Thank you!

What string theory can teach us about discrete symmetries and their anomalies

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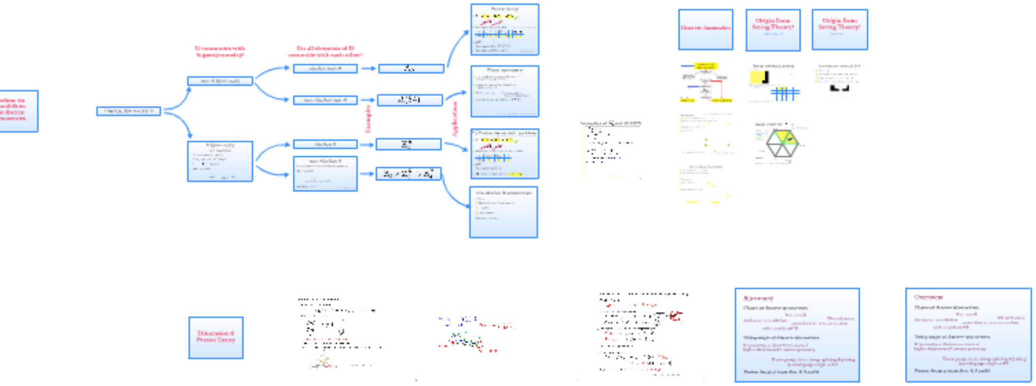
Technische Universität München

Bethe Forum Bonn - April 7th, 2017



Overview

Classes of discrete symmetries:

R vs. non-R

GS mechanism

Abelian vs. non-Abelian

anomalous vs. non-anomalous

with or without CP

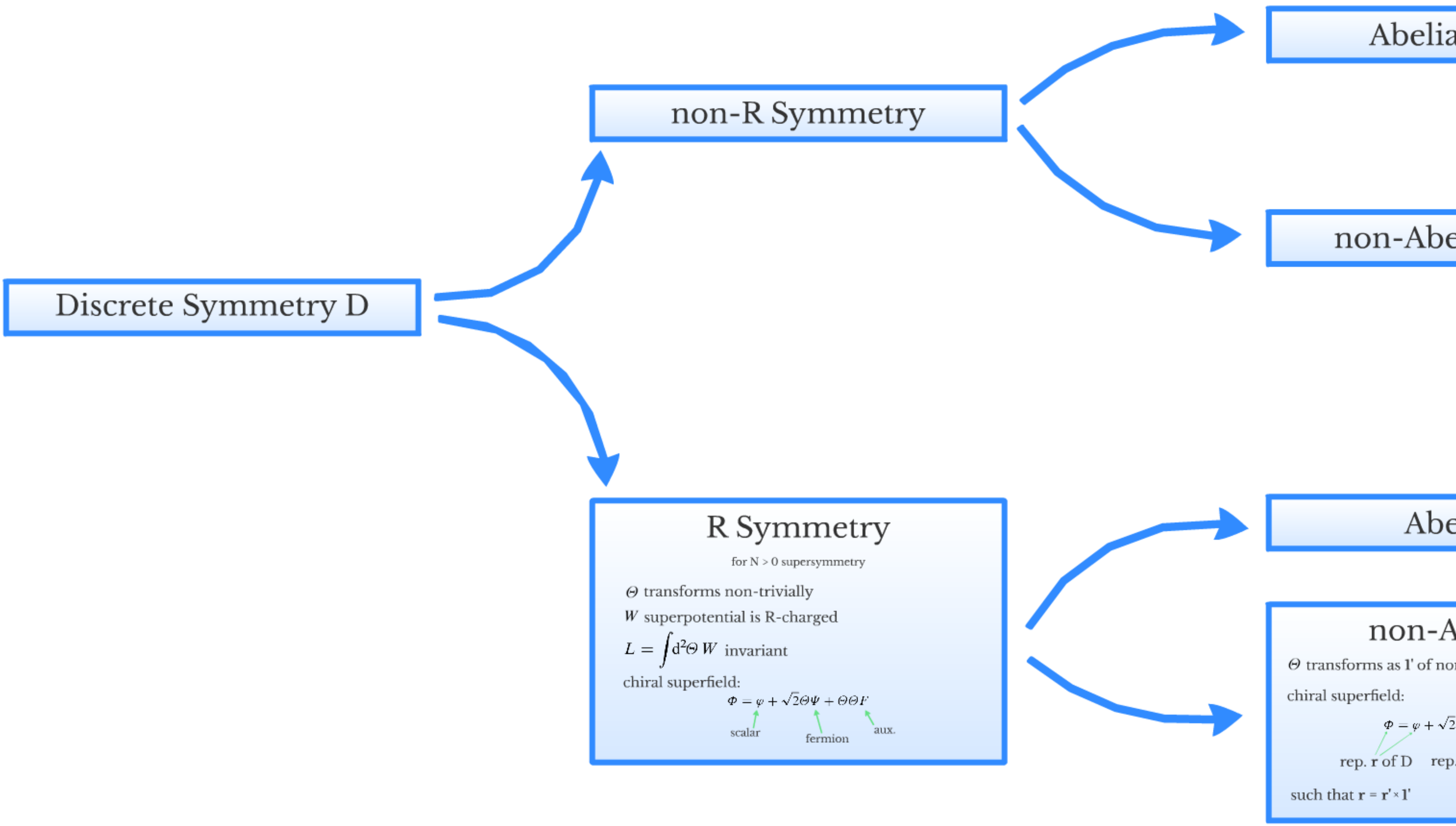
String origin of discrete symmetries:

R symmetry as discrete remnant of
higher dimensional Lorentz symmetry

Flavor groups from strings splitting & joining
(partial) gauge origin at R=1

Proton decay at mass dim. 4, 5 and 6

Bottom-up possibilities for discrete symmetries



D commutes with Supersymmetry?

Do all ele
commute wi

Discrete Symmetry D

non-R Symmetry

Abelia

non-Abe

R Symmetry

for N > 0 supersymmetry

Θ transforms non-trivially

W superpotential is R-charged

$L = \int \mathrm{d}^2\Theta\, W$ invariant

chiral superfield:

$$\underset{\text{scalar}}{\Phi = \varphi} + \sqrt{2}\Theta\underset{\text{fermion}}{\Psi} + \Theta\Theta \underset{\text{aux.}}{F}$$

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Θ transforms as 1' of no

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rep. **r** of D rep.

such that **r** = **r**' × 1'

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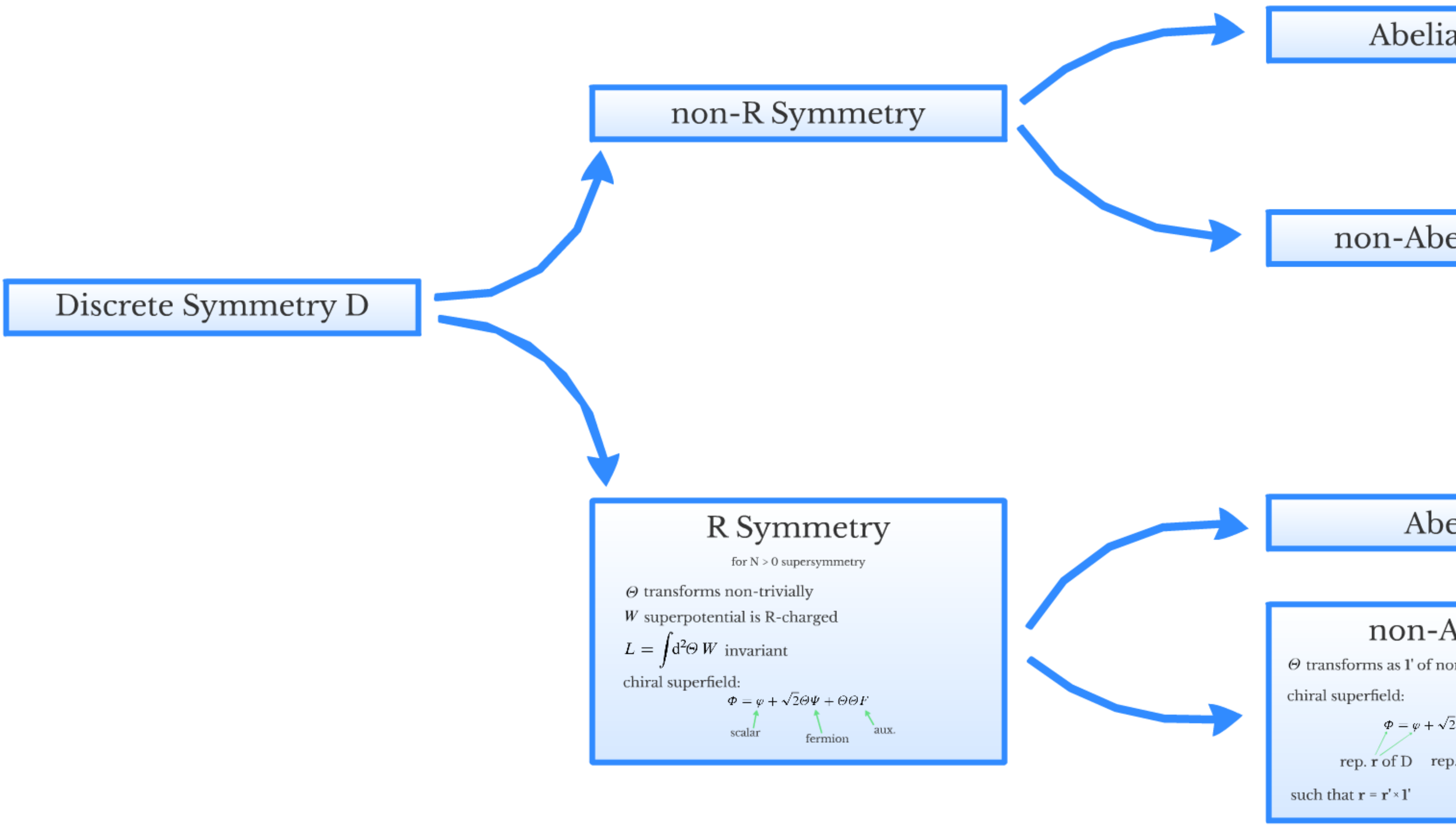

