Some entertaining aspects of Multiple Parton Interactions Physics

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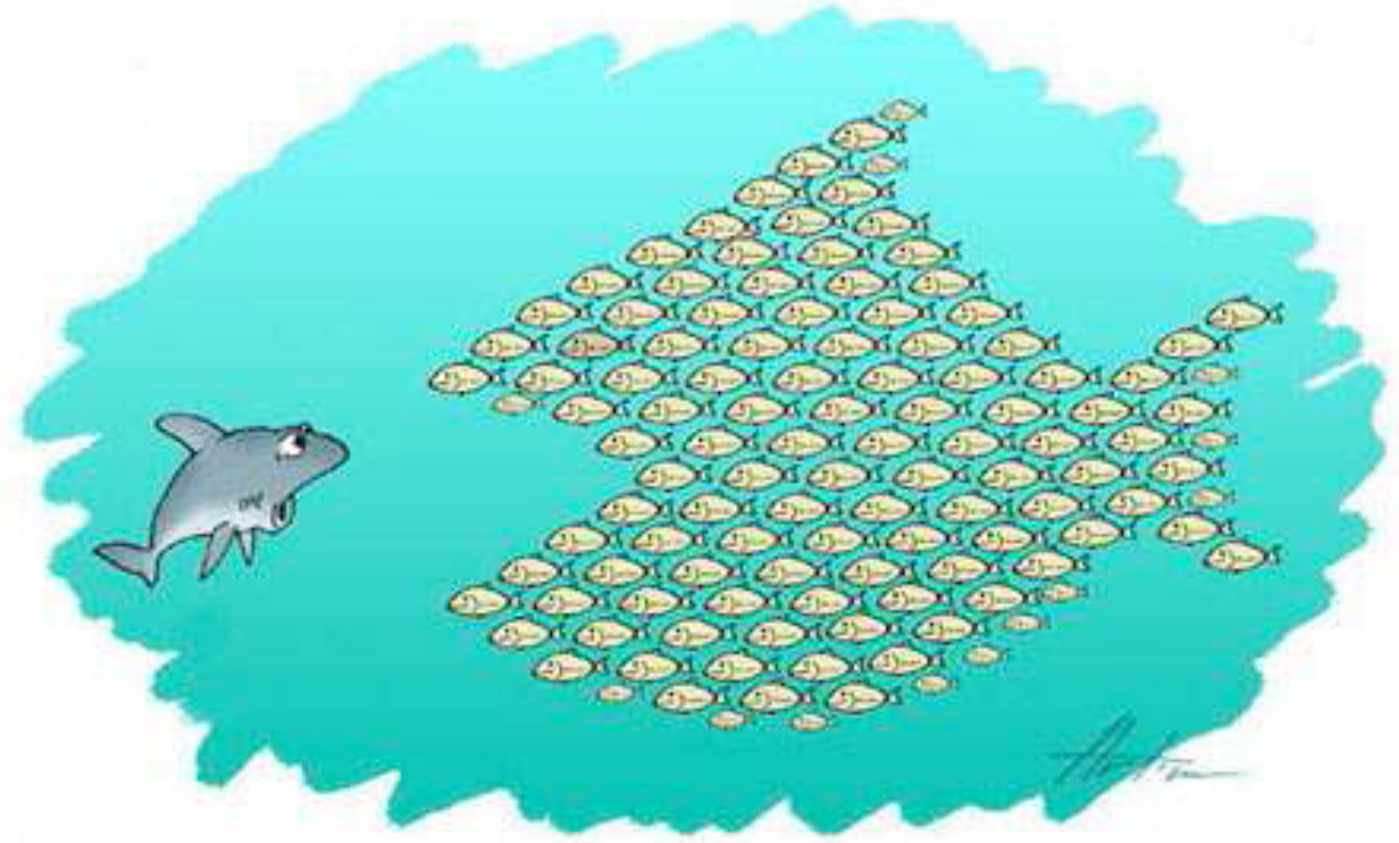
DESY 07 2011

Multi-Parton Interactions



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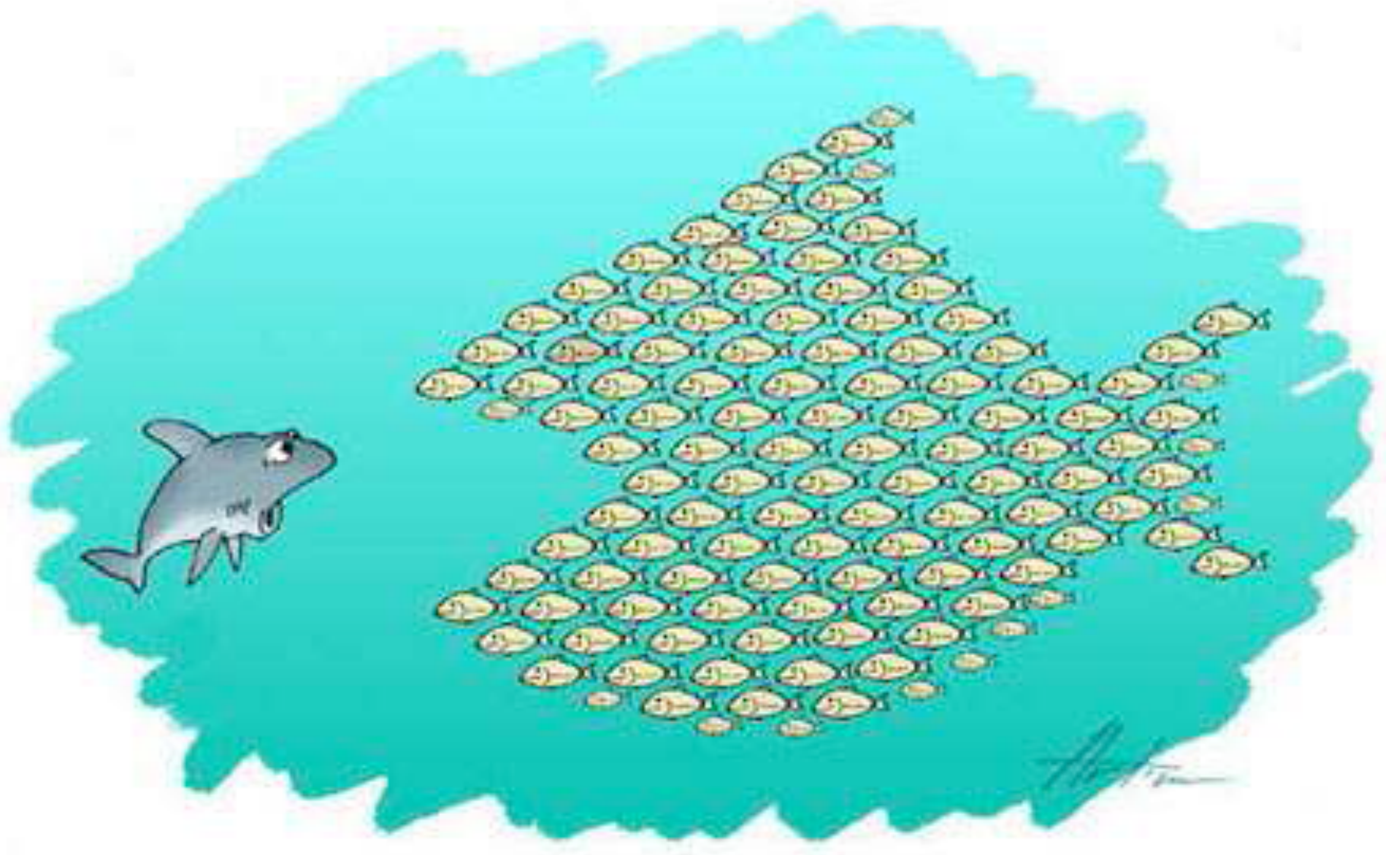

WORK IN COLLABORATION WITH B.BLOK, L.FRANKFURT AND M.STRIKMAN

The Four jet production at LHC and Tevatron in QCD.
Phys. Rev. D83 : 071501, 2011; e-Print: **arXiv:1009.2714** [hep-ph]

pQCD physics of multiparton interactions.
e-Print: **arXiv:1106.5533** [hep-ph]

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prelude

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4-jet production in the back-to-back kinematics

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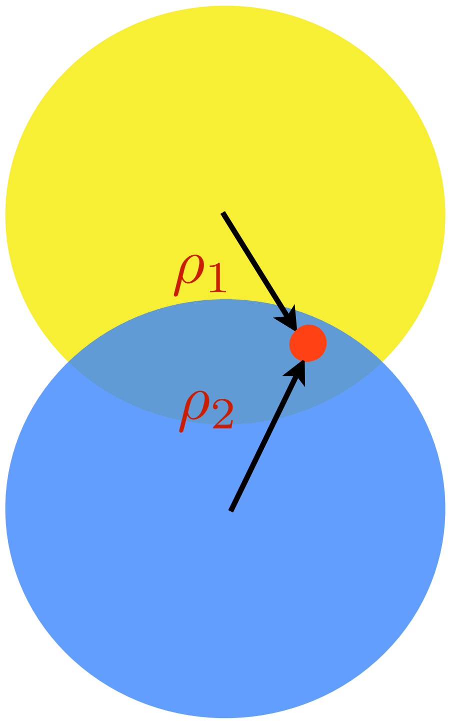

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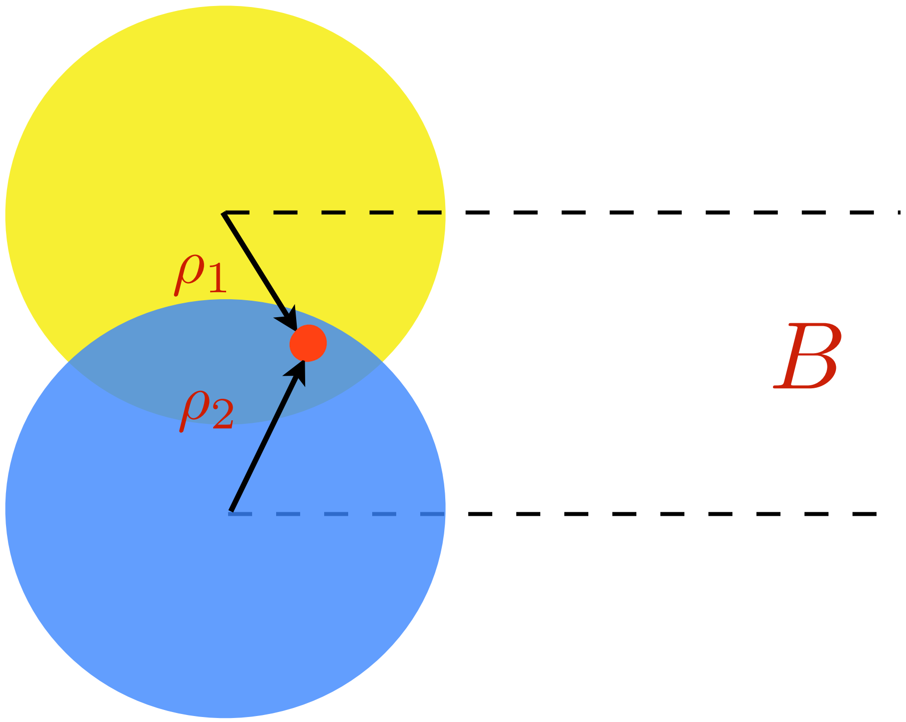

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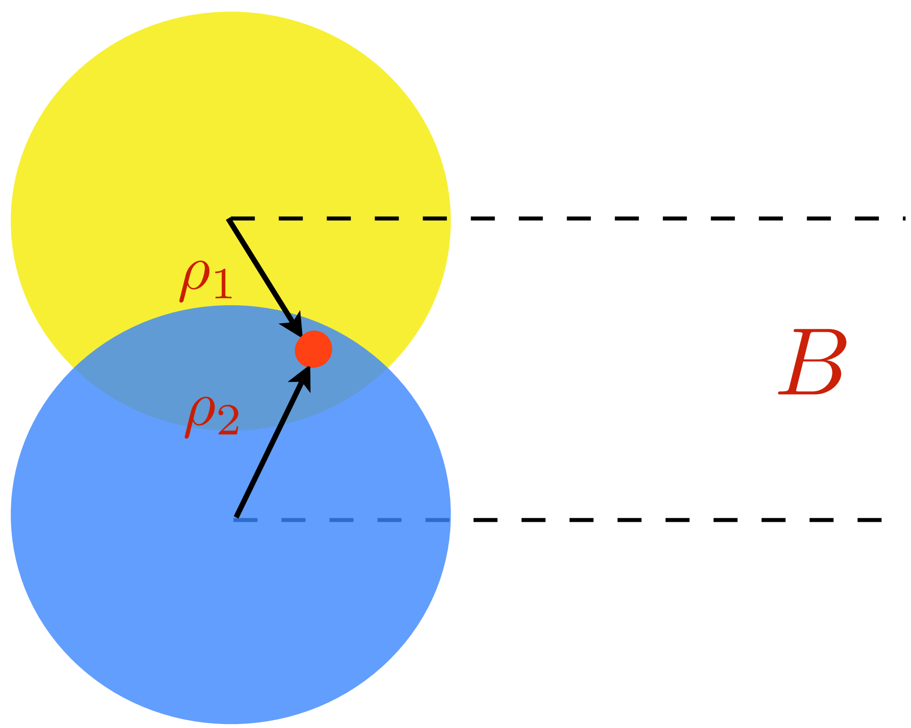

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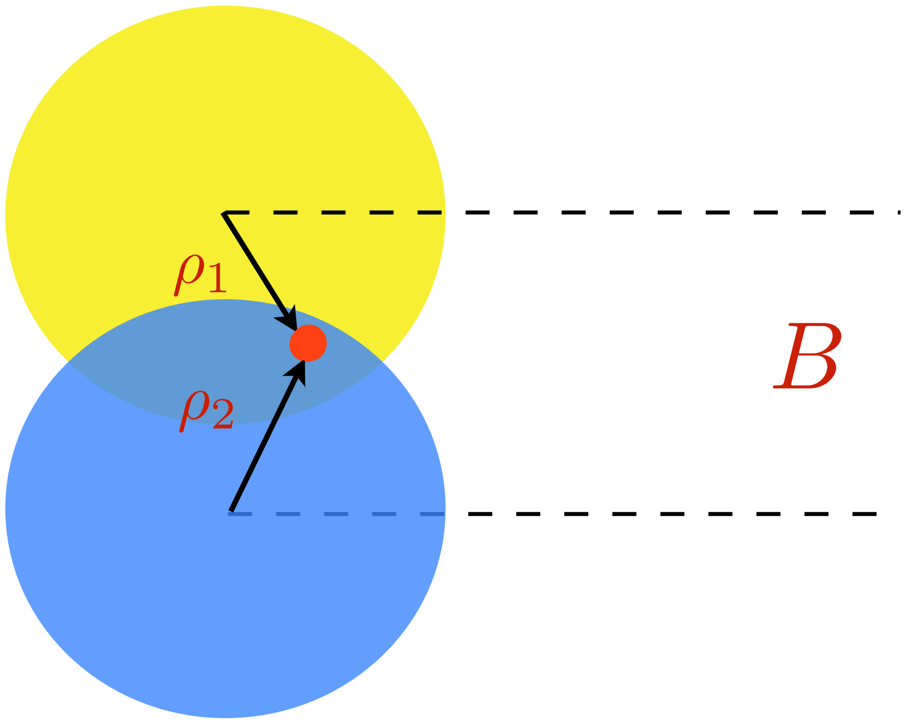

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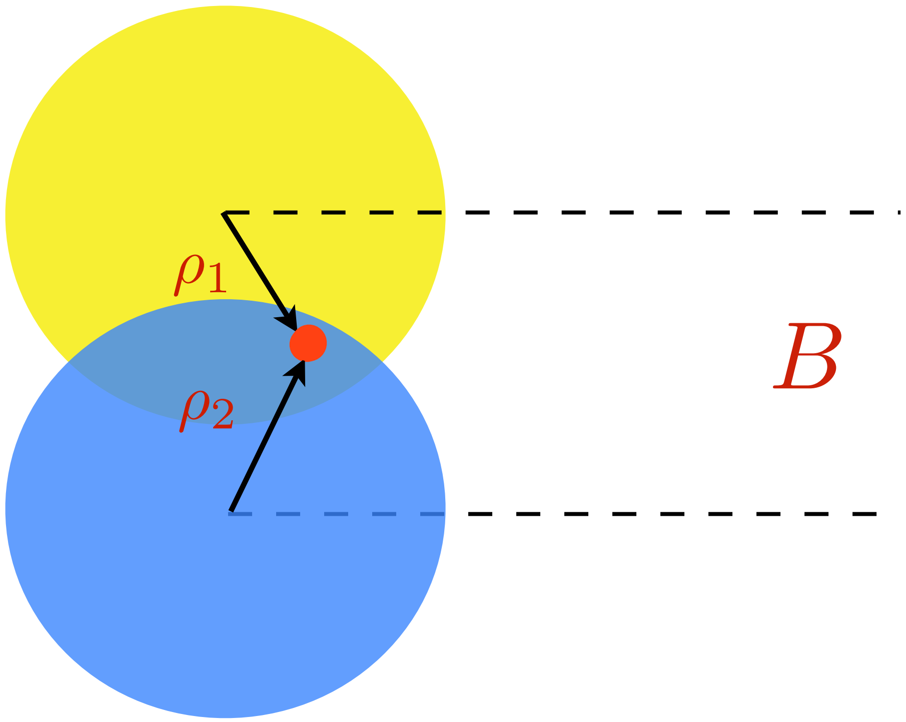

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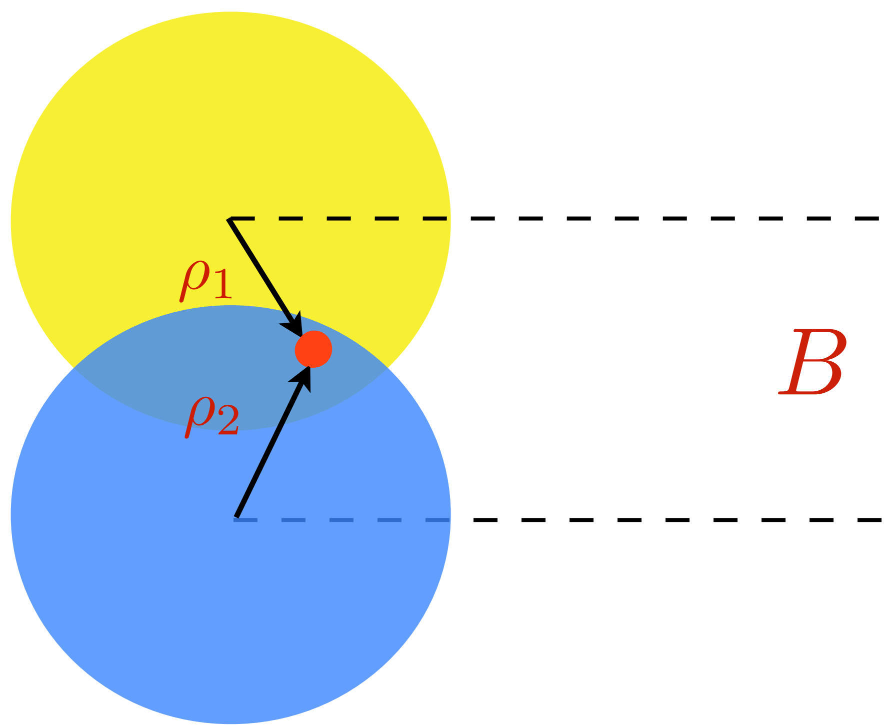

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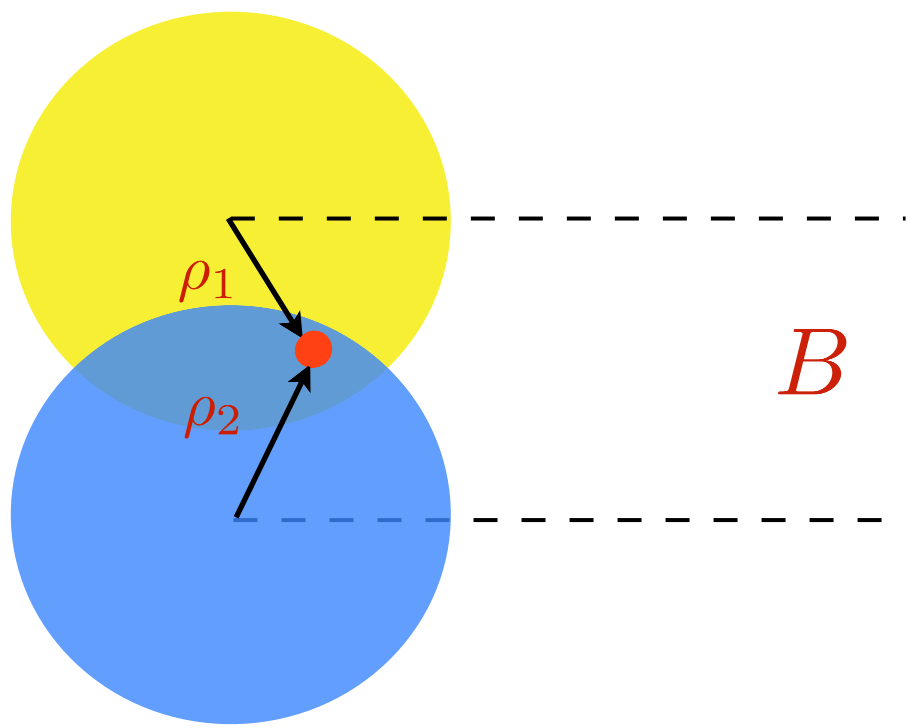

$$\sigma_2 = \int d^2\rho_1 d^2 B \; f(x_1, \vec{\rho}_1, p^2) f(x_2, \vec{B} - \vec{\rho}_1, p^2) \; \frac{d\sigma^h}{d\hat{t}} d\hat{t}$$

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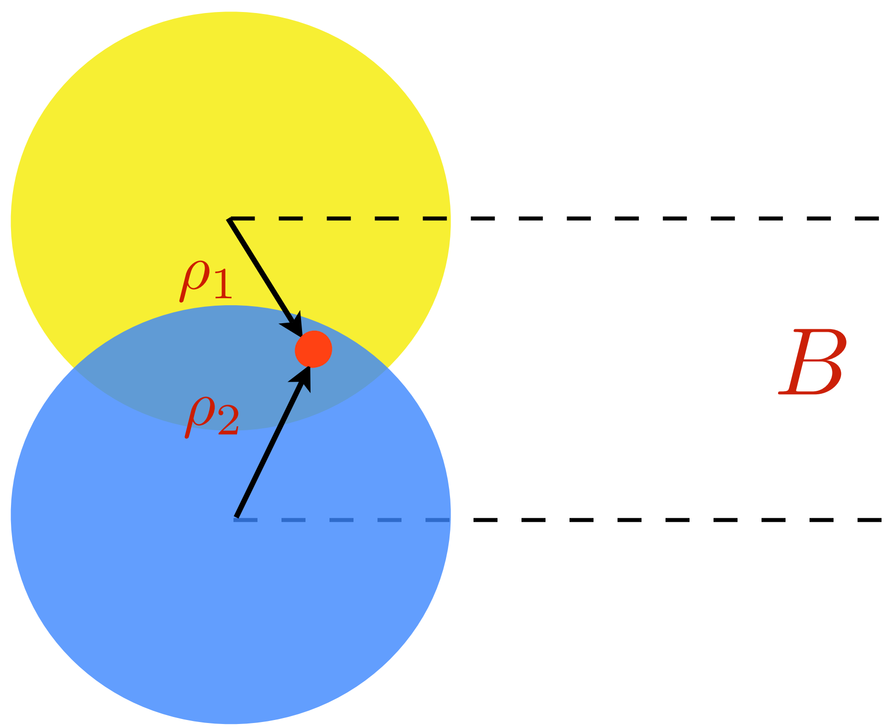

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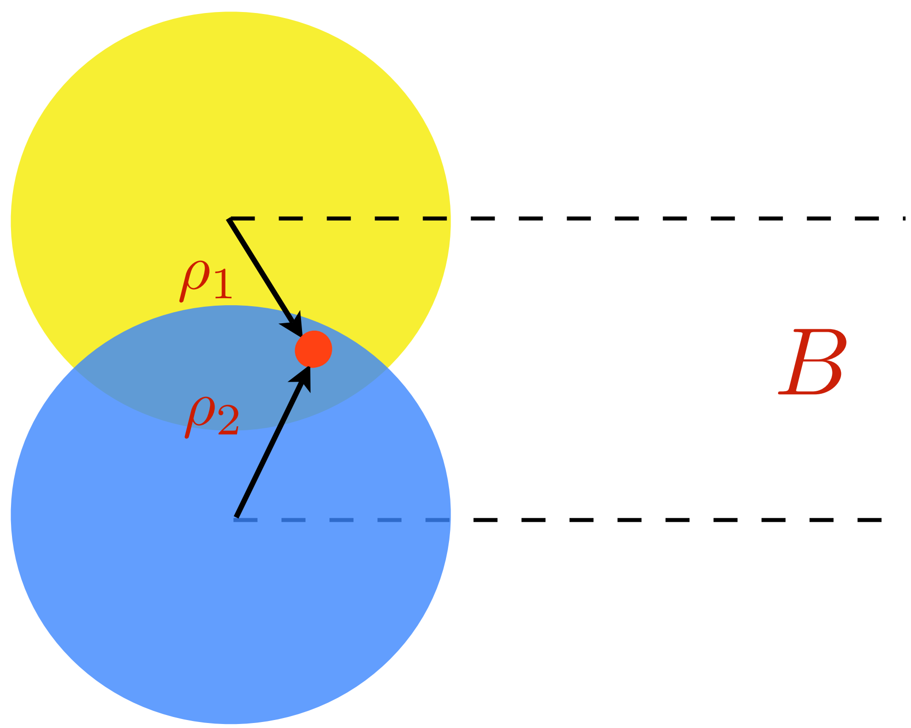

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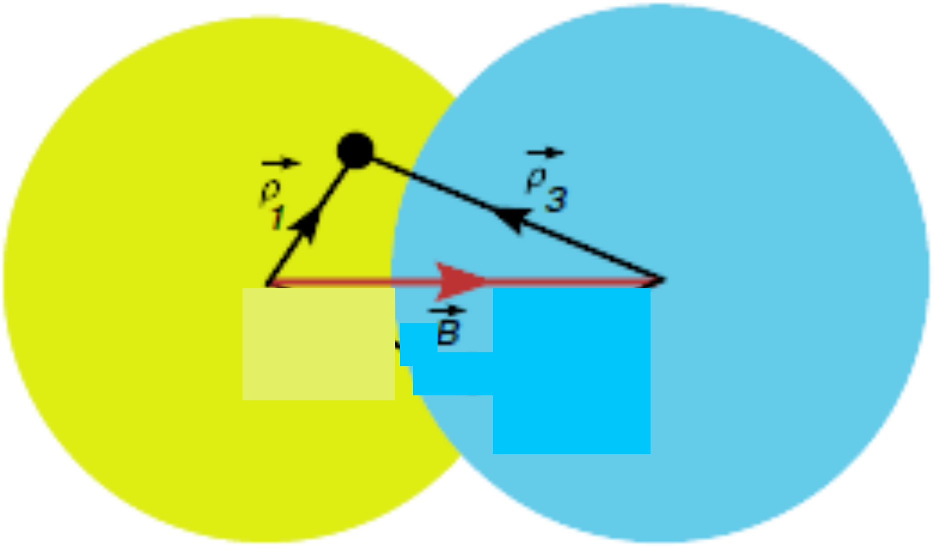

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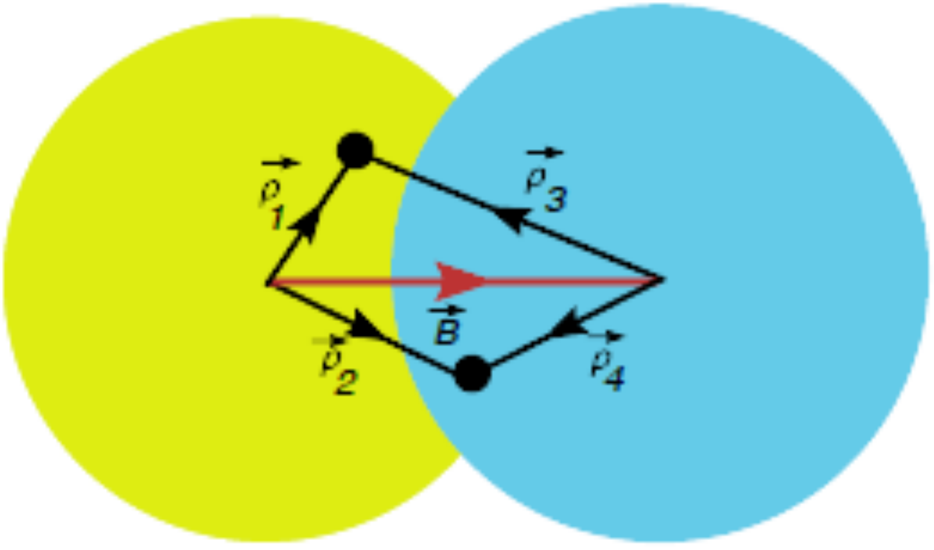

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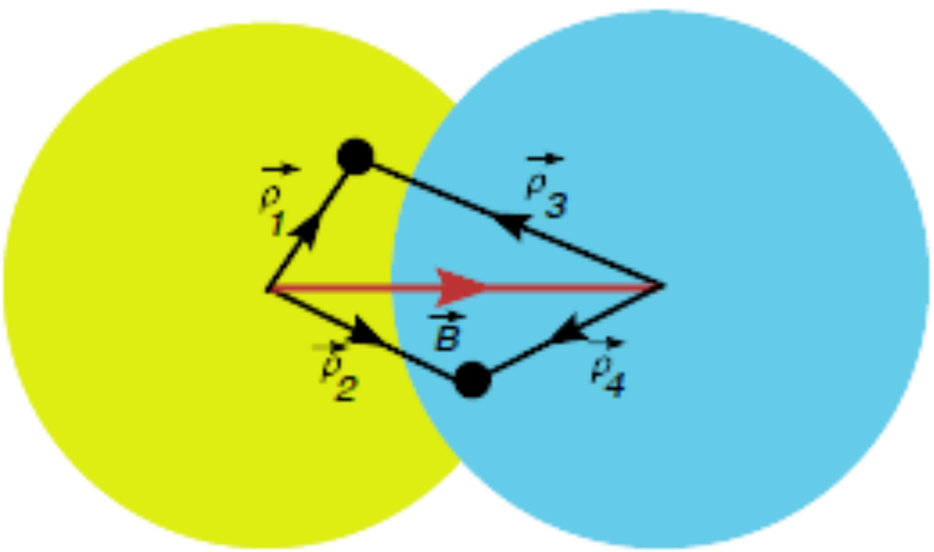

Let us see, what difference does it make to our formulae

Multi-parton wave function

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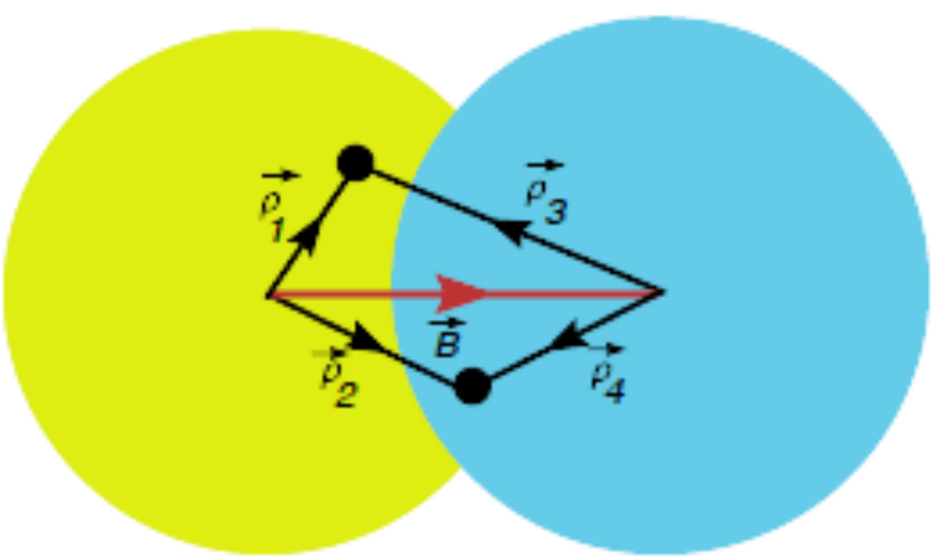

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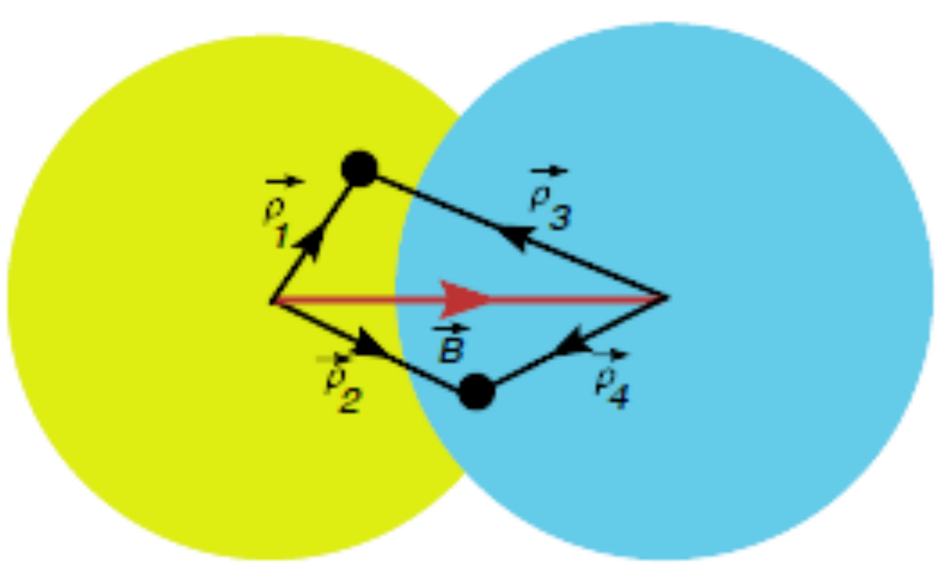

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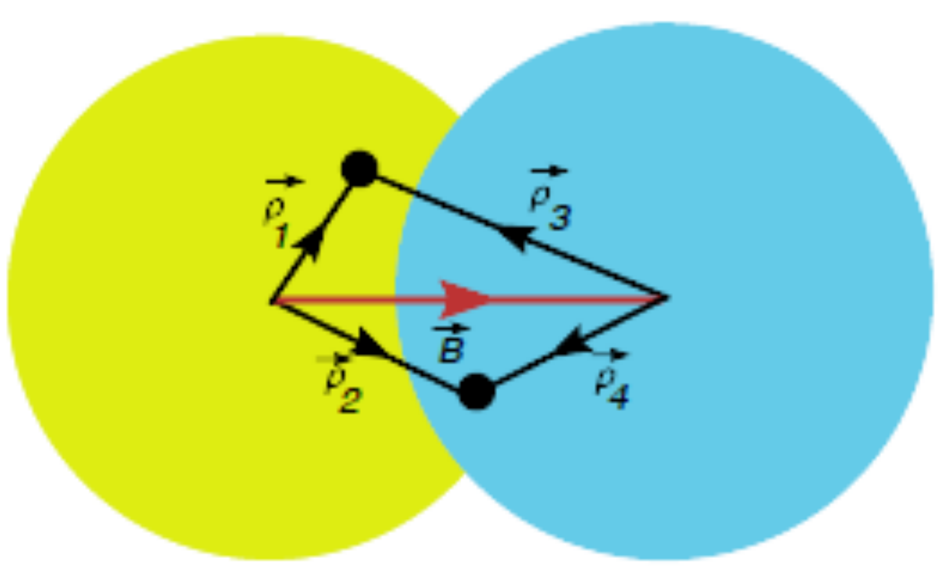

