Neutrino Mass Sum Rules

Martin Spinrath (史馬丁)
National Center for Theoretical Sciences (NCTS), Hsinchu, Taiwan

Bethe Forum "Discrete Symmetries" - 06/04/2017

Based on collaborations with:
Julia Gehrlein, Alexander Merle, Jens P. Oppermann, Daniela Schäfer

Outline

- Introduction
- Neutrino Mass Sum Rules
- Summary and Conclusions

Outline

- **Introduction**
- Neutrino Mass Sum Rules
- Summary and Conclusions

Sum Rules in the Quark Sector

- The "GST" relation

$$\left| \theta_{12} - \frac{\theta_{13}}{\theta_{23}} \mathrm{e}^{\mathrm{i}\ \delta} \right| = \sqrt{\frac{m_d}{m_s}}$$

[not(!) Gatto, Sartori, Tonin '68; …]

- The phase sum rule

$$\alpha \approx \delta_{12}^d - \delta_{12}^u \approx 90^\circ$$

[Antusch, King, Malinsky, MS '09; Antusch, King, Luhn, MS '11], see also [Fritzsch and Xing; Masina and Savoy '06; Harrison, Dallison, Roythorne, Scott '09]

$\Downarrow$

$\boxed{\textbf{!! solution to strong CP problem [Antusch, Holthausen, Schmidt, MS '13] !!}}$

Sum Rules in the Lepton Sector

- Mixing sum rules, e.g.,

$$\sin^2\theta_{12} \approx \frac{1}{3} + \frac{2\sqrt{2}}{3}\sin\theta_{13}\cos\delta$$

[Antusch, Aranda, Ballett, Boudjemaa, de Medeiros-Varzielas, Ding, Gehrlein, Girardi, Hagedorn, He, Hochmuth, King, Luhn, Malinsky, Marzocca, Masina, Merle, Pascoli, Petcov, Rodejohann, Romanino, Schmidt, Stuart, Tanimoto, Titov, Valle, Volkas, ….?]

- Mass sum rules, e.g.,

$$m_1 \mathrm{e}^{-\mathrm{i}\,\phi_1} + m_2 \mathrm{e}^{-\mathrm{i}\,\phi_2} - m_3 = 0$$

[Antusch, Aranda, Ballett, Boudjemaa, de Medeiros-Varzielas, Ding, Gehrlein, Girardi, Hagedorn, He, Hochmuth, King, Luhn, Malinsky, Marzocca, Masina, Merle, Pascoli, Petcov, Rodejohann, Romanino, Schmidt, Stuart, Tanimoto, Titov, Valle, Volkas, ….?]

Outline

- Introduction
- **Neutrino Mass Sum Rules**
  - **General remarks**
  - Corrections
  - What happens in a real model?
- Summary and Conclusions

General remarks

- Not specific to a special family symmetry
- Not related to any residual symmetry [Gehrlein, MS to appear]
- Not specific to any mass mechanism
- Only in common: 3 complex neutrino masses depend on 2 complex parameters
  ➜ 2 Relations

One example

[Gehrlein, Oppermann, Schäfer, MS ’14]

- SU(5) x A_5 GUT Flavour Model
- Type I seesaw with

$$Y \sim \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \text{ and } M_{RR} \sim \begin{pmatrix} 2\sqrt{\frac{2}{3}}(v_2+v_3) & -\sqrt{3}v_2 & -\sqrt{3}v_2 \\ -\sqrt{3}v_2 & \sqrt{6}v_3 & -\sqrt{\frac{2}{3}}(v_2+v_3) \\ -\sqrt{3}v_2 & -\sqrt{\frac{2}{3}}(v_2+v_3) & \sqrt{6}v_3 \end{pmatrix}$$

- gives mass sum rule

$$\frac{\mathrm{e}^{\mathrm{i}\ \alpha_1}}{m_1} + \frac{\mathrm{e}^{\mathrm{i}\ \alpha_2}}{m_2} = \frac{1}{m_3}$$

The Parametrisation

[King, Stuart, Merle ’13; Gehrlein, Merle, MS ’15]

$$s \equiv c_1 \left(m_1 \mathrm{e}^{-\mathrm{i}\,\phi_1}\right)^d \mathrm{e}^{\mathrm{i}\,\Delta\chi_{13}} + c_2 \left(m_2 \mathrm{e}^{-\mathrm{i}\,\phi_2}\right)^d \mathrm{e}^{\mathrm{i}\,\Delta\chi_{23}} + m_3^d \stackrel{!}{=} 0$$

