}, \varphi_2 = 0$ in n=3



- 2^{nd} example: $U_{II,1}$ with $\varphi_3 = 0, \varphi_4 = \frac{\pi}{2}$ in n=2



- 3rd example: : $U_{III,2}$ with $\varphi_5 = \frac{\pi}{2}, \varphi_6 = 0$ in n=2



The effective mass $m_{ee} \geq 1.3 \times 10^{-3}$eV for IO mass spectrum

● 4^{th} example of $U_{IV,2}$ with φ_7=0 in n=2