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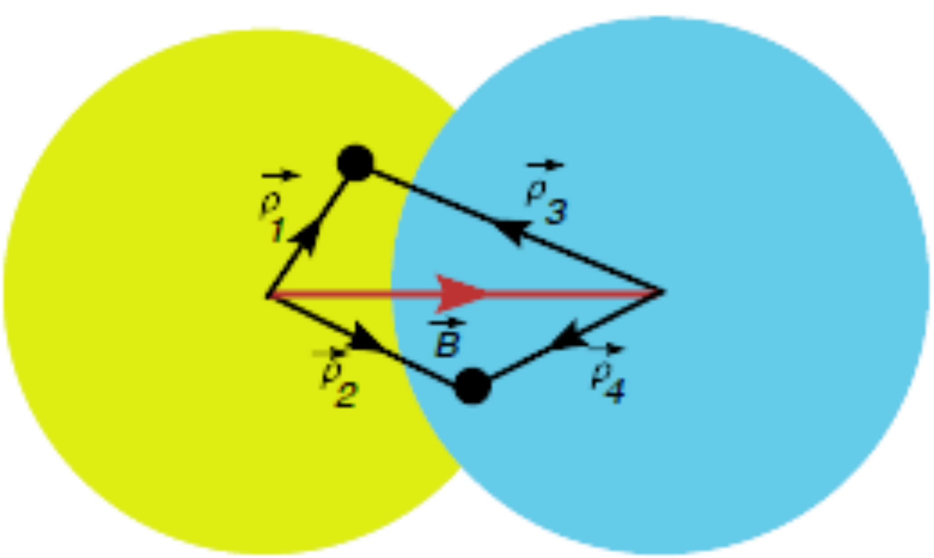

multi-partons

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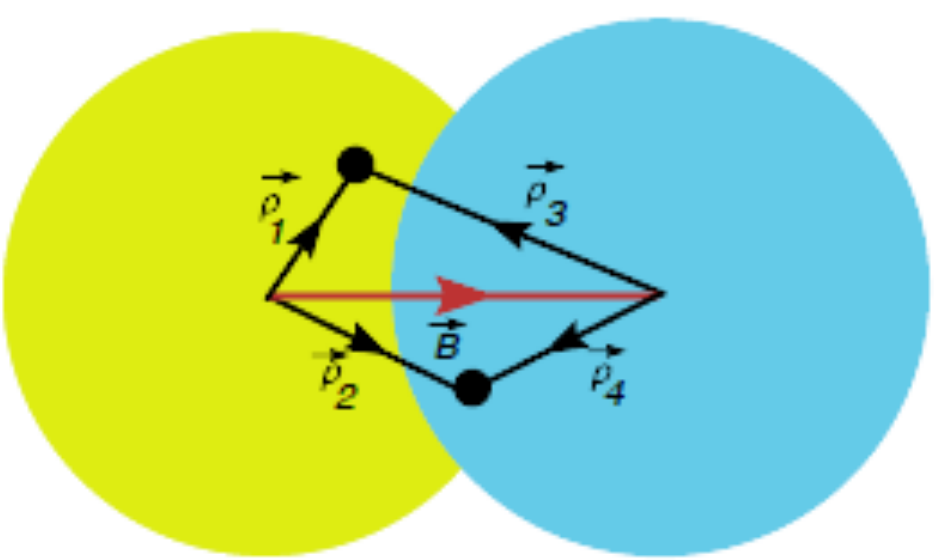
$$\psi_n \ (x_1, \vec{\rho}_1, x_2, \vec{\rho}_2, ...) = \int \prod_{i=1}^{i=n} \frac{d^2 k_i}{(2\pi)^2} \exp(i \sum_{i=1}^{i=n} \vec{k}_i \vec{\rho}_i) \ \psi_n(x_1, \vec{k}_1, x_2, \vec{k}_2, ..)(2\pi)^2 \delta(\sum \vec{k}_i)$$

Inclusive 2-parton probability distribution in the impact parameter space :

$$D \ (x_1, x_2, \vec{\rho}_1, \vec{\rho}_2) = \sum_{n=3}^{n=\infty} \int \prod_{i\geq 3}^{i=n} [dx_i d^2\rho_i] \, \psi_n(x_1, \vec{\rho}_1, x_2, \vec{\rho}_2, ...x_i, \vec{\rho}_i,) \ \psi_n^+(x_1, \vec{\rho}_1, x_2, \vec{\rho}_2, ..., x_i, \vec{\rho}_i, ...)\delta(\sum_{i=1}^{i=n} x_i \vec{\rho}_i)$$

Independent impact parameter integration ⟹ equality of parton momenta in ψ and $\psi^\dagger$

$$k_\perp \ = \ k'_\perp$$



$\rho_1 + \rho_2$

multi-partons

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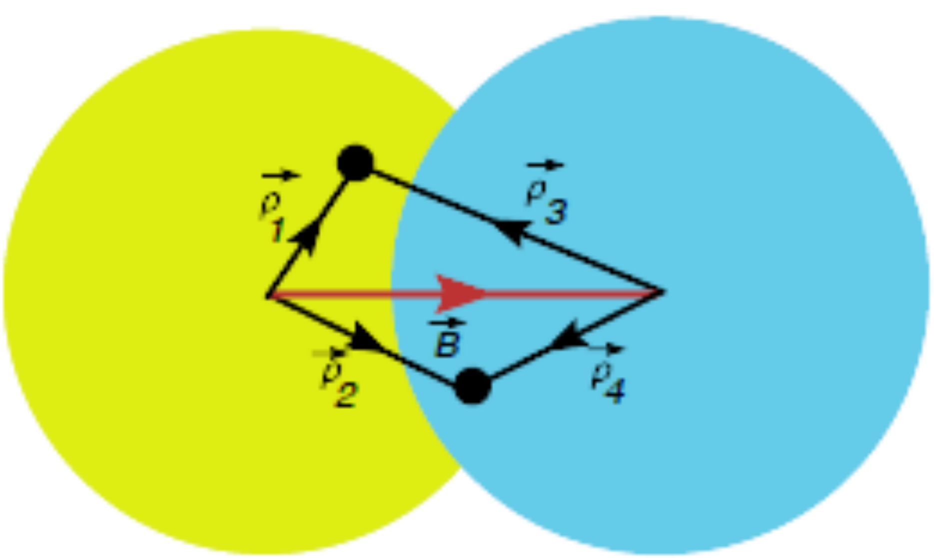

$$\rho_1 + \rho_2 \Longrightarrow k'_1 - k_1 = -(k'_2 - k_2) \equiv \Delta$$

multi-partons

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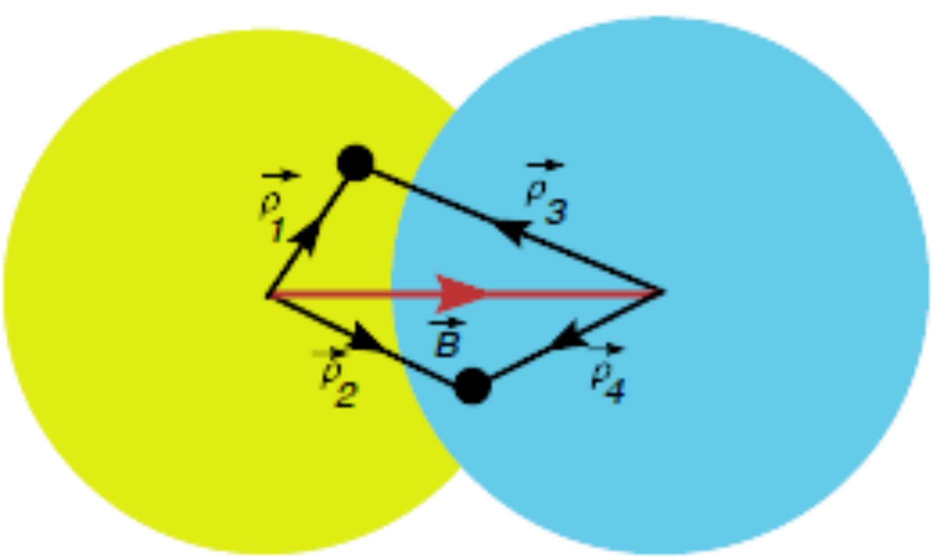

$\rho_1 + \rho_2$

$\rho_3 + \rho_4$

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multi-partons

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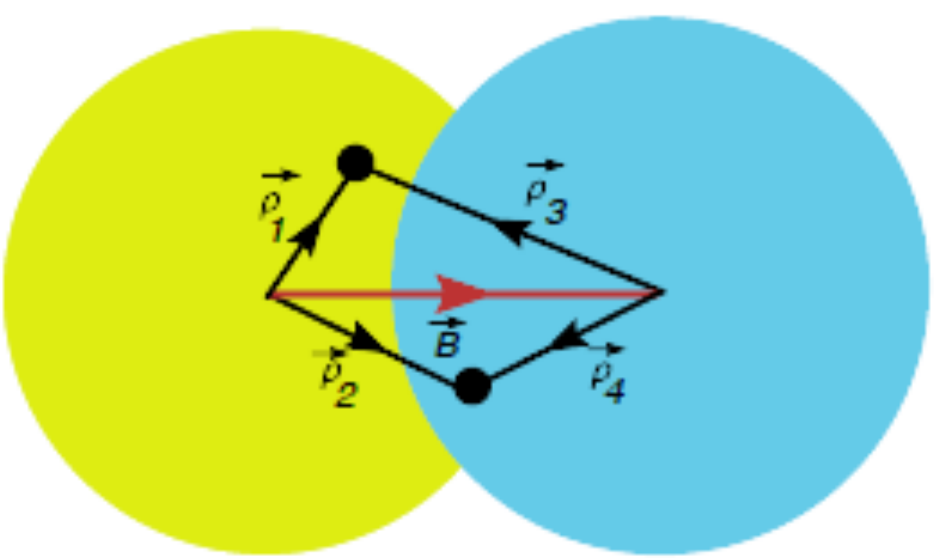

$\rho_1 + \rho_2$ ⇒ $k'_1 - k_1 = -(k'_2 - k_2) \equiv \Delta$

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Multi-parton wave function

$$\psi_n\ (x_1,\vec{\rho}_1,x_2,\vec{\rho}_2,...) = \int\prod_{i=1}^{i=n}\frac{d^2k_i}{(2\pi)^2}\exp(i\sum_{i=1}^{i=n}\vec{k}_i\vec{\rho}_i)\ \psi_n(x_1,\vec{k}_1,x_2,\vec{k}_2,..)(2\pi)^2\delta(\sum\vec{k}_i)$$

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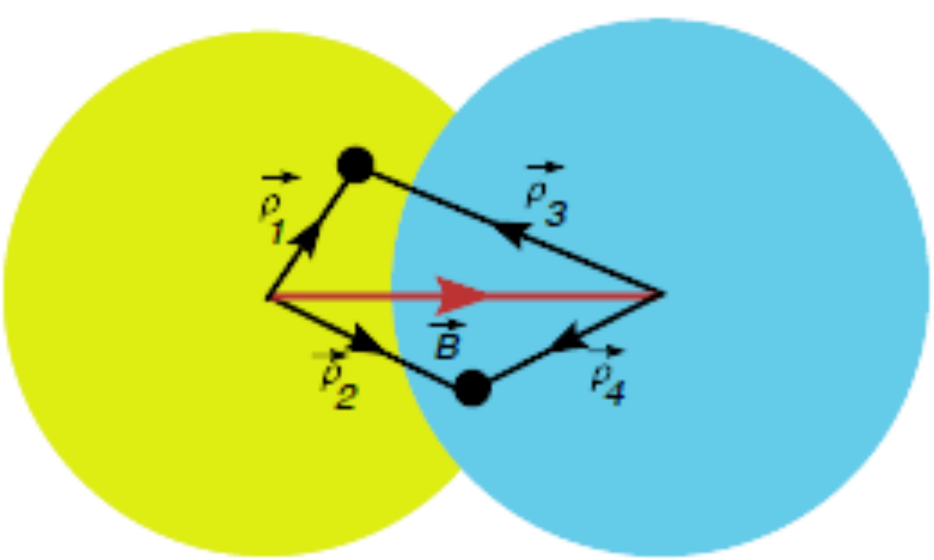

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$(\rho_1-\rho_2)+(\rho_3-\rho_4)$

multi-partons

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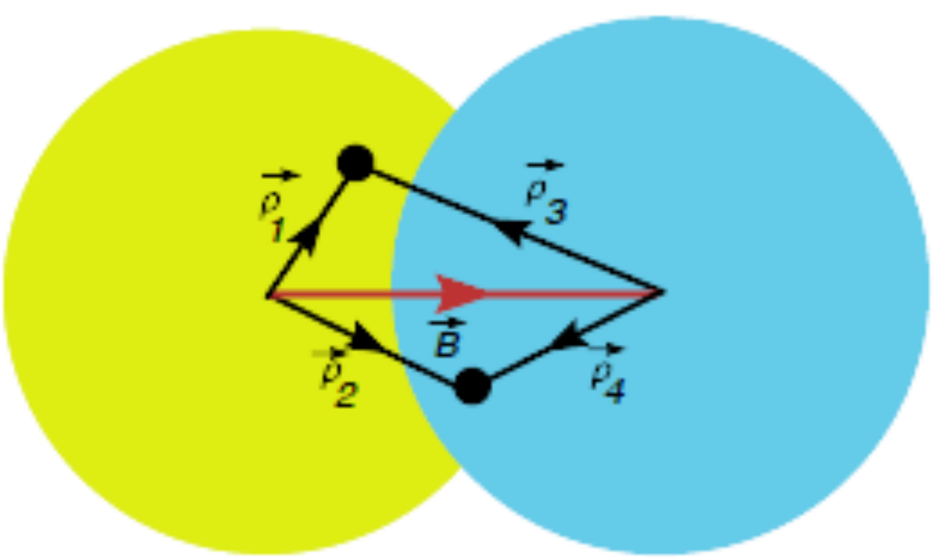

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Independent impact parameter integration → equality of parton momenta in ψ and $\psi^\dagger$

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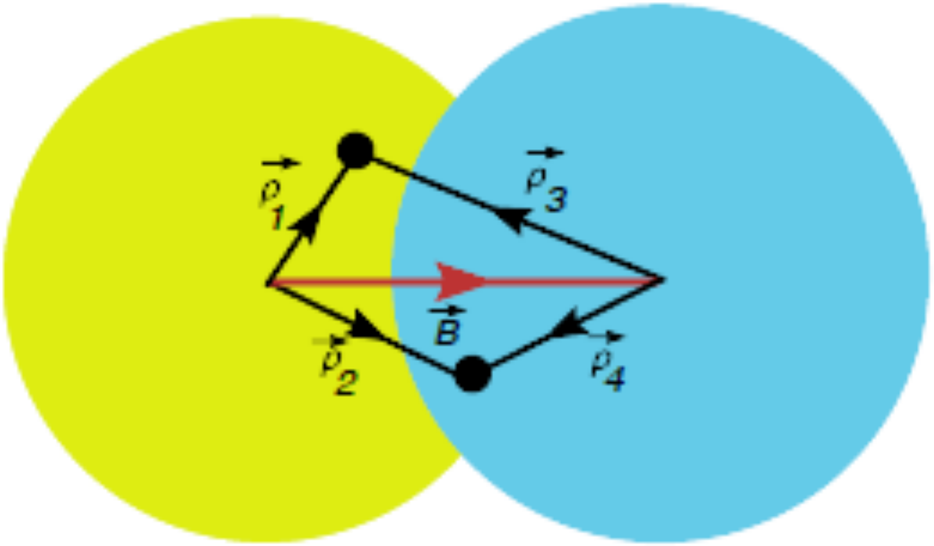

$\rho_1+\rho_2$ → $k'_1-k_1=-(k'_2-k_2)\equiv\Delta$

$\rho_3+\rho_4$ → $k'_3-k_3=-(k'_4-k_4)\equiv\widetilde{\Delta}$

$(\rho_1-\rho_2)+(\rho_3-\rho_4)$ → $\Delta=-\widetilde{\Delta}$

$$\delta\Big((\rho_1-\rho_2)-(\rho_3-\rho_4)\Big)$$

multi-partons

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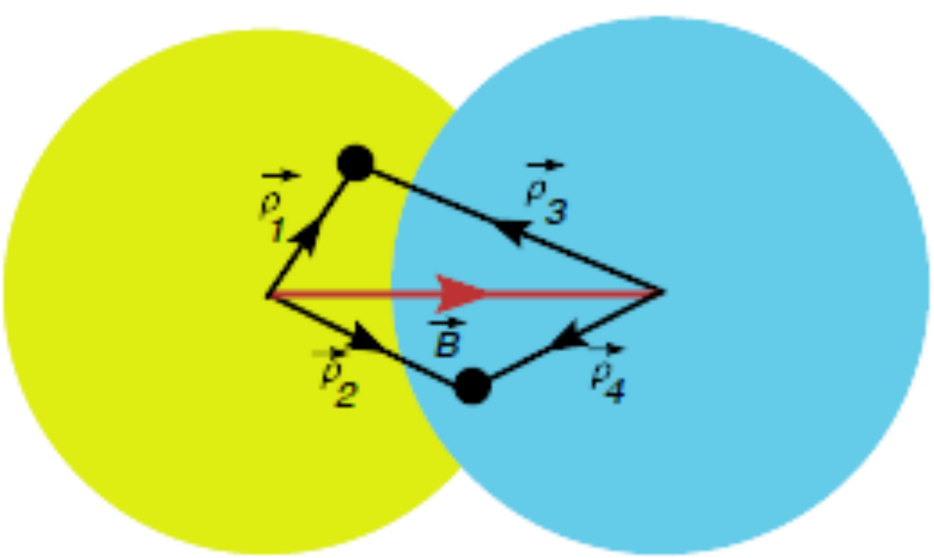

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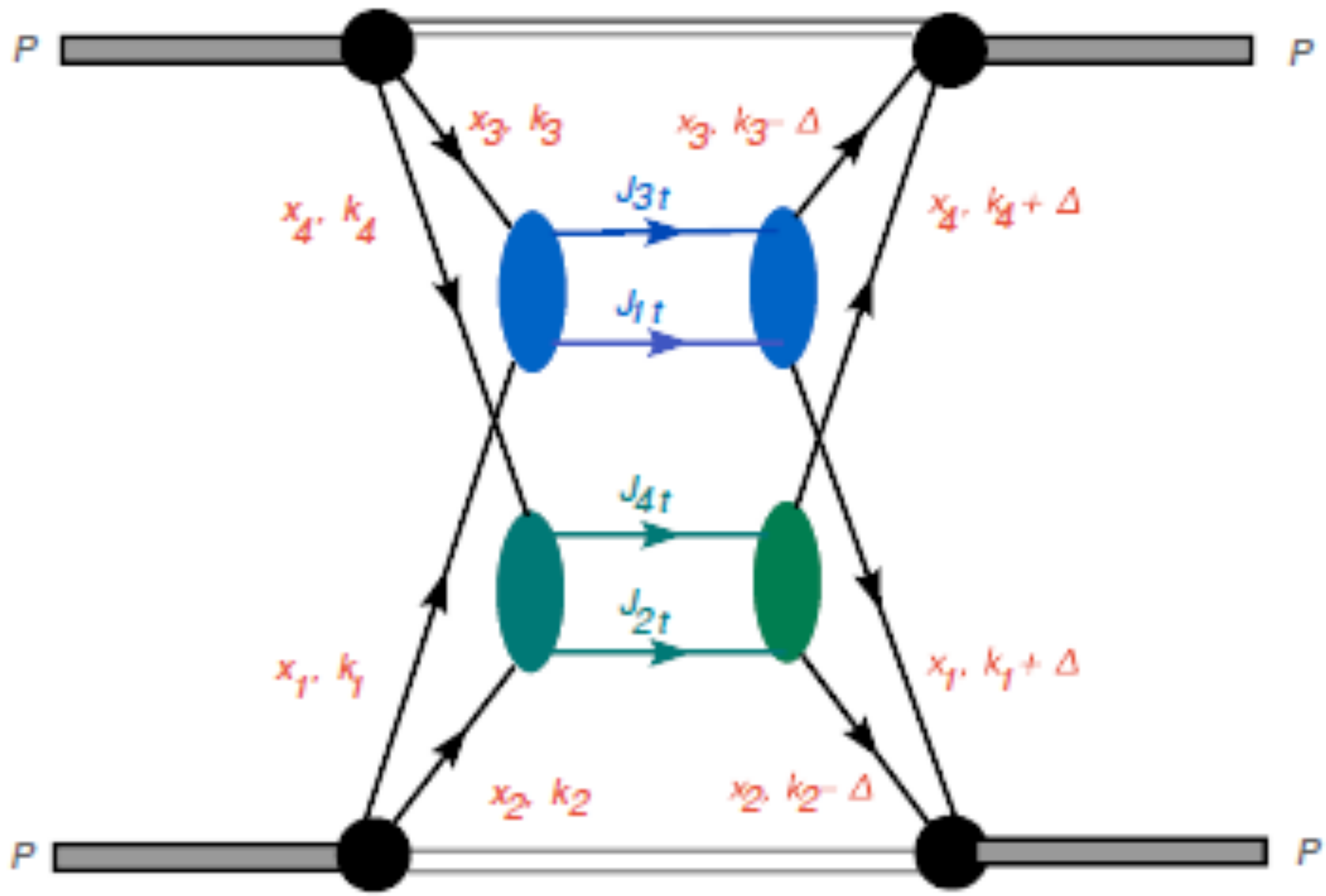
$\rho_3 + \rho_4$ ⟹ $k'_3 - k_3 = -(k'_4 - k_4) \equiv \tilde{\Delta}$

$(\rho_1 - \rho_2) + (\rho_3 - \rho_4)$ ⟹ $\Delta = -\tilde{\Delta}$

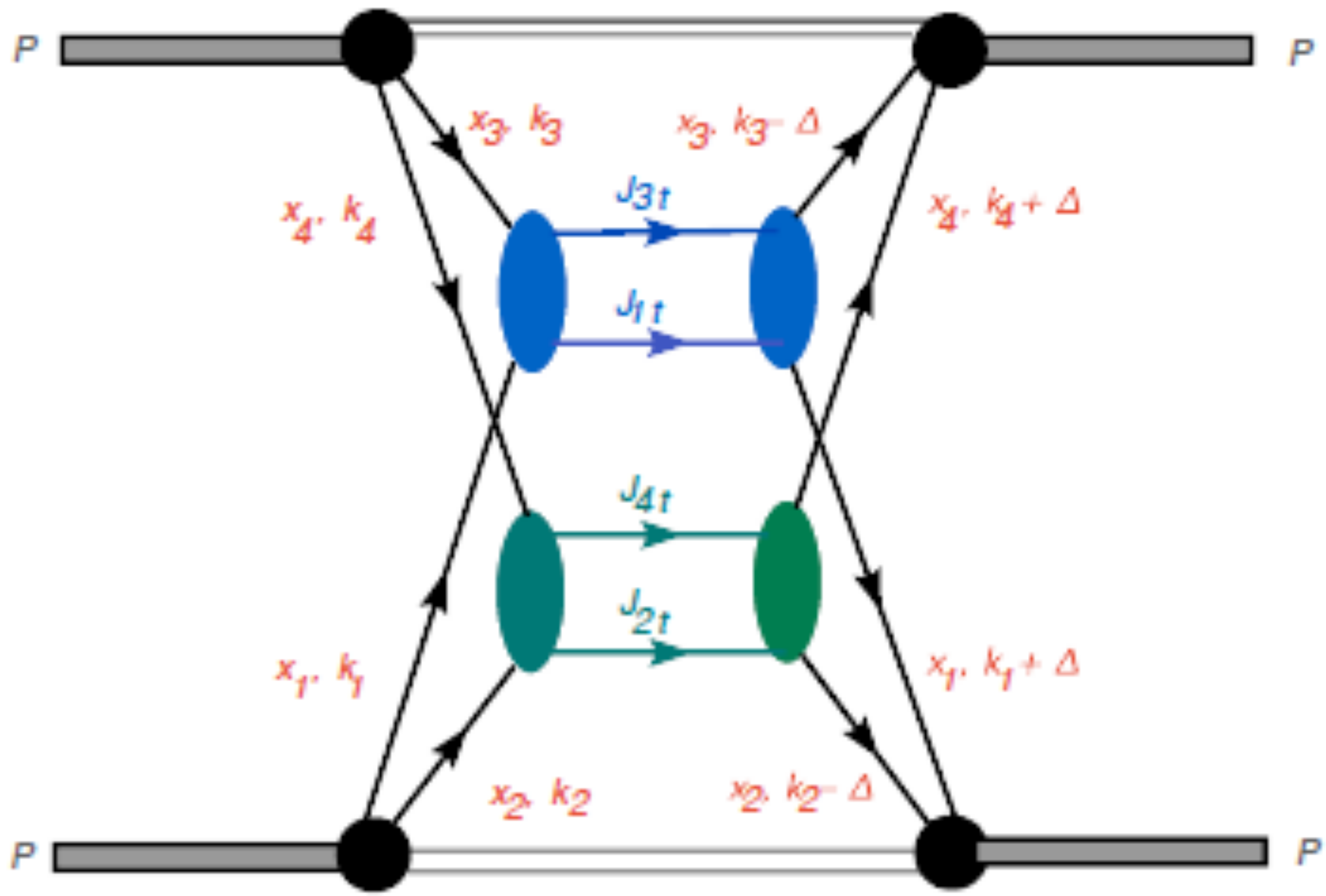
$\delta\big((\rho_1 - \rho_2) - (\rho_3 - \rho_4)\big)$ ⟹ $\vec{\Delta}$ ***arbitrary***

4-parton collision

4-parton collision



4-parton collision



In order to be able to trace the *relative distance between the partons*, one has to use the mixed *longitudinal momentum – impact parameter* representation which, in the momentum language, reduces to introduction of a **mismatch** between the transverse momentum of the parton in the ***amplitude*** and that of the same parton in the ***amplitude conjugated***.

4-parton cross section

4-parton cross section

$$\int \frac{d^2\vec{\Delta}}{(2\pi)^2} D_a(x_1, x_2; \vec{\Delta})\, D_b(x_3, x_4; -\vec{\Delta})$$

4-parton cross section

$$\frac{1}{S} = \int \frac{d^2\vec{\Delta}}{(2\pi)^2} D_a(x_1, x_2; \vec{\Delta})\, D_b(x_3, x_4; -\vec{\Delta})$$

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S - effective parton interaction area

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S - effective parton interaction area

$$\frac{d\sigma(x_1, x_2, x_3, x_4)}{d\hat{t}_1 \, d\hat{t}_2} = \frac{d\sigma^{13}}{d\hat{t}_1} \frac{d\sigma^{24}}{d\hat{t}_2}$$

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D is a ***generalized double parton distribution*** - a new object we know little about.

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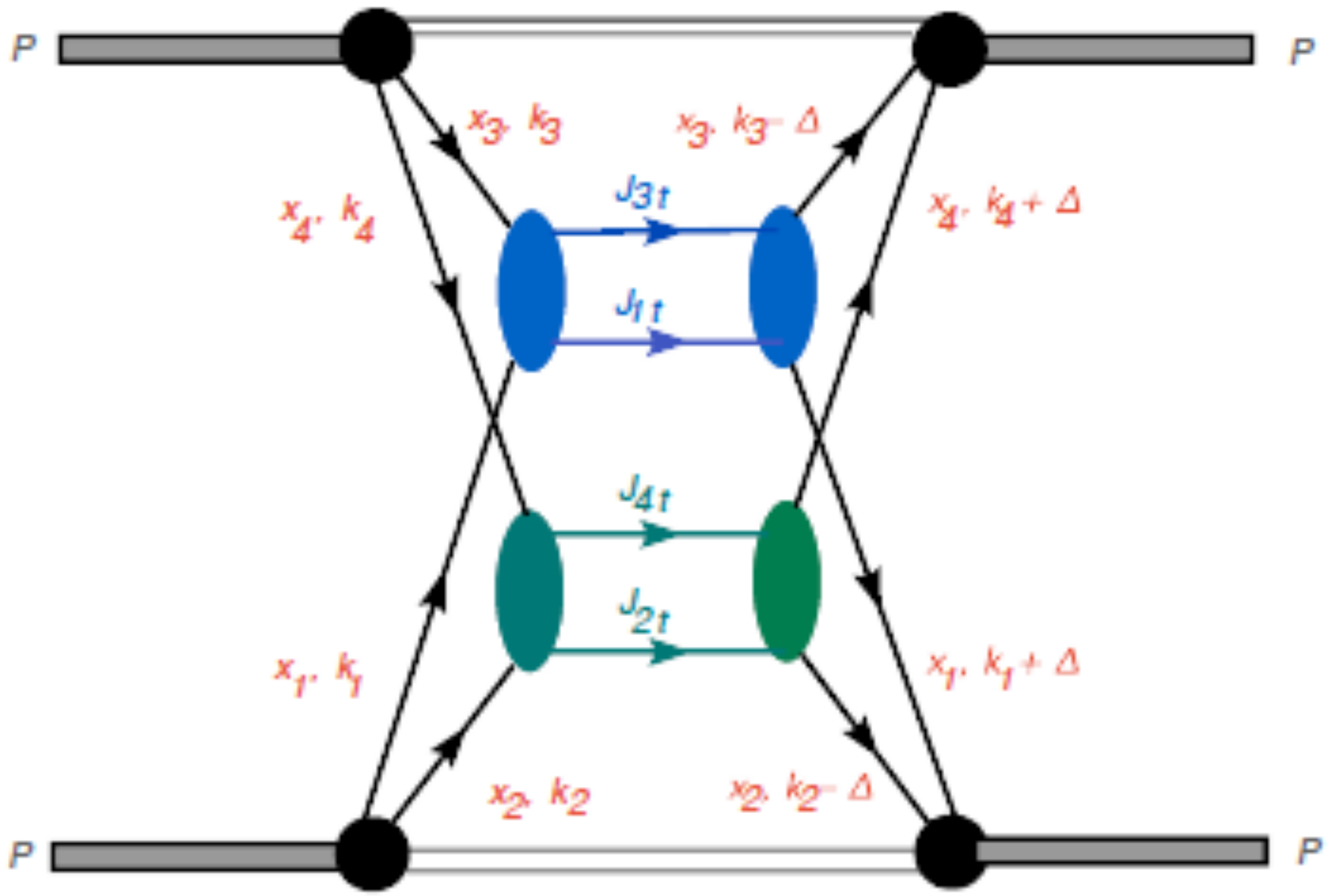
D is a ***generalized double parton distribution*** - a new object we know little about.

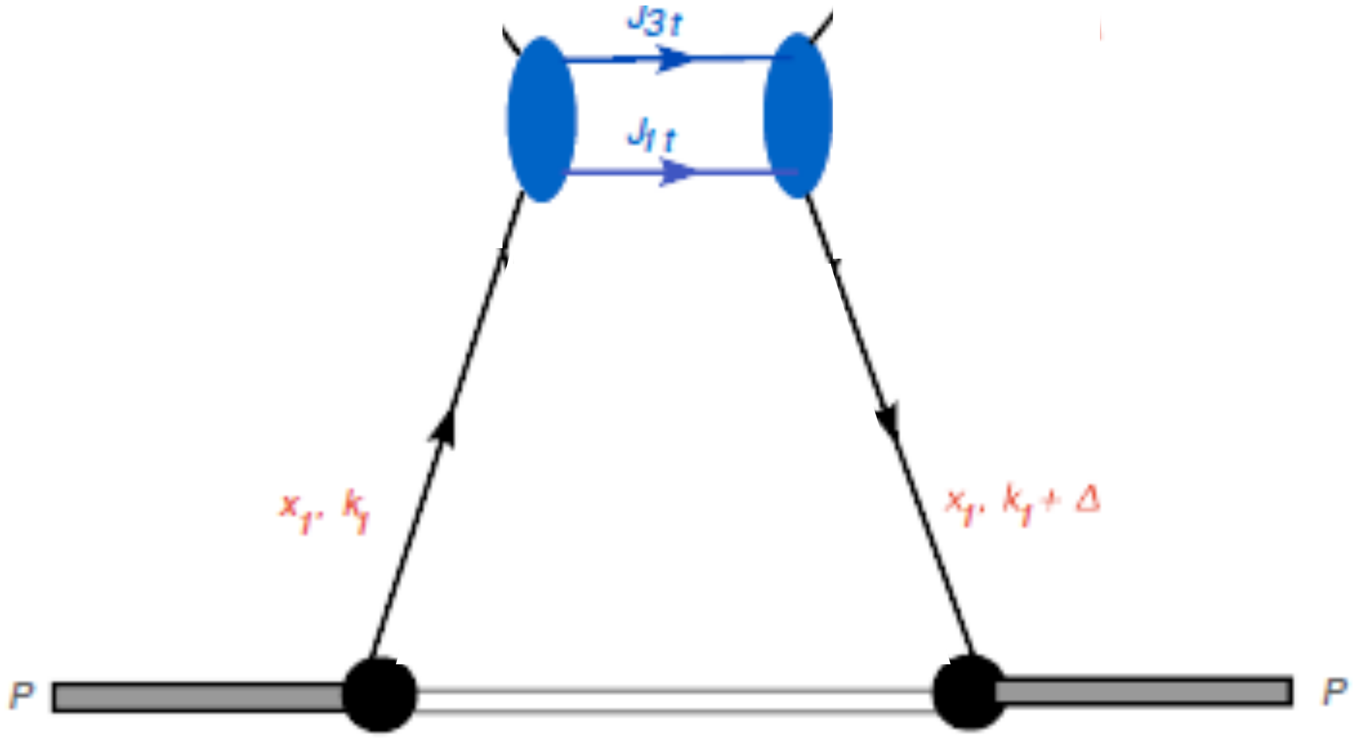
Can it be modeled, for lack of anything better ?