- c_1, c_2, d, $\Delta\chi_{13}$ and $\Delta\chi_{23}$ fixed by the model
- For instance, in the previous example,

$$c_1 = c_2 = 1,\ d = -1,\ \Delta\chi_{13} = \Delta\chi_{23} = \pi$$

$$\Rightarrow \frac{\mathrm{e}^{\mathrm{i}\ \alpha_1}}{m_1} + \frac{\mathrm{e}^{\mathrm{i}\ \alpha_2}}{m_2} = \frac{1}{m_3}$$

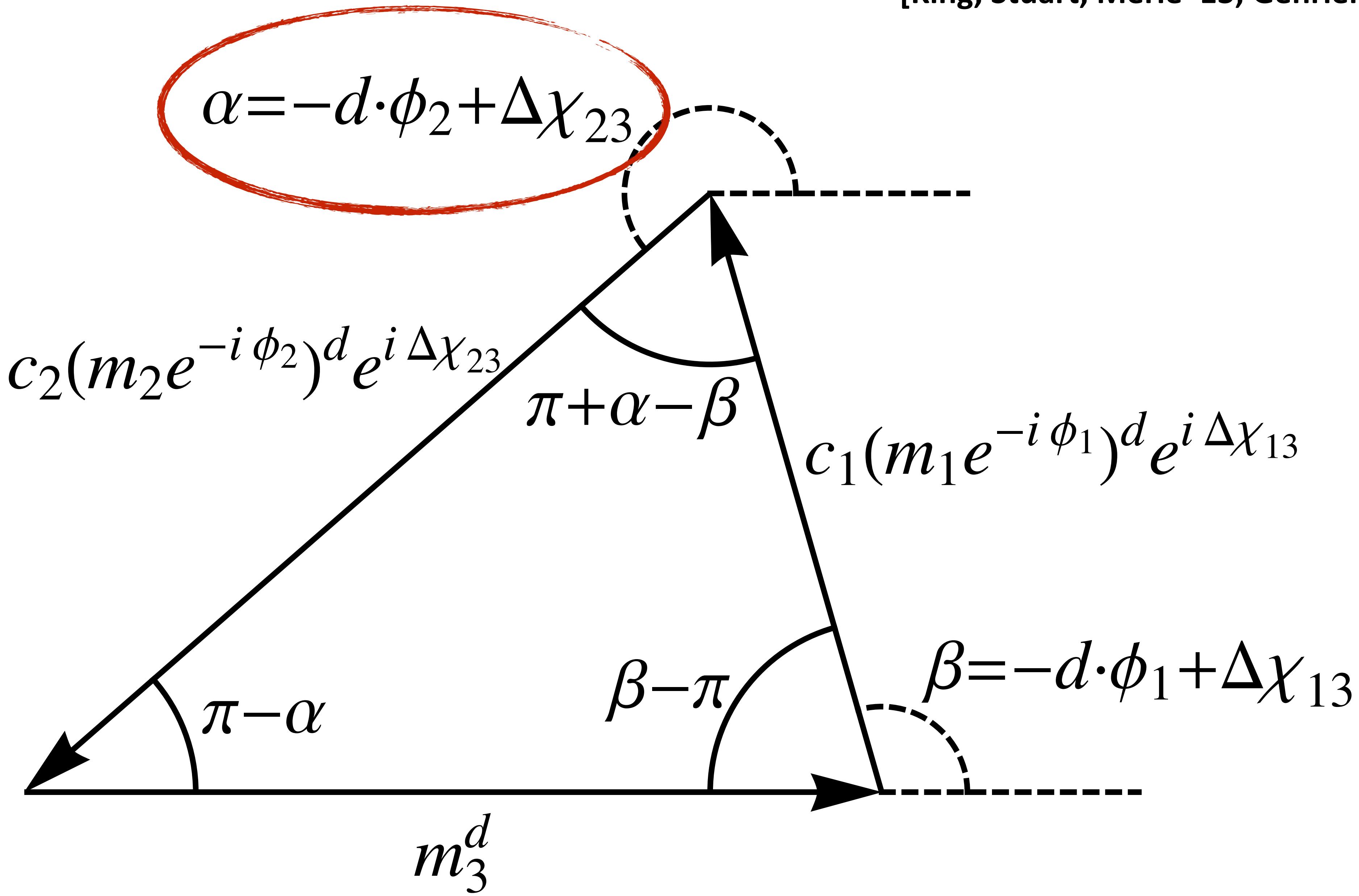
Known Rules

[King, Stuart, Merle ’13; Gehrlein, Merle, MS ’15]

Sum rule	c_1	c_2	d	$\Delta\chi_{13}$	$\Delta\chi_{23}$
1	1	1	1	π	π
2	1	2	1	π	π
3	1	2	1	π	0
4	1/2	1/2	1	π	π
5	$\frac{2}{\sqrt{3}+1}$	$\frac{\sqrt{3}-1}{\sqrt{3}+1}$	1	0	π
6	1	1	-1	π	π
7	1	2	-1	π	0
8	1	2	-1	0	π
9	1	2	-1	π	$\pi/2, 3\pi/2$
10	1	2	1/2	$\pi, 0, \pi/2$	$0, \pi, \pi/2$
11	1/3	1	1/2	π	0
12	1/2	1/2	$-1/2$	π	π