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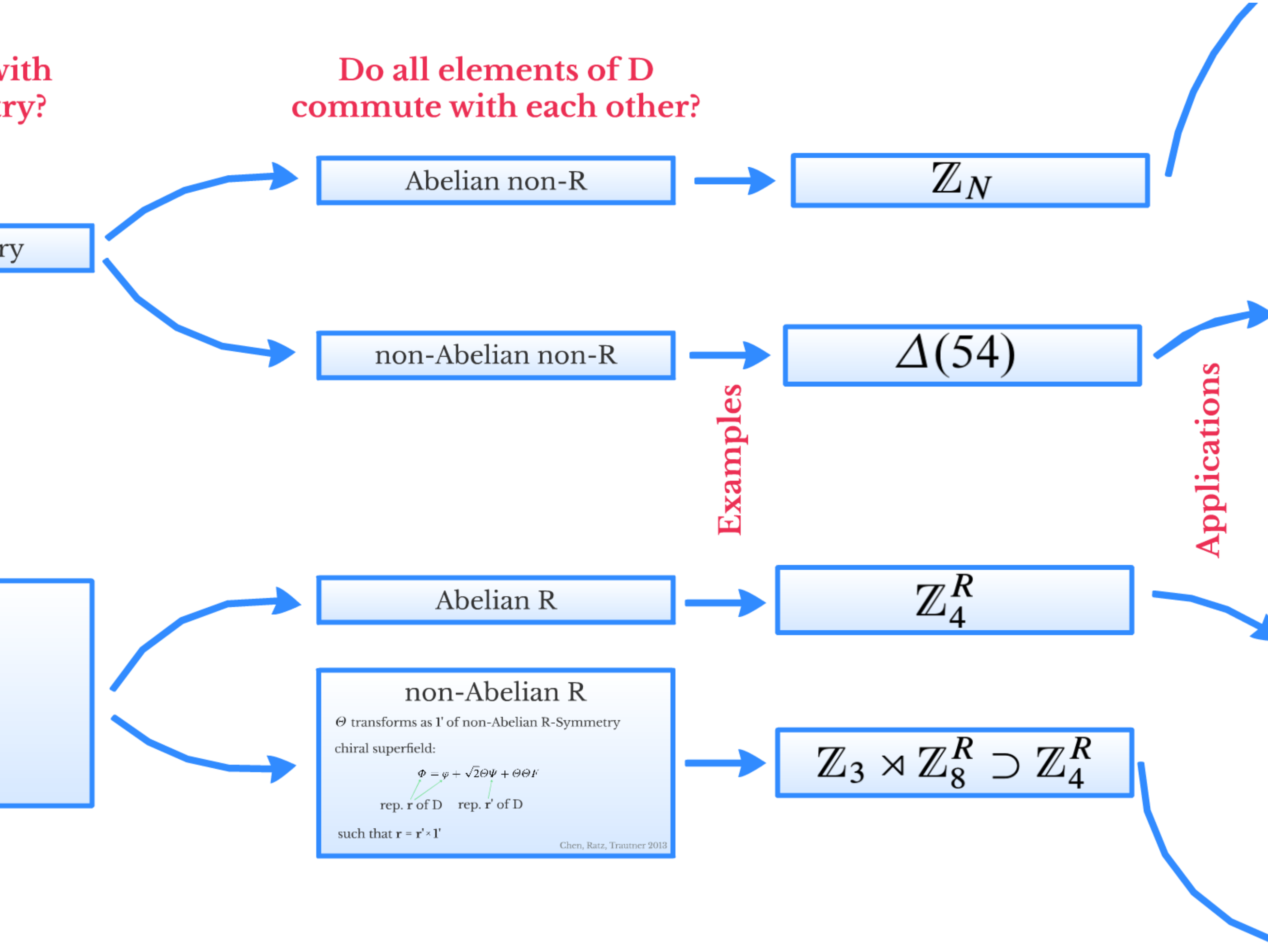

vith
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Abelian non-R

$\mathbb{Z}_N$

non-Abelian non-R

$\Delta(54)$

Examples

Applications

Abelian R

$\mathbb{Z}_4^R$

non-Abelian R

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Chen, Ratz, Trautner 2013

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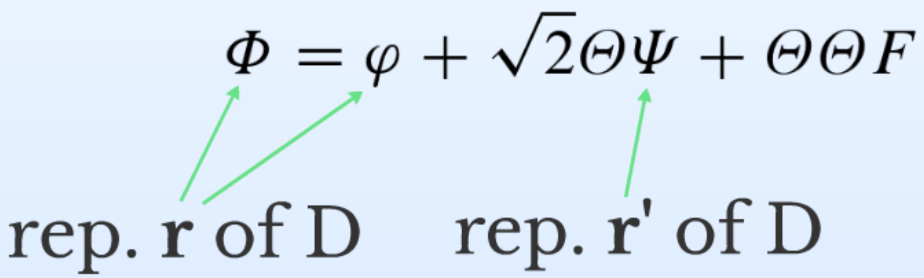

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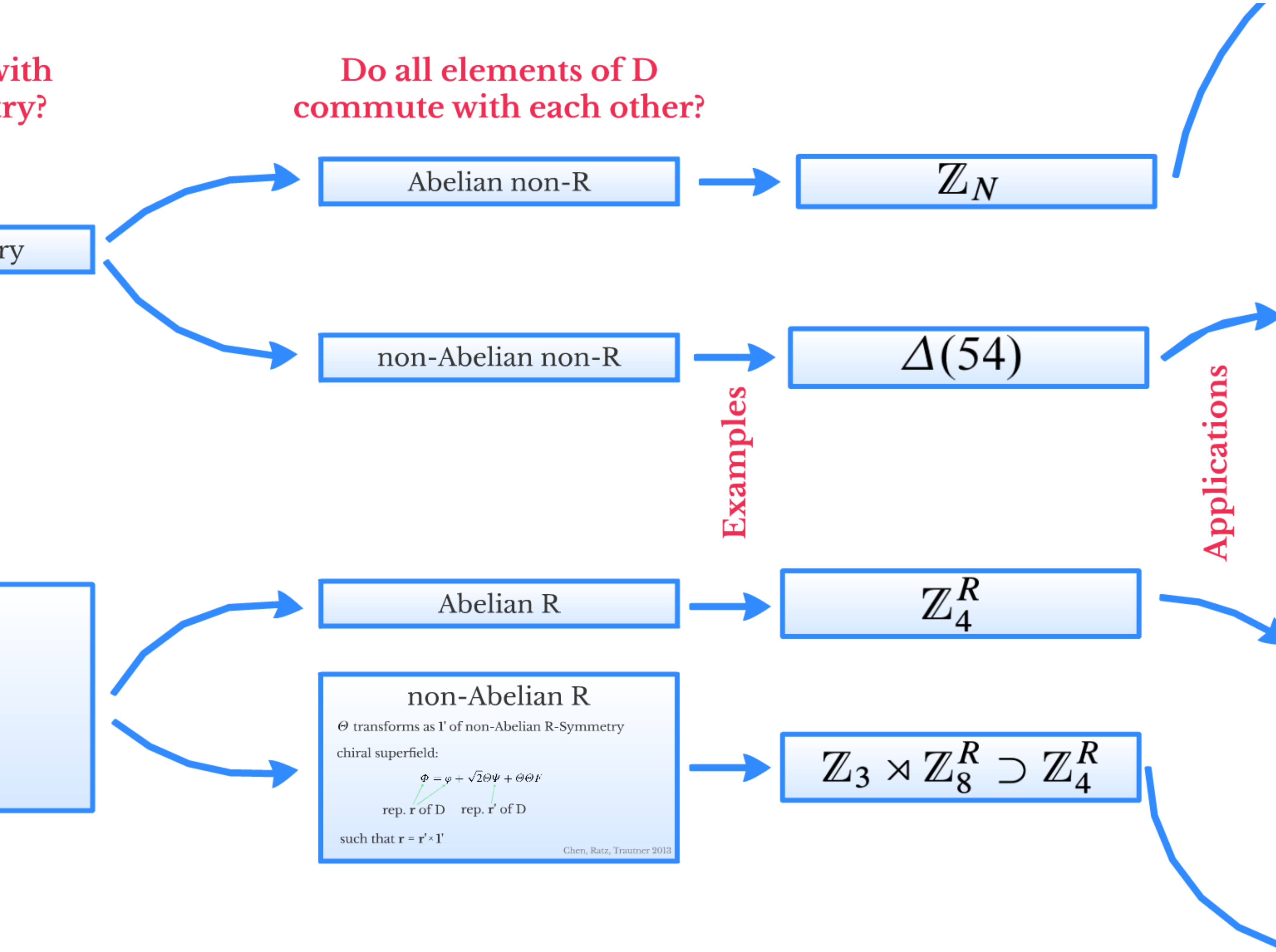