GPD

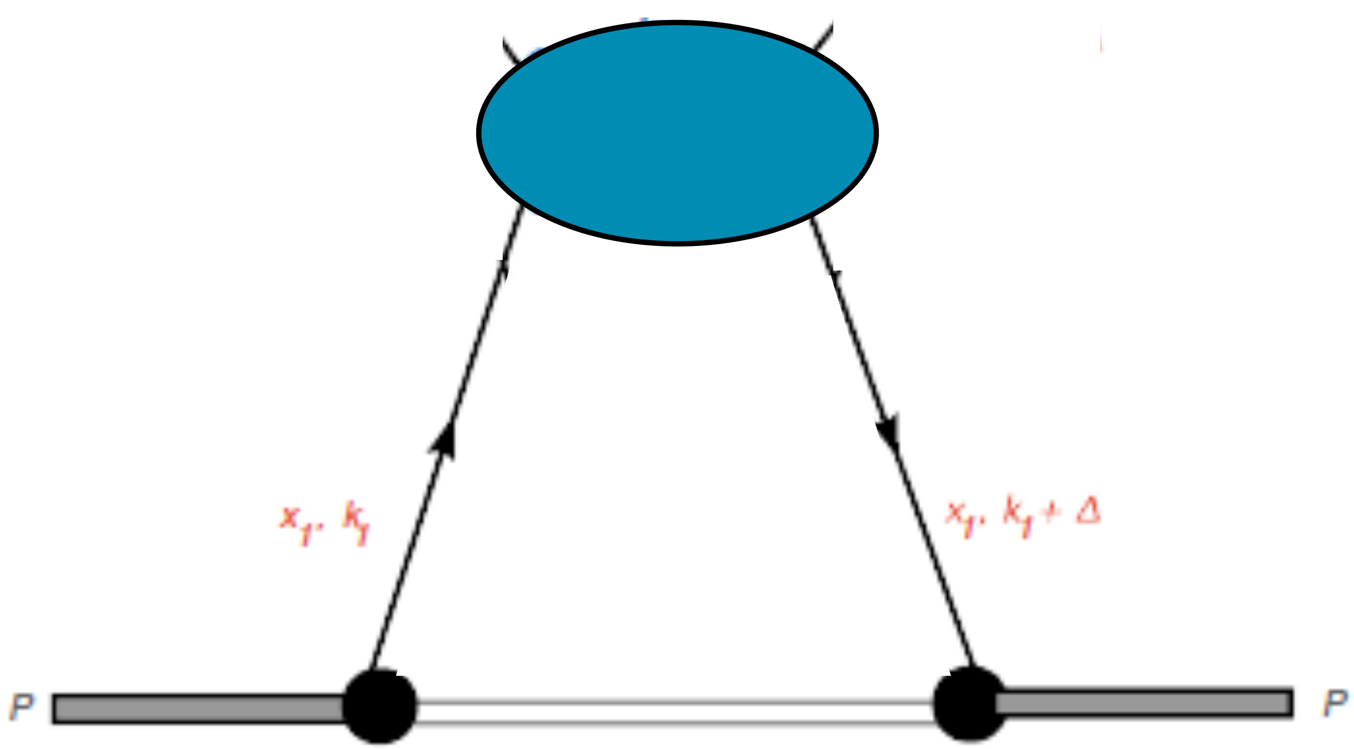
G P D


P
P
x3, k3
x3, k3 − Δ
x4, k4
x4, k4 + Δ
J3t
J1t
J4t
J2t
x1, k1
x1, k1 + Δ
x2, k2
x2, k2 − Δ
P
P

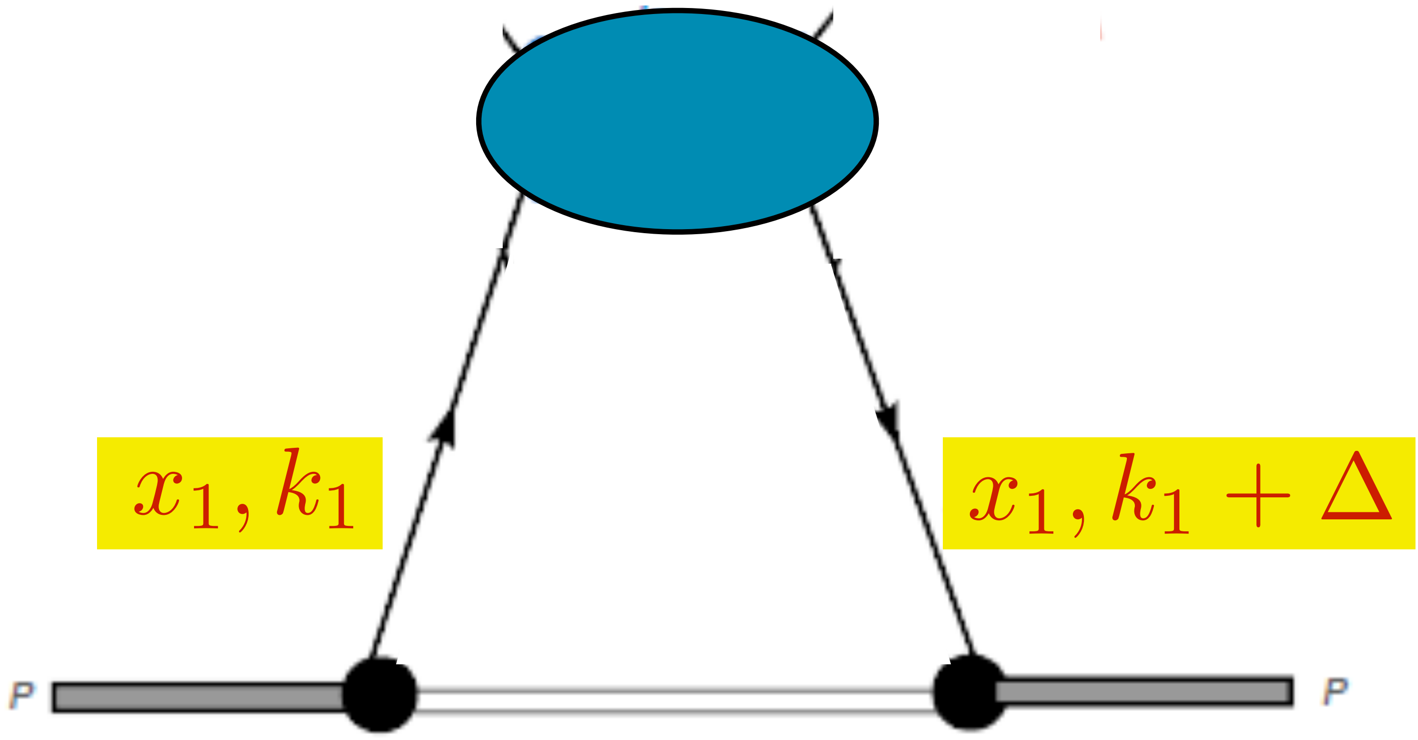
G P D



G P D



GPD



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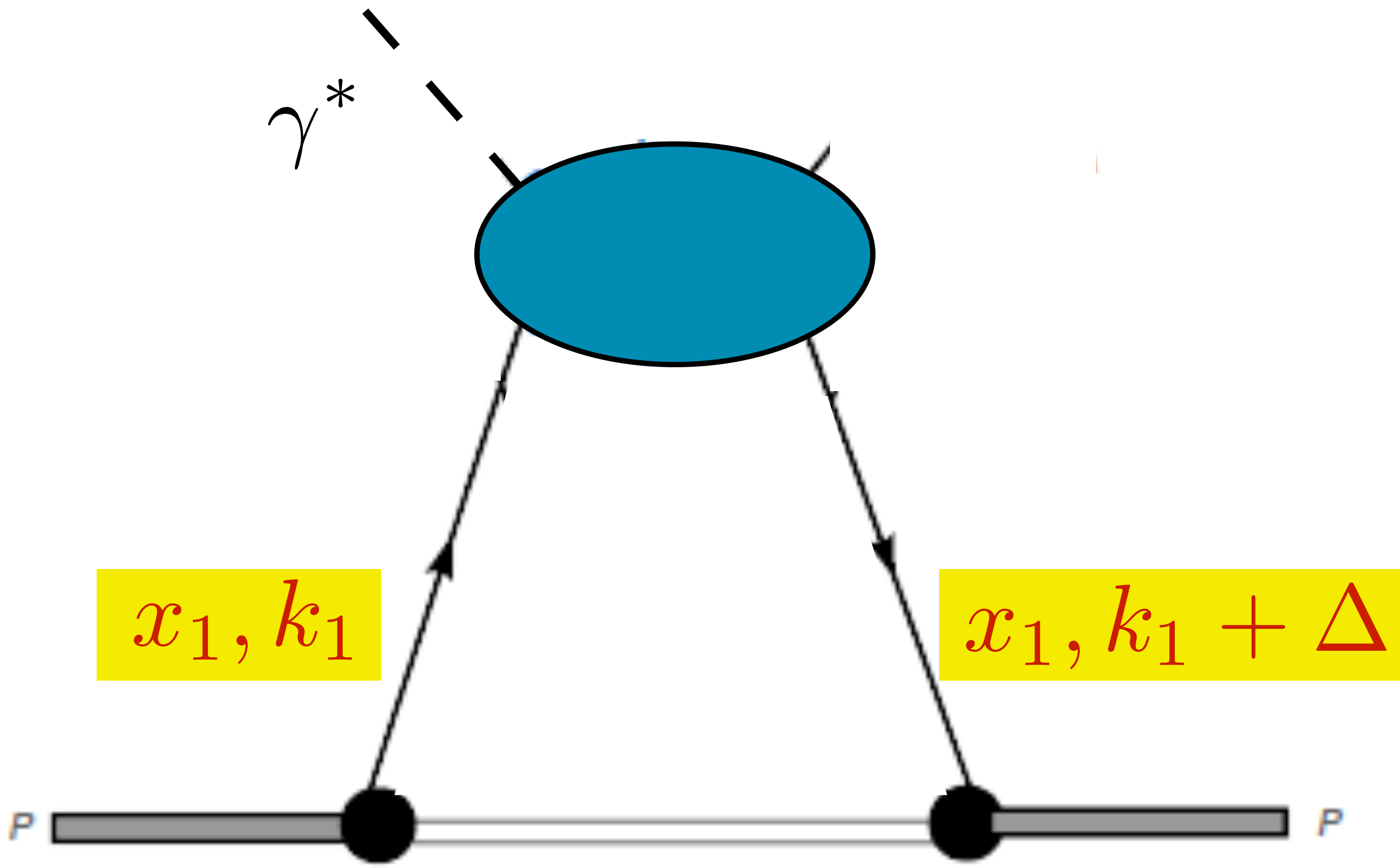

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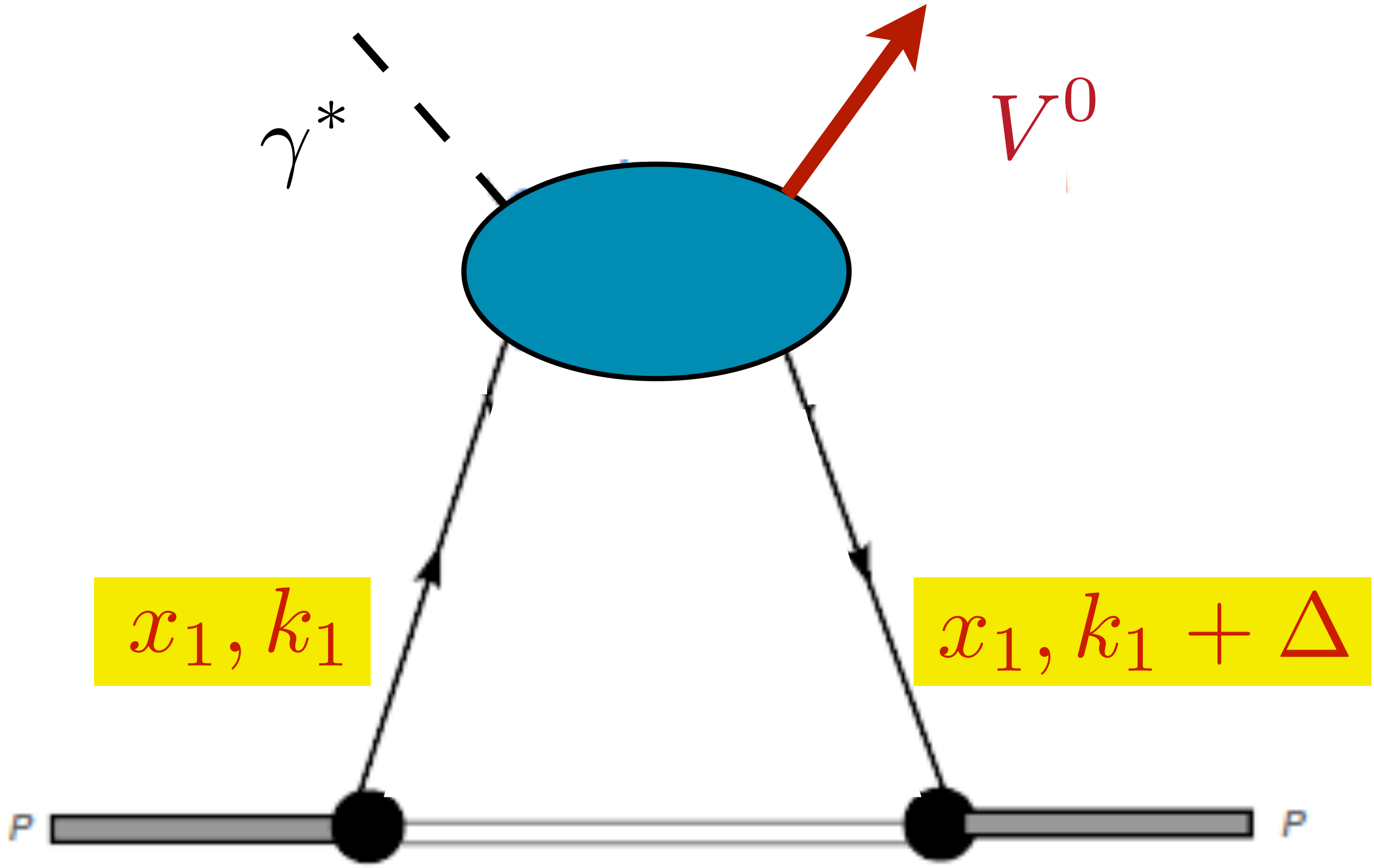

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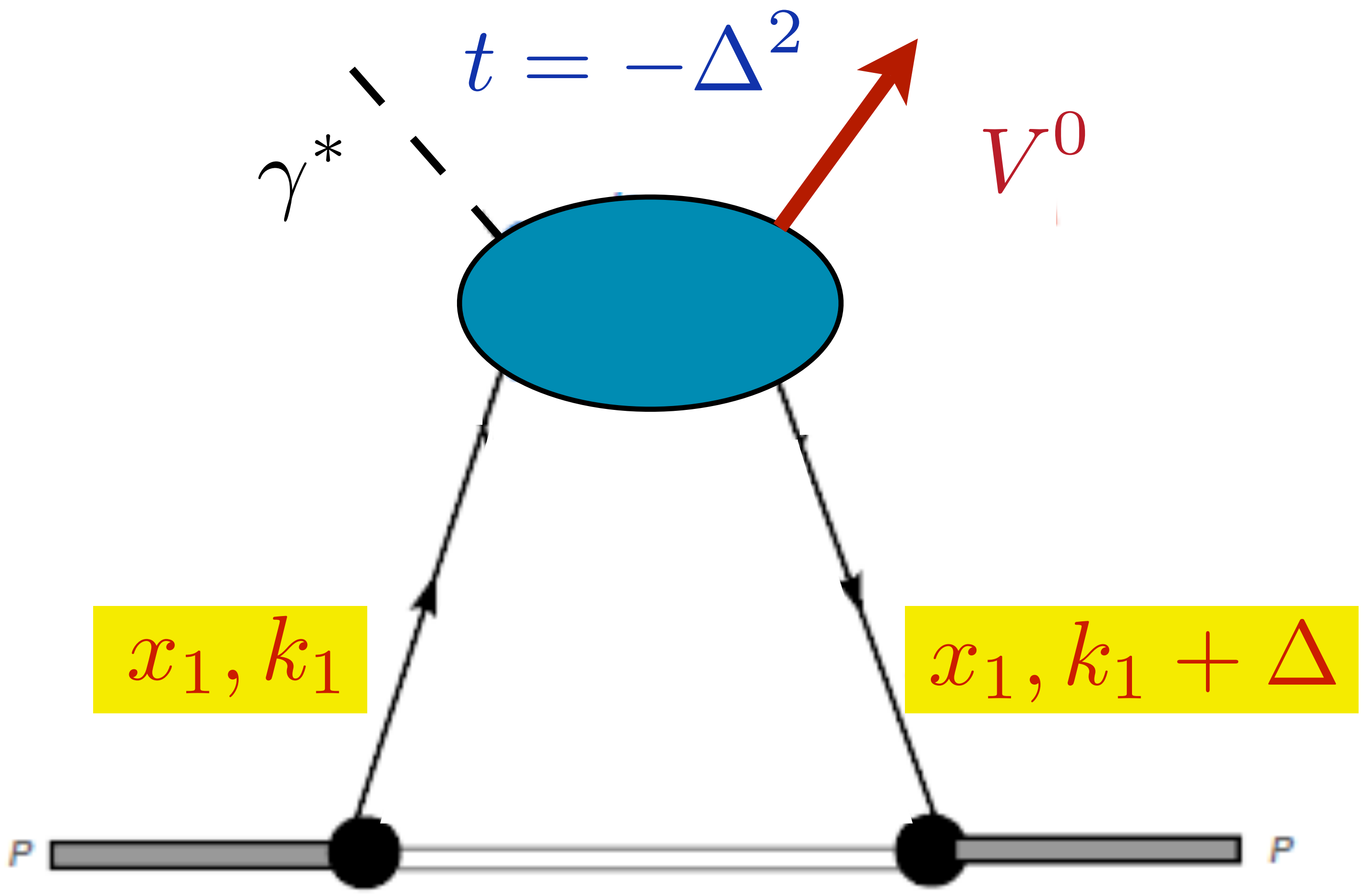

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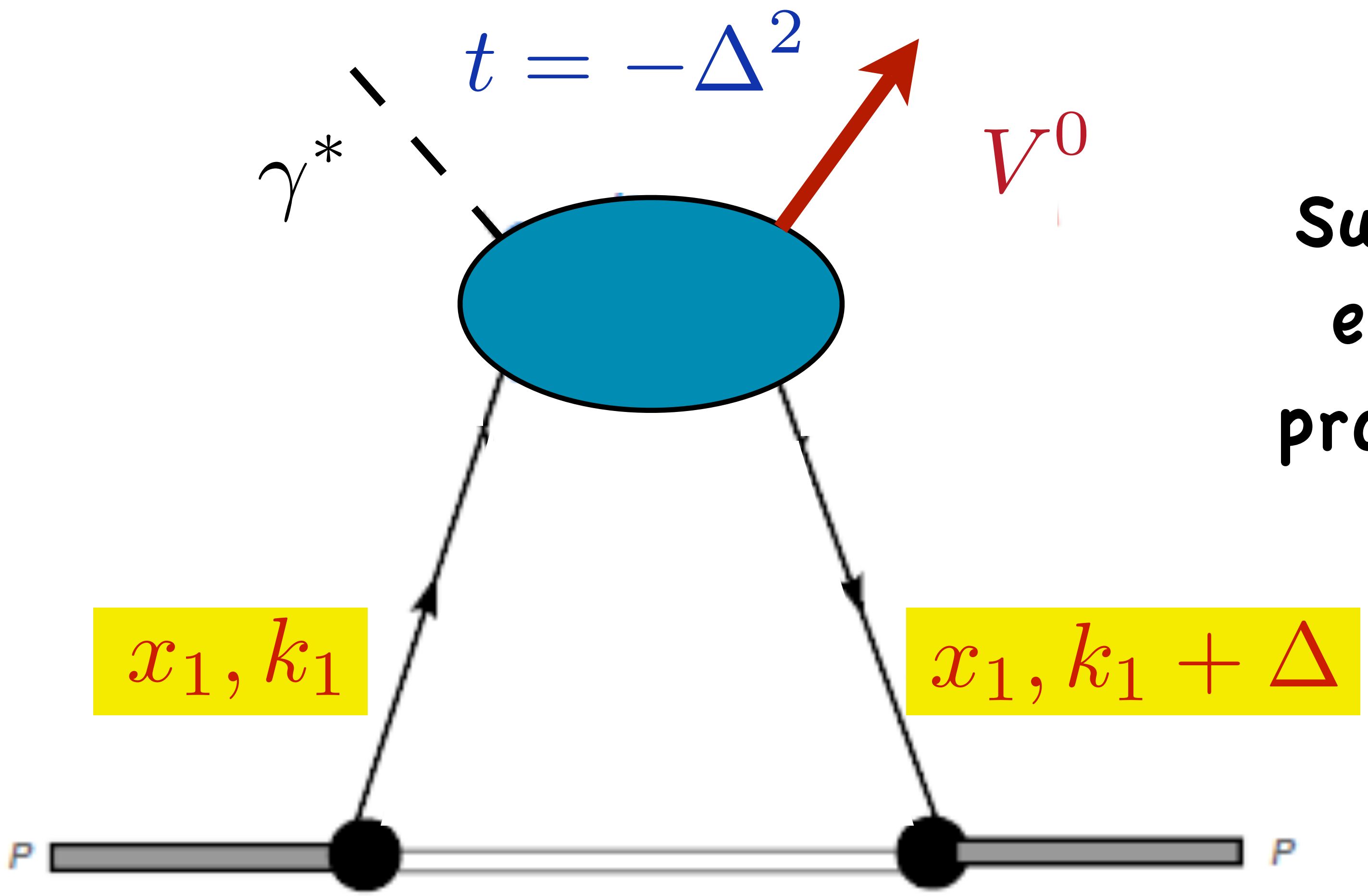
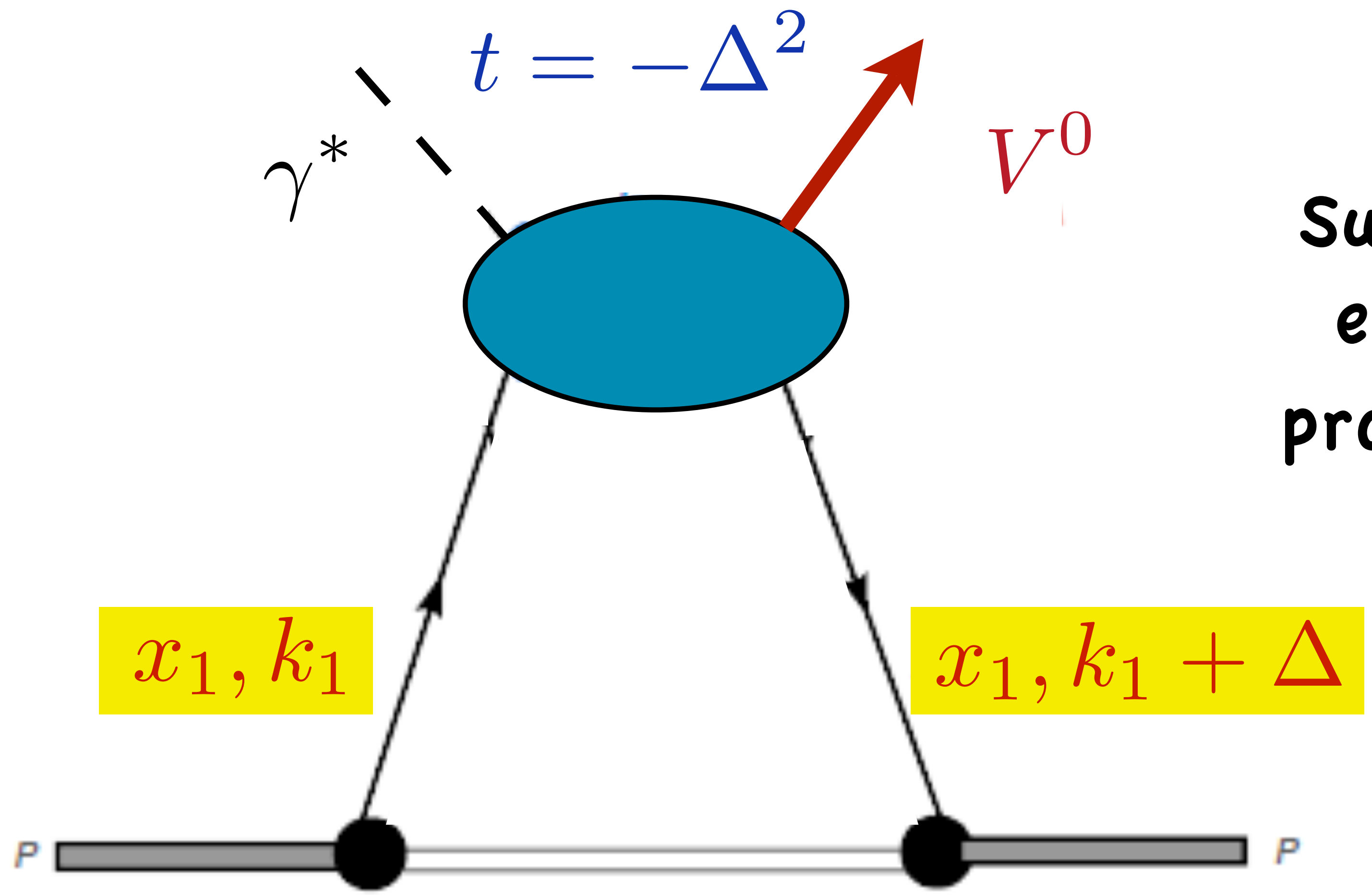


Such an amplitude describes exclusive photo-(/electro-) production of vector mesons at HERA !



G P D

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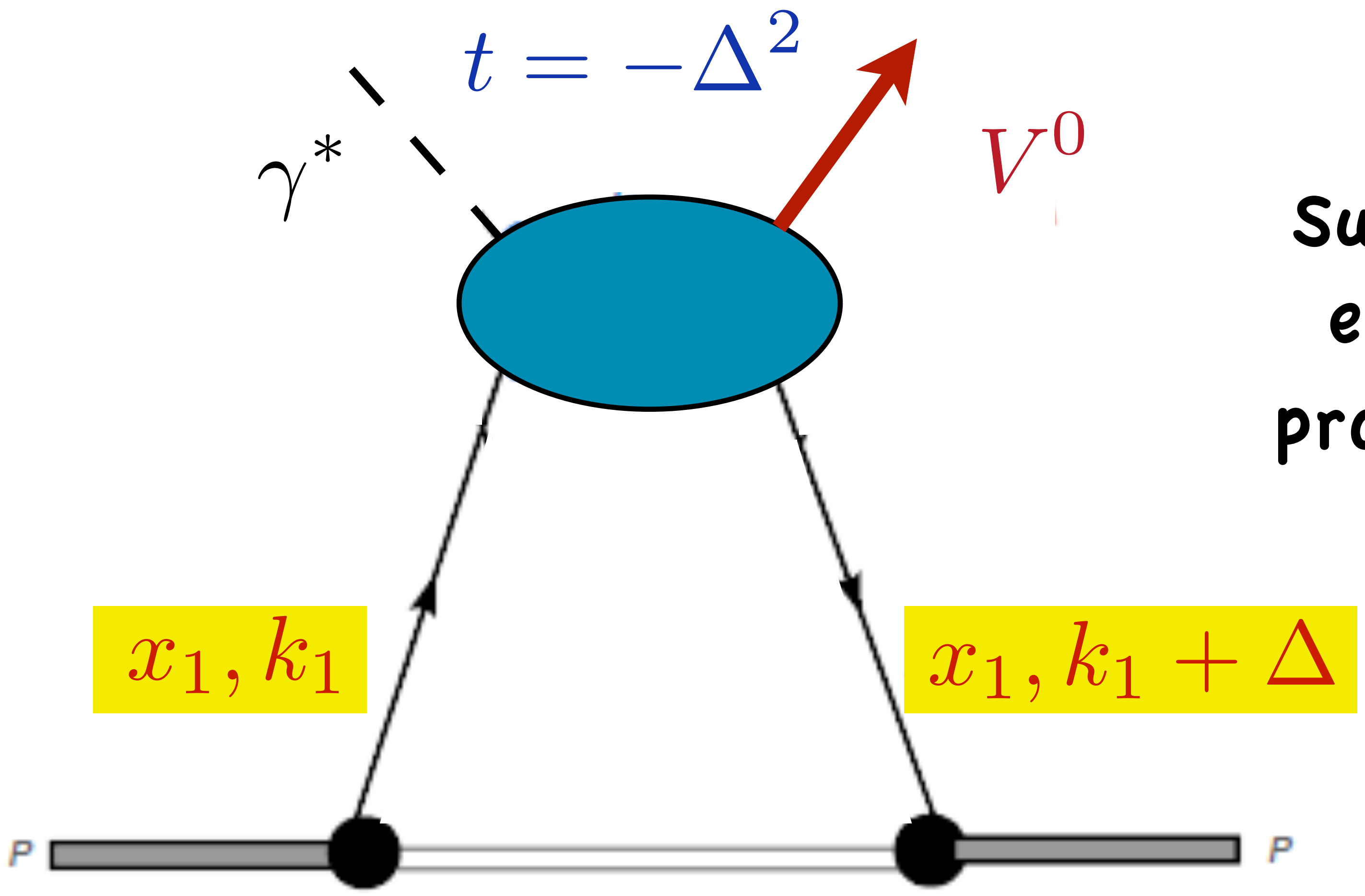
Generalized parton distribution : $G_N(x, Q^2, \vec{\Delta}) =$

G P D



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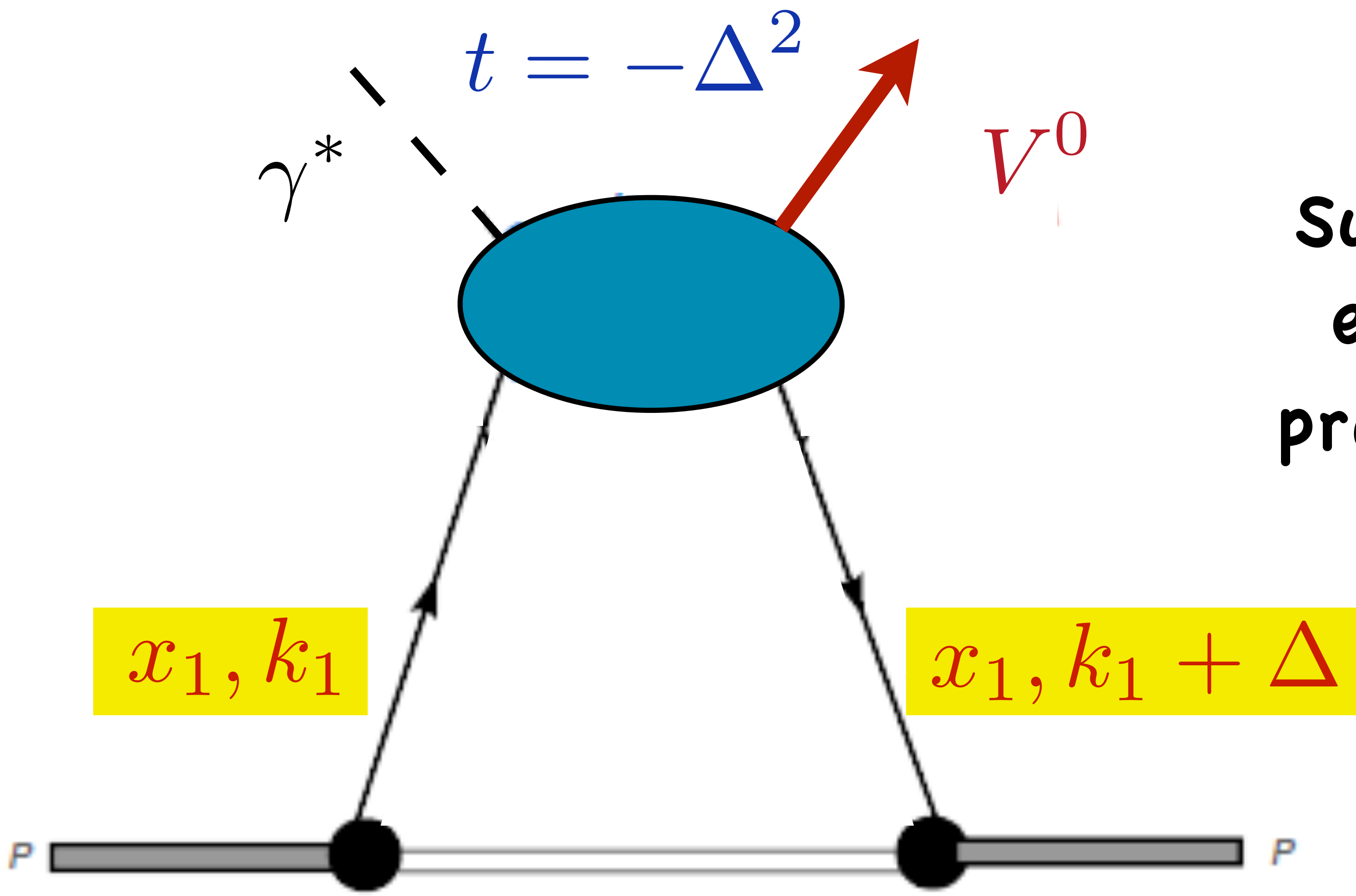
Generalized parton distribution :

$$G_N(x, Q^2, \vec{\Delta}) = G_N(x, Q^2) F_{2g}(\Delta)$$

G P D



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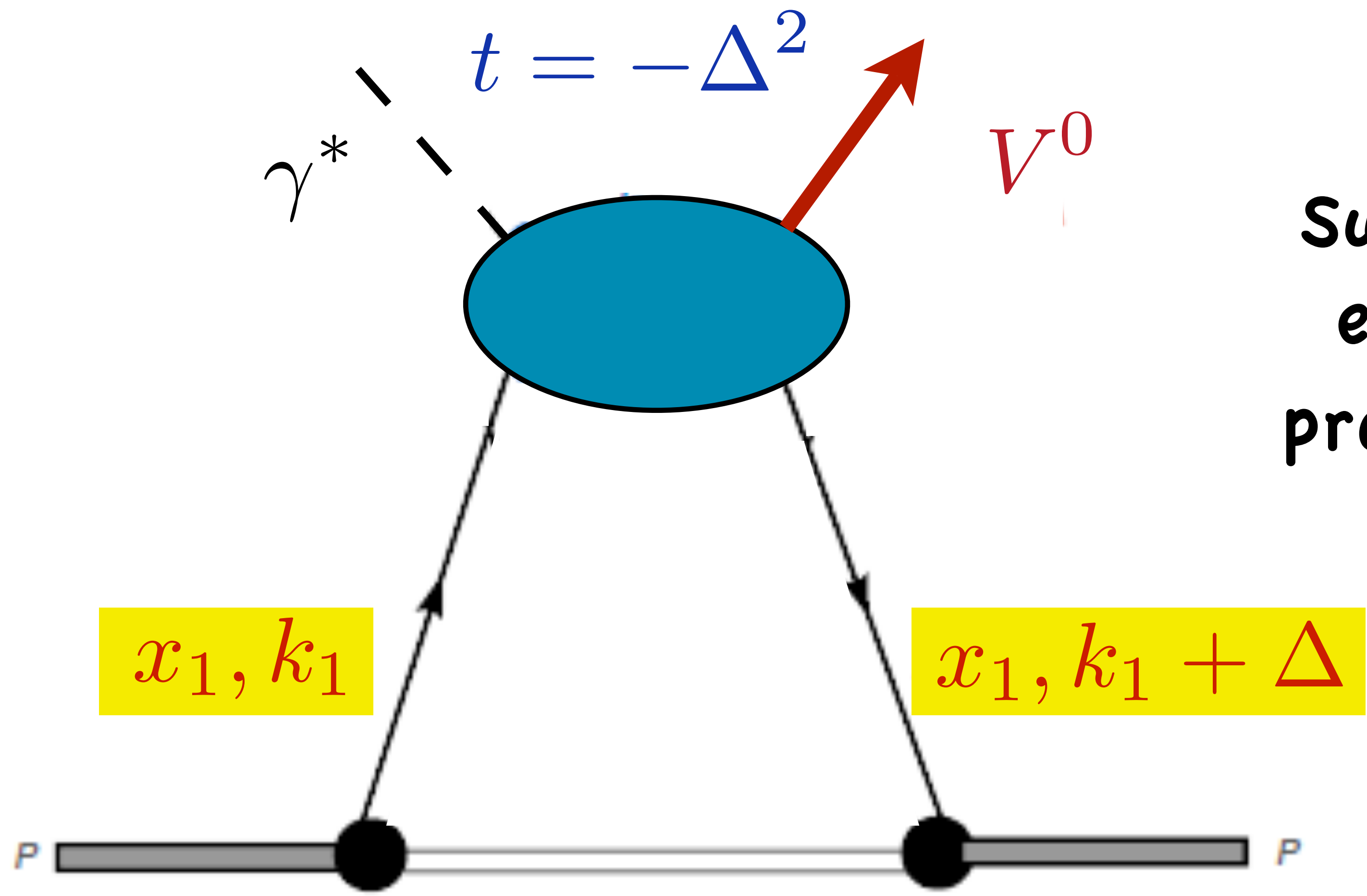
$$G_N(x, Q^2, \vec{\Delta}) \;=\; G_N(x, Q^2) F_{2g}(\Delta)$$

G - the usual 1-parton distribution (determining DIS structure functions)

G P D



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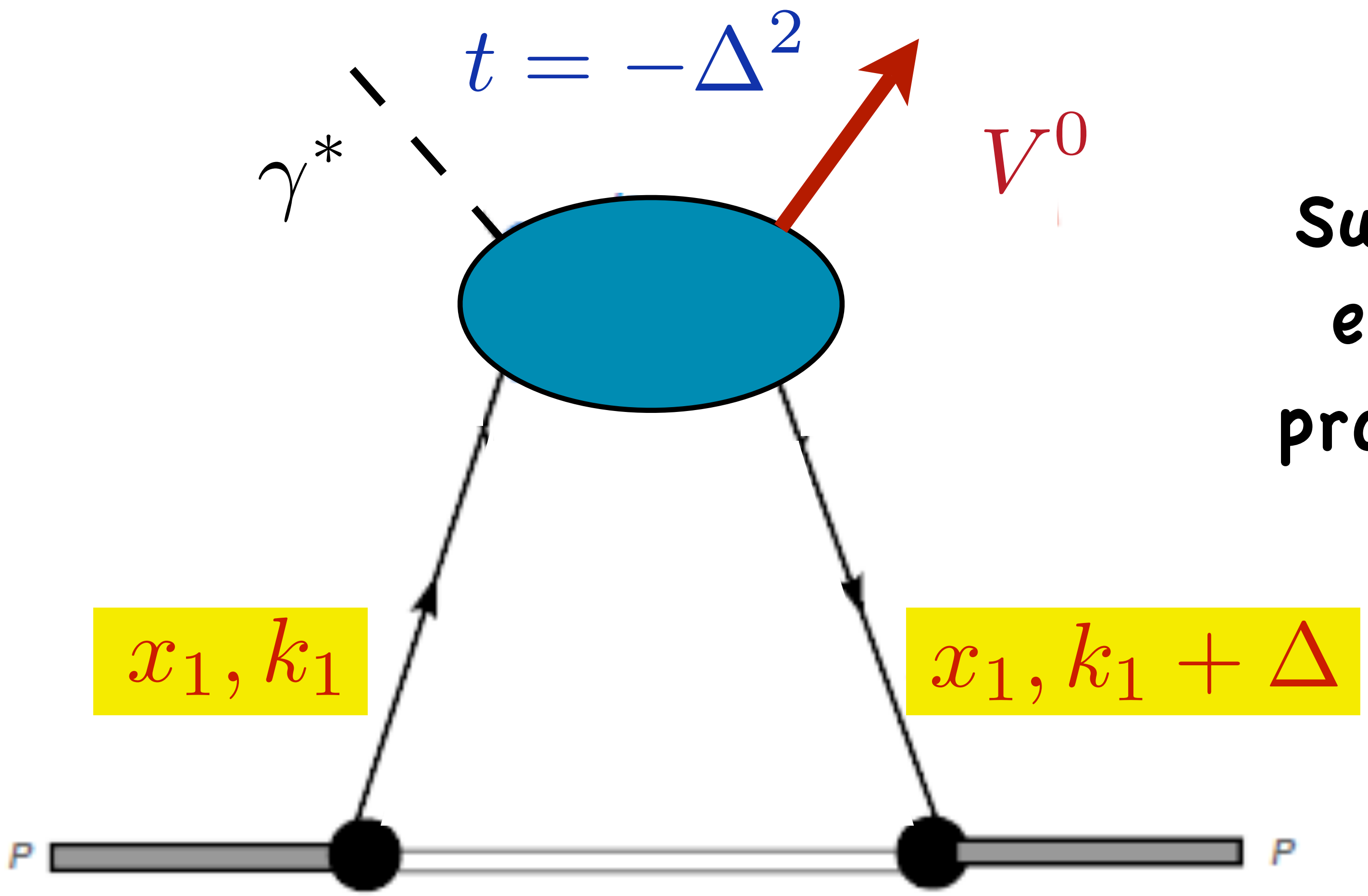