Geometrical Interpretation

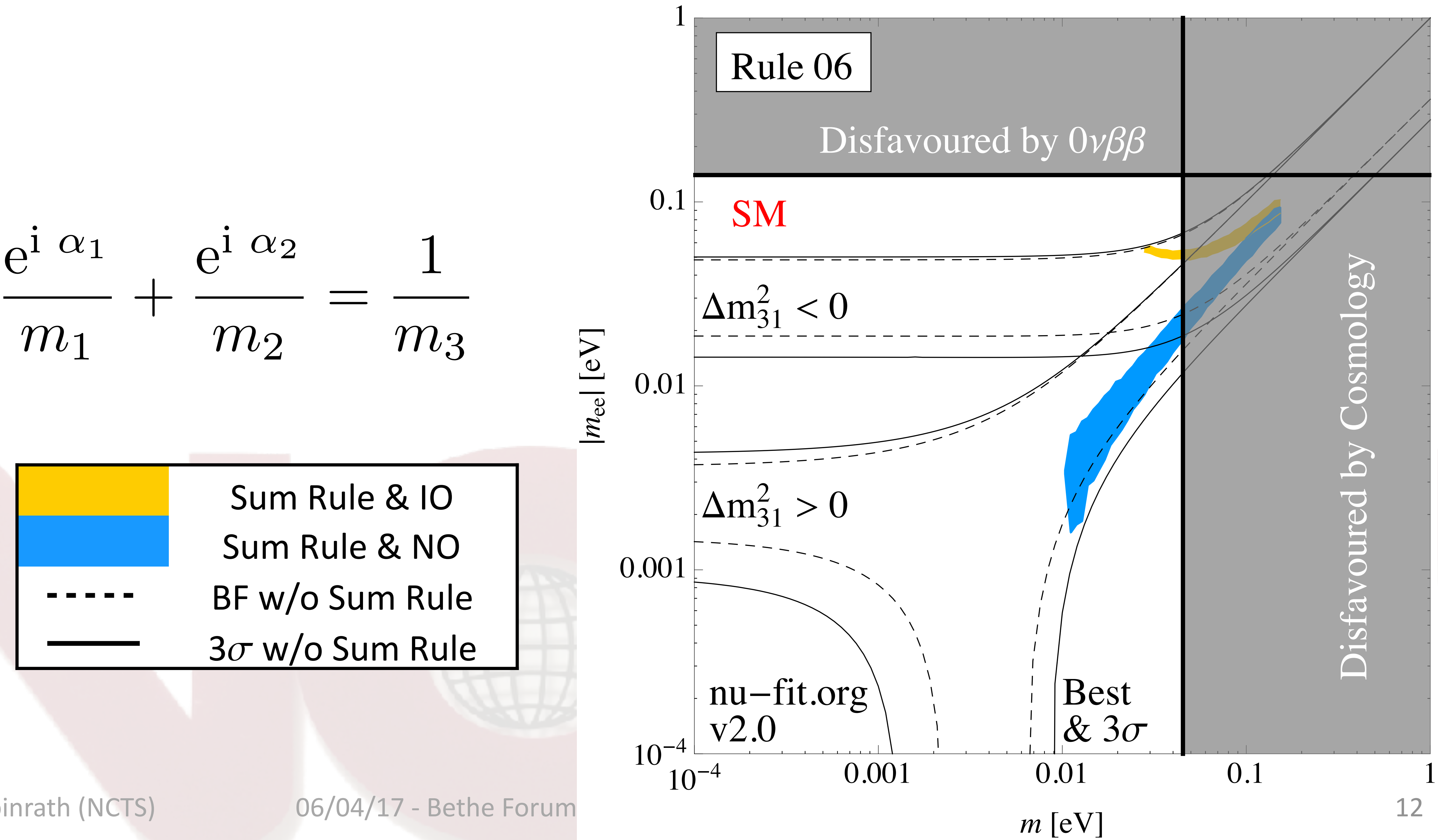
[King, Stuart, Merle '13; Gehrlein, Merle, MS '15]



Phenomenological Implications

[Gehrlein, Merle, MS '15]