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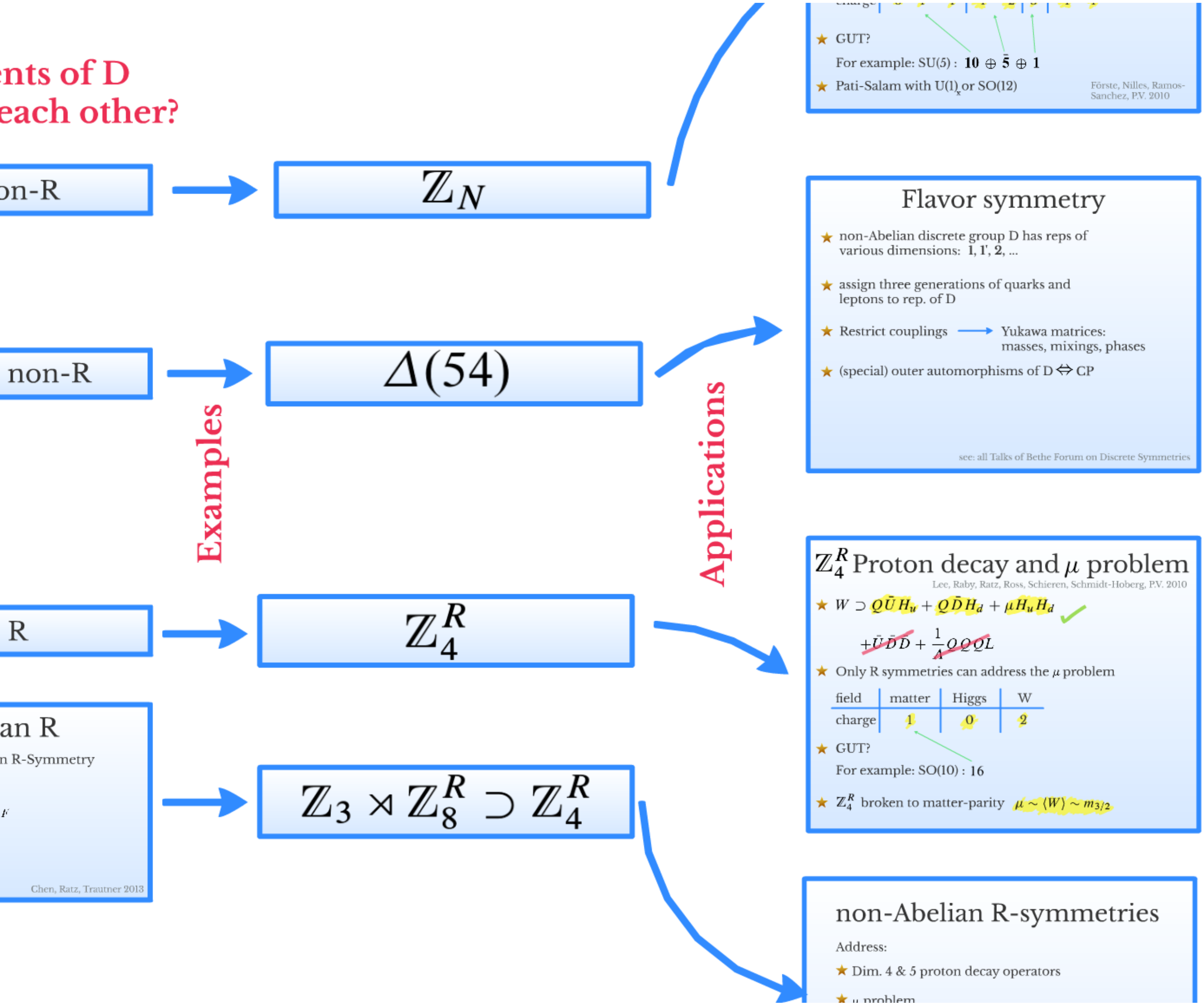

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- Pati-Salam with $U(1)_x$ or SO(12)

Förste, Nilles, Ramos-Sanchez, P.V. 2010

Flavor symmetry

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see: all Talks of Bethe Forum on Discrete Symmetries

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  $+\cancel{\bar{U}\bar{D}\bar{D}} + \cancel{\frac{1}{\Lambda}QQQL}$
- Only R symmetries can address the μ problem

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- ★ For example: Proton Hexality P_6 Dreiner, Luhn, Thormeier 2006

field	Q	$\bar{U}$	$\bar{E}$	$\bar{D}$	L	ν	H_u	H_d
charge	0	1	1	-1	-2	3	-1	1

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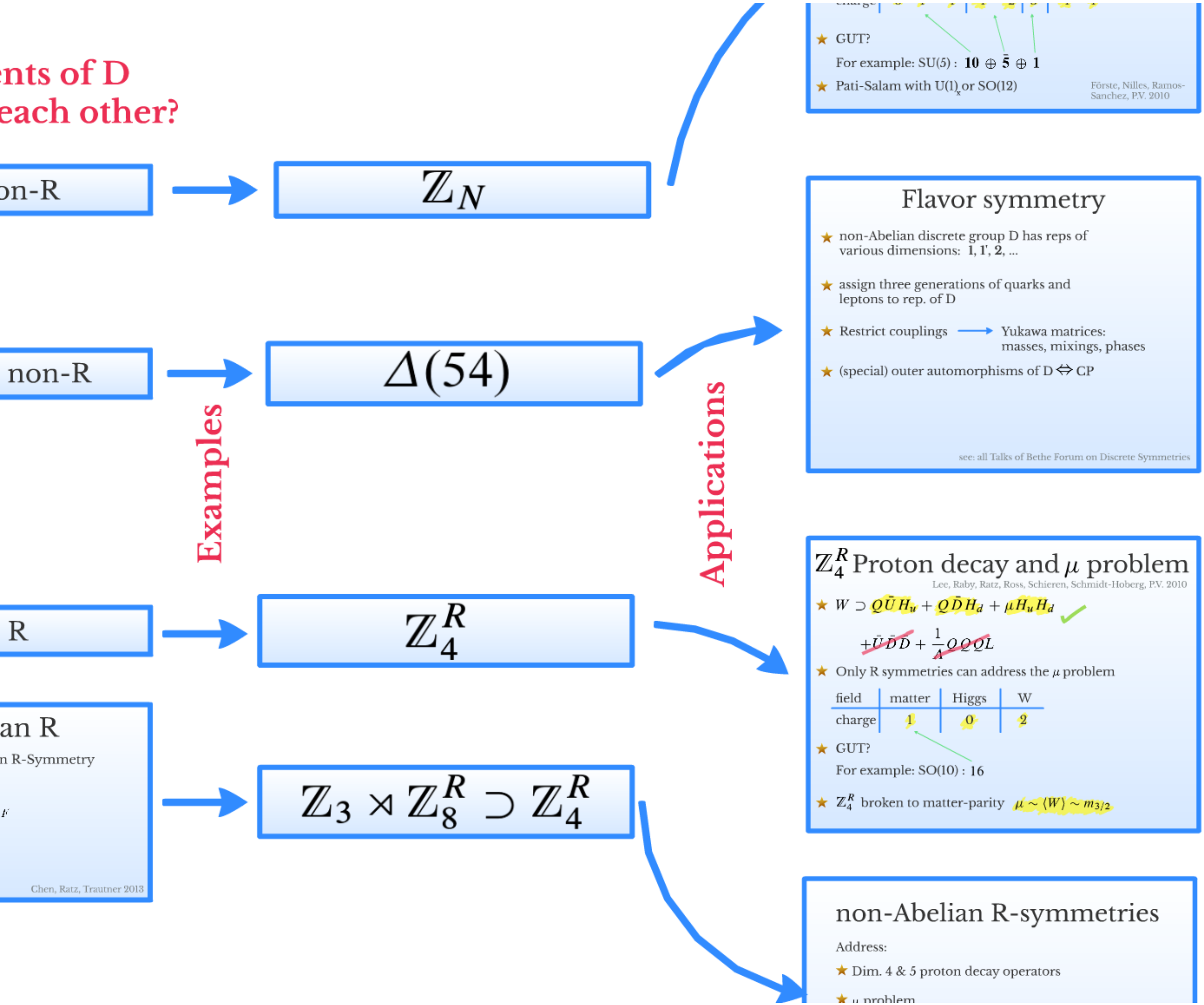