G P D

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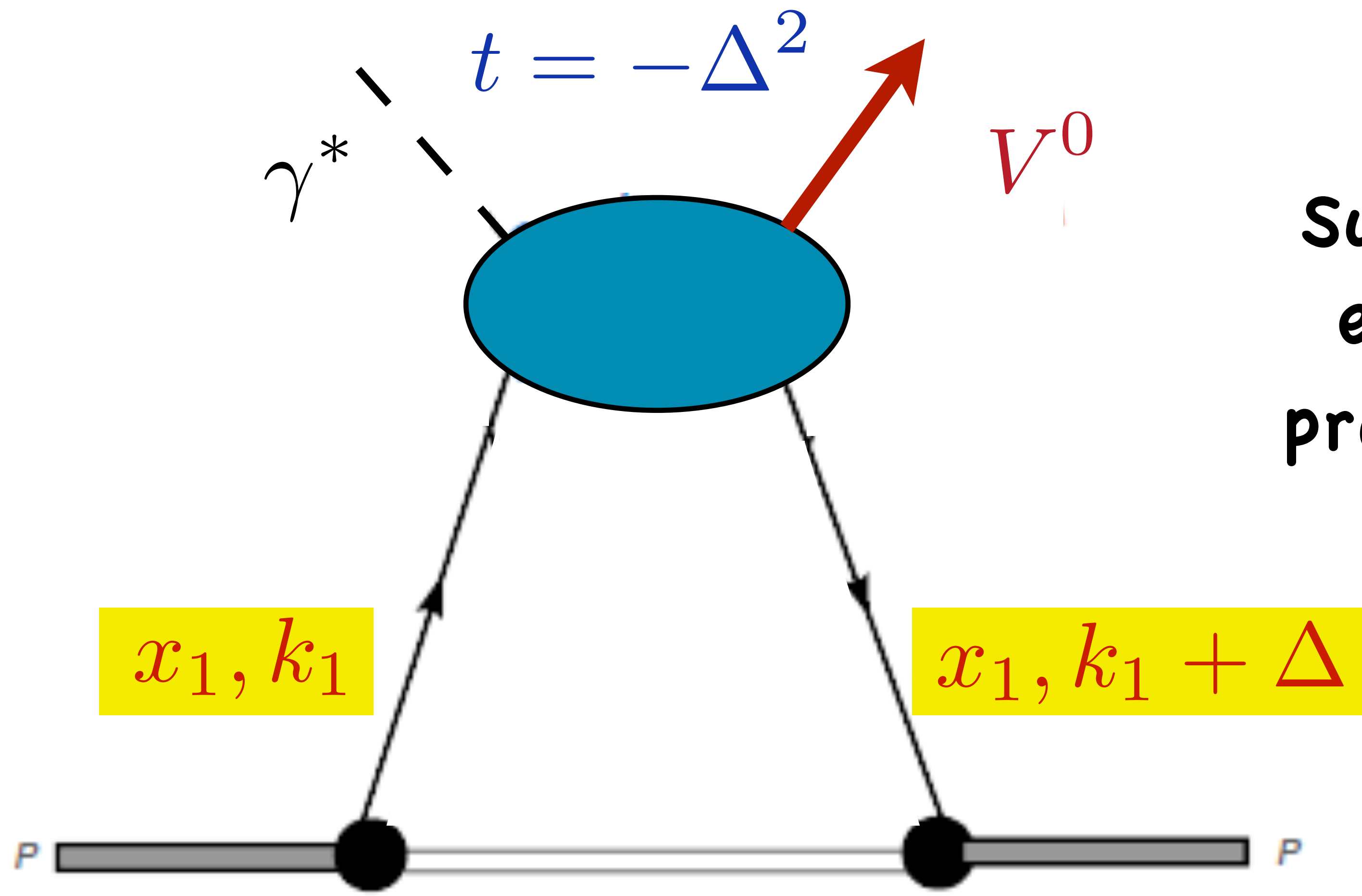
the dipole fit :
$$F_{2g}(\Delta) \simeq \frac{1}{\left(1 + \Delta^2/m_g^2\right)^2}$$

G P D



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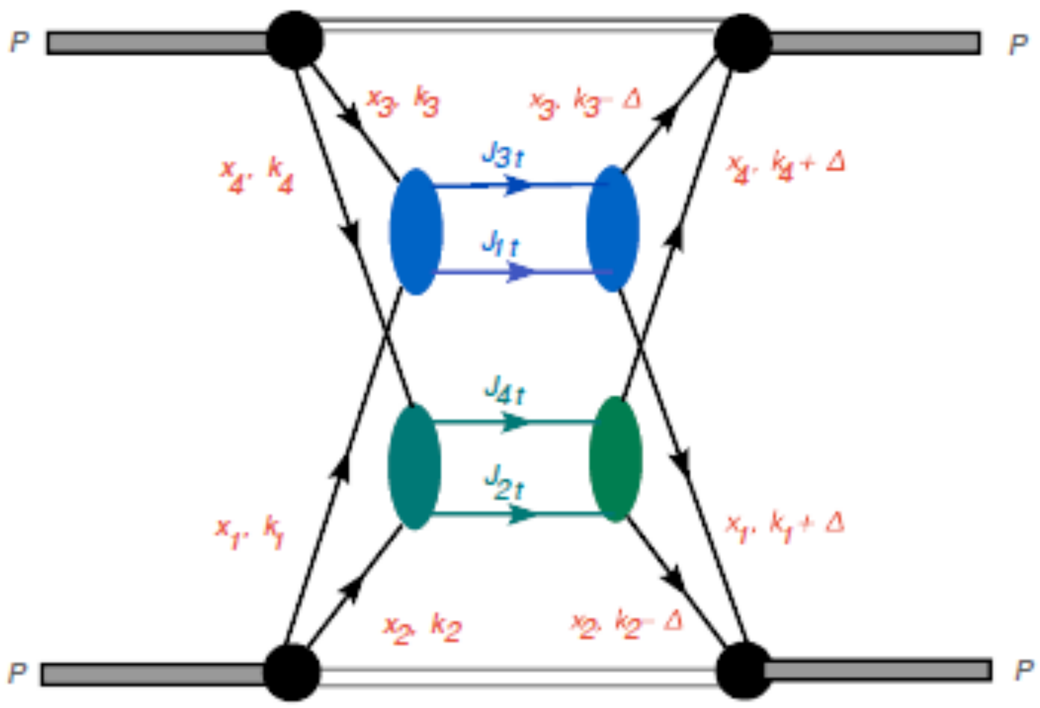
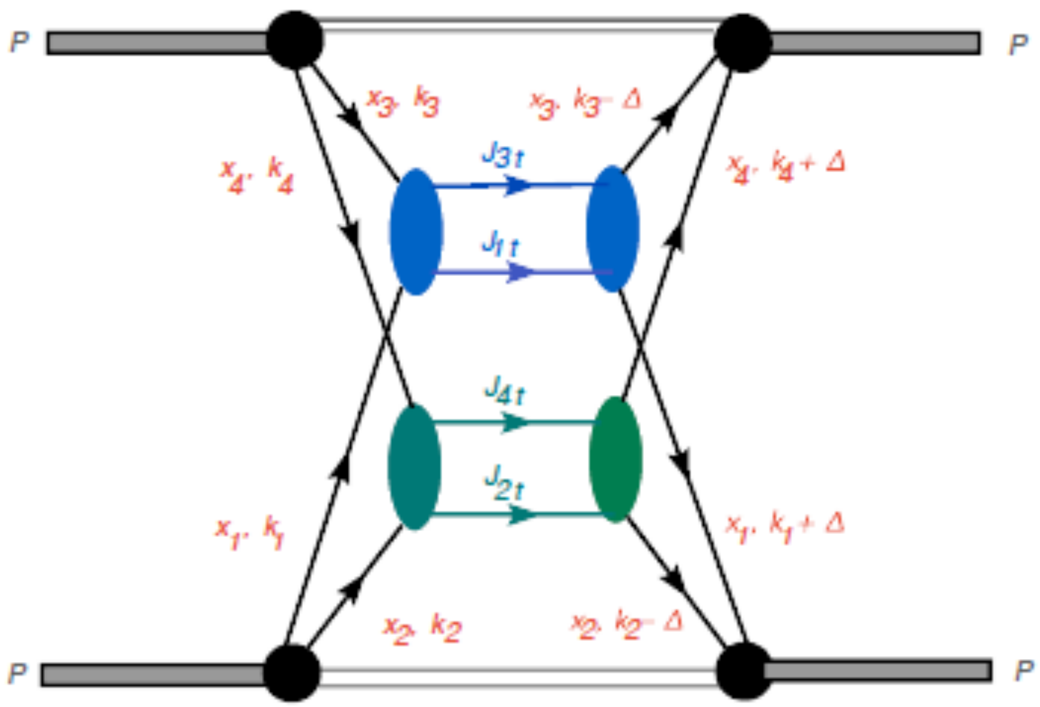
$m_g^2(x \sim 0.03, Q^2 \sim 3\text{GeV}^2)$
$\simeq 1.1\text{GeV}^2$



P
P
x3, k3
x3, k3 − Δ
x4, k4
x4, k4 + Δ
J3t
J1t
J4t
J2t
x1, k1
x1, k1 + Δ
x2, k2
x2, k2 − Δ
P
P



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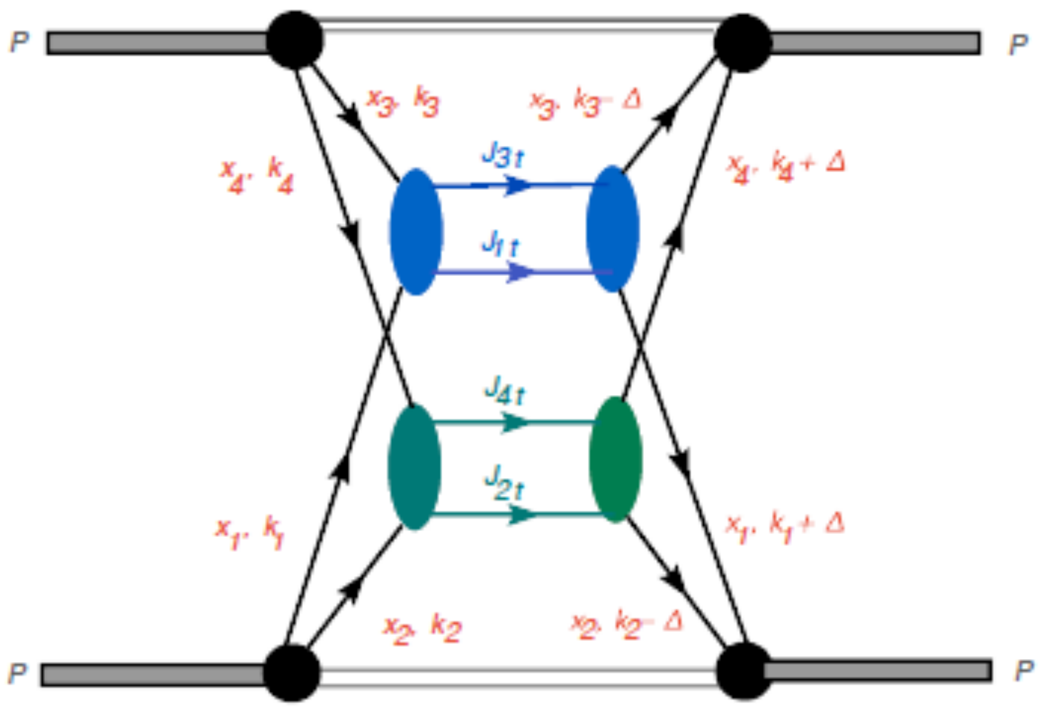

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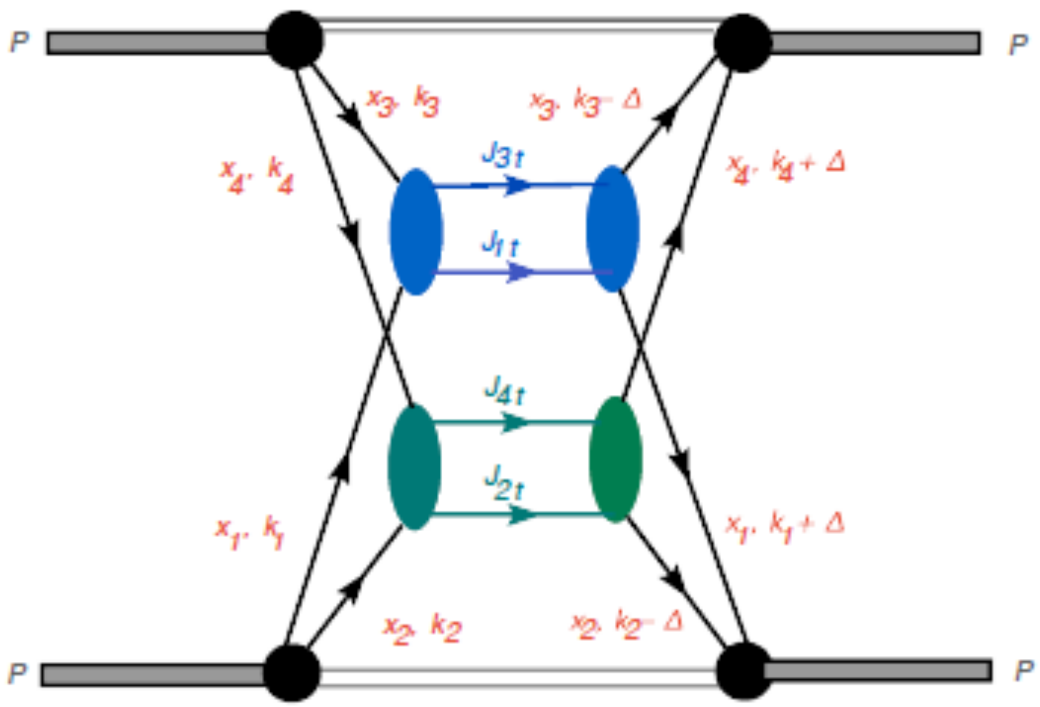

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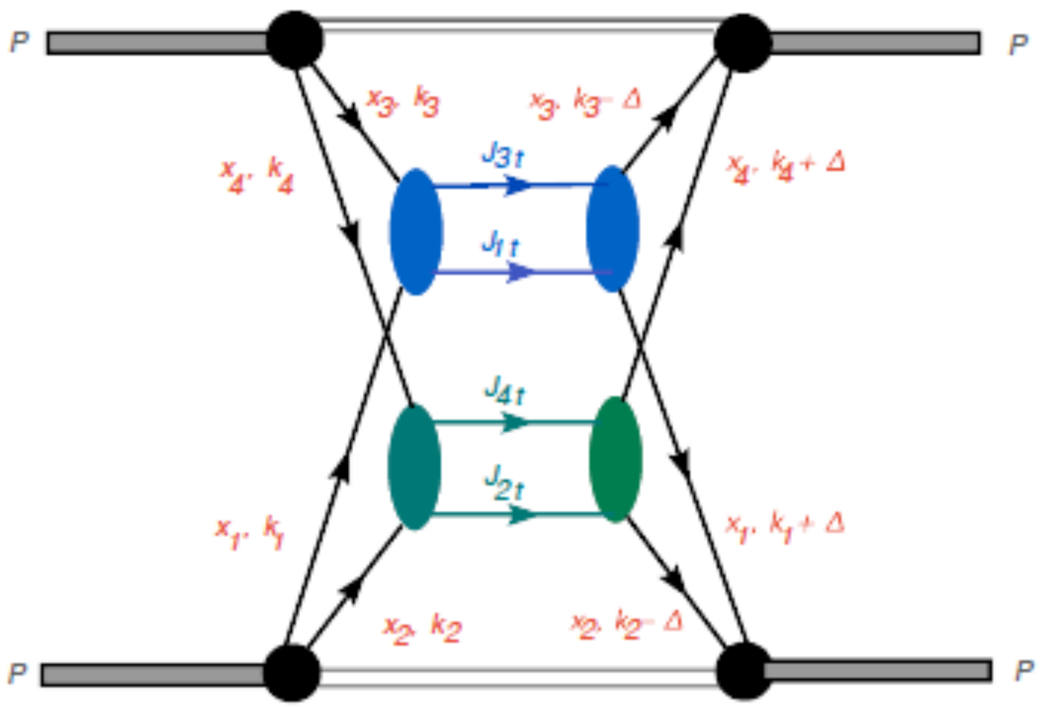

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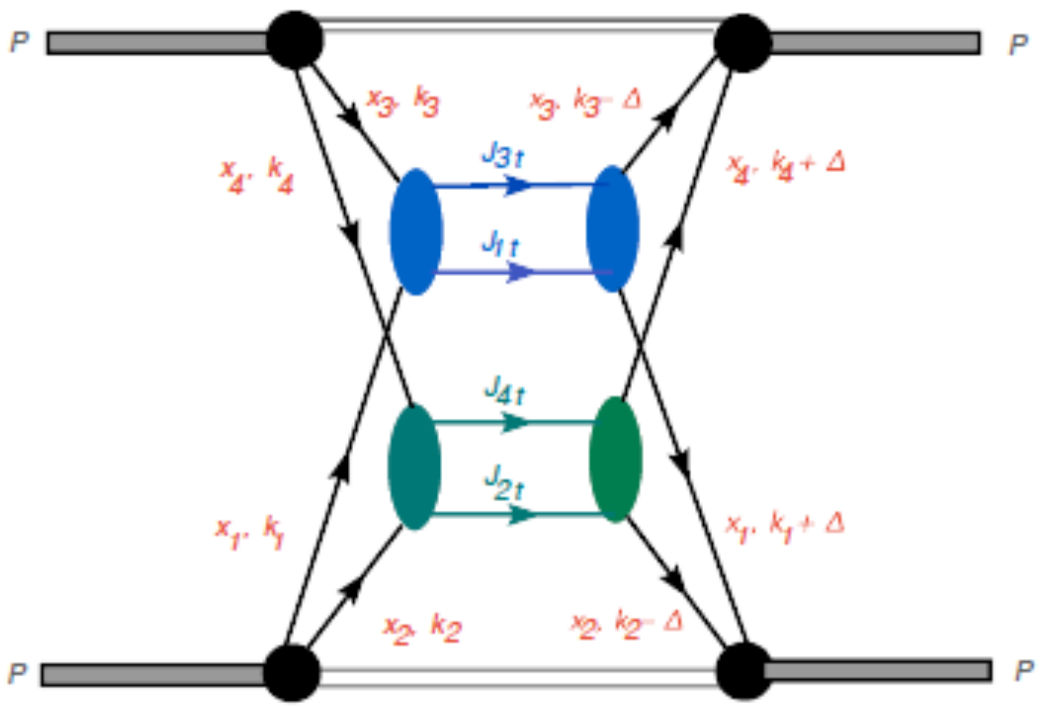

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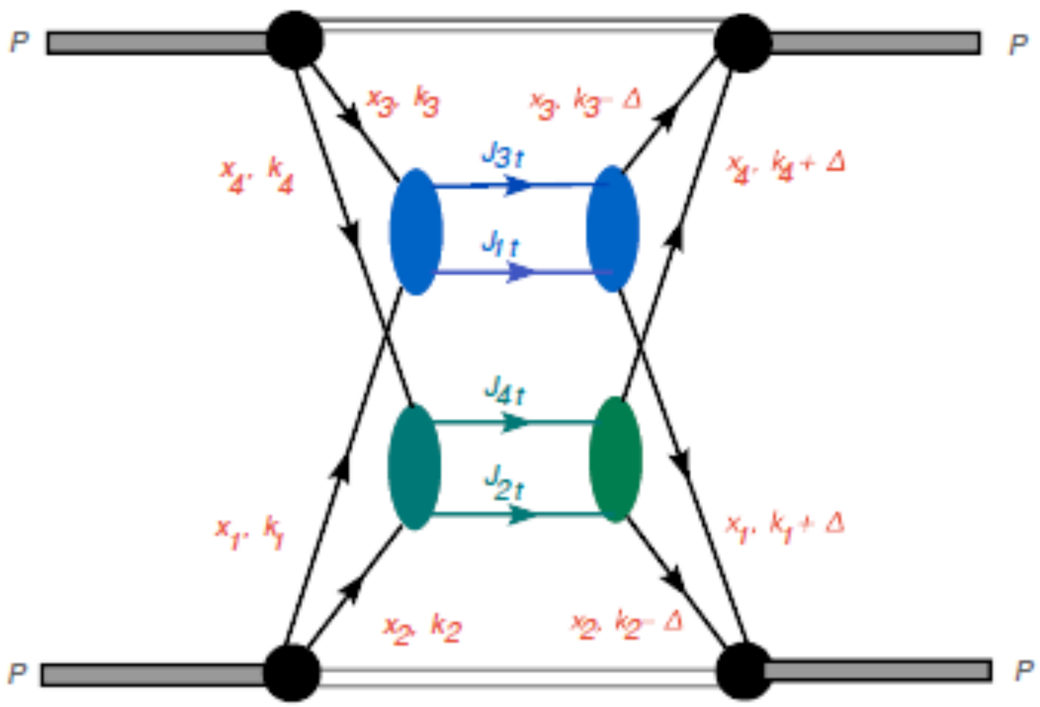

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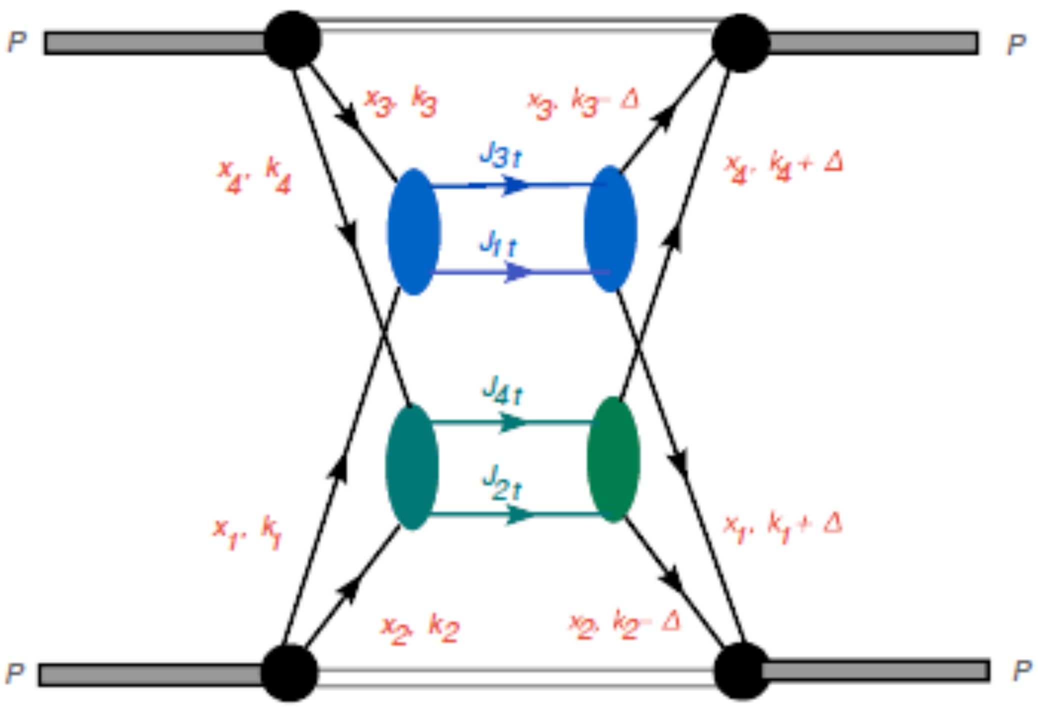

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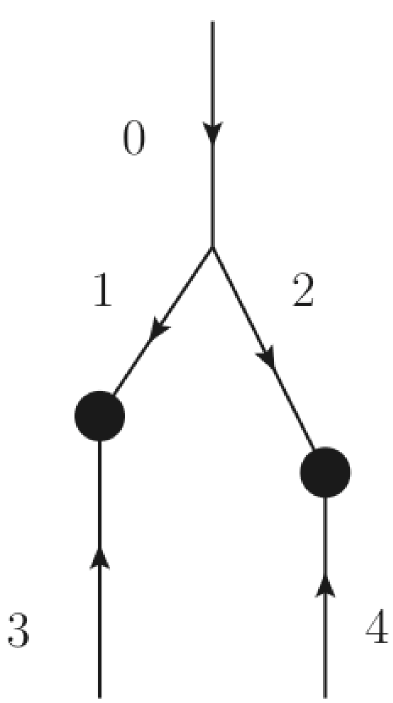

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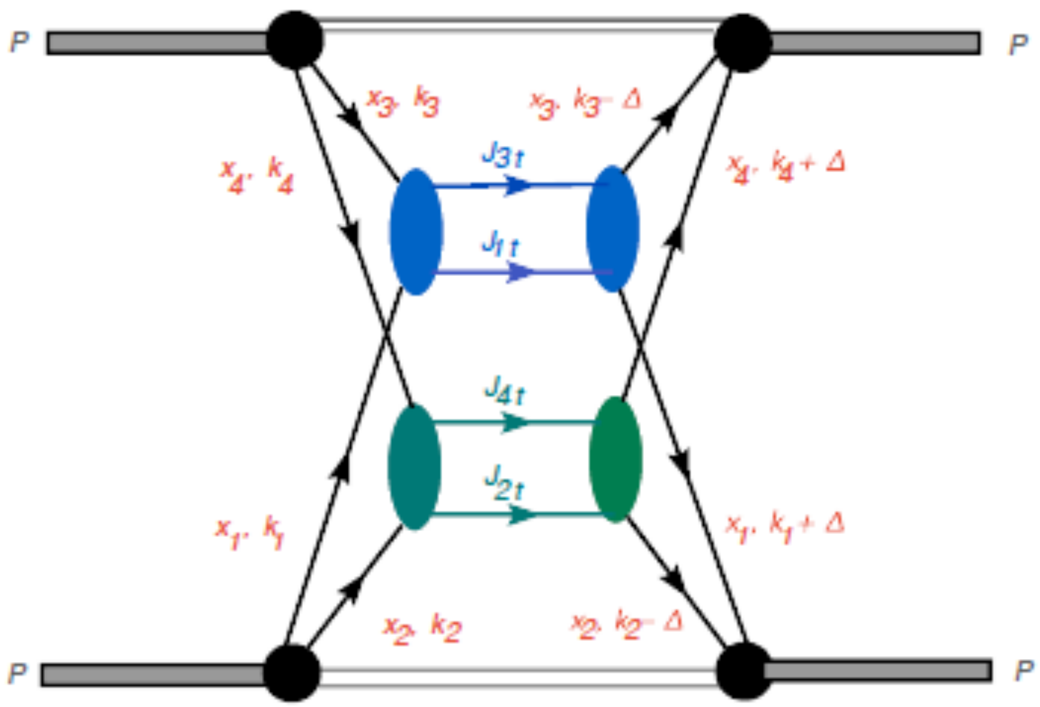

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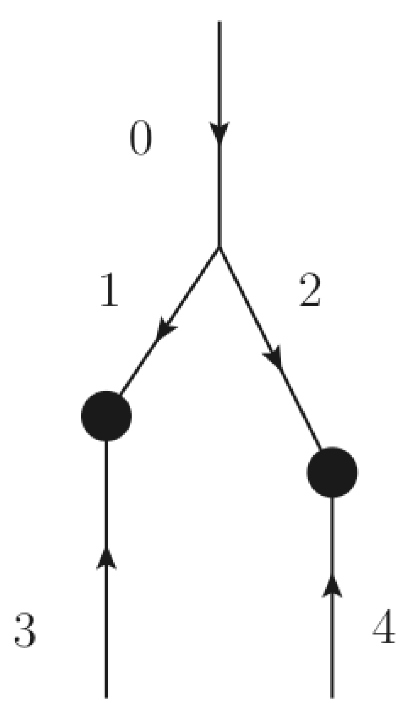

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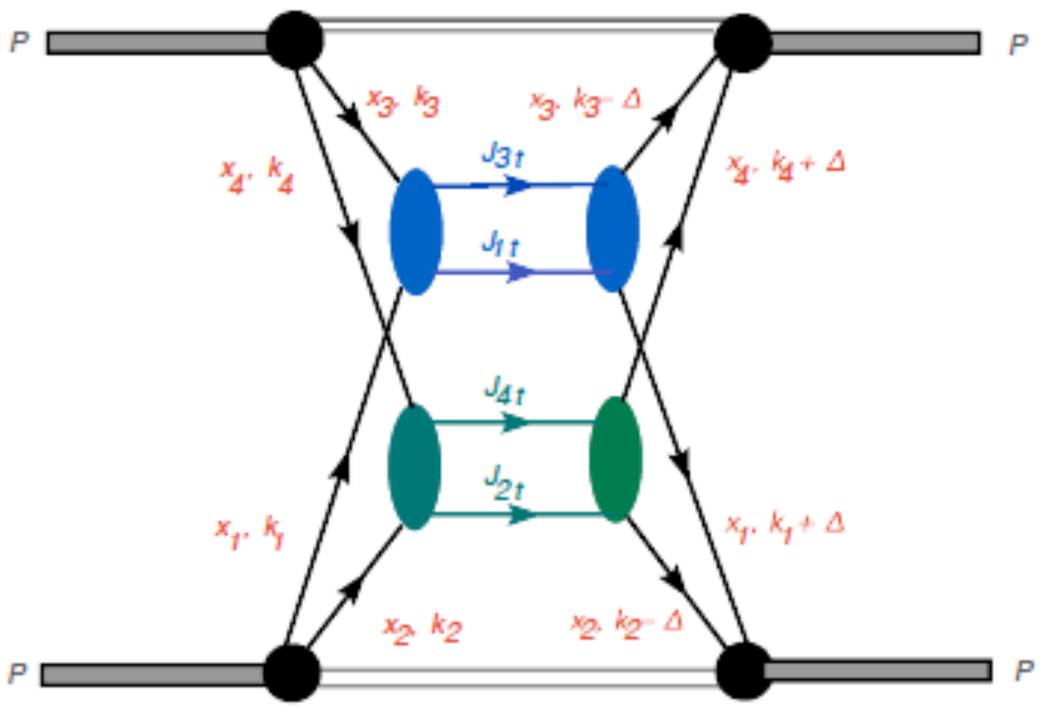

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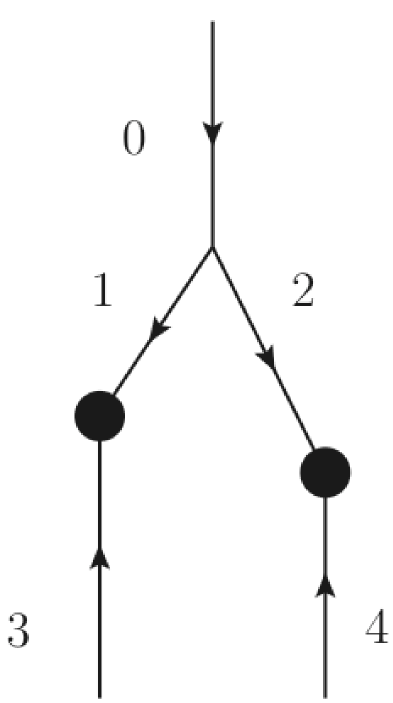

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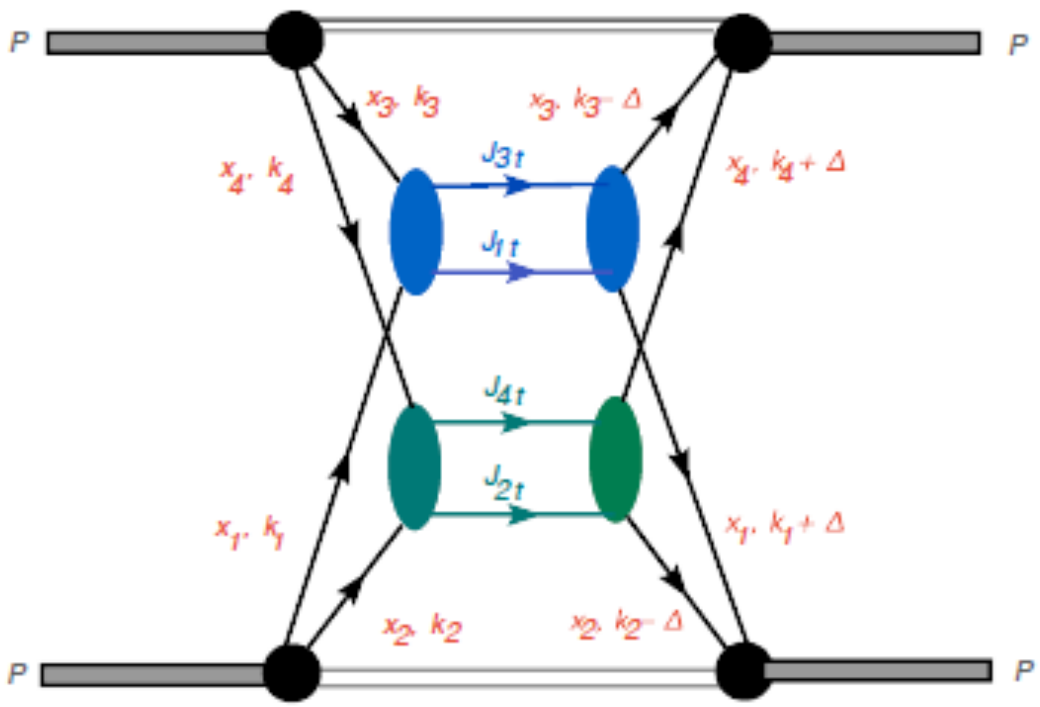

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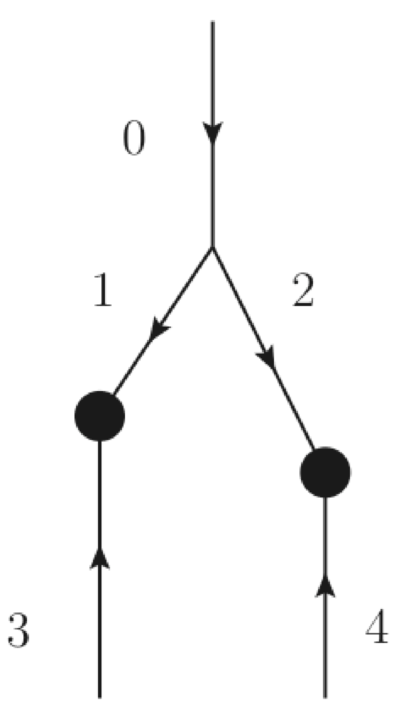

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3-> 4 contribution vs. **4-> 4** is enhanced by a factor

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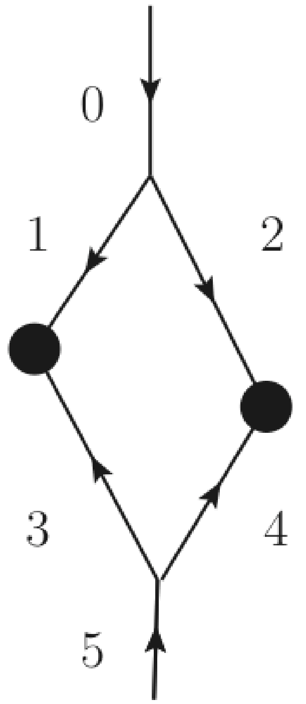

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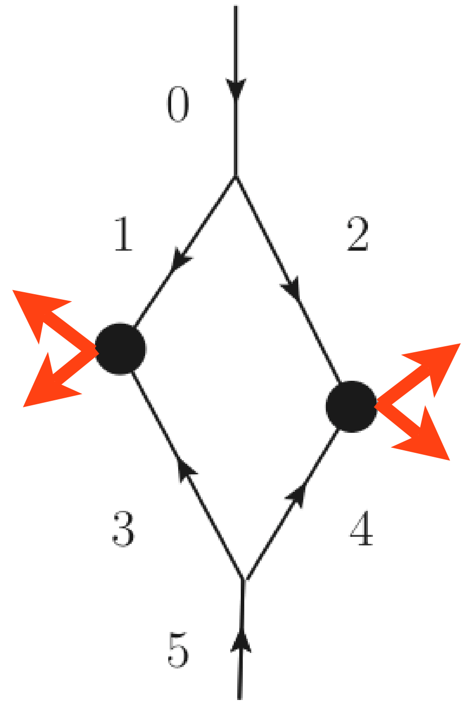

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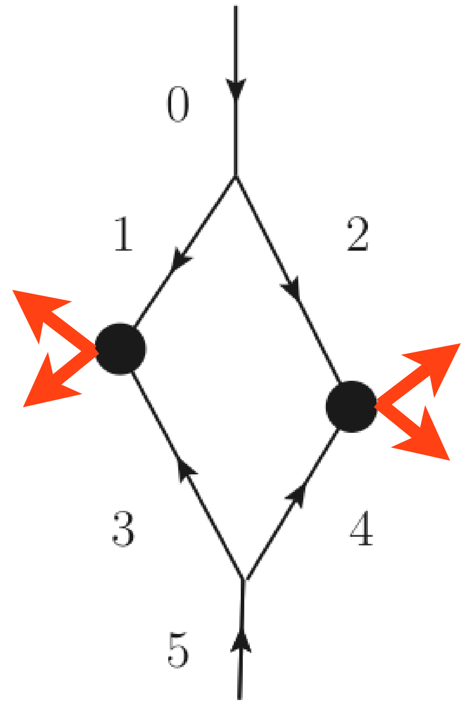