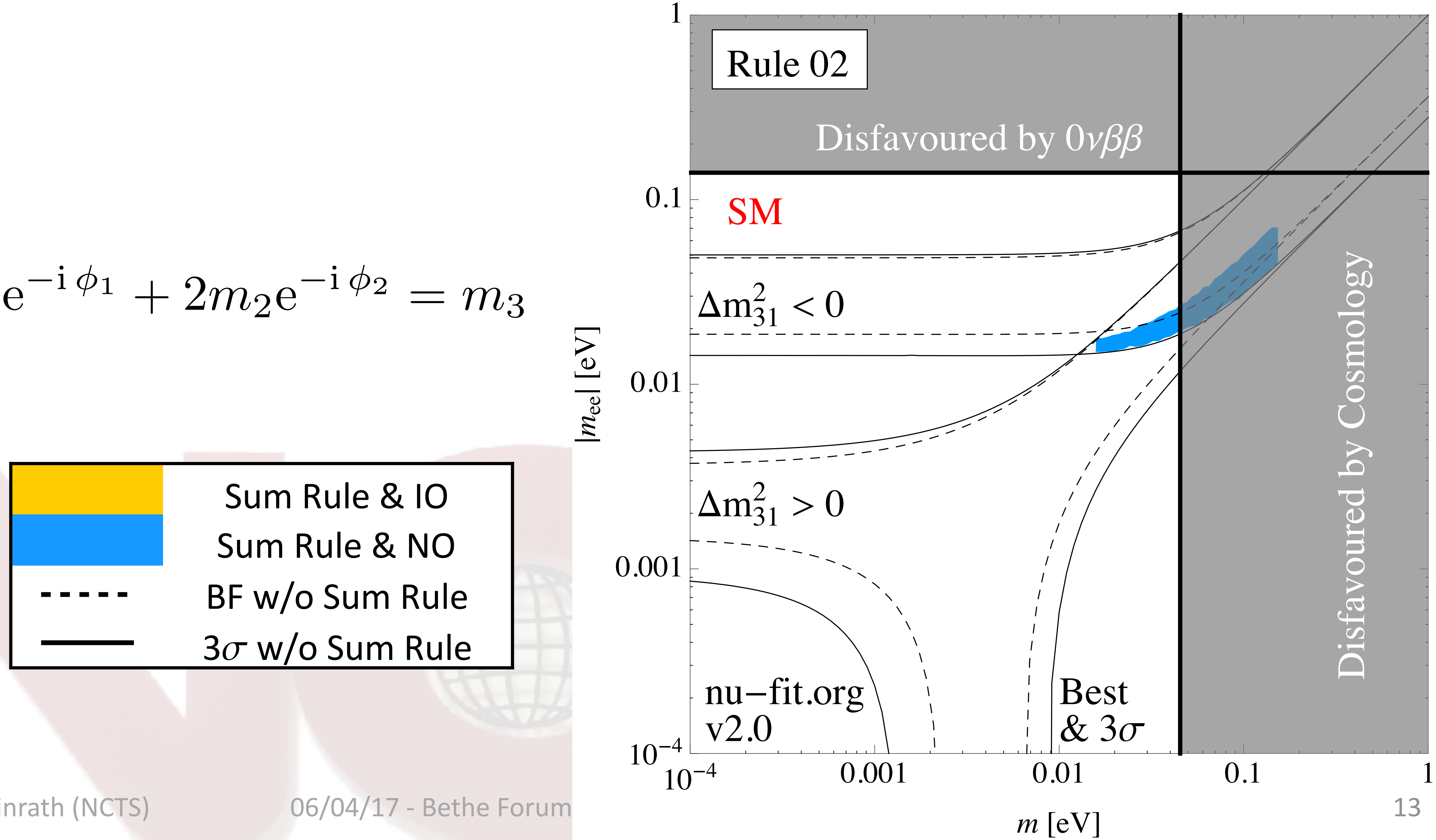
$$\frac{e^{\mathrm{i}\,\alpha_1}}{m_1} + \frac{e^{\mathrm{i}\,\alpha_2}}{m_2} = \frac{1}{m_3}$$



Phenomenological Implications

[Gehrlein, Merle, MS ’15]

$$m_1 \mathrm{e}^{-\mathrm{i}\,\phi_1} + 2 m_2 \mathrm{e}^{-\mathrm{i}\,\phi_2} = m_3$$



Outline

- Introduction
- **Neutrino Mass Sum Rules**
  - General remarks
  - **Corrections**
  - What happens in a real model?
- Summary and Conclusions

Corrections

- Generic: RGE corrections

- Specific: Higher-dimensional operators, charged lepton sector, corrections to vacuum alignment, …

Questions

- Can we reconstitute forbidden orderings?
- How big is the impact on neutrinoless double beta decay?