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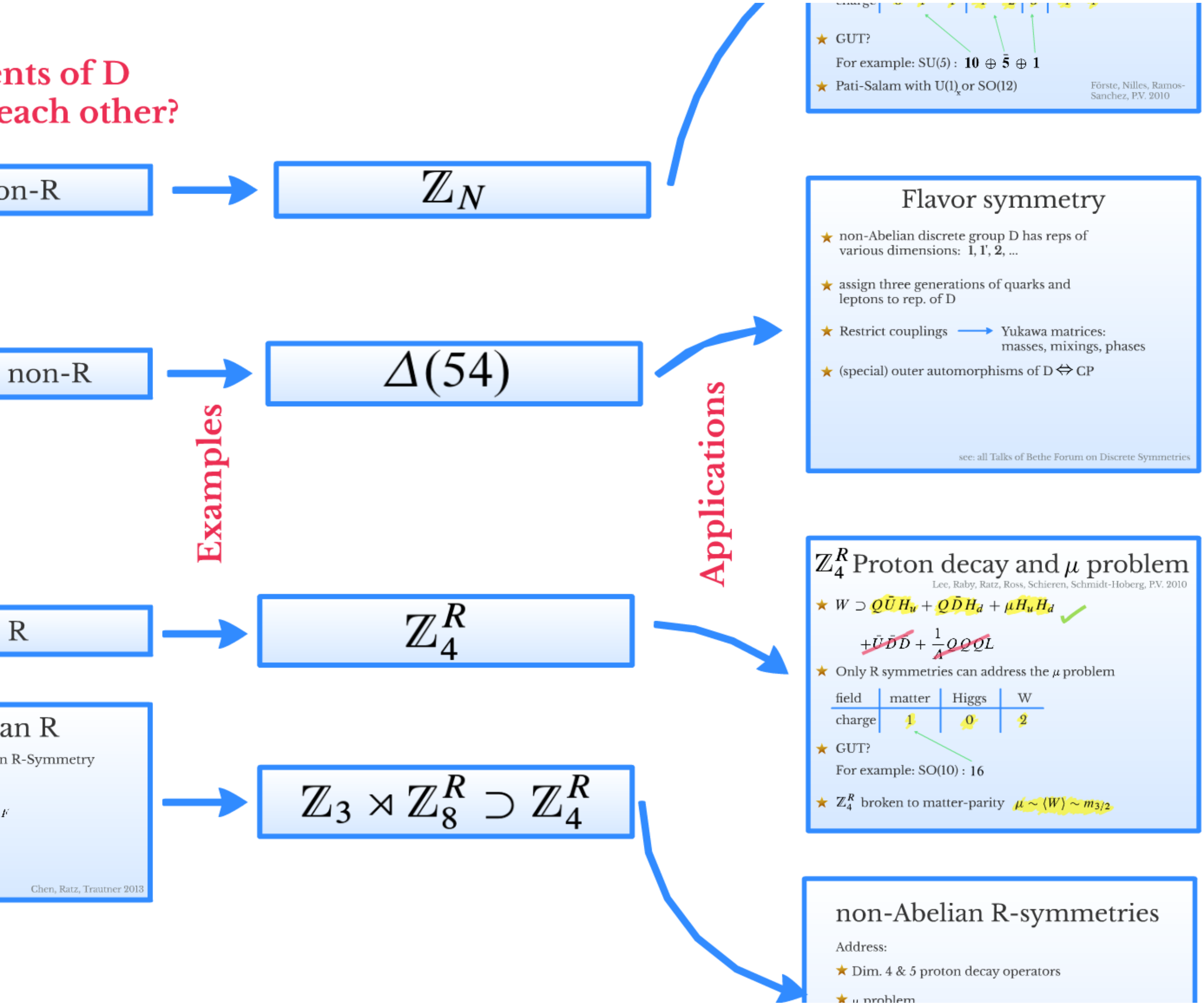

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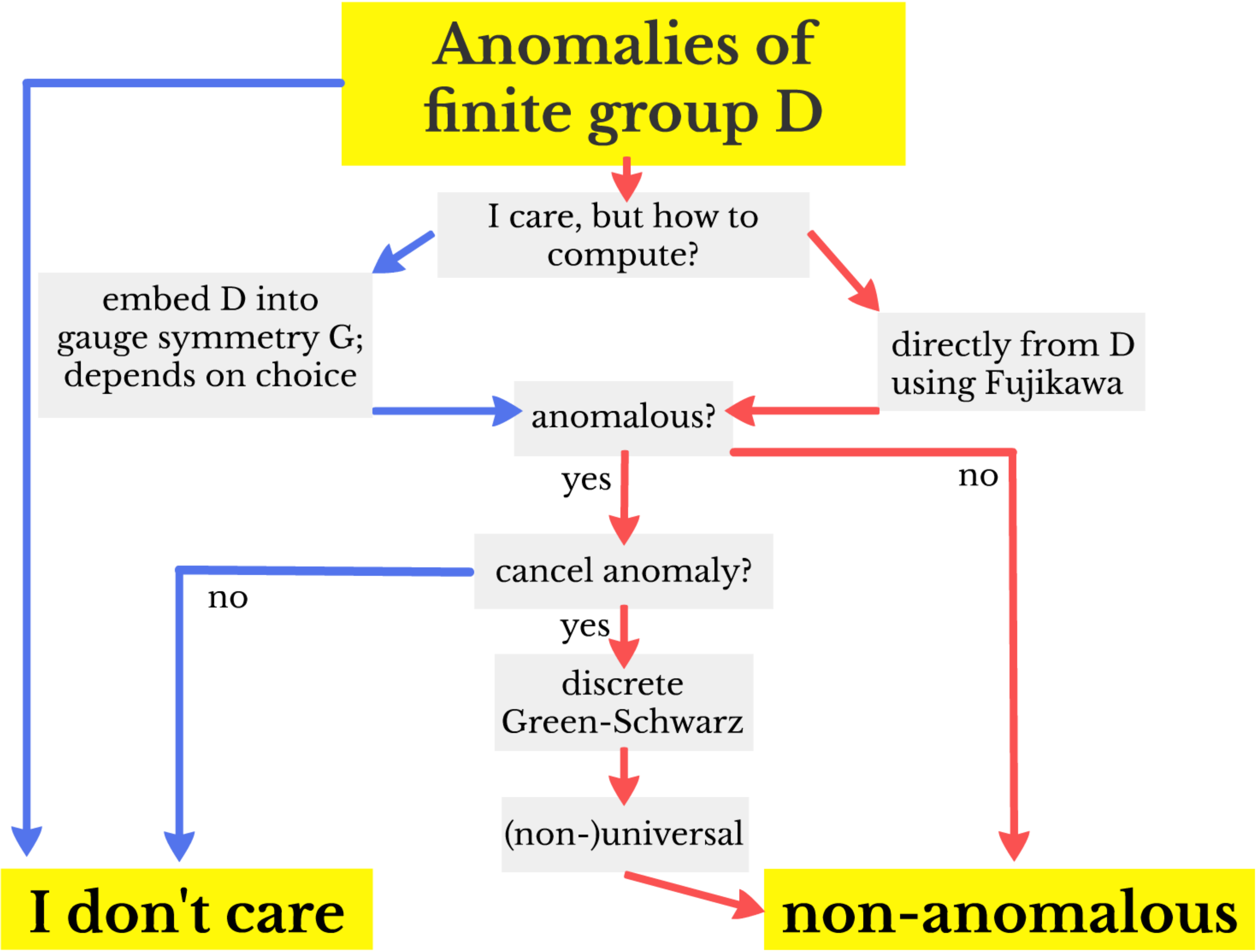
Discrete Anomalies


Anomalies of finite group D
I care, but how to compute?
embed D into gauge symmetry G; depends on choice
directly from D using Fujikawa
anomalous?
yes
no
cancel anomaly?
no
yes
discrete Green-Schwarz
(non-)universal
I don't care
non-anomalous

Anomalies

e.g. Araki, Kobayashi, Kubo, Ramos-Sanchez, Ratz, P.V. 2008

- Symmetry D of classical action $\Psi \mapsto \exp\left(iq\alpha\right)\Psi$ $\quad S[\Psi,\bar{\Psi}] = \int \mathrm{d}^4x\, \mathcal{L}[\Psi,\bar{\Psi}]$
- Anomaly = path integral measure not invariant

$$\int D\Psi D\bar{\Psi} \exp\left(-S[\Psi,\bar{\Psi}]\right) \qquad D\Psi D\bar{\Psi} \mapsto J(\alpha) D\Psi D\bar{\Psi} \qquad J(\alpha) = \exp\left(-\int \mathrm{d}^4x\, \alpha \mathcal{A}(x)\right)$$

- If D gauge group: current J^μ and $\mathcal{A}(x) = \langle D_\mu J^\mu \rangle$
- For general D:

$$\mathcal{A}(x) \sim \mathcal{A}_{\mathrm{SU}(N)-\mathrm{SU}(N)-D}\, \mathrm{tr} F^{\mathrm{SU}(N)}_{\mu\nu} \tilde{F}^{\mathrm{SU}(N)\,\mu\nu} + \frac{1}{6} \mathcal{A}_{\mathrm{U}(1)-\mathrm{U}(1)-D}\, \mathrm{tr} F^{\mathrm{U}(1)}_{\mu\nu} \tilde{F}^{\mathrm{U}(1)\,\mu\nu}$$

$$+\frac{1}{24} \mathcal{A}_{\mathrm{grav.}-\mathrm{grav.}-D}\, \mathrm{tr} R_{\mu\nu} \tilde{R}^{\mu\nu} \overset{!}{=} 0 \;\Leftrightarrow\; \mathcal{A}_{\dots} \overset{!}{=} 0$$

$$\mathcal{A}_{\mathrm{SU}(N)-\mathrm{SU}(N)-D} = \sum_{\mathrm{rep.}\mathbf{r}^{(f)}} \ell(\mathbf{r}^{(f)})\, q^{(f)} \qquad \ell(\mathbf{N}) = \frac{1}{2} \text{ for } \mathrm{SU}(N)$$

$$\mathcal{A}_{\mathrm{U}(1)-\mathrm{U}(1)-D} = \sum_{\mathrm{ferm.}f} \left(q^{(f)}_{\mathrm{U}(1)}\right)^2 q^{(f)} \qquad \mathcal{A}_{\mathrm{grav.}-\mathrm{grav.}-D} = \sum_{\mathrm{ferm.}f} q^{(f)}$$

D continuous : α continuous

D discrete : $\alpha = \frac{k}{N} \Rightarrow \mathcal{A}_{\dots} \overset{!}{=} 0 \bmod N \qquad q^{(f)} = \frac{N}{2\pi i} \ln(\det(U^{(f)}))$

Perfect groups: free of anomalies $J(\alpha) \sim \mathbf{1_0}$ Chen, Fallbacher, Ratz, Trautner, P.V. 2015

Green Schwarz mechanism

- D not symmetry of classical action $\Psi \mapsto \exp\left(iq\alpha\right)\Psi \quad S[\Psi,\bar{\Psi}] = \int \mathrm{d}^4x\, \mathcal{L}[\Psi,\bar{\Psi}]$
- Path integral measure not invariant