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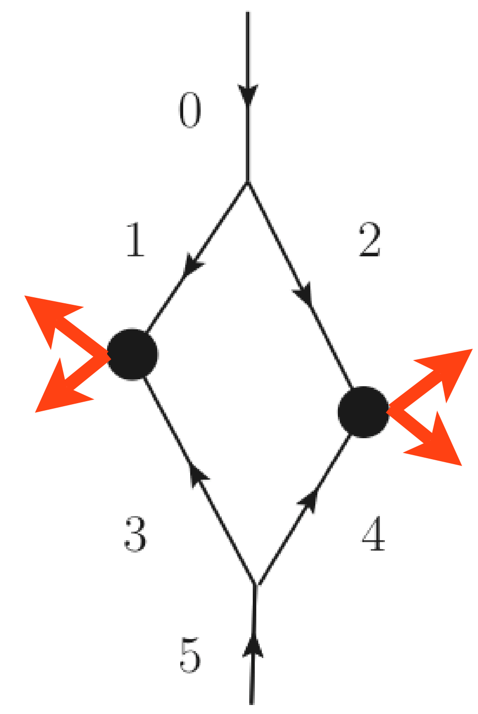

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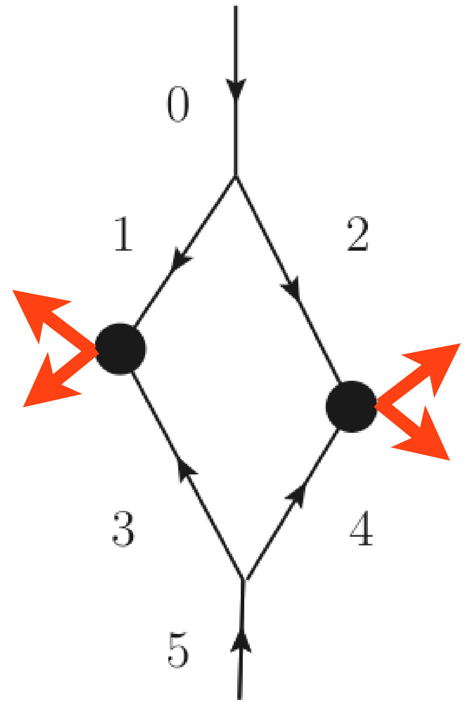

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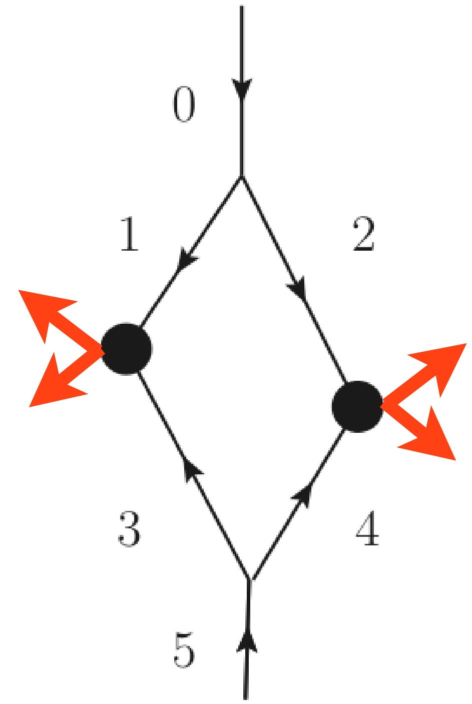

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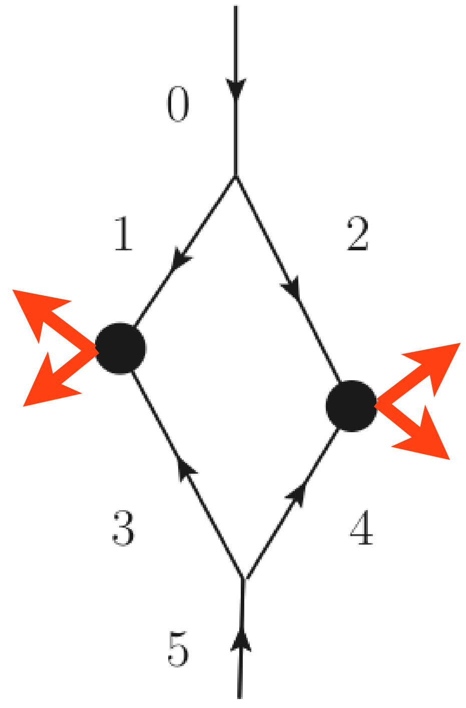

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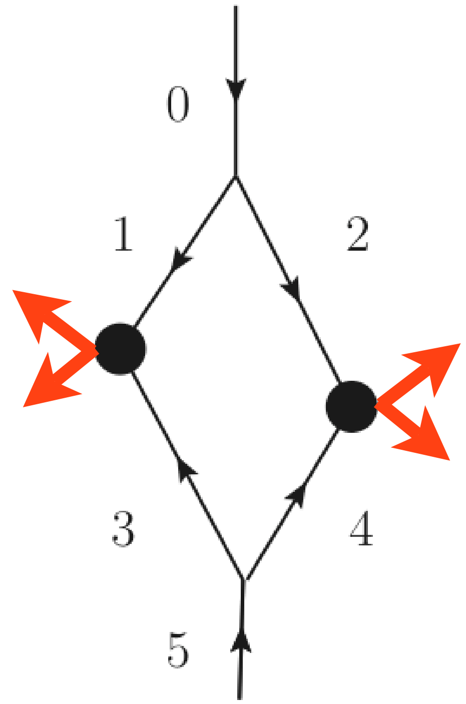

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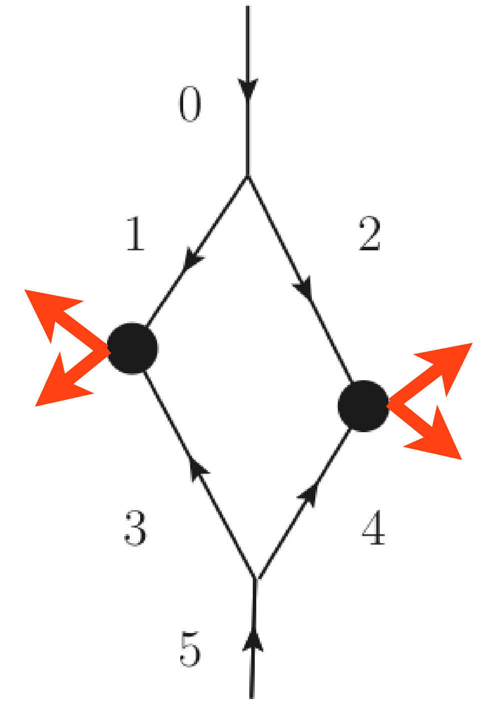

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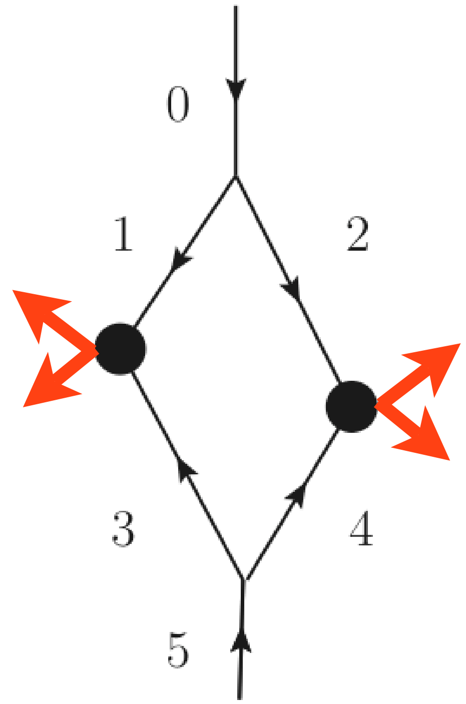

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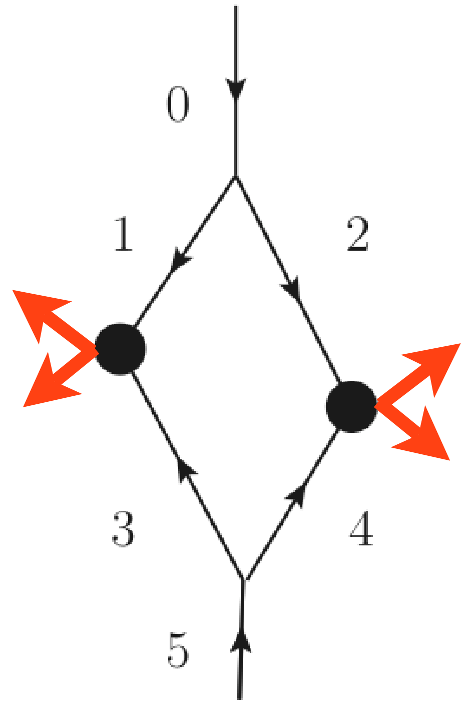

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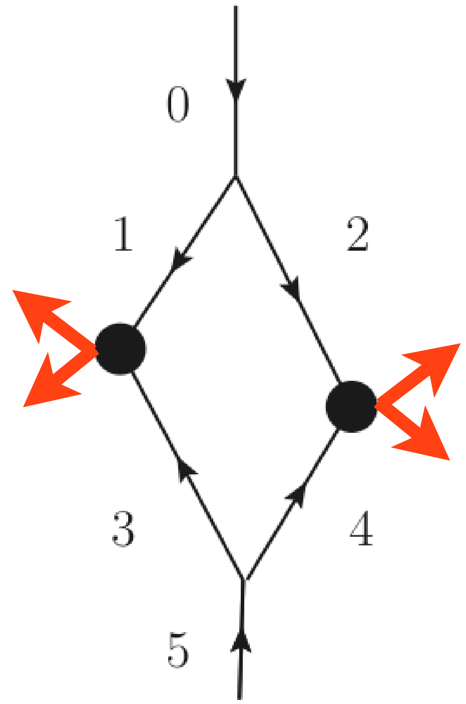

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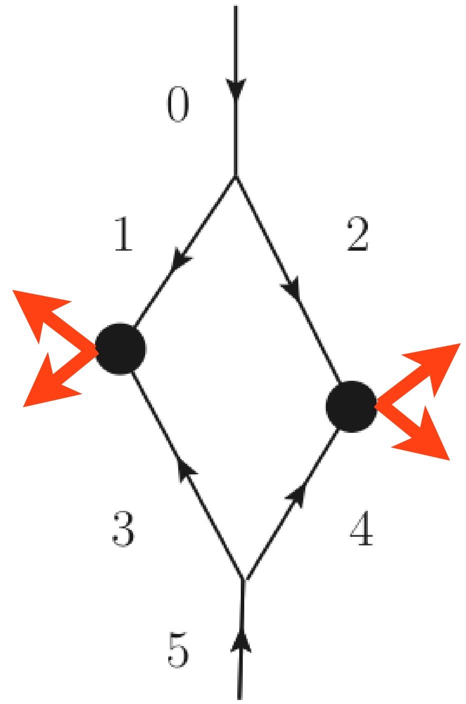

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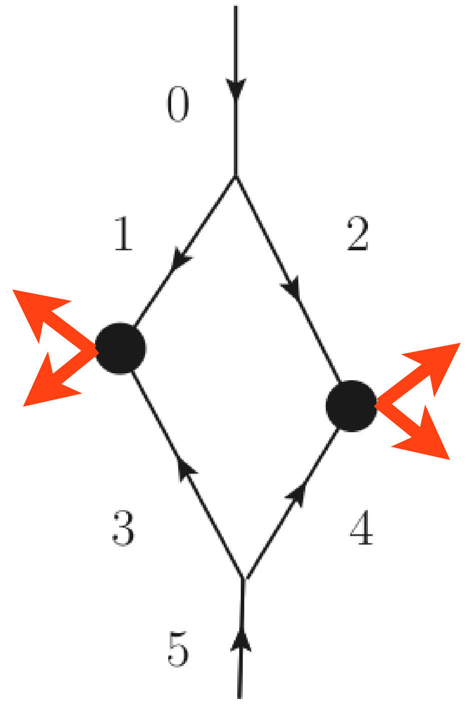

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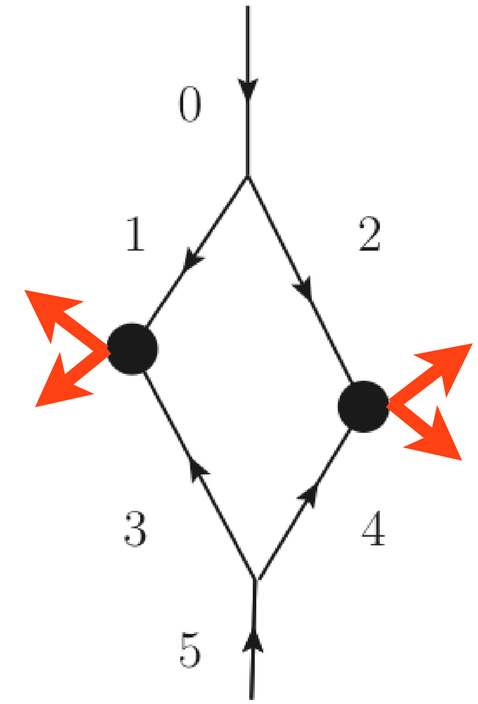

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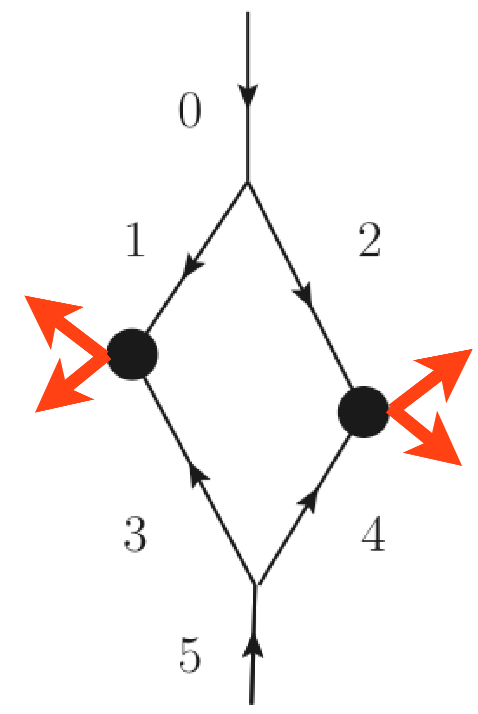

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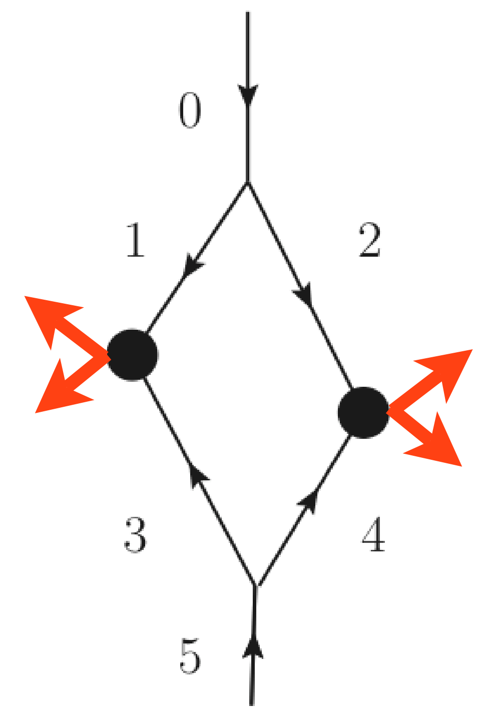

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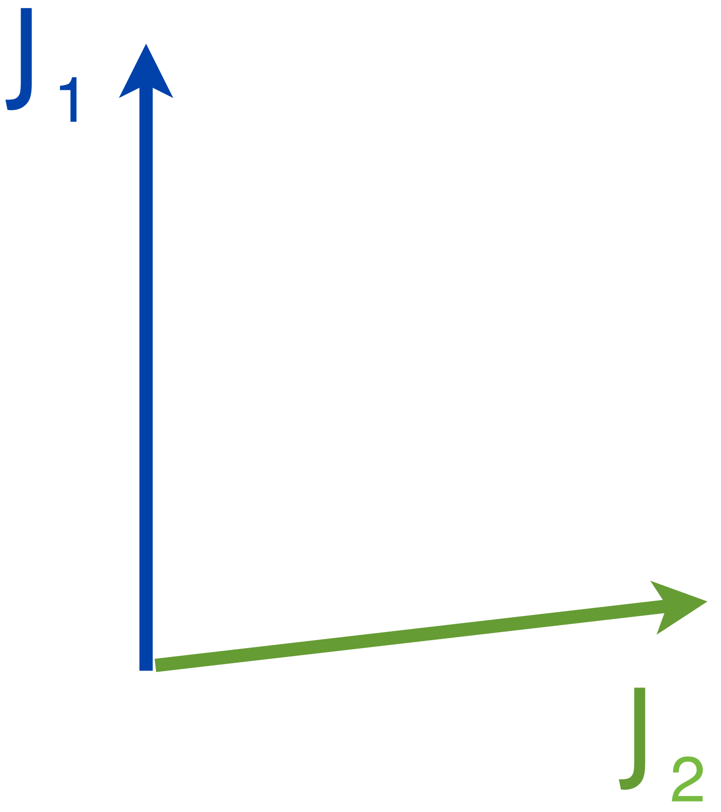
What distinguishes "double hard collisions" is the differential jet spectrum

back-to-back kinematics



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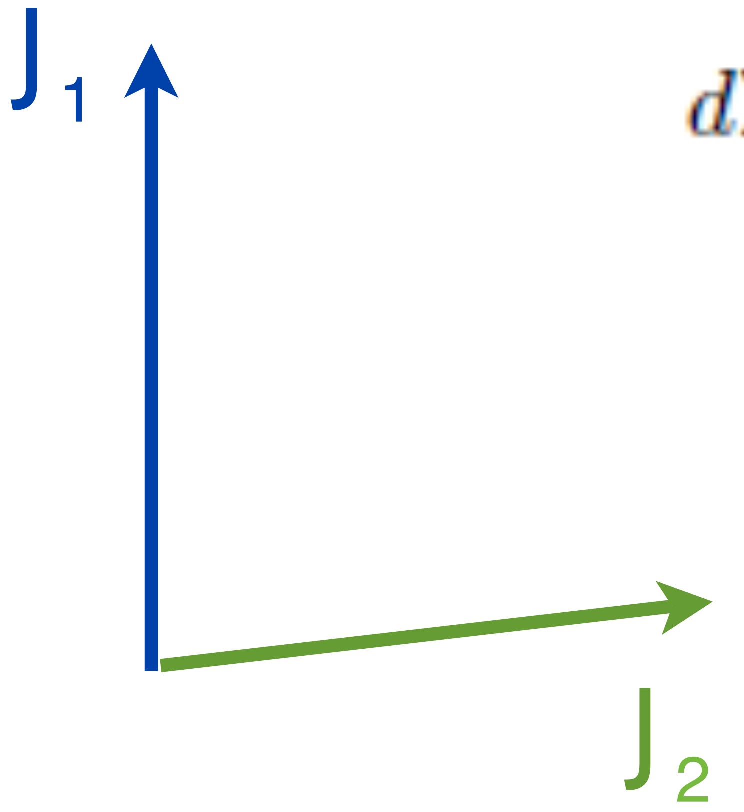
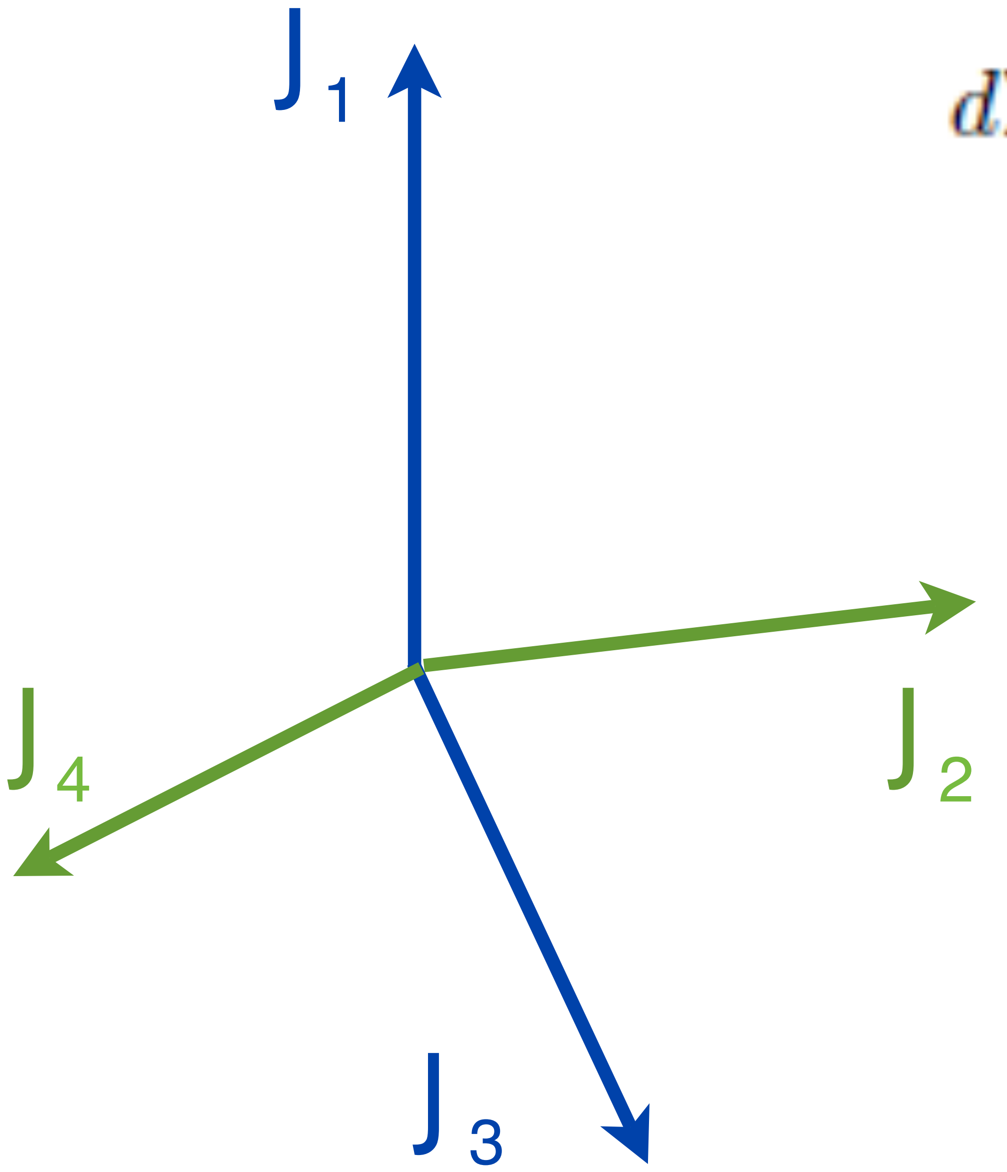


$$d\Sigma(\hat{t}_1, \hat{t}_2)$$

back-to-back kinematics



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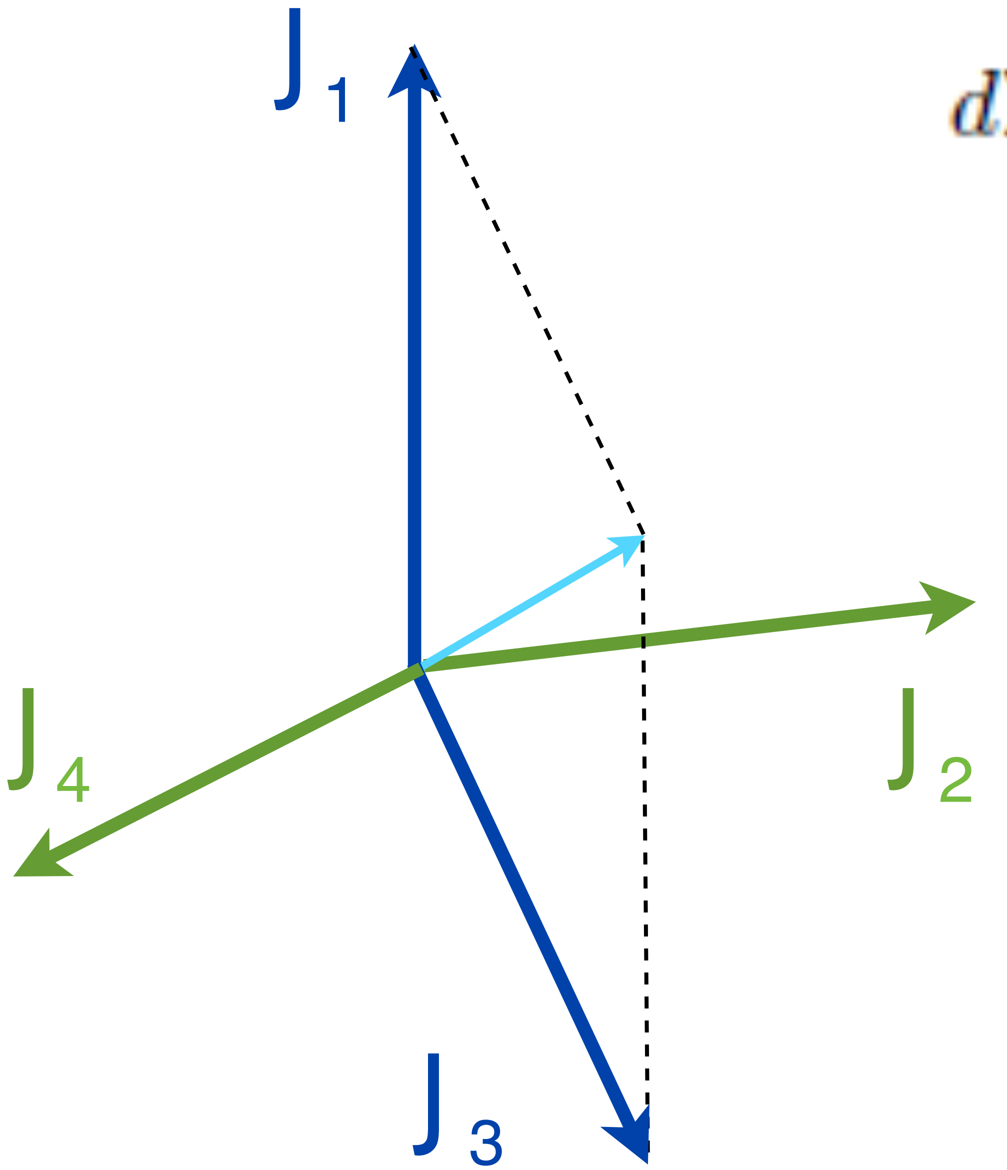

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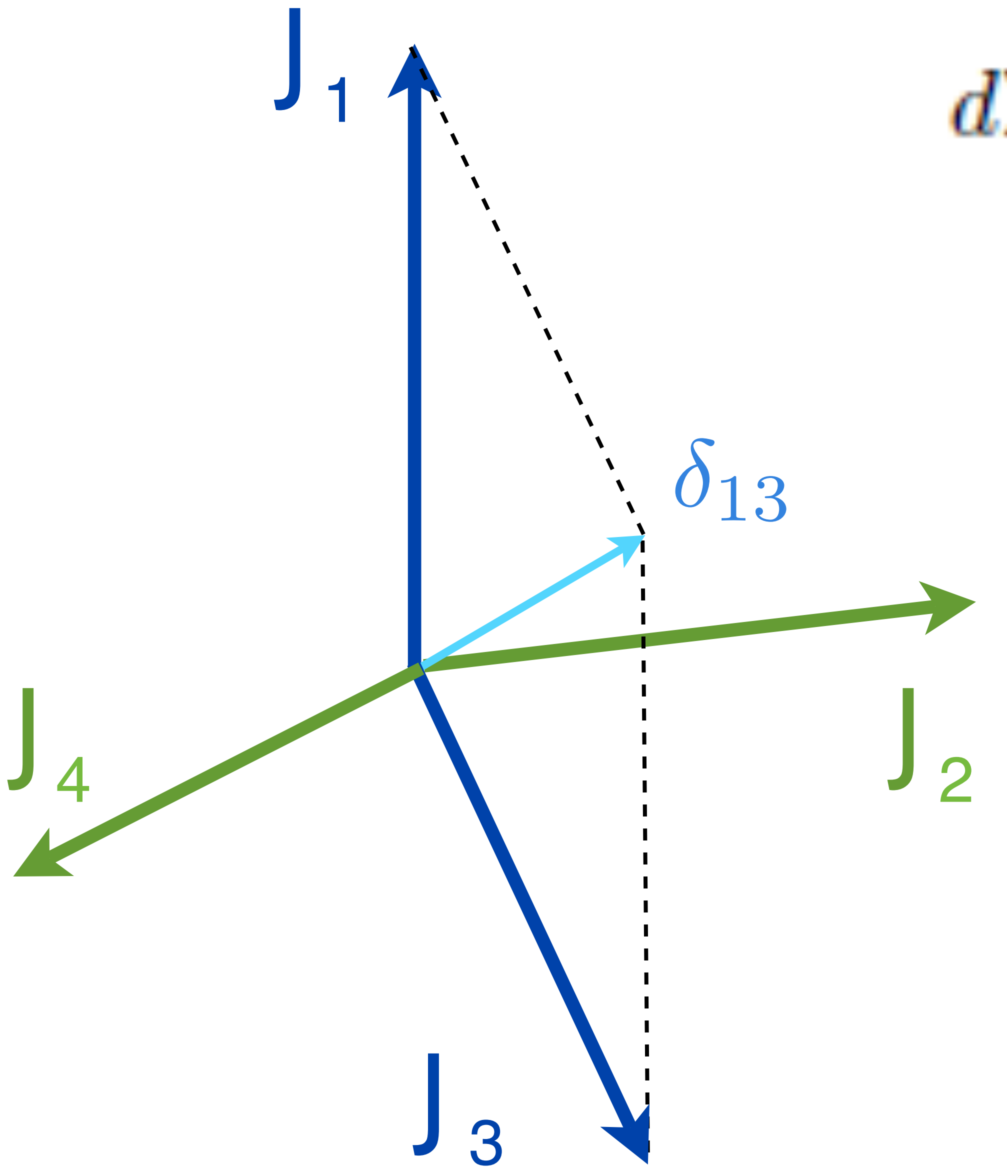

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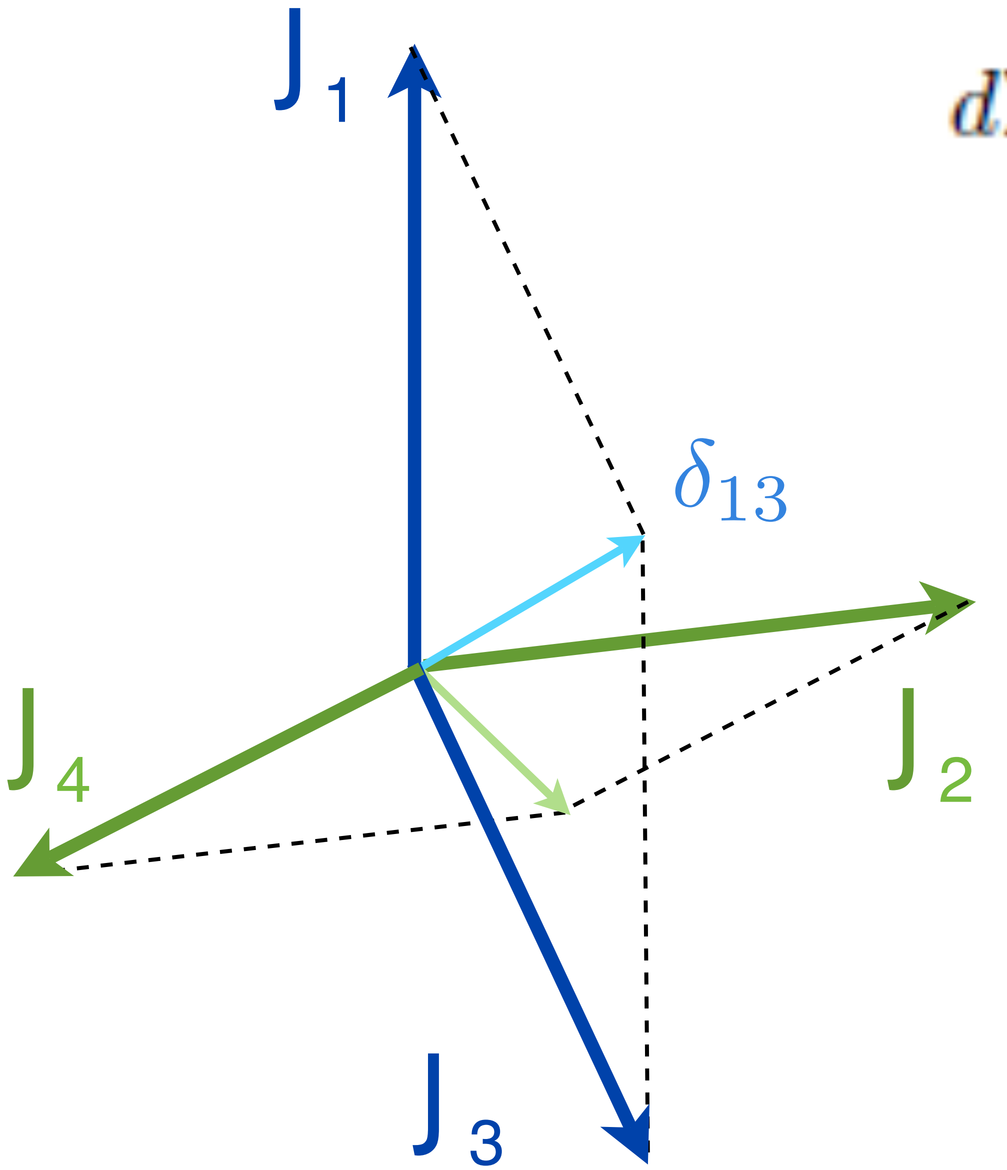

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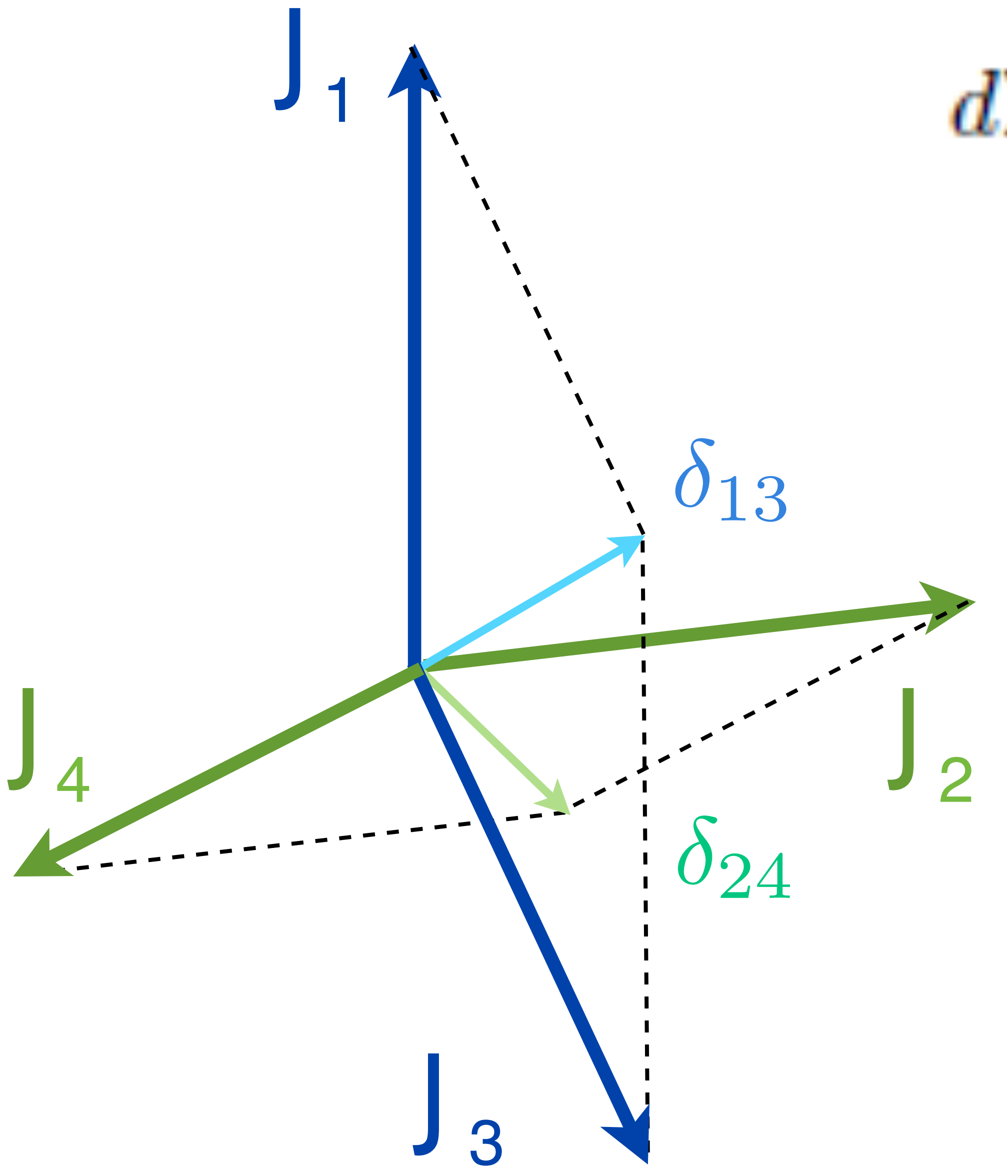