Forbidden Orderings

[Gehrlein, Merle, MS ’15]

- Sum Rule 2

$$m_1 \mathrm{e}^{-\mathrm{i}\,\phi_1} + 2 m_2 \mathrm{e}^{-\mathrm{i}\,\phi_2} - m_3 = 0$$

- From geometry and inverted ordering

$$\cos\alpha^{\text{tree}} = \frac{m_1^2 - 4m_2^2 - m_3^2}{4 m_2 m_3} < -\frac{1}{4}\left(3\frac{m_2^2}{m_3^2} + 1\right) < -1$$

- No inverted ordering on tree level

Forbidden Orderings

[Gehrlein, Merle, MS ’15]

- RGE corrections in MSSM

$$\delta(\cos\alpha)^{\mathrm{RGE}} \approx -\underbrace{\frac{C y_\tau^2}{192\pi^2}}_{>0}\underbrace{\frac{2.8 m_1^2 - 0.4 m_2^2 + 0.1 m_3^2}{m_2 m_3}}_{>0}\underbrace{\log\frac{M_S}{M_Z}}_{>0}$$

- Forbidden orderings possibly allowed only in very extreme regions of parameter space

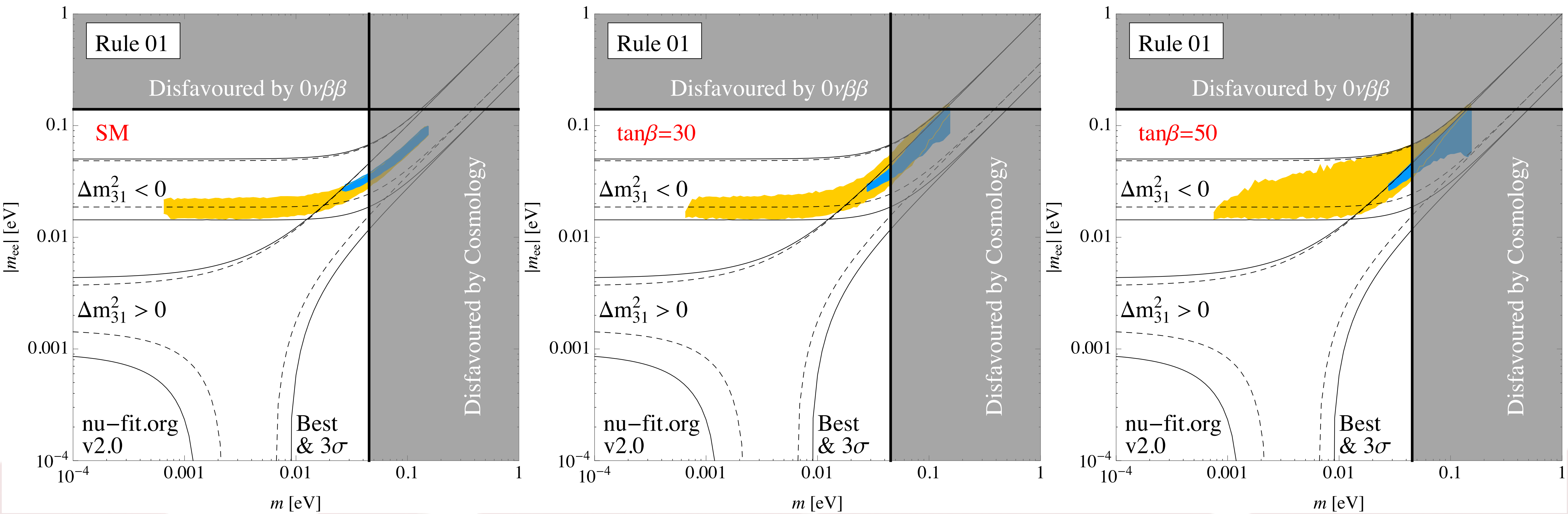
Numerical Procedure

[Gehrlein, Merle, MS ’15]

- Running with REAP [Antusch, Kersten, Lindner, Ratz, Schmidt ’15]
- Mixing parameters from nu-fit v2.0 [nu-fit.org]
- $M_S = 10^{13}$ GeV
- 3 Benchmarks (SM + 2 MSSM)

Numerical Results

[Gehrlein, Merle, MS ’15]



$$m_1 \mathrm{e}^{-\mathrm{i}\,\phi_1} + m_2 \mathrm{e}^{-\mathrm{i}\,\phi_2} = m_3$$