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- Add terms: $\mathcal{L}_{\mathrm{GS}} \sim a\, \mathrm{tr} F^{\mathrm{SU}(N)}_{\mu\nu} \tilde{F}^{\mathrm{SU}(N)\,\mu\nu} + a\, \mathrm{tr} F^{\mathrm{U}(1)}_{\mu\nu} \tilde{F}^{\mathrm{U}(1)\,\mu\nu} + a\, \mathrm{tr} R_{\mu\nu} \tilde{R}^{\mu\nu}$
- Field a shifts under D: $a \overset{D}{\mapsto} a + \alpha\, \delta_{\mathrm{GS}}$

$$S[\Psi,\bar{\Psi},a] \overset{D}{\mapsto} S[\Psi,\bar{\Psi},a] + \alpha \int \mathrm{d}^4x\, \Delta\mathcal{L}_{\mathrm{GS}}(\delta_{\mathrm{GS}})$$

- Path integral transforms as

$$\int D\Psi D\bar{\Psi} \exp\left(-S[\Psi,\bar{\Psi},a]\right) \overset{D}{\mapsto} \int D\Psi D\bar{\Psi} \exp\left(-S[\Psi,\bar{\Psi},a]\right) \times \underbrace{\exp\left(-\alpha \int \mathrm{d}^4x\, \mathcal{A}(x) - \Delta\mathcal{L}_{\mathrm{GS}}(\delta_{\mathrm{GS}})\right)}_{=1}$$

$$\mathcal{A}(x) \sim \mathcal{A}_{\mathrm{SU}(N)-\mathrm{SU}(N)-D}\, \mathrm{tr} F^{\mathrm{SU}(N)}_{\mu\nu} \tilde{F}^{\mathrm{SU}(N)\,\mu\nu} + \frac{1}{6} \mathcal{A}_{\mathrm{U}(1)-\mathrm{U}(1)-D}\, \mathrm{tr} F^{\mathrm{U}(1)}_{\mu\nu} \tilde{F}^{\mathrm{U}(1)\,\mu\nu}$$

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- ★ $W \supset Q\bar{U}H_u + Q\bar{D}H_d + \mu H_u H_d$ ✓

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  For example: SO(10) : 16

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Anomalies of $\mathbb{Z}_4^R$ and 4D GUTs

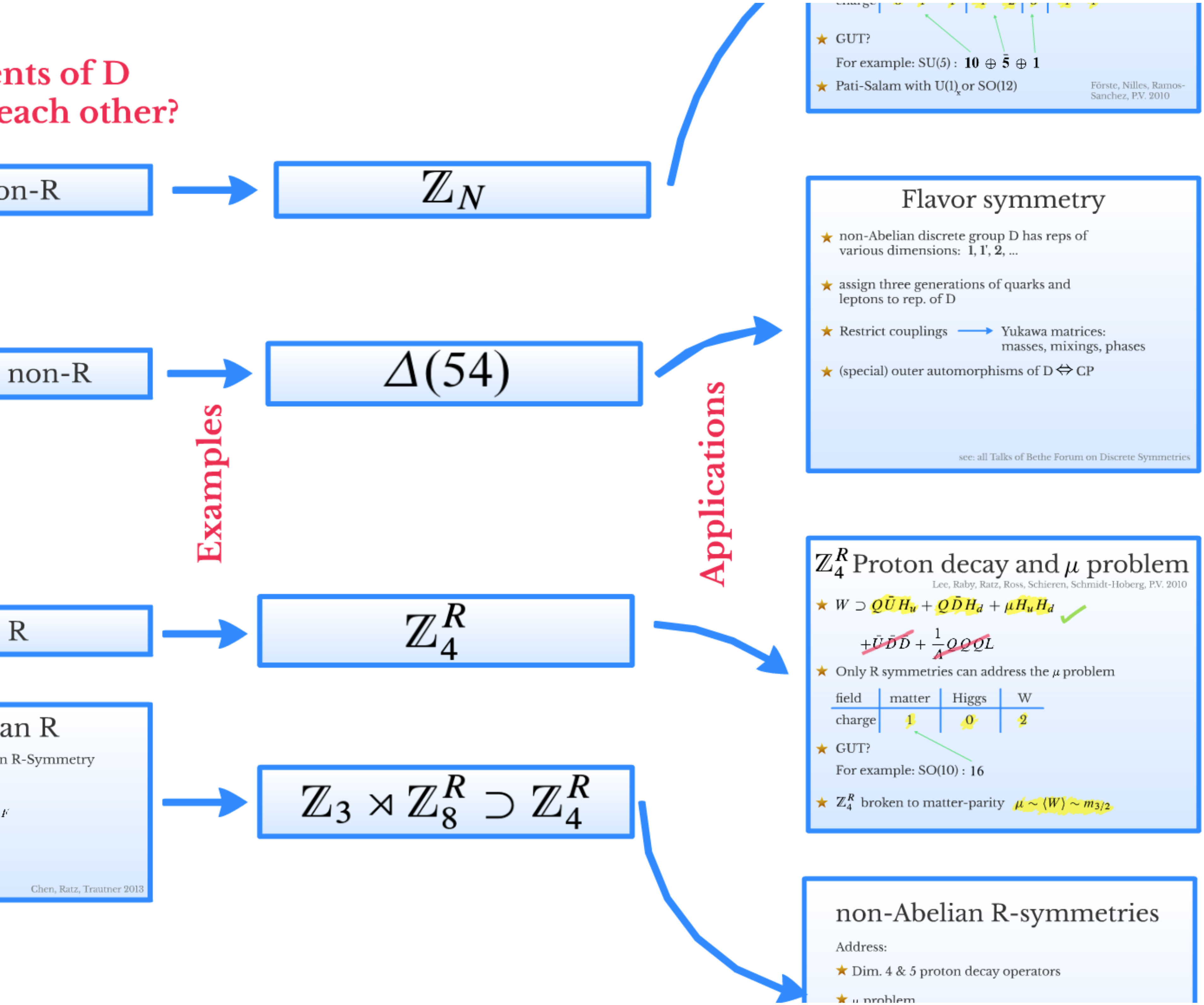
$\mathbb{Z}_M^R$ and GUTs:

- assume $\mathrm{SU}(5) \times \mathbb{Z}_M^R$
- break $\mathrm{SU}(5)$ but keep R symmetry
- basic obstruction:

$$\mathbf{24}_2 = \underbrace{(\mathbf{8},\mathbf{1})_0}_{S_2} \oplus \underbrace{(\mathbf{1},\mathbf{3})_0}_{T_2} \oplus \underbrace{(\mathbf{1},\mathbf{1})_0}_{T'_2} \oplus \underbrace{(\mathbf{3},\mathbf{2})_{-5/6}}_{X_2} \oplus \underbrace{(\overline{\mathbf{3}},\mathbf{2})_{5/6}}_{Y_2}$$

- $q_R(\mathbf{24}_0) = 0 \bmod M \Rightarrow \quad \mathbb{Z}_M^R$
- $q_R(\mathcal{W}) = 2 \bmod M \;\Rightarrow\; S_0$ and T_0 massless
- add mass partner S_2 and T_2, i.e. $\mathbf{24}_2$

$\Rightarrow X_2$ and Y_2 massless



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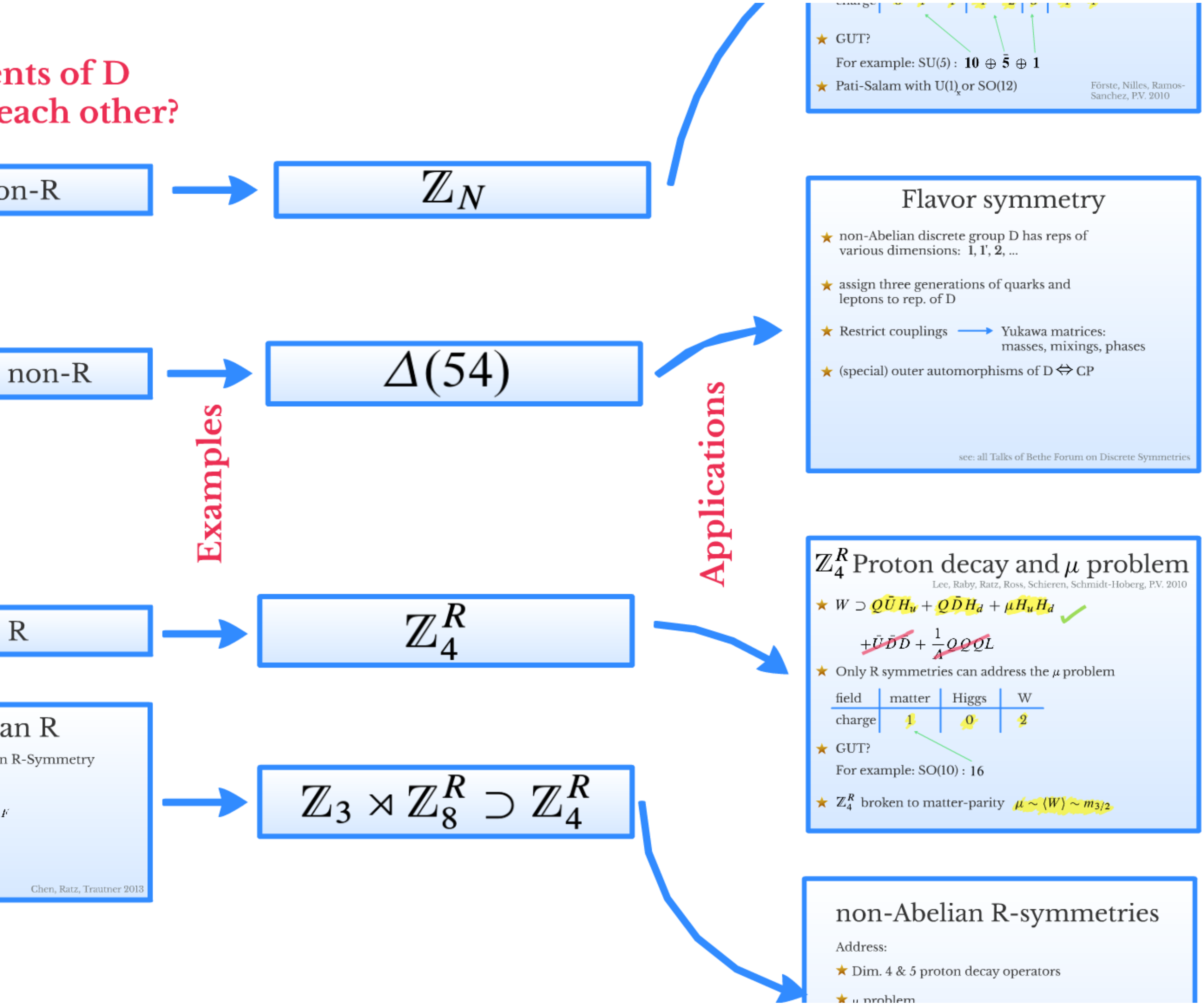
non-Abelian R-symmetries