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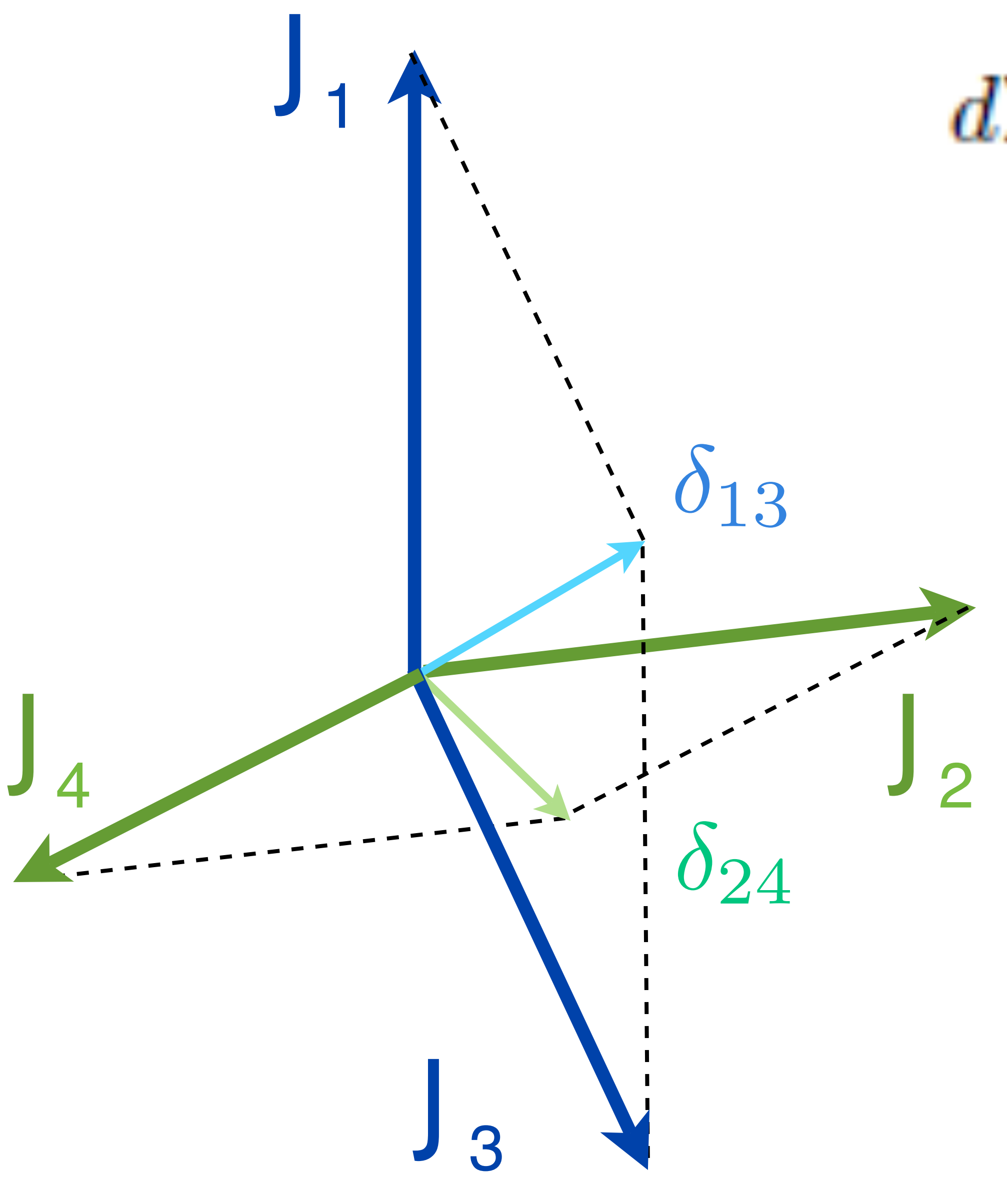

$$d\Sigma(\hat{t}_1, \hat{t}_2)$$

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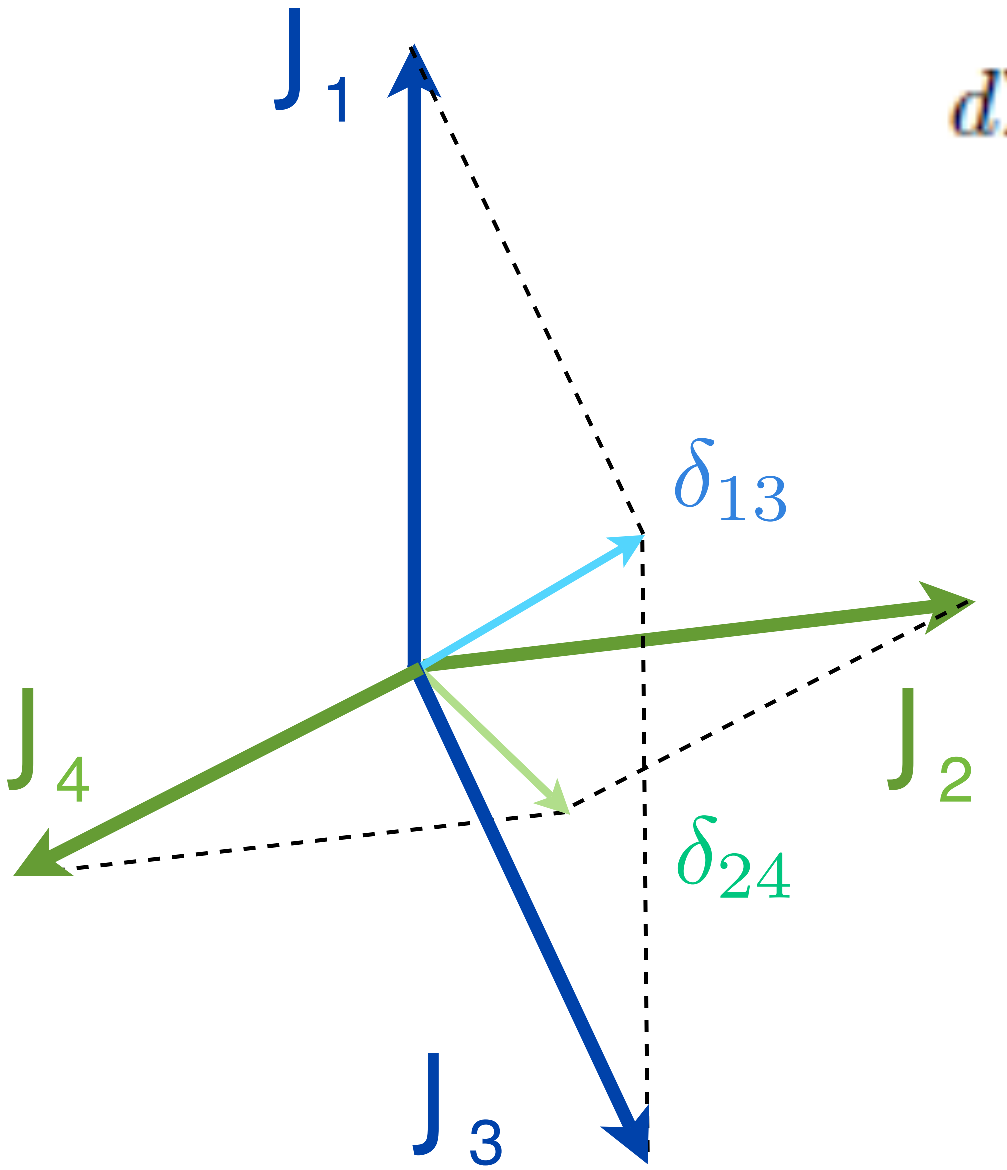
back-to-back kinematics

$$\delta_{13}^2, \delta_{24}^2 \ll J_{i\perp}^2$$



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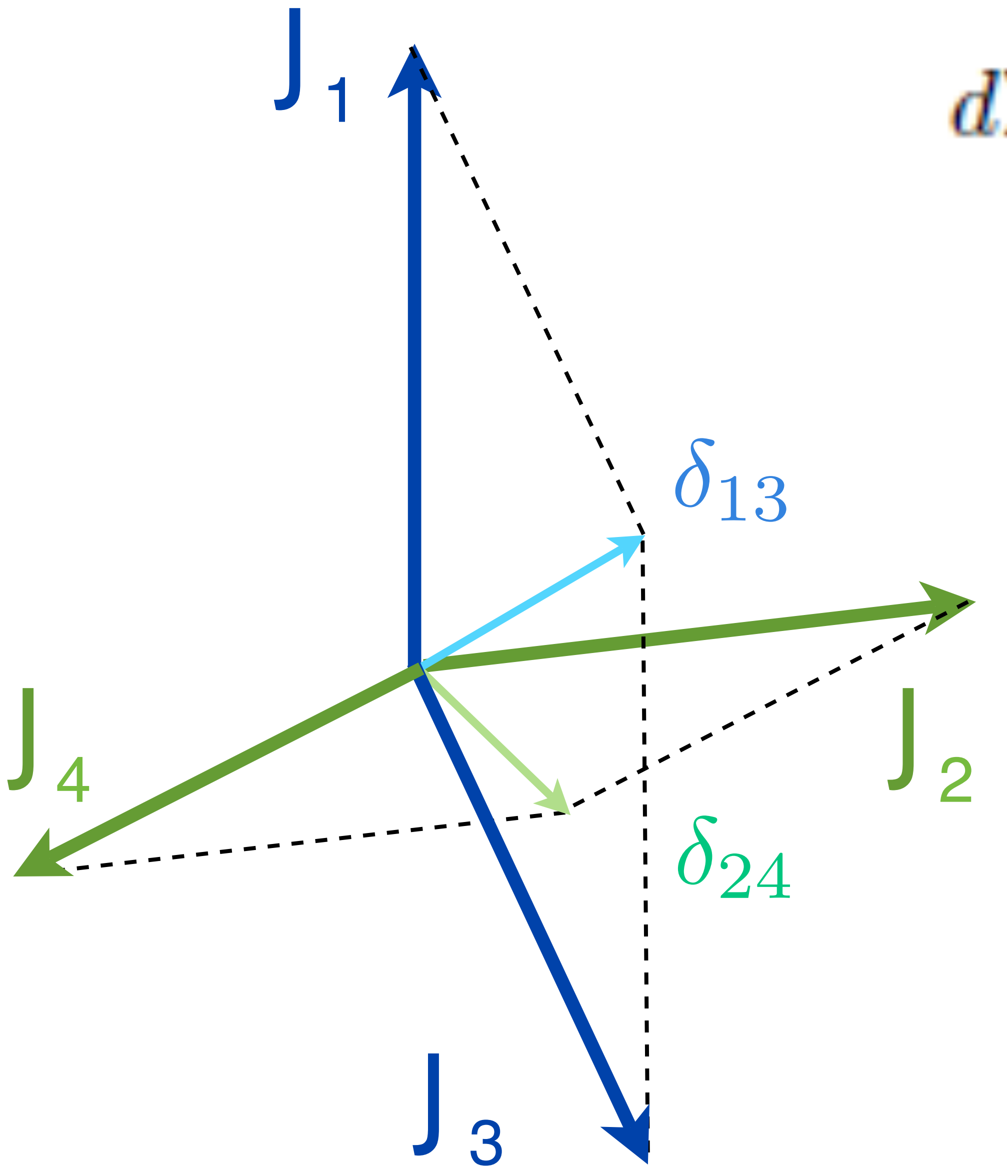

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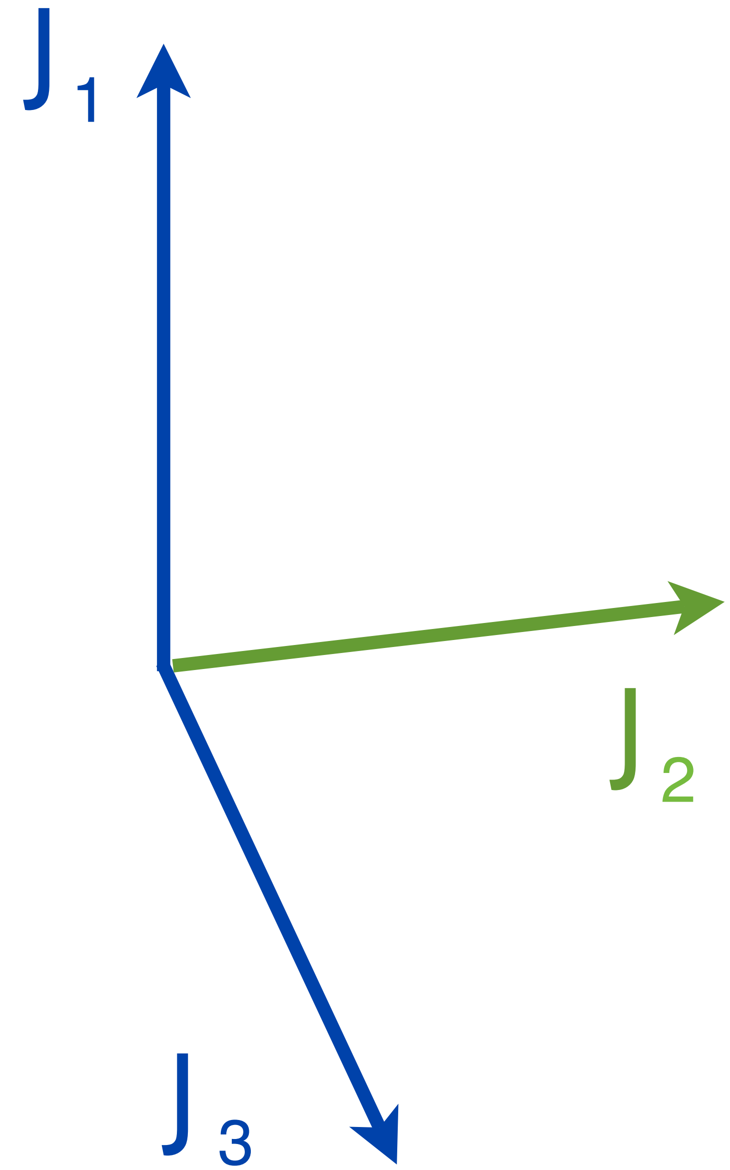

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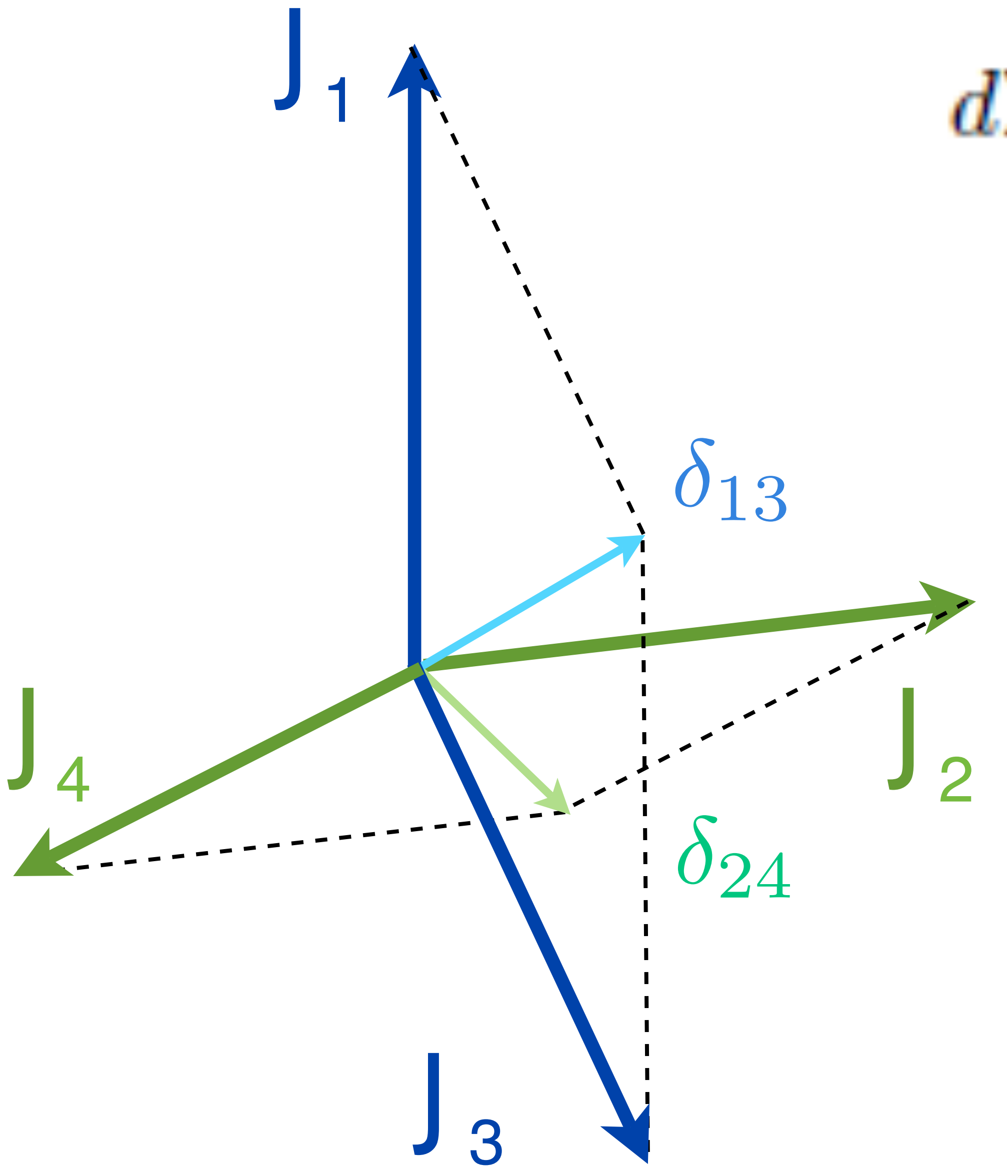

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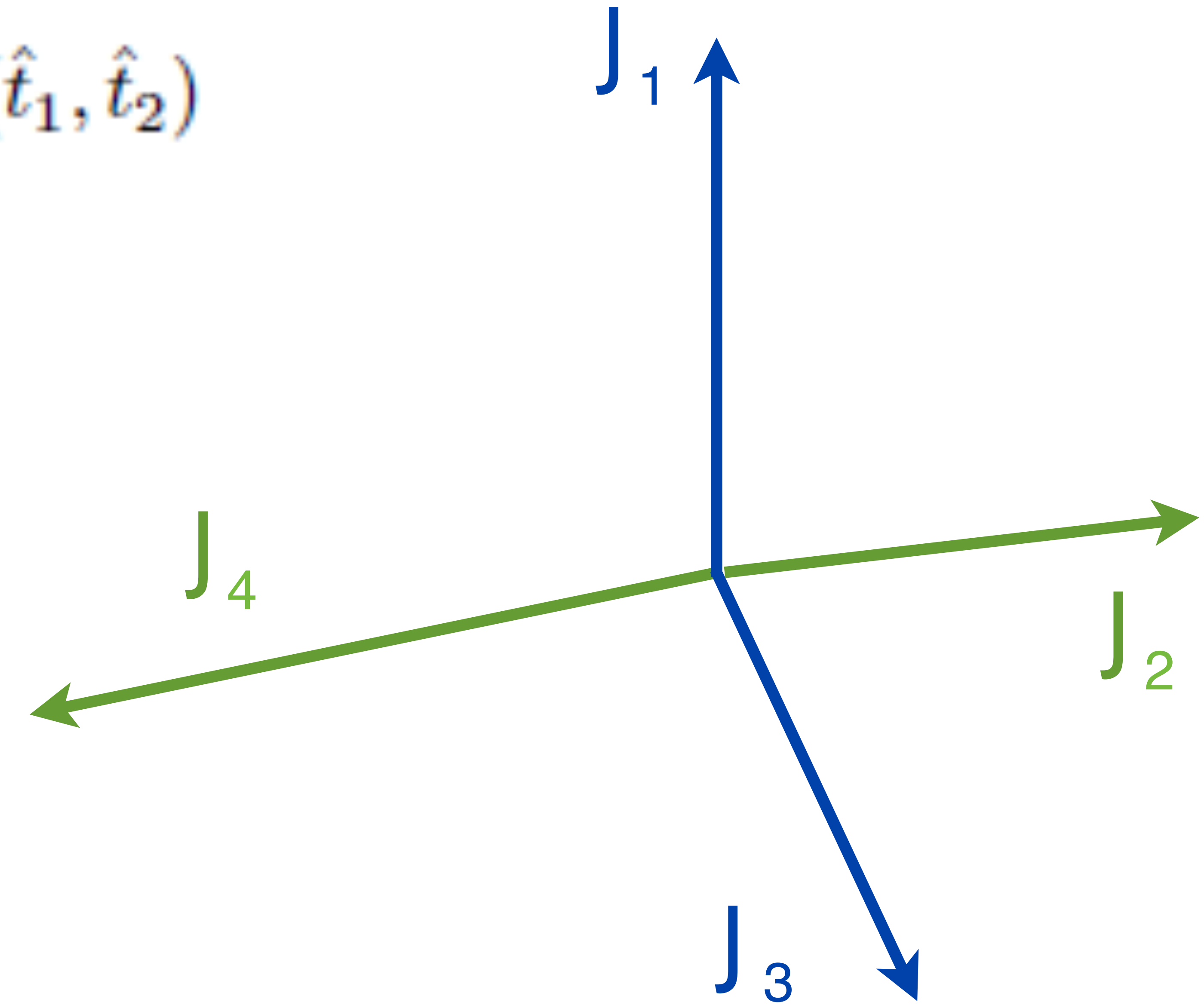

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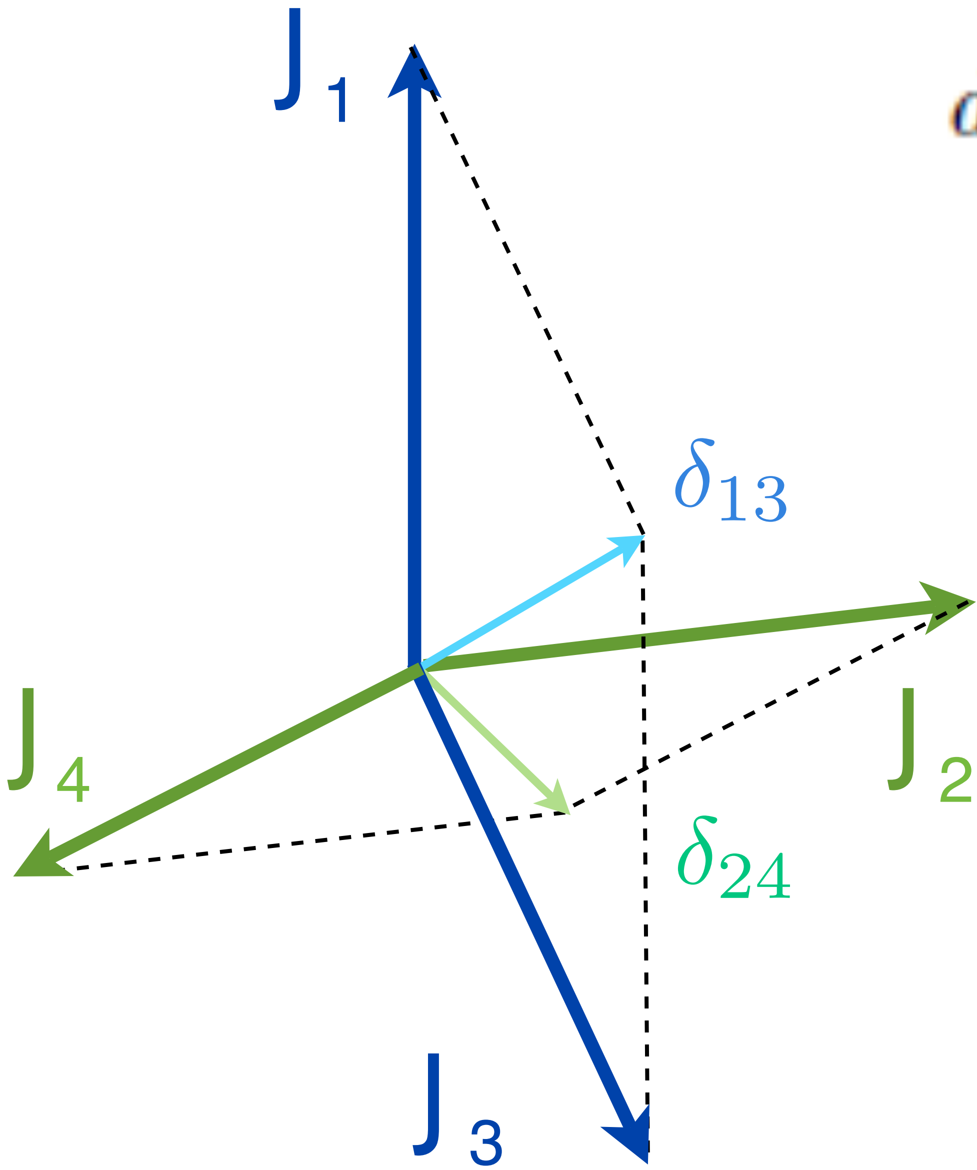

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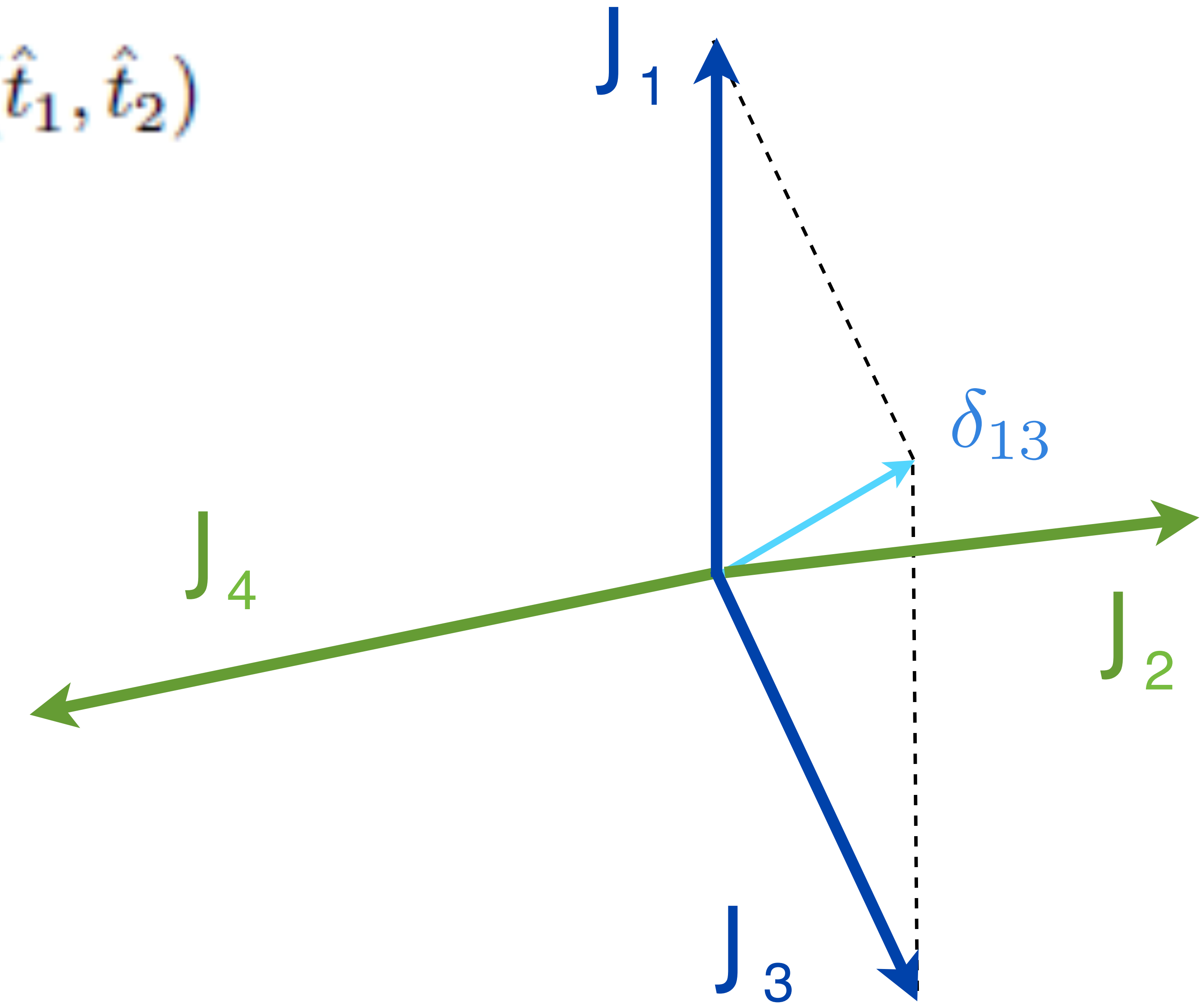

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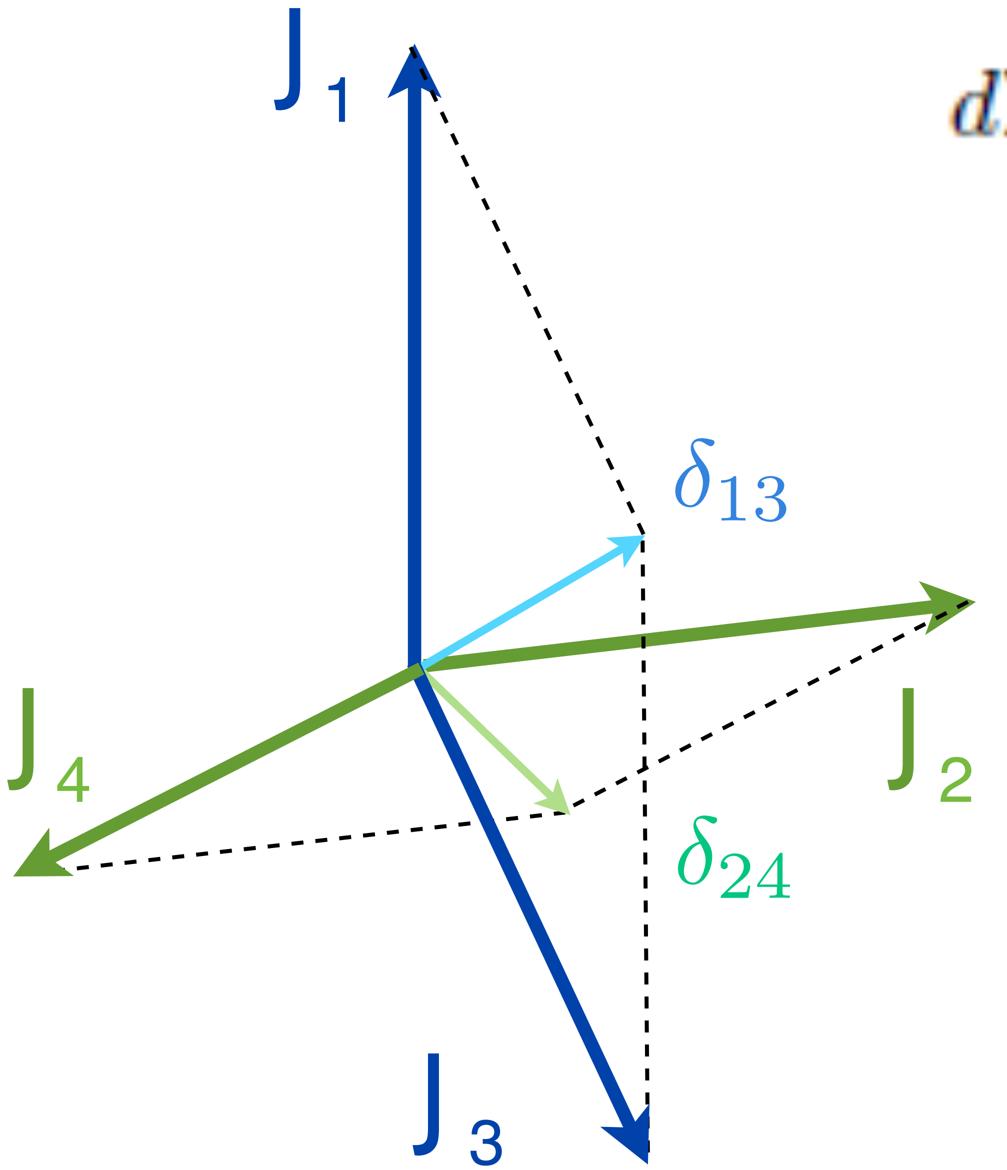

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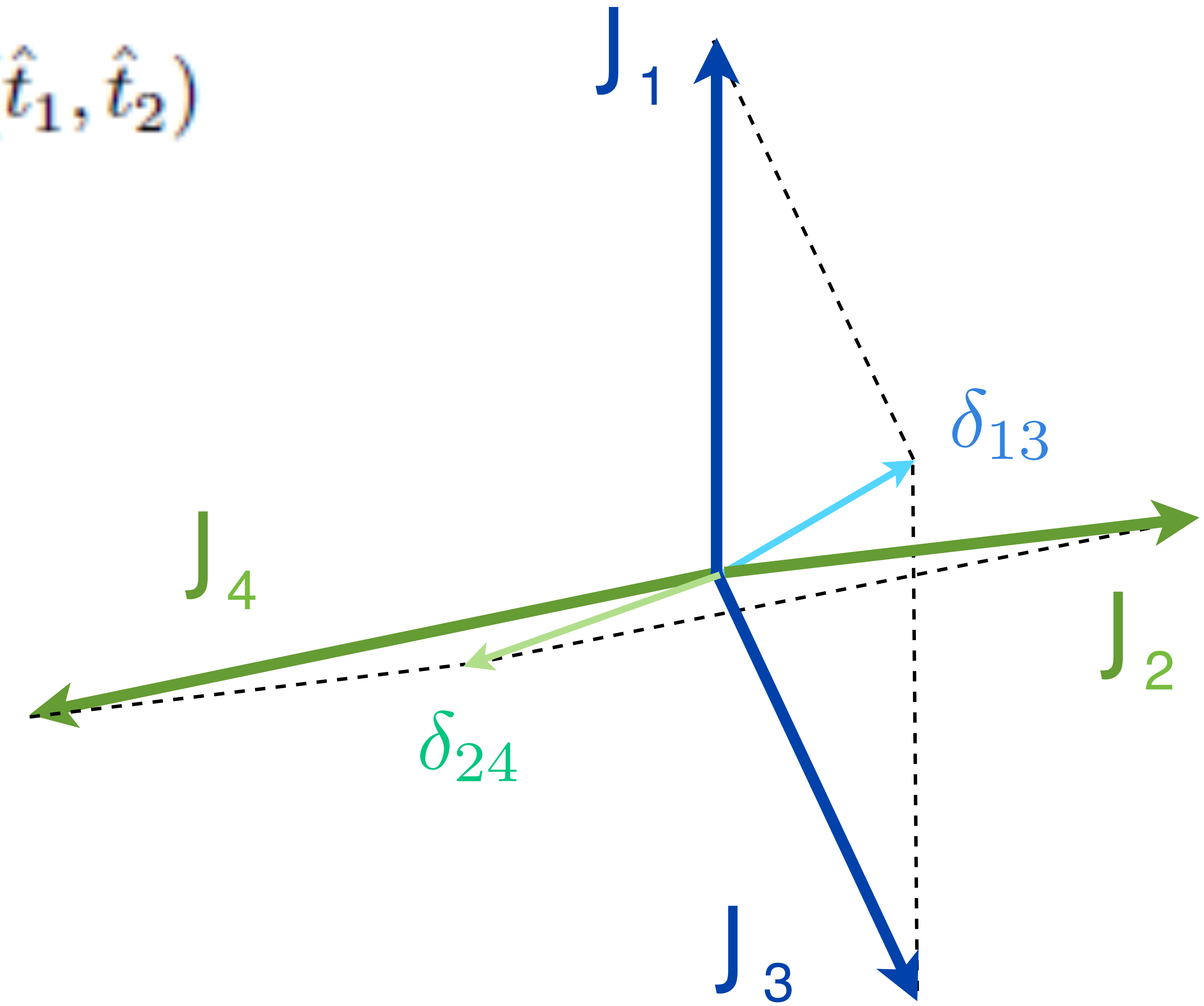