Sum Rule & IO

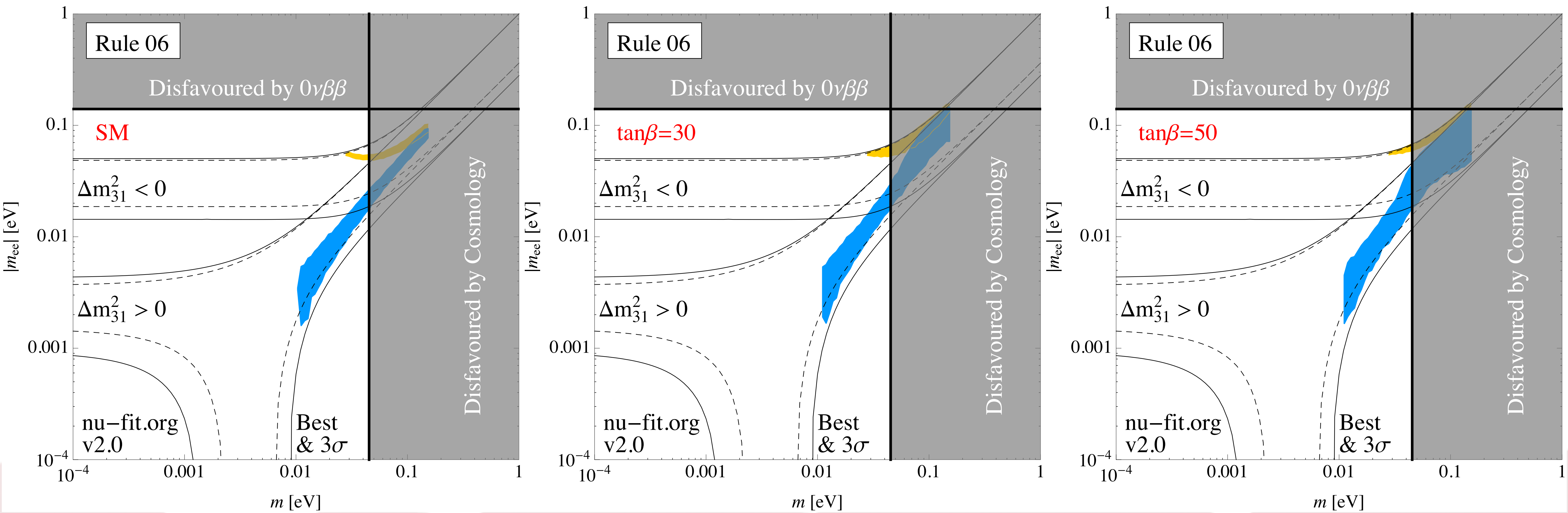
Sum Rule & NO

BF w/o Sum Rule

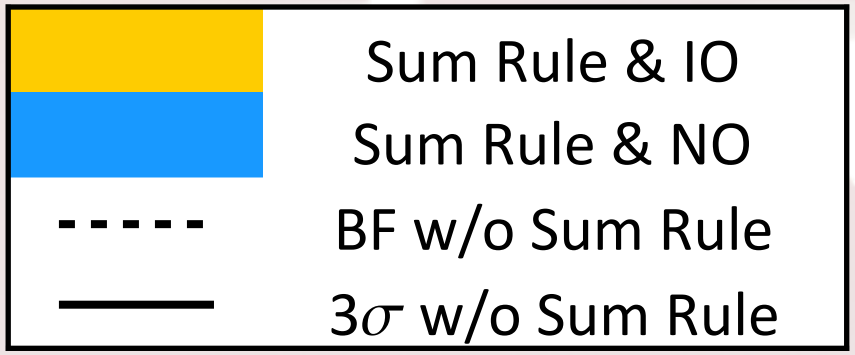
3σ w/o Sum Rule

Numerical Results

[Gehrlein, Merle, MS '15]