Address:

- ★ Dim. 4 & 5 proton decay operators
- ★ μ problem
- ★ flavor structure

More studies needed...



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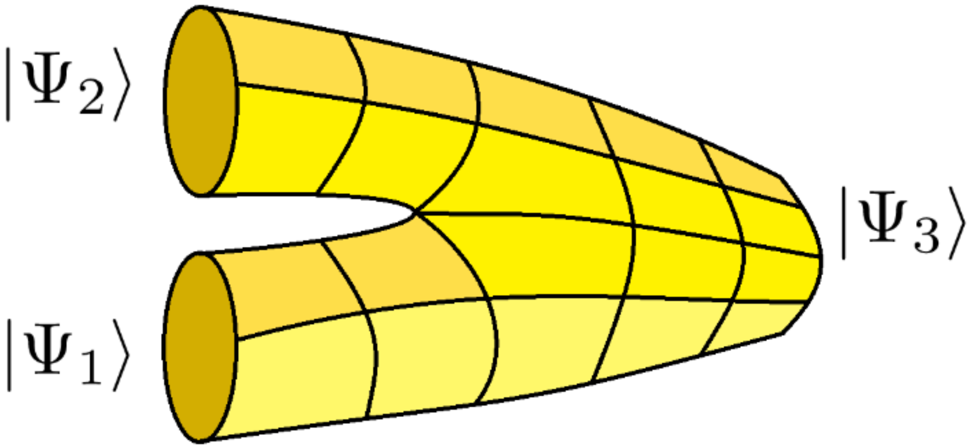
Origin from String Theory?

Part 1: non-R

Strings splitting & joining

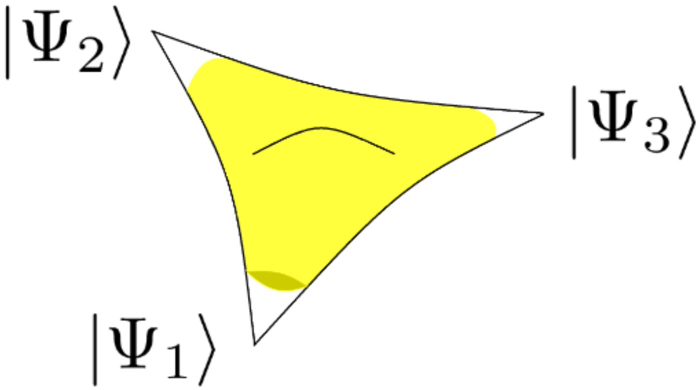
closed strings



$$|\Psi_1\rangle\,|\Psi_2\rangle \longrightarrow |\Psi_3\rangle$$

Strings live in 10D → compactification

closed strings on orbifolds



	$\mathbb{Z}_3^{\mathrm{PG}}$	$\mathbb{Z}_3^{\mathrm{SG}}$	S_3	$\Delta(54)$
$\|\Psi_\mathrm{b}\rangle$	0	0	1	1
$\|\Psi_1\rangle$	1	0	$\mathbf{3} = \mathbf{2} + \mathbf{1}$	$\mathbf{3}$
$\|\Psi_2\rangle$	1	1		
$\|\Psi_3\rangle$	1	2		
	from	string		

e.g. "Origin of family symmetries" by Nilles, Ratz & P.V. 2012

Gauge origin for $T^2/\mathbb{Z}_3$

Beye, Kobayashi, Kuwakino 2014
Biermann, Ratz, P.V. unpublished

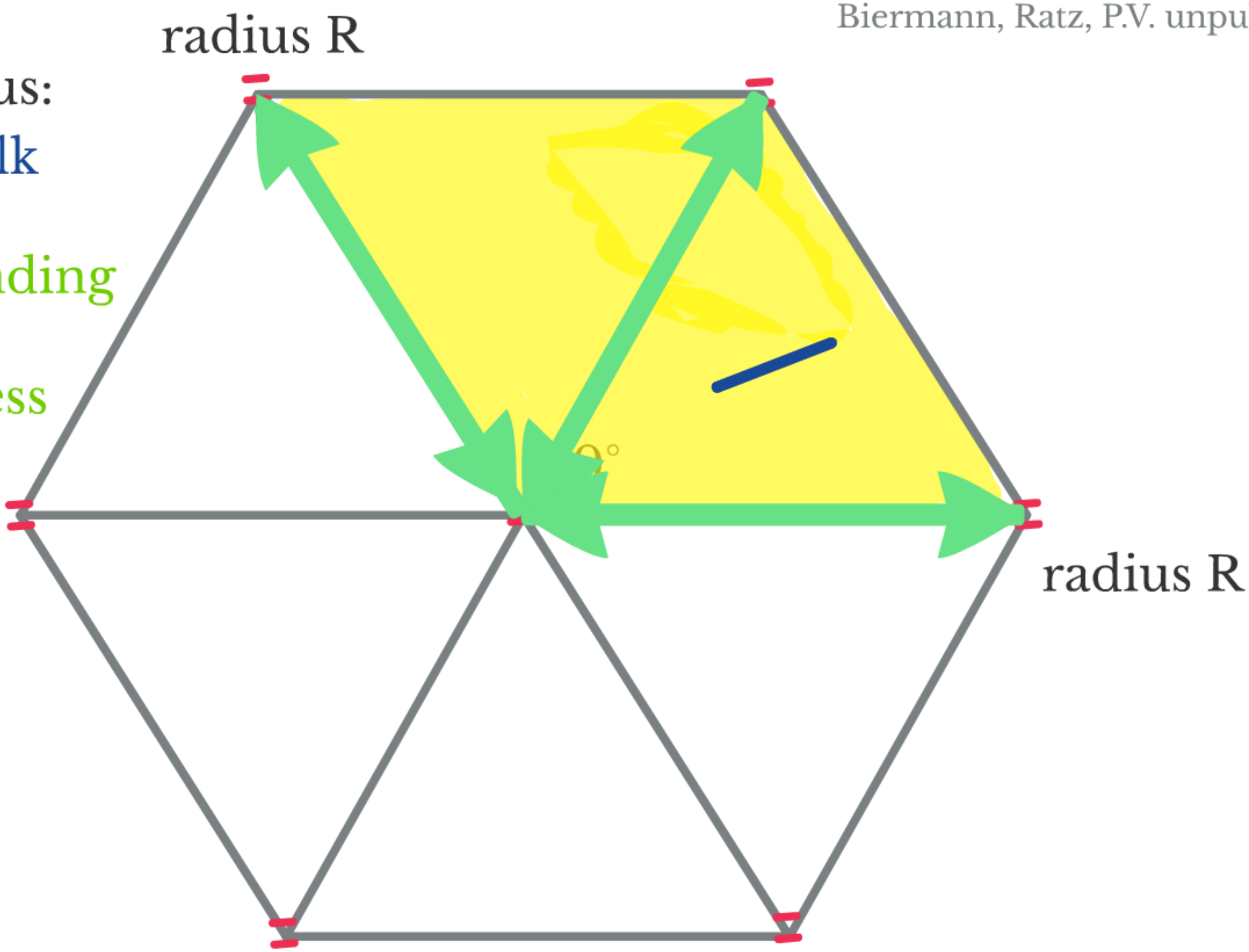
special T^2 torus:

2 massless bulk strings

6 massive winding strings

at R=1: massless

$\mathrm{SU}(3) \rtimes \mathbb{Z}_2$



$T^2/\mathbb{Z}_3$

120° rotation

$\mathrm{U}(1)^2 \rtimes S_3$

Summary: $\mathrm{SU}(3) \rtimes \mathbb{Z}_2 \longrightarrow \mathrm{U}(1)^2 \rtimes S_3 \xrightarrow[\text{change R}]{} (\mathbb{Z}_3 \times \mathbb{Z}_3) \rtimes S_3 = \Delta(54)$