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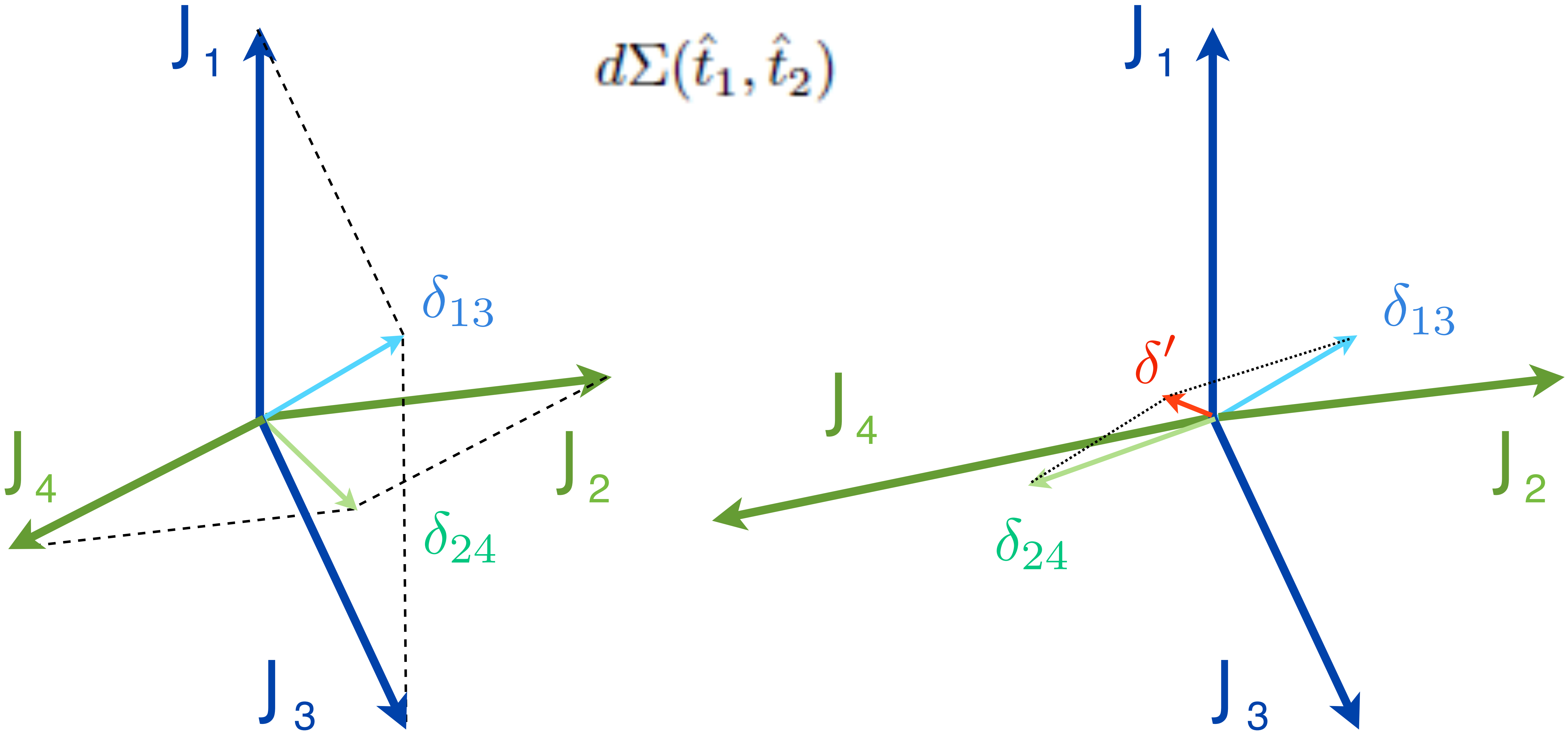

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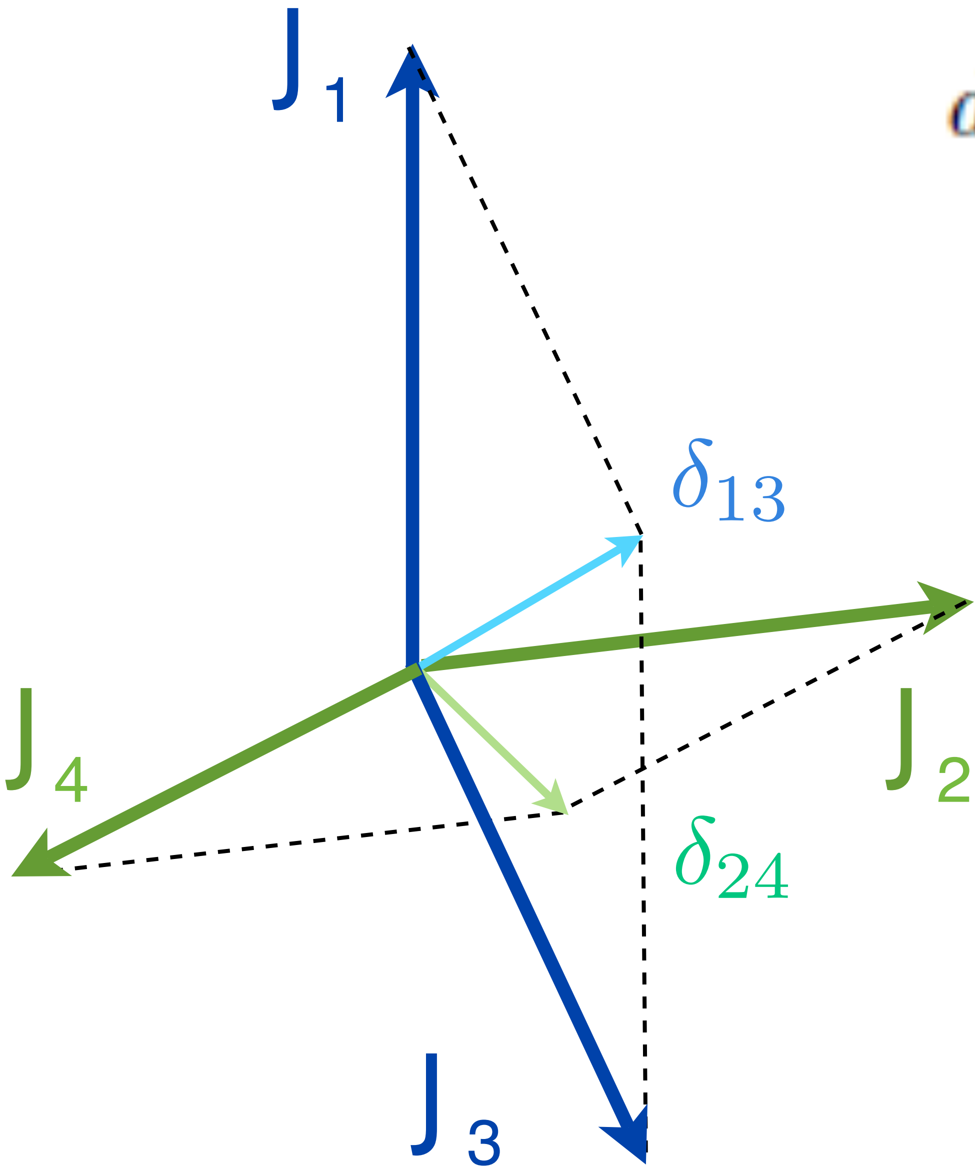

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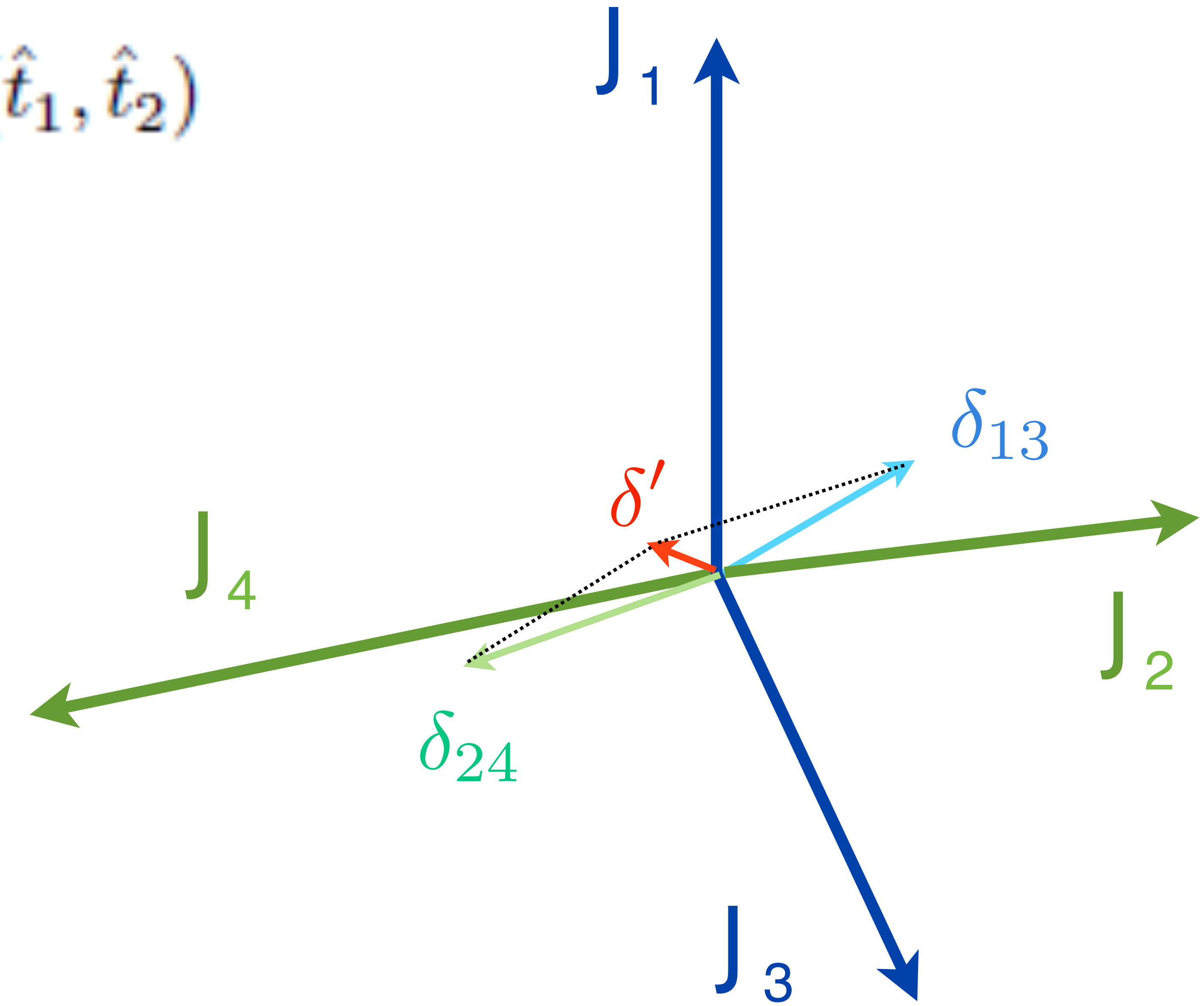

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$$\delta'^2 \ll \delta_{13}^2 \simeq \delta_{24}^2 \ll J_{i\perp}^2$$

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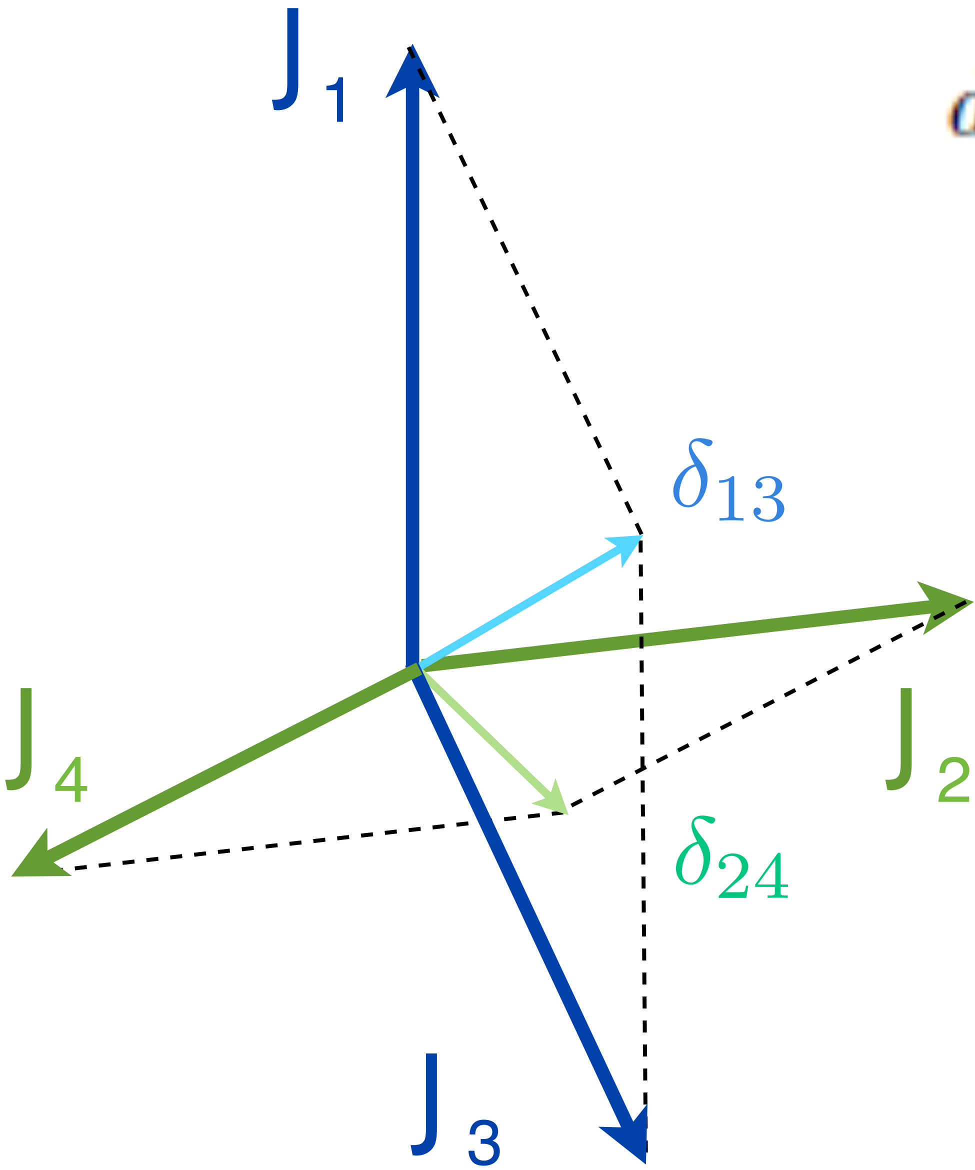

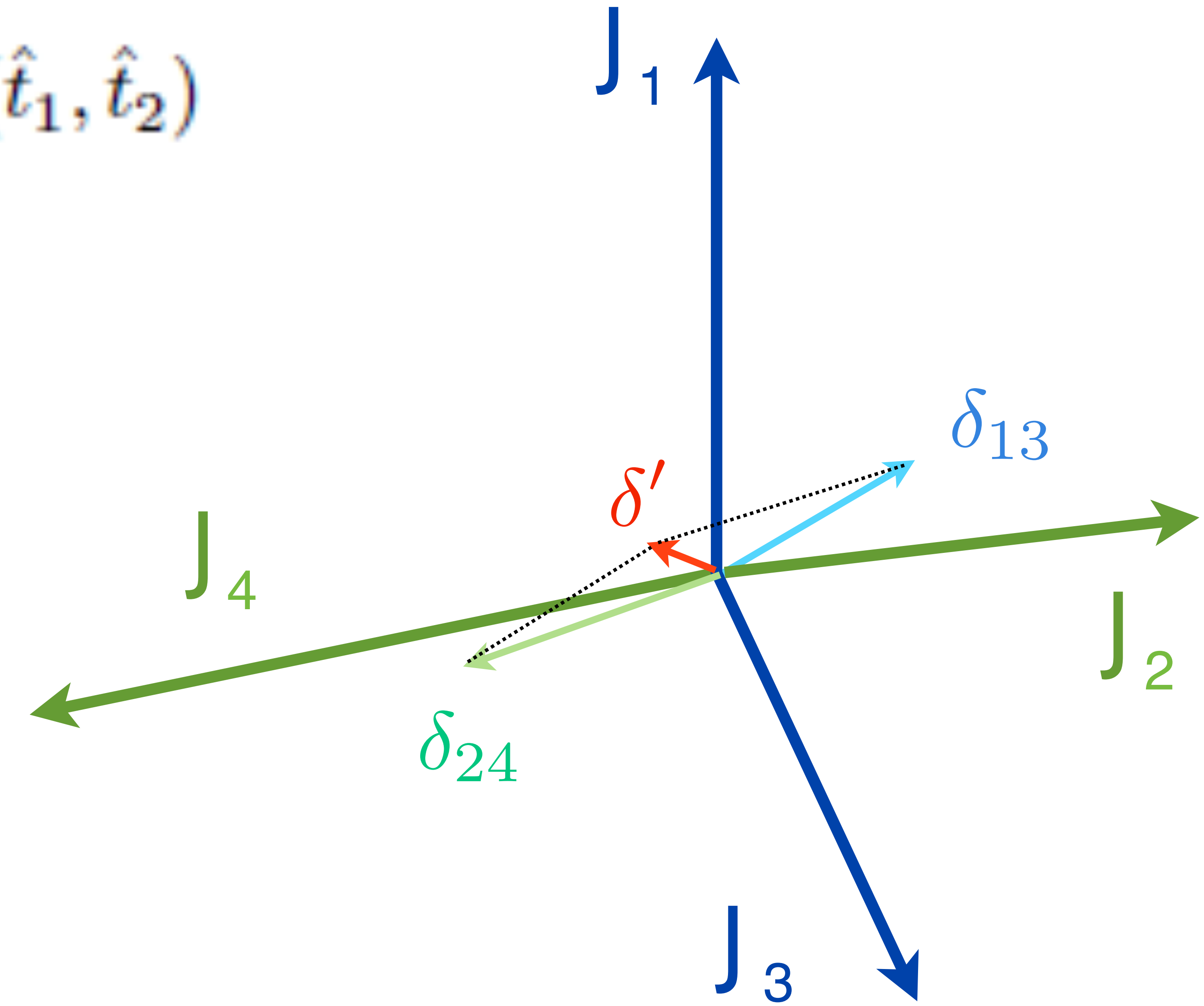

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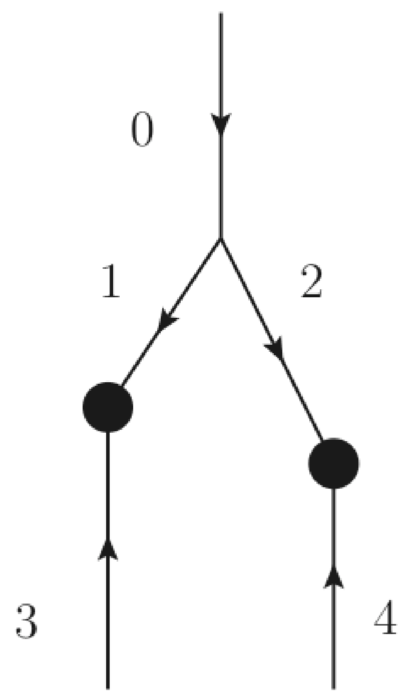
Underwater stones of the MPI analysis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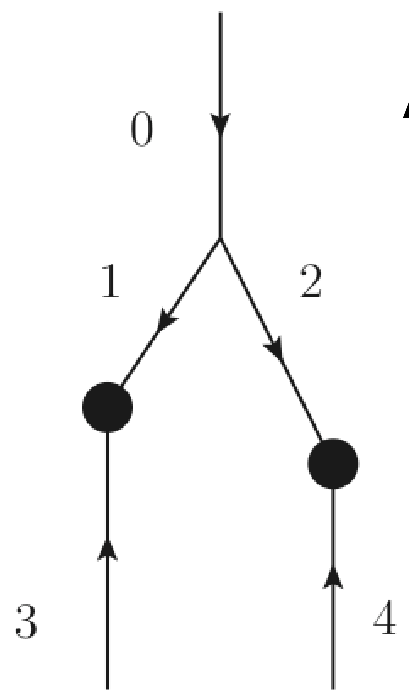
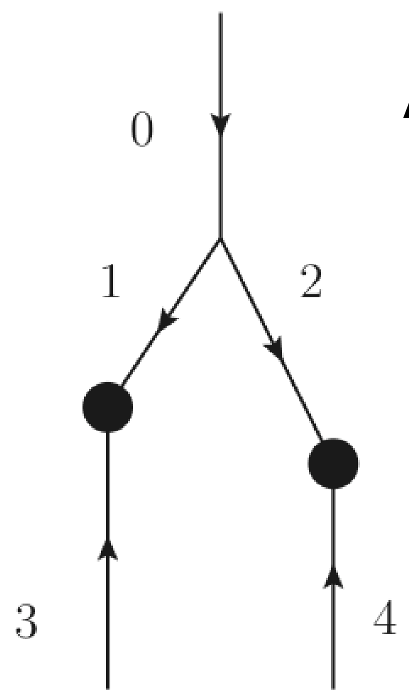
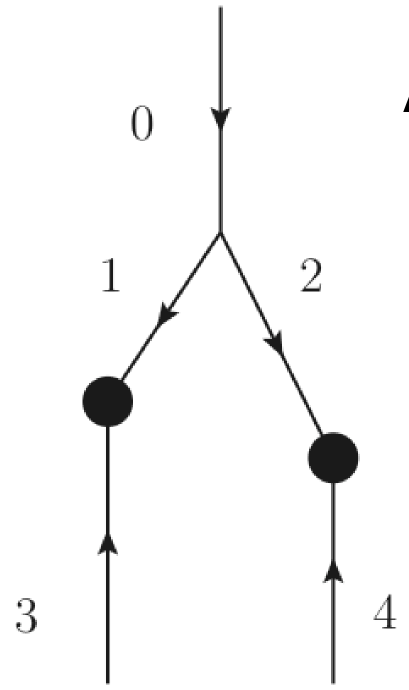


A tree Feynman diagram.



A tree Feynman diagram.   Momenta of internal parton lines are fixed ...



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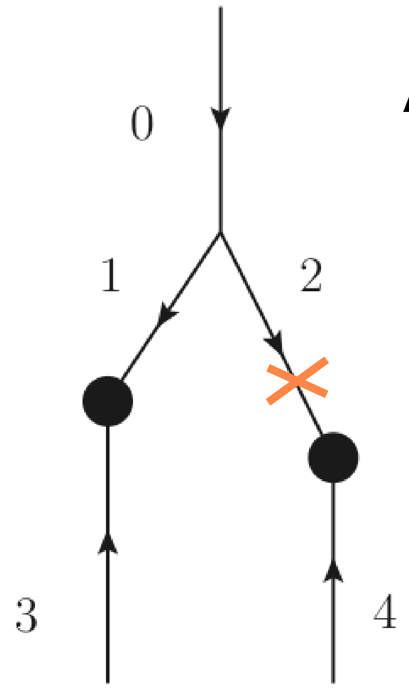
Singularities in the physical region of parton momenta !



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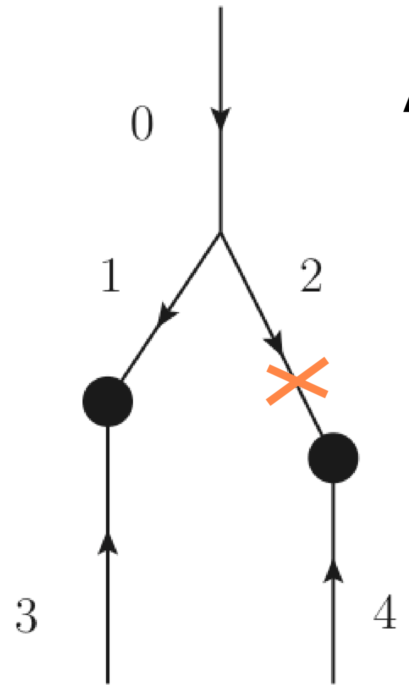

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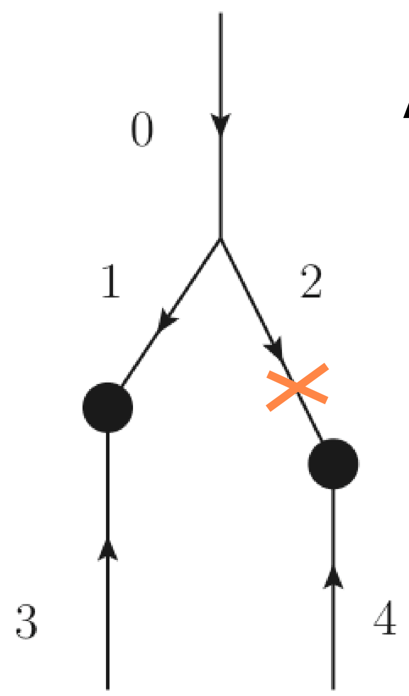

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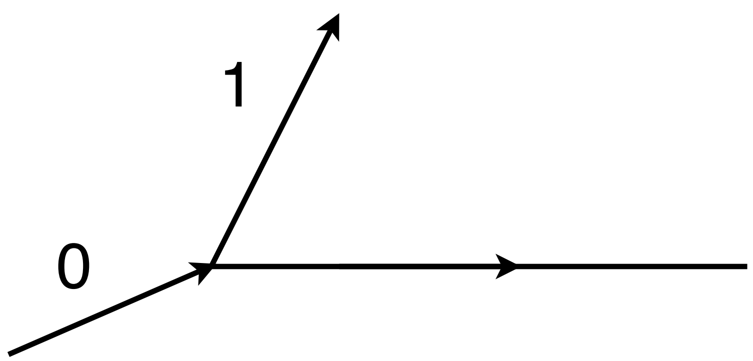

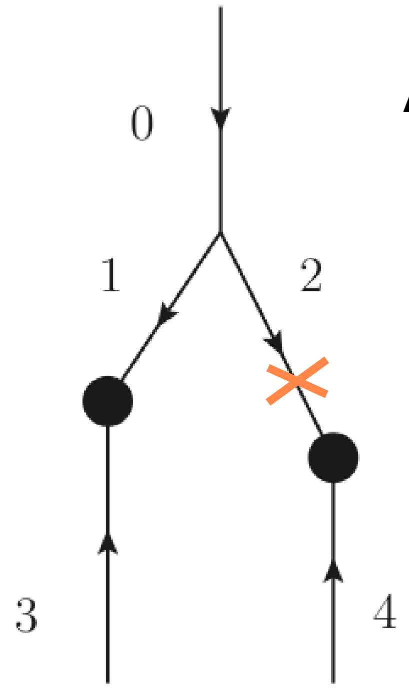

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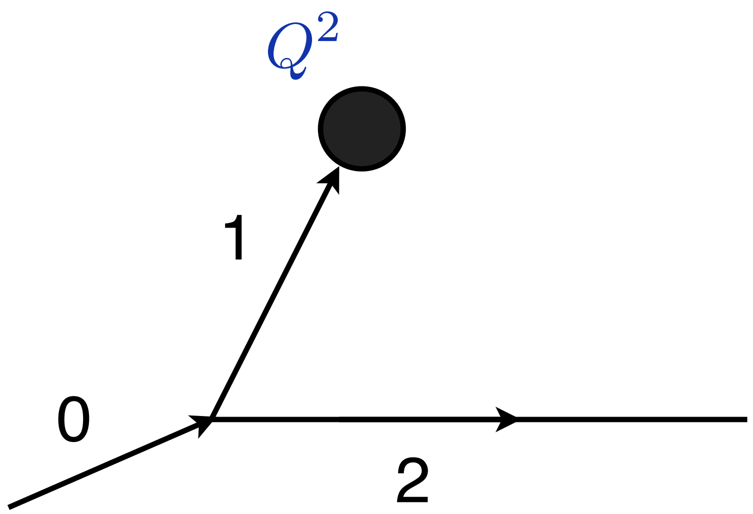

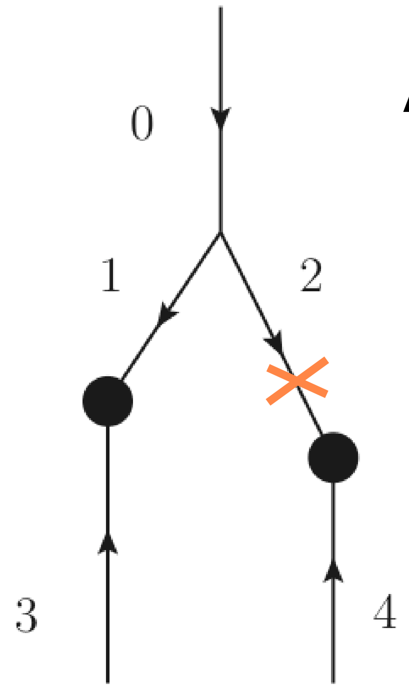

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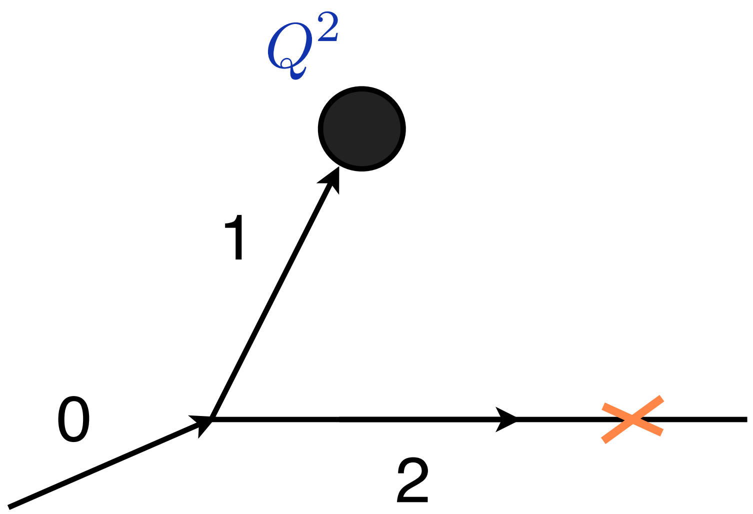

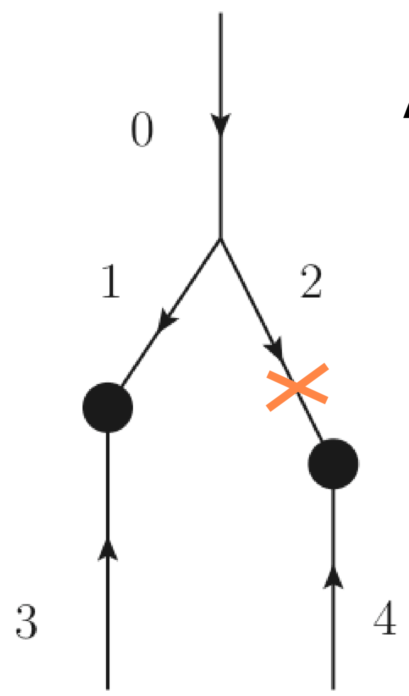

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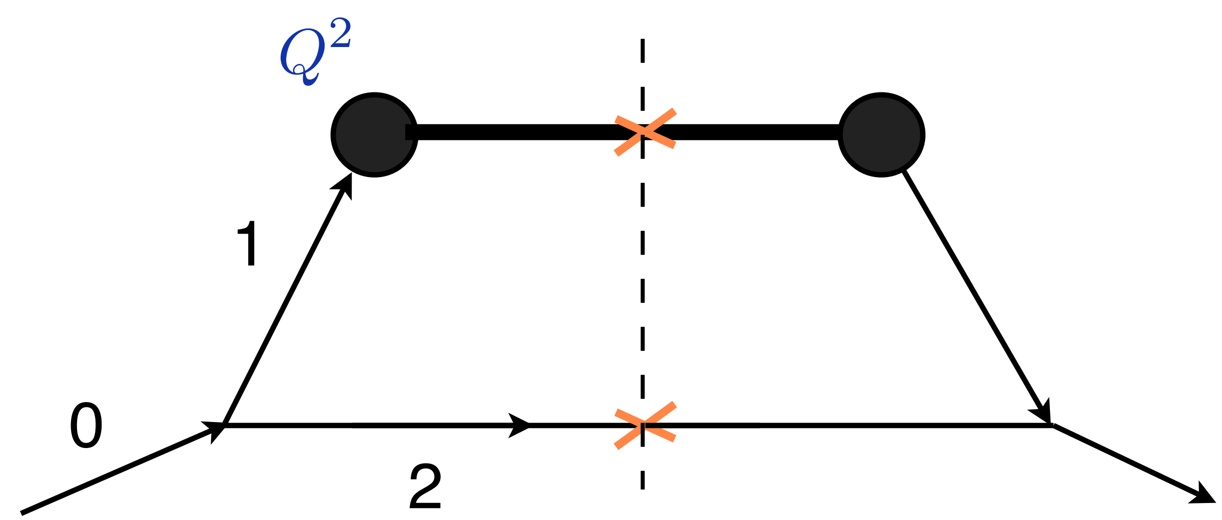

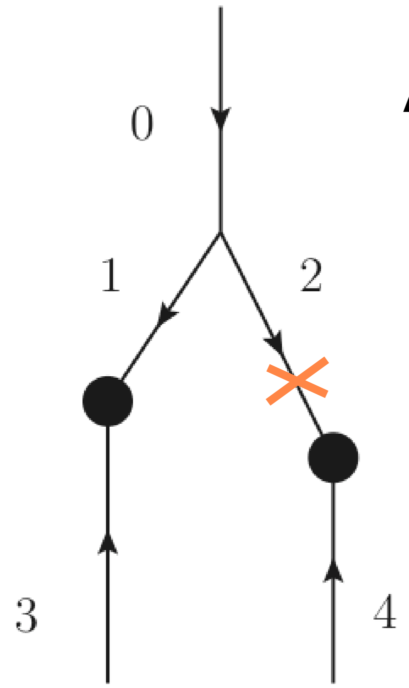

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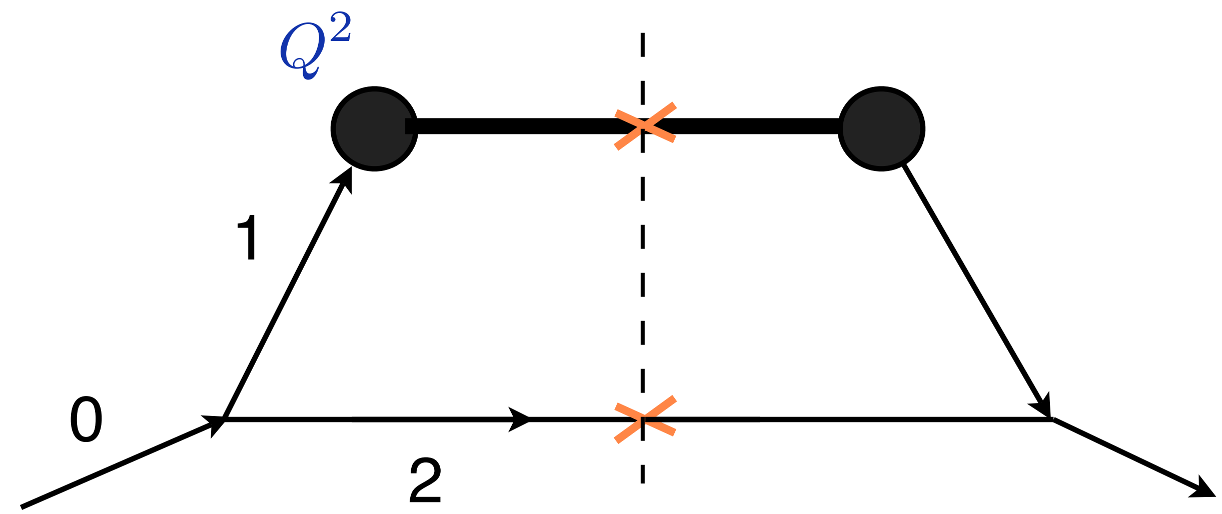

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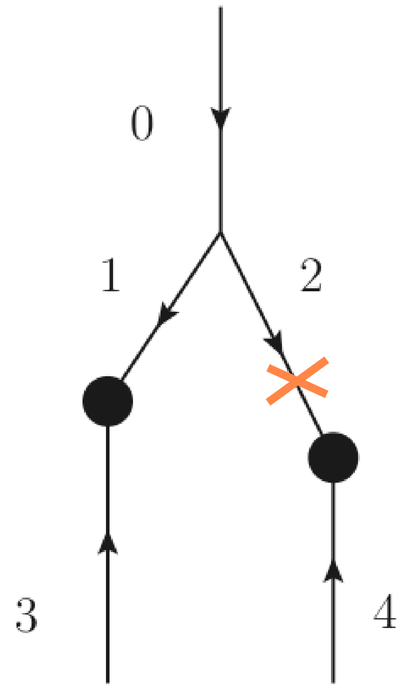