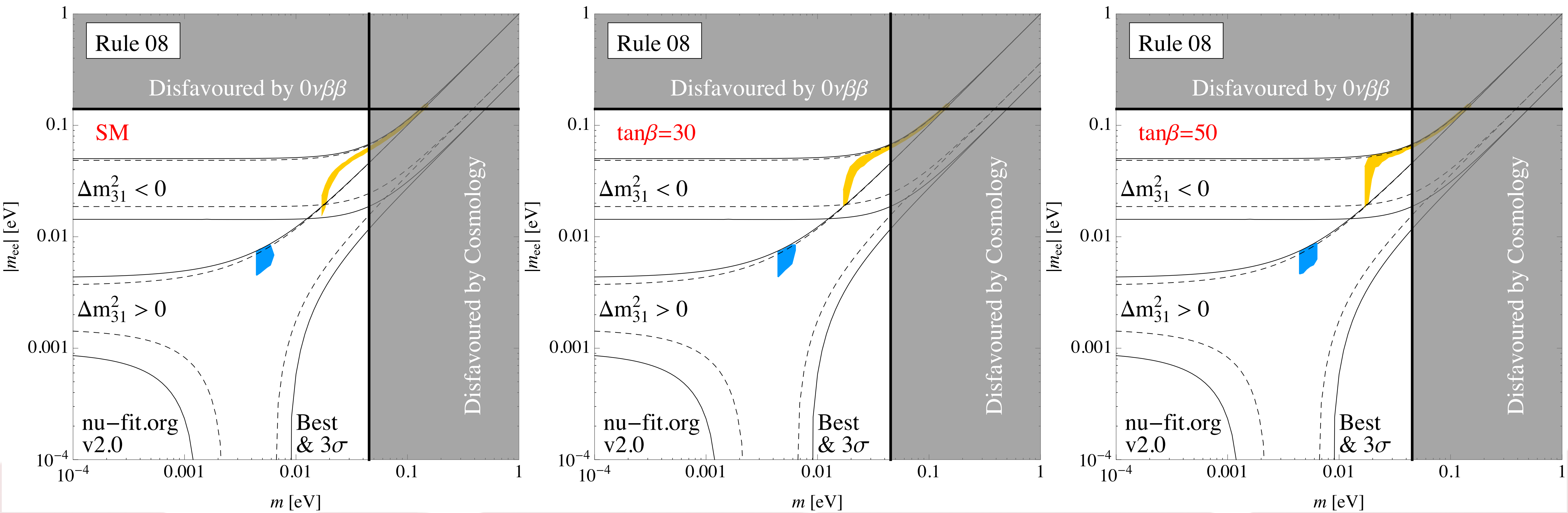


$$\frac{e^{i\,\alpha_1}}{m_1} + \frac{e^{i\,\alpha_2}}{m_2} = \frac{1}{m_3}$$



Numerical Results

[Gehrlein, Merle, MS '15]



$$\frac{e^{i\,\alpha_1}}{m_1} - 2\frac{e^{i\,\alpha_2}}{m_2} = -\frac{1}{m_3}$$

(yellow)	Sum Rule & IO
(blue)	Sum Rule & NO
- - - - -	BF w/o Sum Rule
——	3σ w/o Sum Rule

Model Specific Corrections

[Gehrlein, Merle, MS ’16]

- Add perturbation to the masses

$$m_i \mathrm{e}^{-\mathrm{i}\phi_i} = m_i^{(0)} \mathrm{e}^{-\mathrm{i}\phi_i^{(0)}} + \delta m_i \; \mathrm{e}^{\mathrm{i}\delta\phi_i}$$

- Measure for deviation

$$\frac{s}{m_n^d} = \hat{s} \approx \frac{s^{(0)}}{m_n^d} + \frac{\delta s}{m_n^d}$$

- By assumption $s^{(0)}$ vanishes

Model Specific Corrections

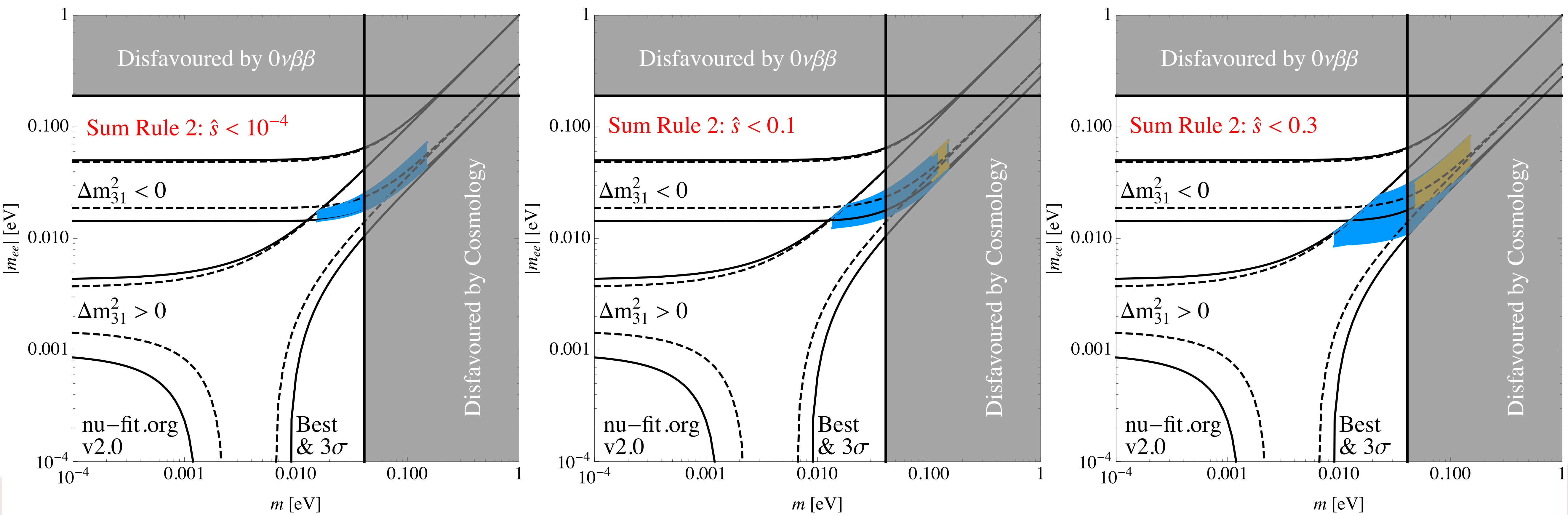
[Gehrlein, Merle, MS ’16]