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strings

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rings

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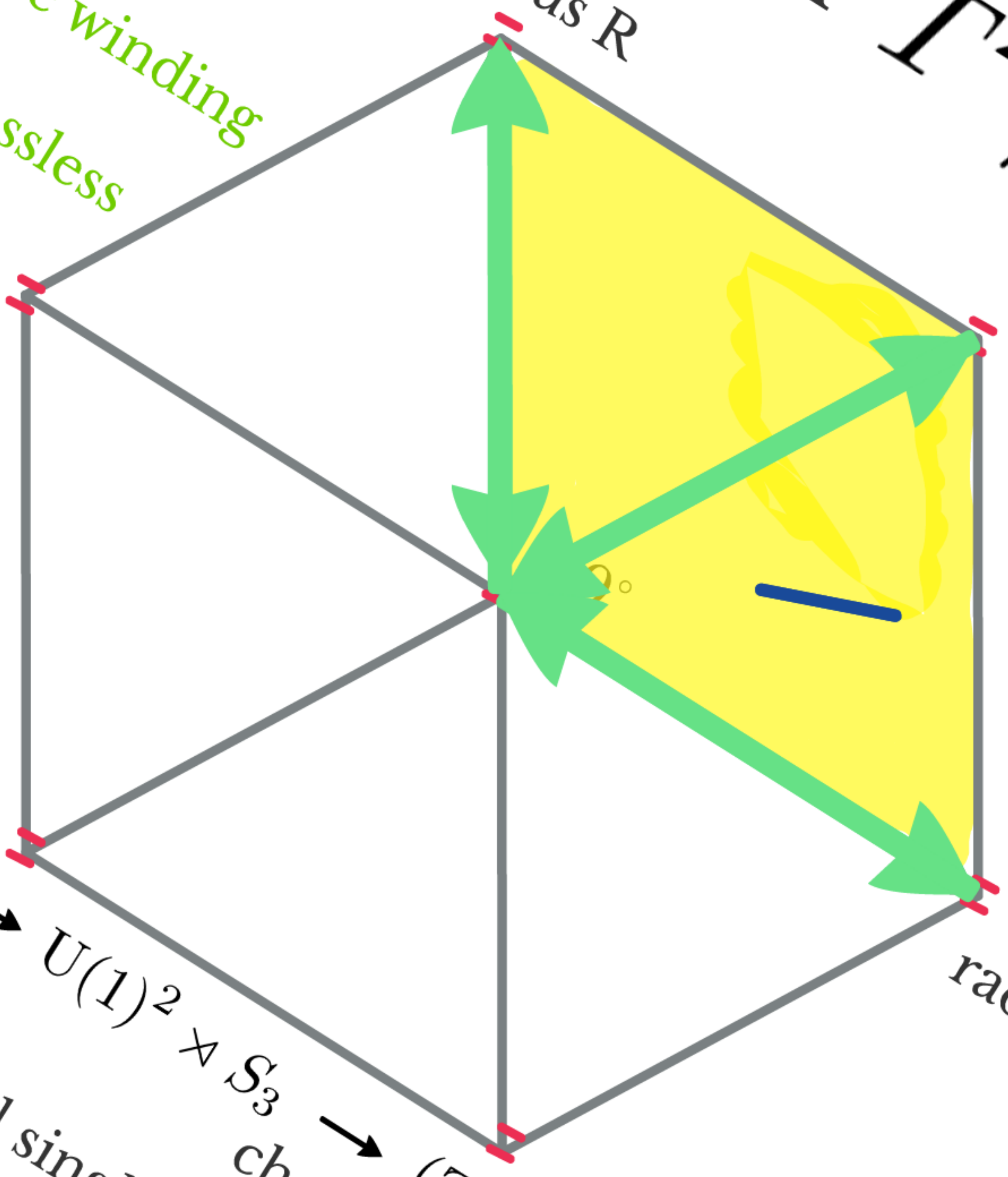

$\rtimes \mathbb{Z}_2 \rightarrow U(1)^2 \rtimes S_3 \rightarrow (\mathbb{Z}_3 \times \mathbb{Z}_3) \rtimes S_3 = \Delta(5$

change R

t: trivial singlet and $\mathbf{3}, \bar{\mathbf{3}}$

Gauge origin for $T^2/\mathbb{Z}_3$

Beye, Kobayashi, Kuwakino 2014
Biermann, Ratz, P.V. unpublished

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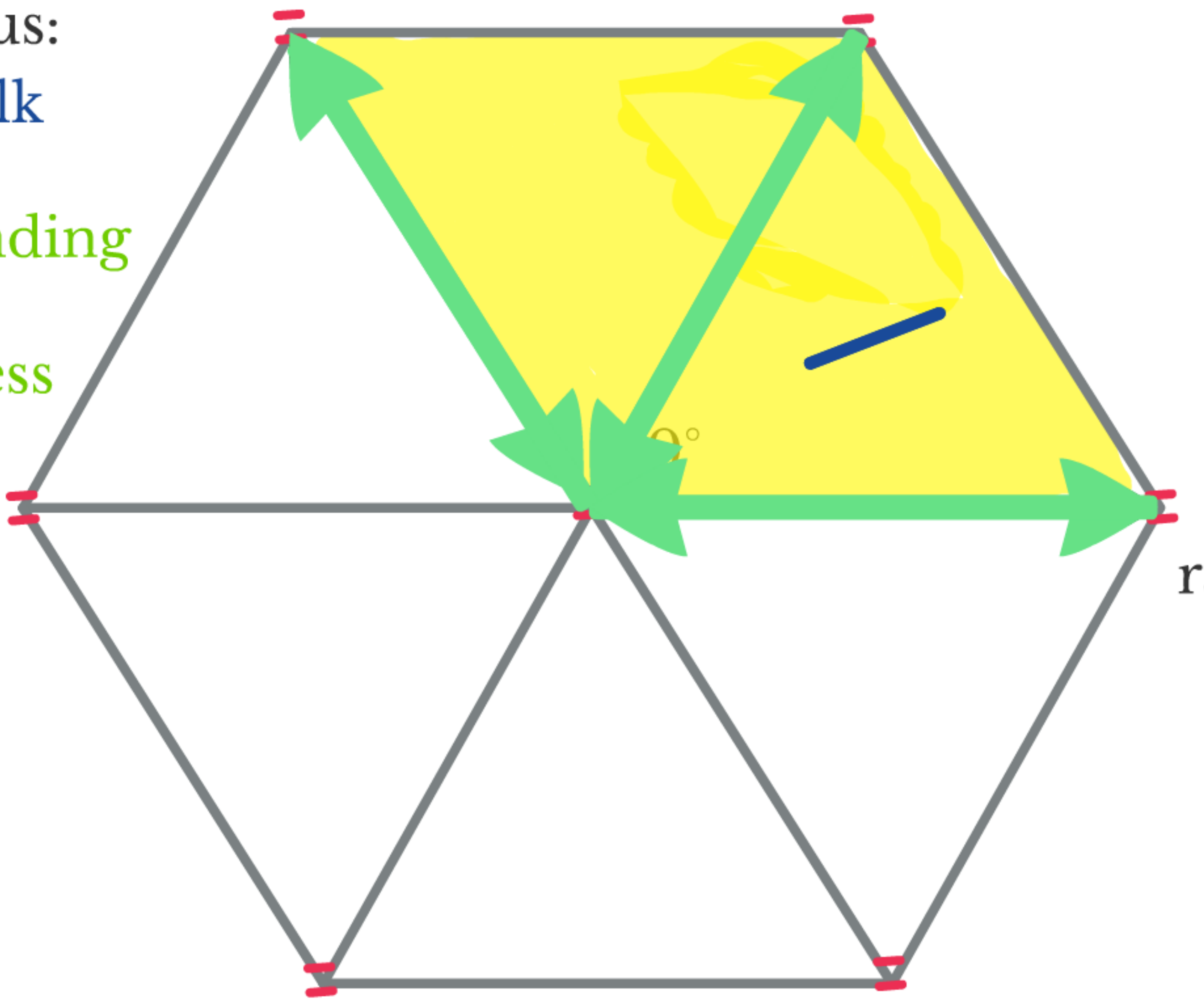

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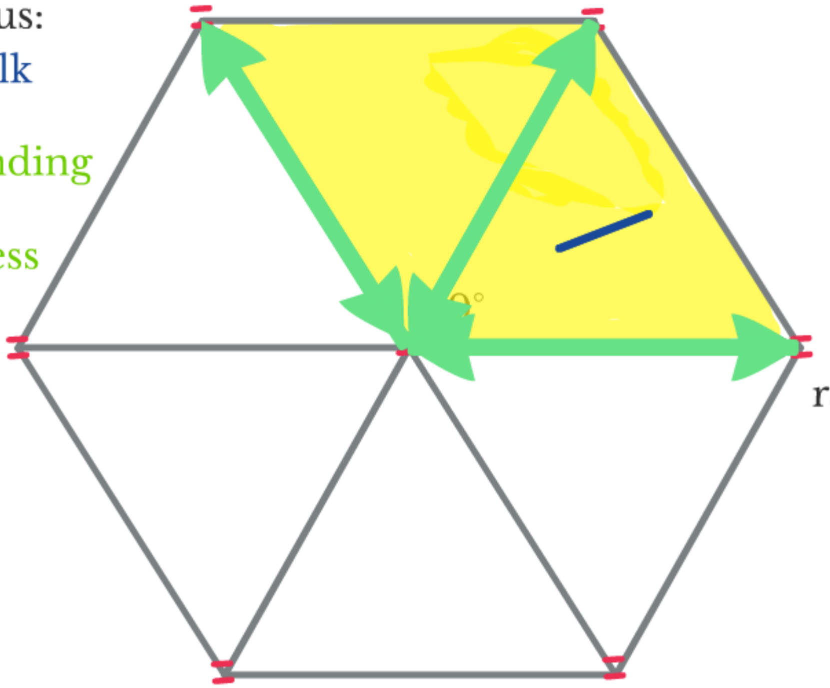

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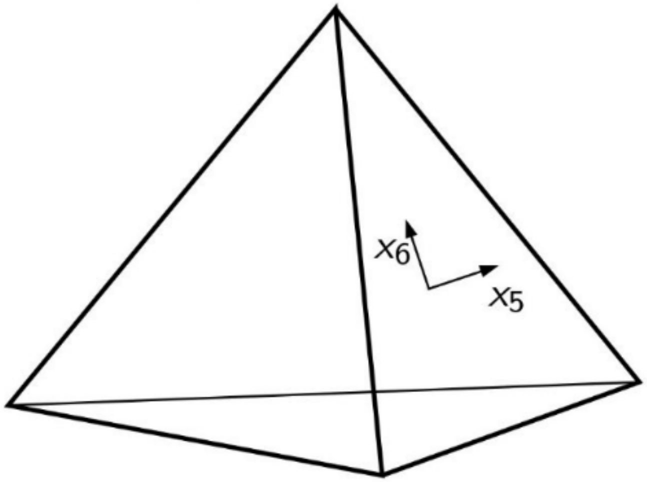
Representation content: trivial singlet and $\mathbf{3}$, $\bar{\mathbf{3}}$

Origin from String Theory?