Sum rule	forbidden ordering	$\delta\hat{s}_r(m_{\text{lightest}})$		
		$m_{\text{lightest}} = 0$ eV	$m_{\text{lightest}} = 0.001$ eV	$m_{\text{lightest}} = 0.05$ eV
2	IO	1.02	1.00	0.30
3	IO	1.02	1.00	0.30
4	NO	0.92	0.90	0.30
5	NO	0.95	0.94	0.30
10	IO	1.01	0.87	0.16
12	IO	1	0.86	0.16

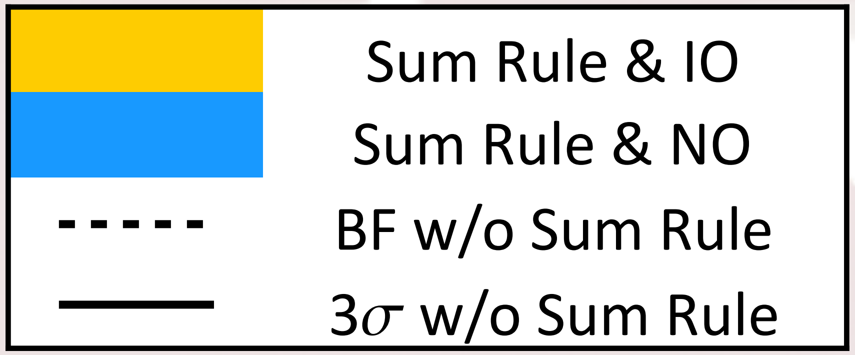
Table: Estimated minimal correction to make a forbidden ordering appear.

Model Specific Corrections

[Gehrlein, Merle, MS '16]



$$m_1 e^{-i\phi_1} + 2 m_2 e^{-i\phi_2} - m_3 = 0$$



Map of Orderings

Rule	1	2	3	4	5	6	7	8	9	10	11	12
NO	✓	✓	✓	X	X	✓	✓	✓	✓	✓	✓	✓
IO	✓	X	X	✓	✓	✓	✓	✓	✓	X	✓	X

Outline

- Introduction
- **Neutrino Mass Sum Rules**
  - General remarks
  - Corrections
  - **What happens in a real model?**
- Summary and Conclusions

The Model

[Gehrlein, Oppermann, Schäfer, MS 2014]

- SU(5) x A_5 GUT Flavour Model
- Golden Ratio Neutrino Mixing

$$\tan\theta_{12}^{\nu} = \frac{2}{1+\sqrt{5}}\ , \quad \theta_{23}^{\nu} = \pi/4\ , \quad \theta_{13}^{\nu} = 0$$

- Cabibbo-like 1-2 charged lepton mixing

$$\theta_{12}^{e} \approx \theta_C \Rightarrow \theta_{13} \approx \frac{\theta_C}{\sqrt{2}} \approx 9^\circ$$

Model Constraints

[Gehrlein, Oppermann, Schäfer, MS 2014]

- Mixing sum rule

$$\theta_{12}^{\mathrm{PMNS}} \approx \theta_{12}^{\nu} + \theta_C \cos\delta$$

- Mass sum rule

$$\frac{\mathrm{e}^{\mathrm{i}\ \alpha_1}}{m_1} + \frac{\mathrm{e}^{\mathrm{i}\ \alpha_2}}{m_2} = \frac{1}{m_3}$$

- From fermion mass ratios

$$\tan\beta \gtrsim 30$$

Implications

[Gehrlein, Oppermann, Schäfer, MS 2014]

- From the mixing sum rule

$$24^\circ \lesssim \theta_{12}^{\mathrm{PMNS}}(M_S) \lesssim 39^\circ$$

- From the mass sum rule

$$\begin{array}{ll} 0.011\ \mathrm{eV} \lesssim m_1 & \text{for NO} \\ 0.028\ \mathrm{eV} \lesssim m_3 \lesssim 0.454\ \mathrm{eV} & \text{for IO} \end{array}$$

- From RGEs

$$\begin{array}{l} \theta_{12}^{\mathrm{PMNS}}(M_S) \lesssim 33.5^\circ \text{ for NO} \\ \theta_{12}^{\mathrm{PMNS}}(M_S) \lesssim \ \ 5.7^\circ \text{ for IO} \Rightarrow ☠ \end{array}$$

Outline

- Introduction
- Neutrino Mass Sum Rules
- **Summary and Conclusions**

Summary and Conclusions

- Constraints on CP violation, mass scale (and ordering)
- Corrections do not affect the qualitative behavior
- Mixing and mass sum rules combined are very powerful!

Thanks a lot for your attention!