Part 2: R

Lorentz symmetry in D>4

- ★ Strings in 10D
- ★ Lorentz group SO(1,9): 10D fermions vs. 10D vector
- ★ Compactification SO(1,9) → SO(1,3)×SO(6)
- ★ SO(6) breaks to rotational symmetry of compact space



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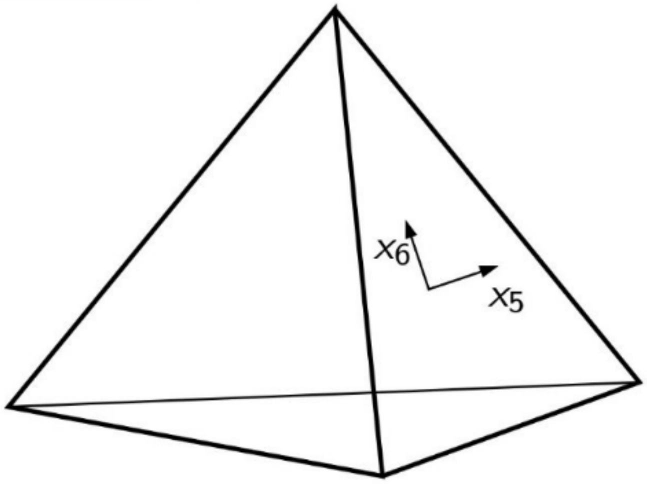

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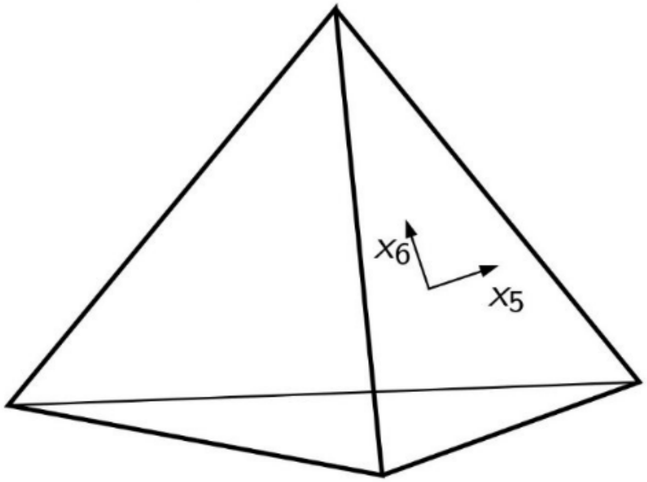


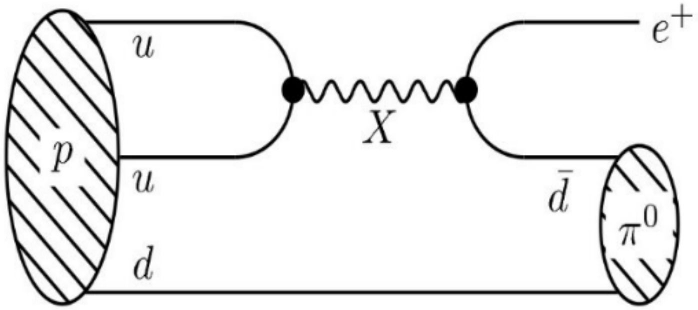
Dimension 6
Proton Decay

Grand Unified Theories in 4D

Dimension 6 Operators

$\mathcal{L} \supset g \sum_i \bar{\ell}_i \gamma^\mu X_\mu \bar{d}_i$ i.e. extra gauge boson exchange

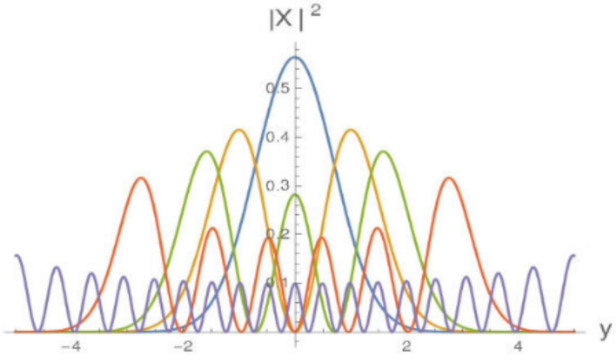
- Decay mode: $p \to \pi^0 e^+$



Smoking gun signature of GUTs ⇒ No!

- Attempt: ~~avoid this in extra dimensions~~

  Localize X boson wavefunction in extra dimension, $n = 0, 1, 2, 5, 20$

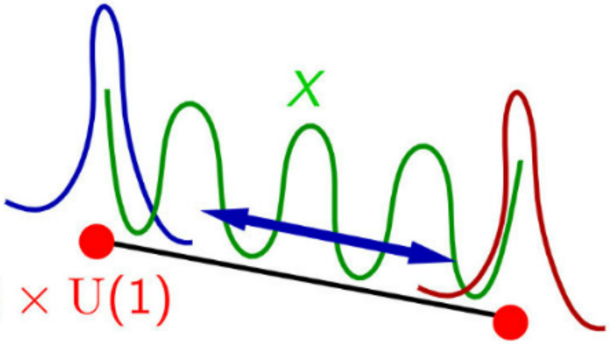


Hebecker, Unwin 2014

$$\Psi_1(x,y) \;=\; \begin{pmatrix} \ell_1(x,y) \\ \bar{d}_1(x,y) \end{pmatrix}$$

$\pi_1 = \mathbb{Z}_2$ & WL

$$\Psi_2(x,y) \;=\; \begin{pmatrix} \ell_2(x,y) \\ \bar{d}_2(x,y) \end{pmatrix}$$



$\mathrm{SU}(5) \times \mathrm{U}(1)$

$\mathrm{SU}(5) \times \mathrm{U}(1)$

Non-local GUT breaking and dimension 6 proton decay:

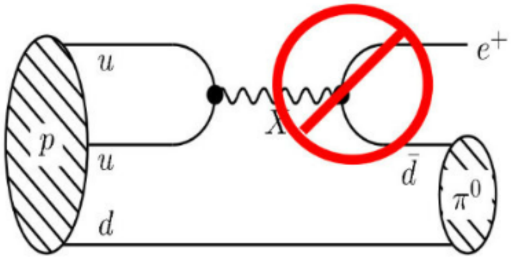
- Matter $\overline{\mathbf{5}}$–plets:

$$\Psi_1(x,y) \overset{\tau}{\mapsto} \Psi_1(x,y+\tau) \overset{!}{=} P\,\Psi_2(x,y)$$

- τ–invariant combinations

$$\ell(x,y) = \frac{1}{\sqrt{2}}\left[\ell_1(x,y) - \ell_2(x,y)\right]$$

$$\bar{d}(x,y) = \frac{1}{\sqrt{2}}\left[\bar{d}_1(x,y) + \bar{d}_2(x,y)\right]$$



- Orthogonal combinations projected out

$$\ell^{(\perp)}(x,y) = \frac{1}{\sqrt{2}}\left[\ell_1(x,y) + \ell_2(x,y)\right]$$

$$\bar{d}^{(\perp)}(x,y) = \frac{1}{\sqrt{2}}\left[\bar{d}_1(x,y) - \bar{d}_2(x,y)\right]$$

- $n_G = 6 \Rightarrow$ 3 generations

- X boson exchange

$$\mathcal{L}_{\text{eff}} \supset \int \mathrm{d}y\, g_{5\text{D}} \left[\bar{\ell}(x,y)\,\gamma^\mu\, X_\mu(x,y)\, \bar{d}^{(\perp)}(x,y) + \bar{\ell}^{(\perp)}(x,y)\,\gamma^\mu\, X_\mu(x,y)\,\bar{d}(x,y)\right]$$

- $\mathcal{L}_{\text{eff}} \not\supset \bar{\ell}\,\gamma^\mu\, X_\mu\, \bar{d} \;\Rightarrow$ No X boson exchange!

Mütter, Ratz and P.V. 2016

Summary

Classes of discrete symmetries:

R vs. non-R

GS mechanism

Abelian vs. non-Abelian

anomalous vs. non-anomalous

with or without CP

String origin of discrete symmetries:

R symmetry as discrete remnant of higher dimensional Lorentz symmetry

Flavor groups from strings splitting & joining
(partial) gauge origin at R=1

Proton decay at mass dim. 4, 5 and 6

Thank you!