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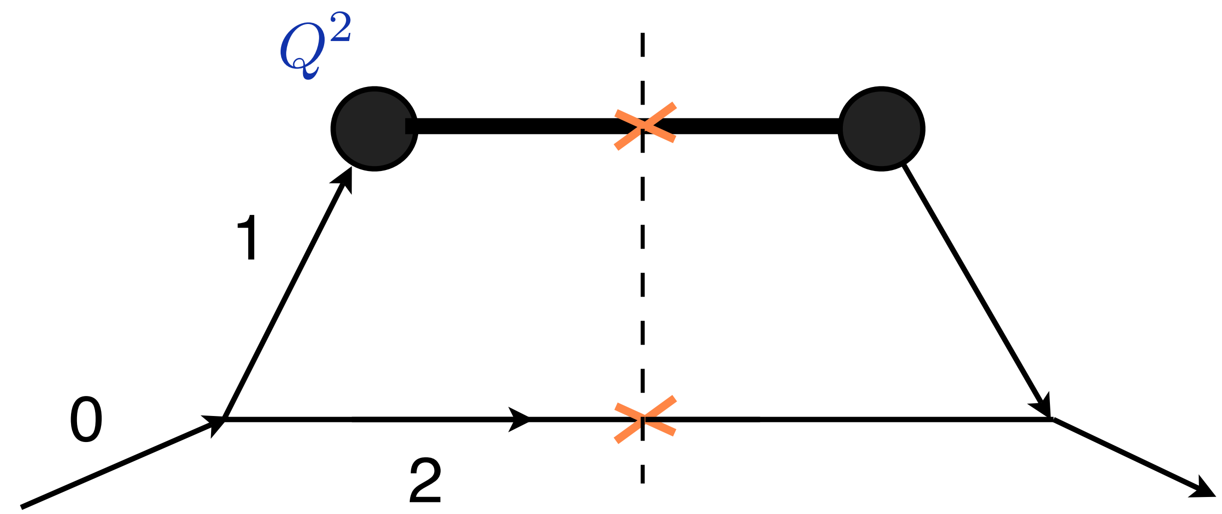

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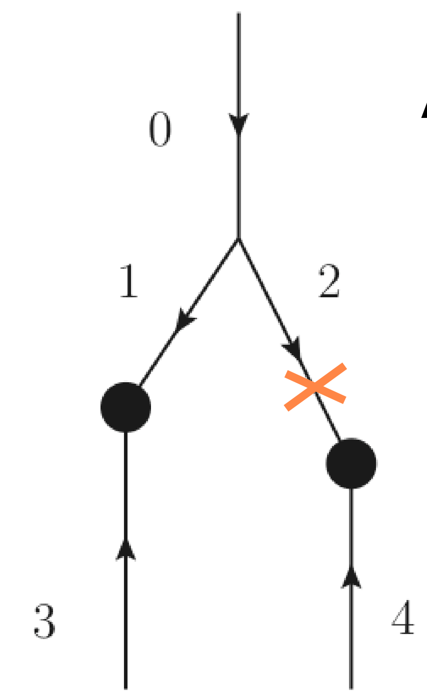

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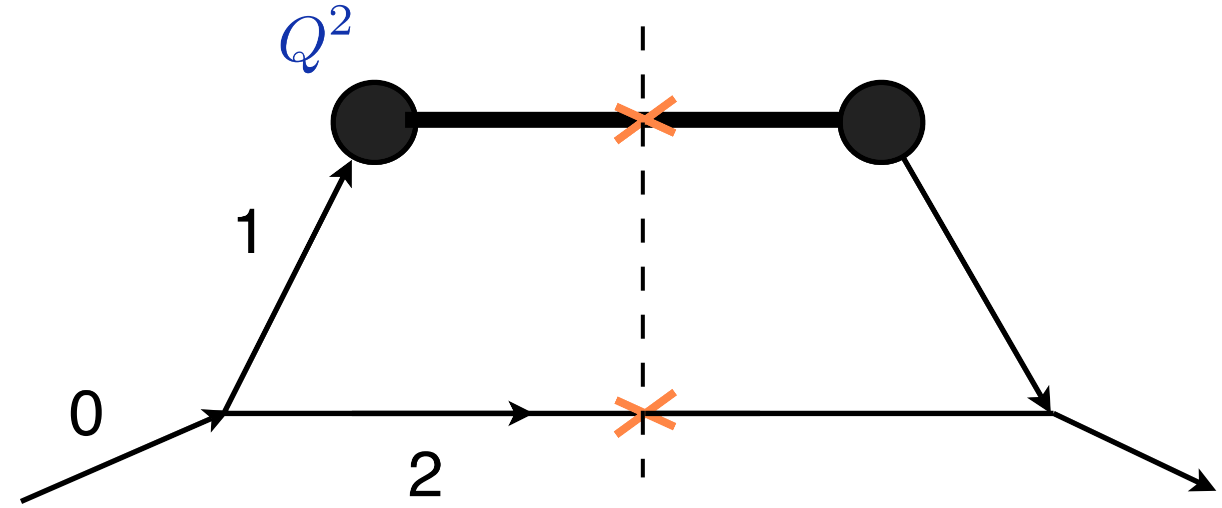

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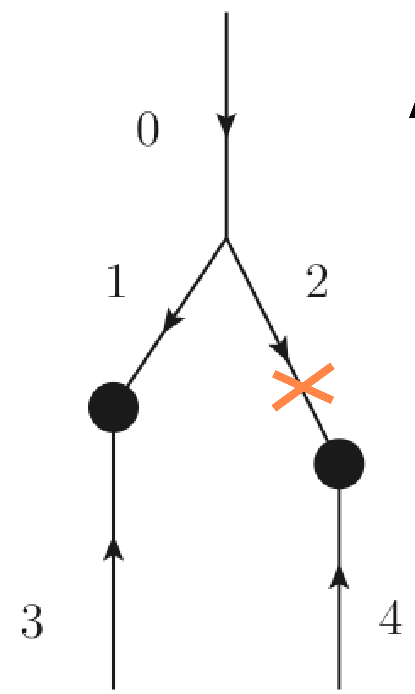

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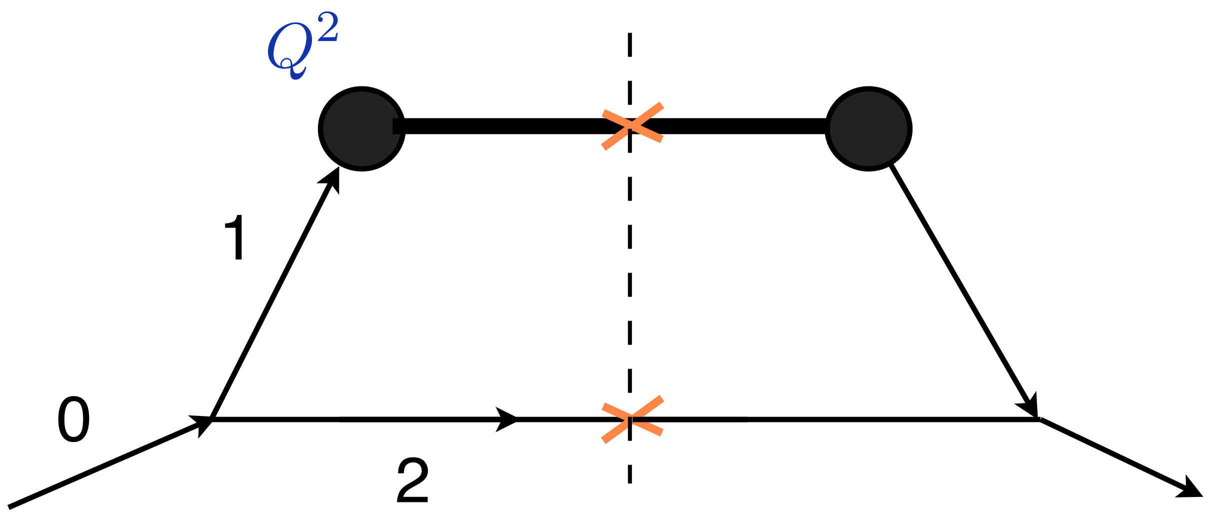

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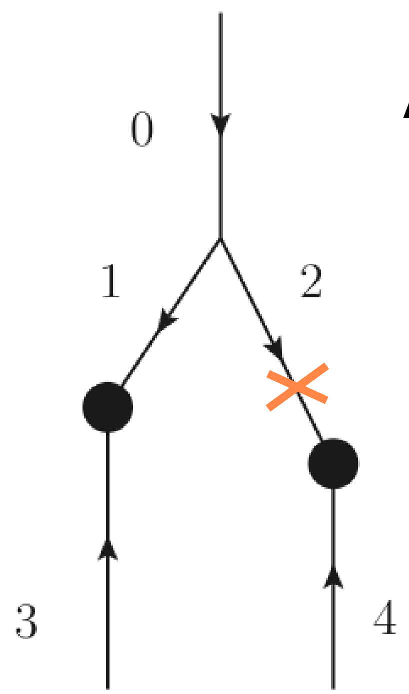

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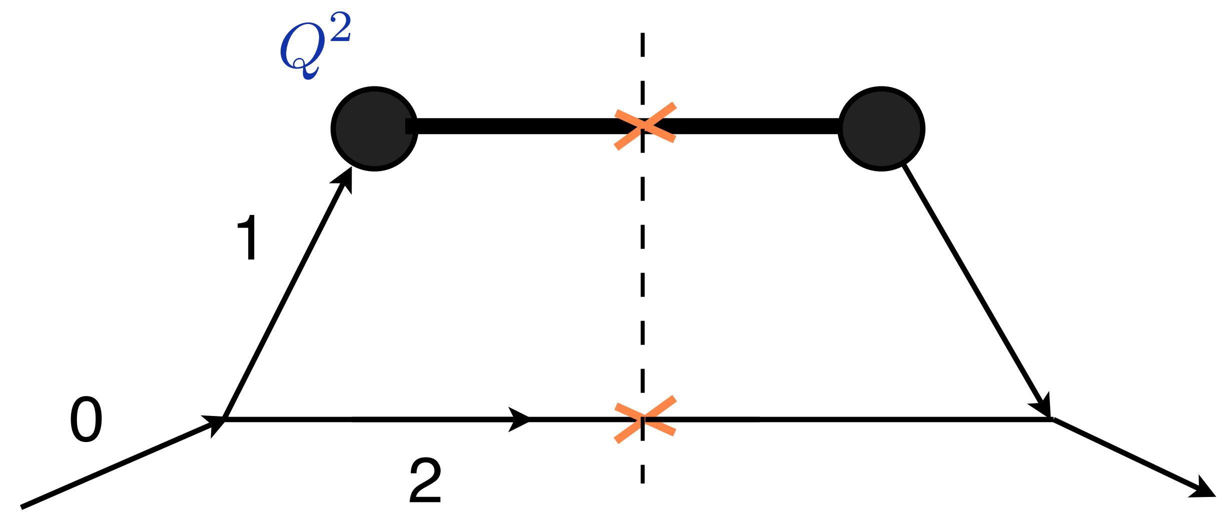

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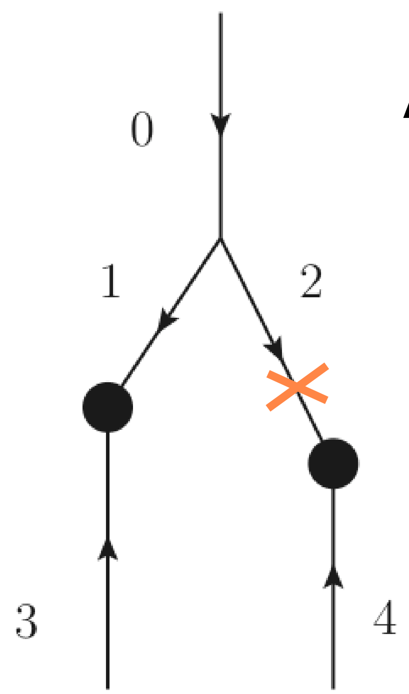

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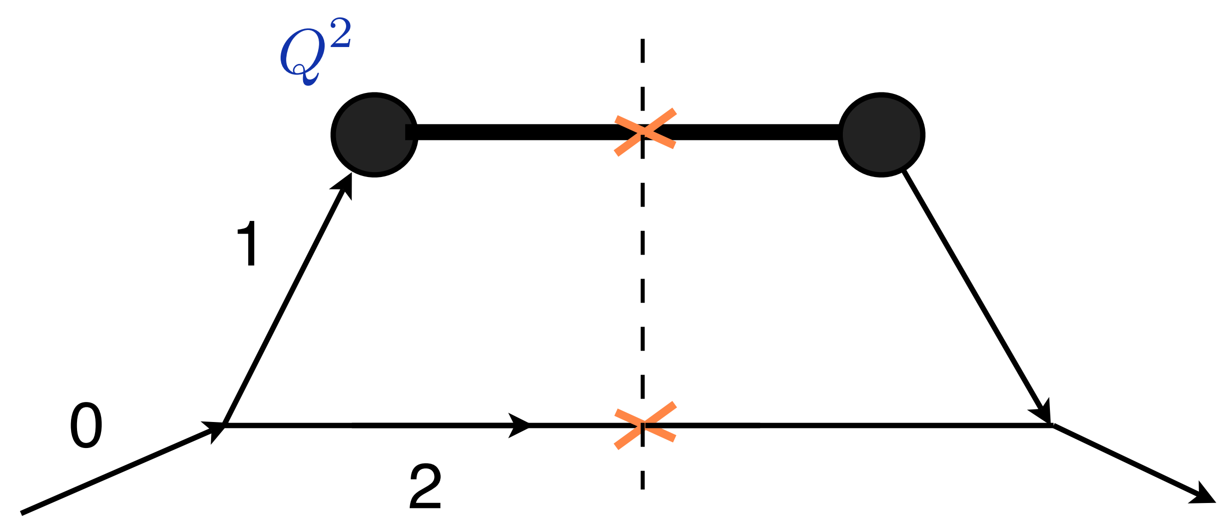

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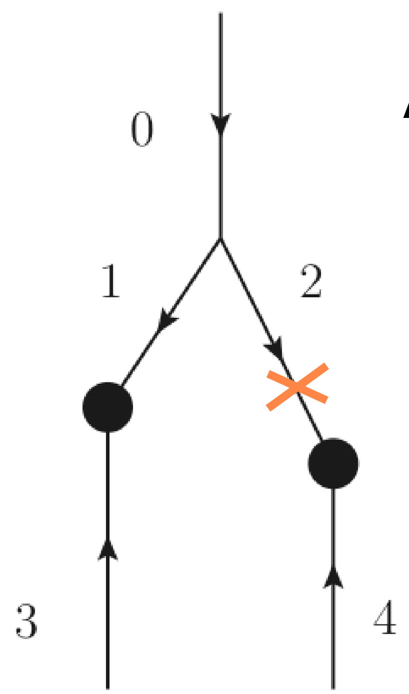

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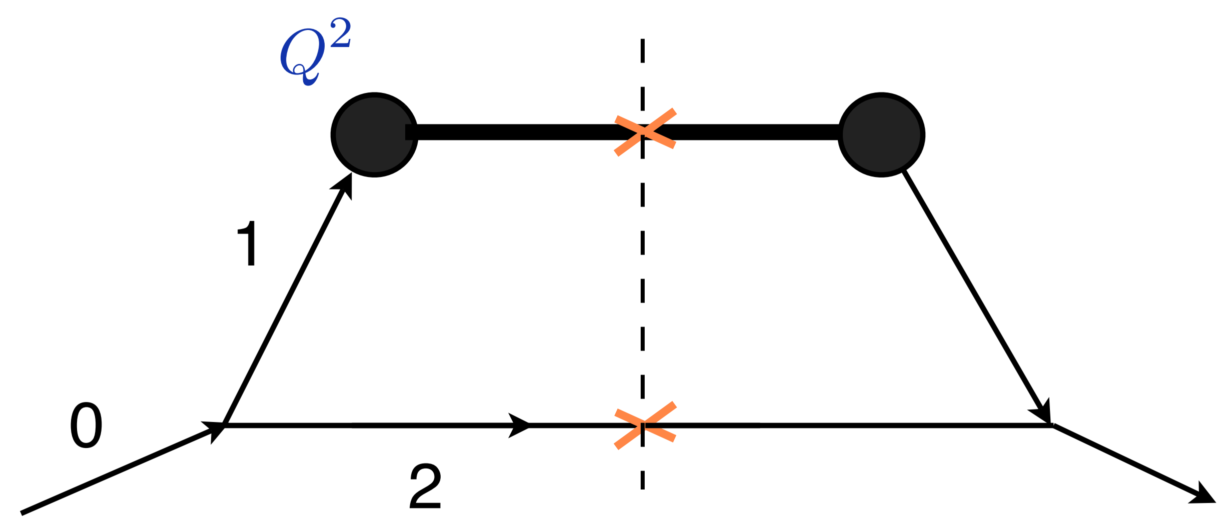

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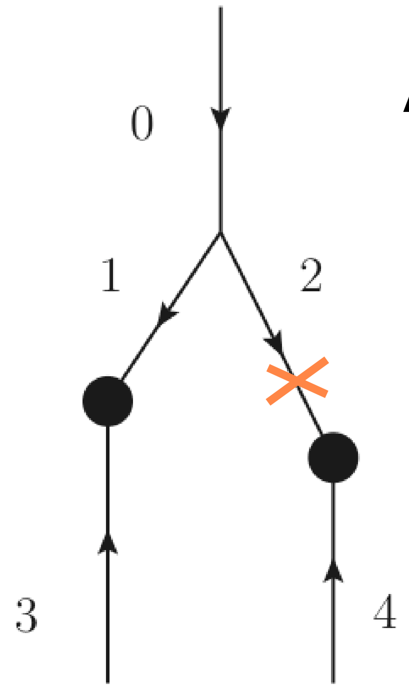

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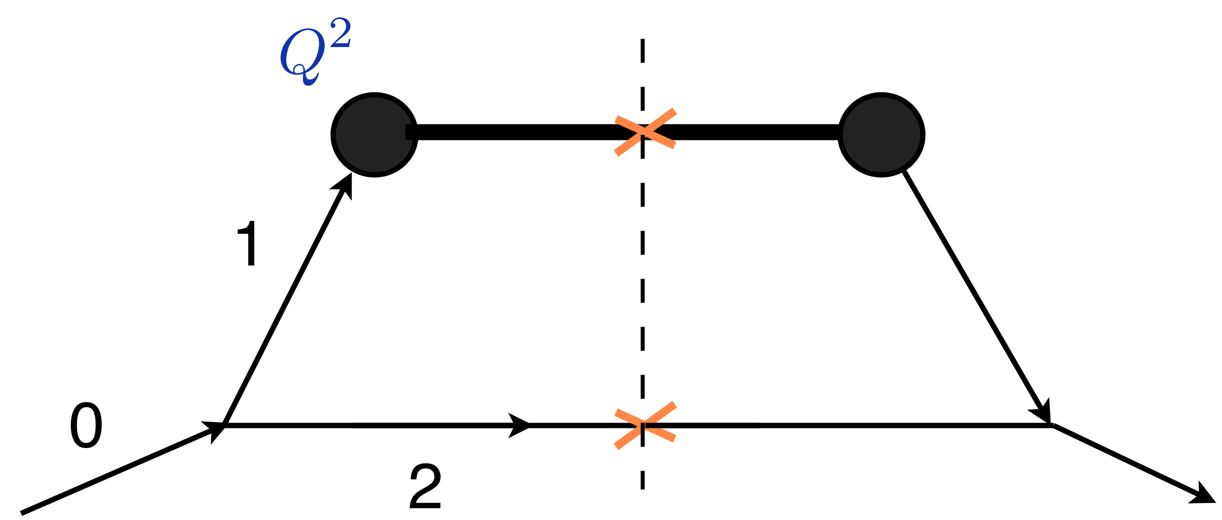

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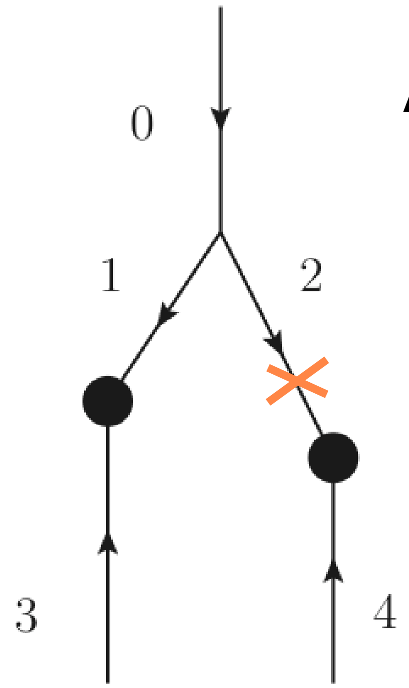
$k_{3+} + k_{4+}$ fixed by hard scattering kinematics



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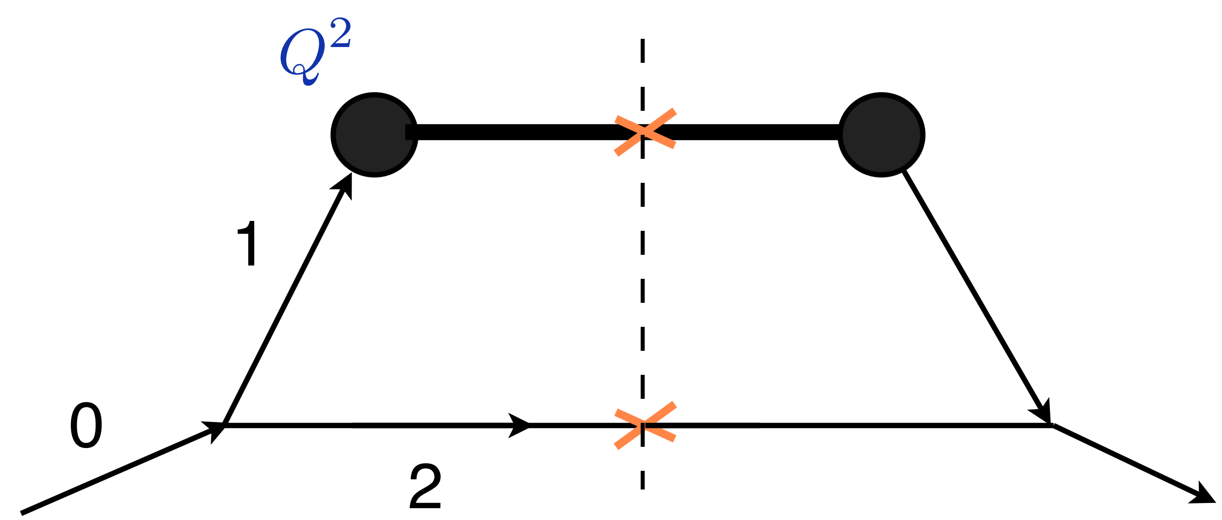


In DIS we trace the fate of **1** but *integrate* over "histories" of the accompanying parton **2**.

Now we want #**2** to enter 2nd hard interaction.

In the above picture it does it "*in the next room*". Or on the Moon, for that matter ...

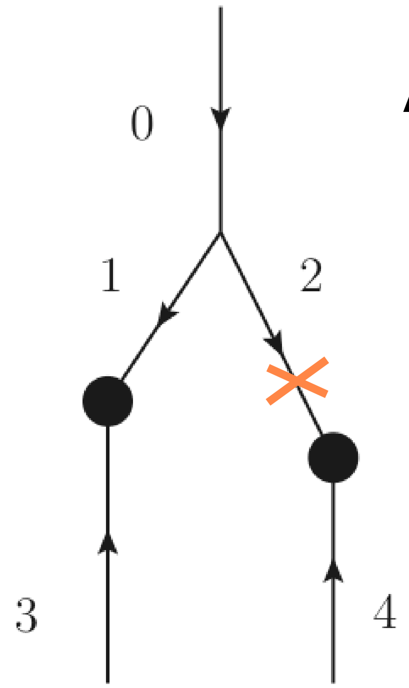
In fact, partons **3** and **4** ***cannot be*** represented by plainly independent ***plane waves***: they belong to ***one hadron***, and therefore, are ***localized within the hadron pancake***...

Remedy: introduce wave packet smearing (longitudinal momentum fraction integral).

Importantly, this has to be done at the ***amplitude level*** !

$k_{3+} + k_{4+}$ fixed by hard scattering kinematics

$k_{3+} - k_{4+}$



A tree Feynman diagram. Momenta of internal parton lines are fixed ...

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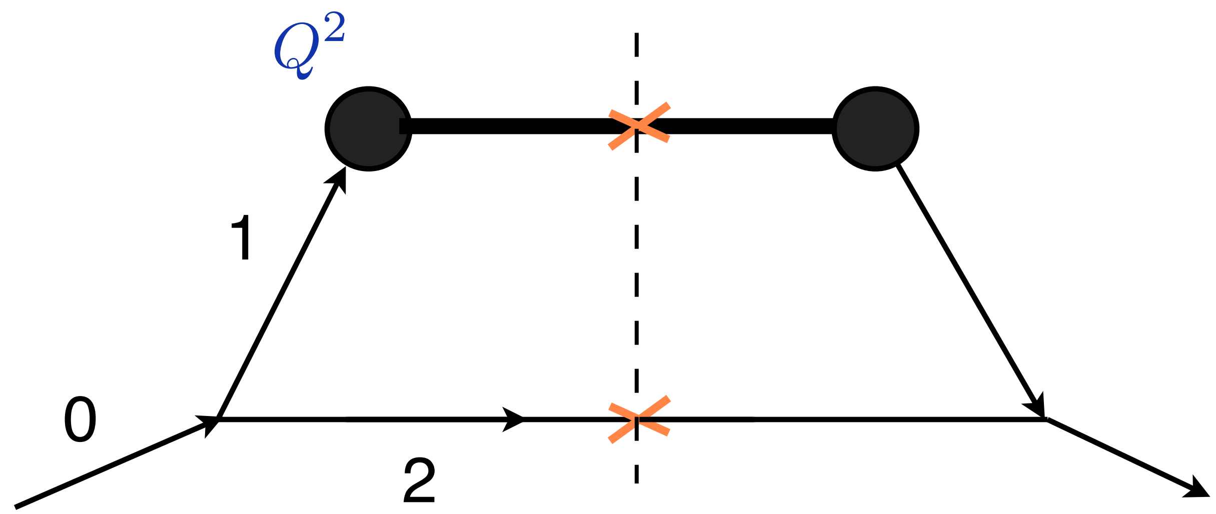

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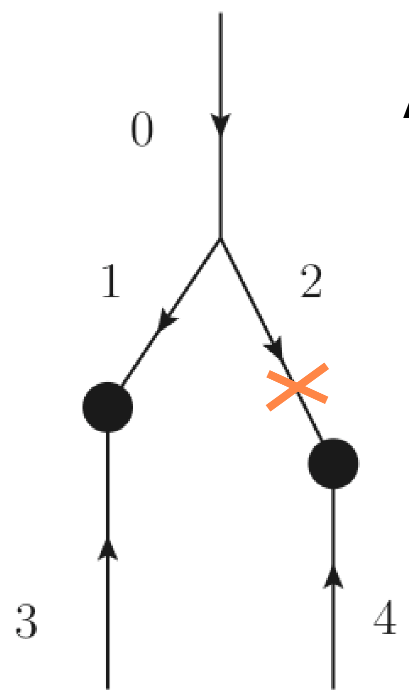

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not anymore

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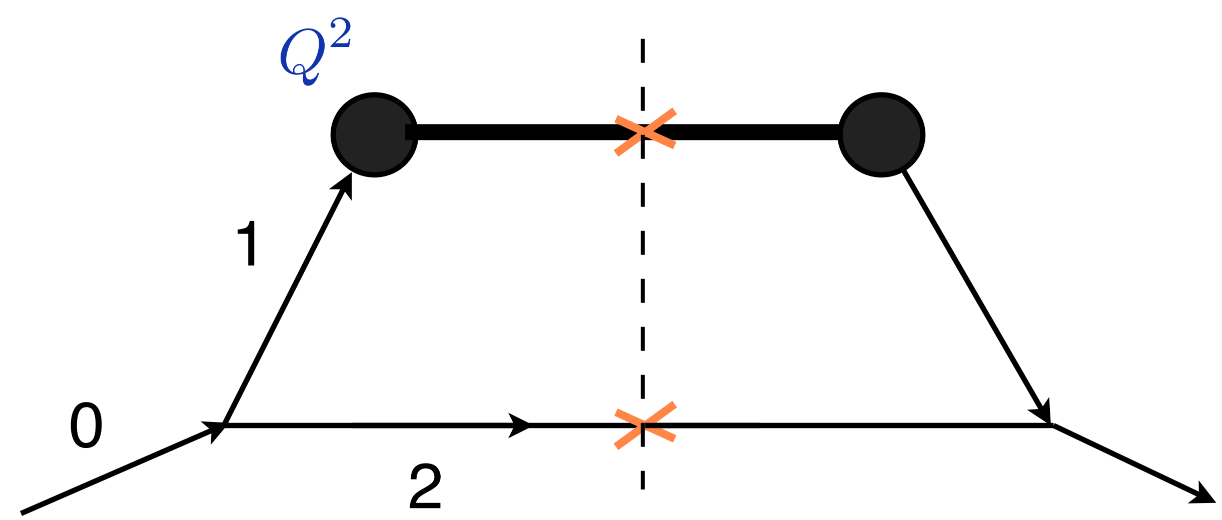

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Drell-Yan process

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Parton splitting probabilities

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4-jet diff. spectrum

Generalization of the DDT-formula for back-to-back 4-jet production spectrum

4-jet diff. spectrum

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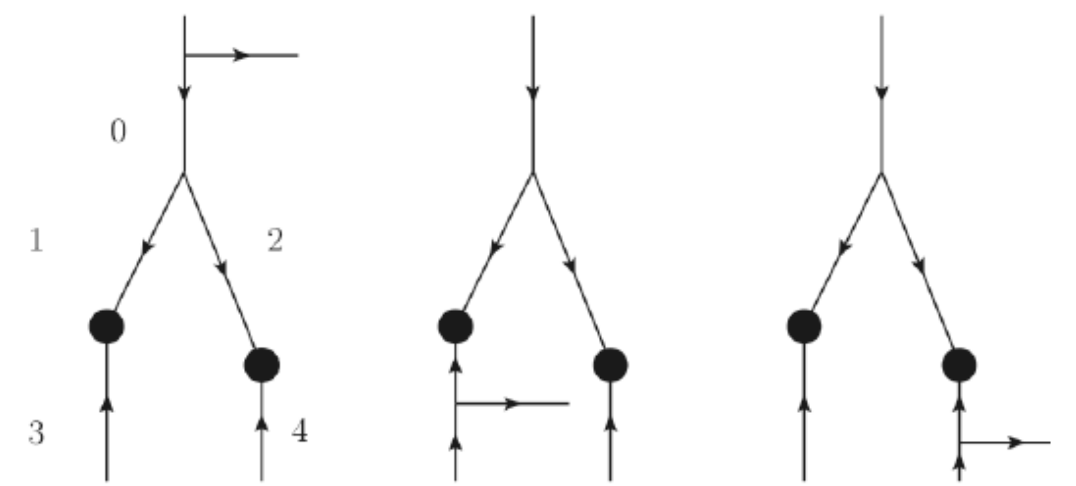
Additional 3 -> 4 contribution :



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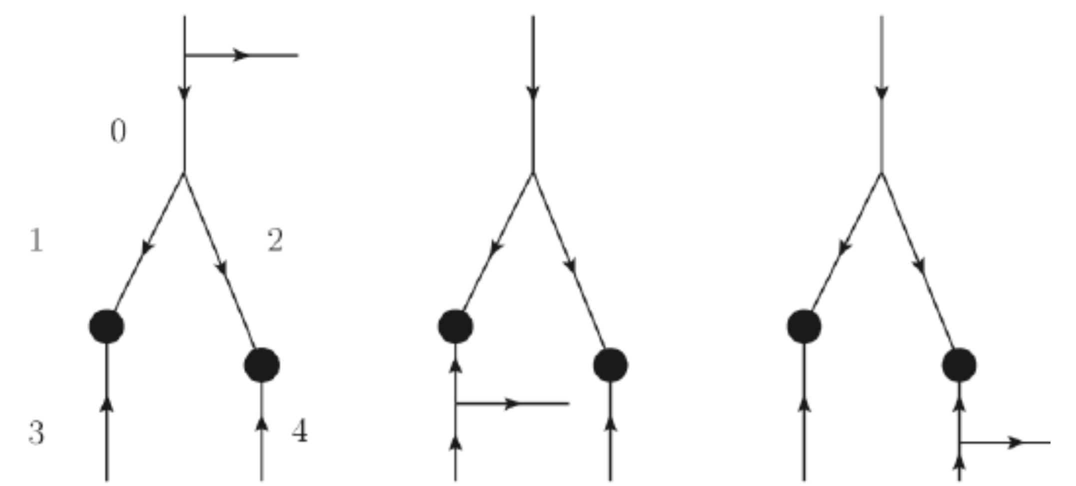

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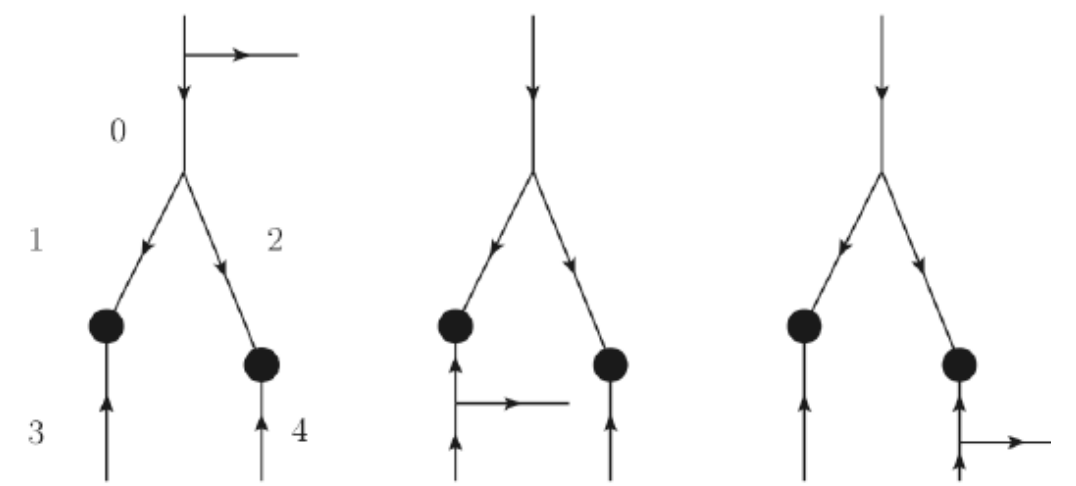

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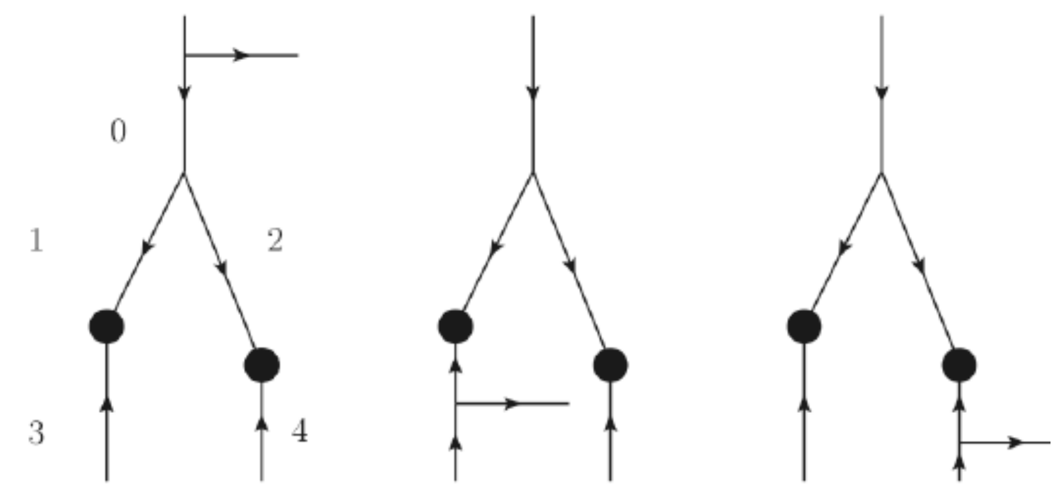

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$$S_1(Q^2,\delta^2)\, S_2(Q^2,\delta^2)\, \frac{\partial}{\partial\delta'^2} \Big\{ S_c(\delta^2,\delta'^2) \frac{G_a^c(x_1+x_2;\delta'^2, Q_0^2)}{x_1+x_2} S_3(Q^2,\delta'^2) S_4(Q^2,\delta'^2) \times {}_{[2]}D_b^{3,4}(x_3, x_4; \delta'^2, \delta'^2) \Big\}$$

Effective interaction areas for **4 -> 4** and **3 -> 4** collisions

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$$+ {}_{[2]}D_a(x_1,x_2;Q^2,Q^2;\vec{\Delta})\, {}_{[1]}D_b(x_3,x_4;Q^2,Q^2;-\vec{\Delta}) + {}_{[1]}D_a(x_1,x_2;Q^2,Q^2;\vec{\Delta})\, {}_{[2]}D_b(x_3,x_4;Q^2,Q^2;-\vec{\Delta}) \Big\}$$

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$$+ {}_{[2]}D_a(x_1,x_2;Q^2,Q^2;\vec{\Delta})\, {}_{[1]}D_b(x_3,x_4;Q^2,Q^2;-\vec{\Delta}) + {}_{[1]}D_a(x_1,x_2;Q^2,Q^2;\vec{\Delta})\, {}_{[2]}D_b(x_3,x_4;Q^2,Q^2;-\vec{\Delta}) \Big\}$$

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$$\times \frac{G_a^c(x_1+x_2,\delta^2,Q_0^2)}{x_1+x_2}\, {}_{[2]}D_b^{3,4}(x_3,x_4;\delta^2,\delta^2;\vec{\Delta}) \qquad + \quad (a \leftrightarrow b; 1,2 \leftrightarrow 3,4)$$

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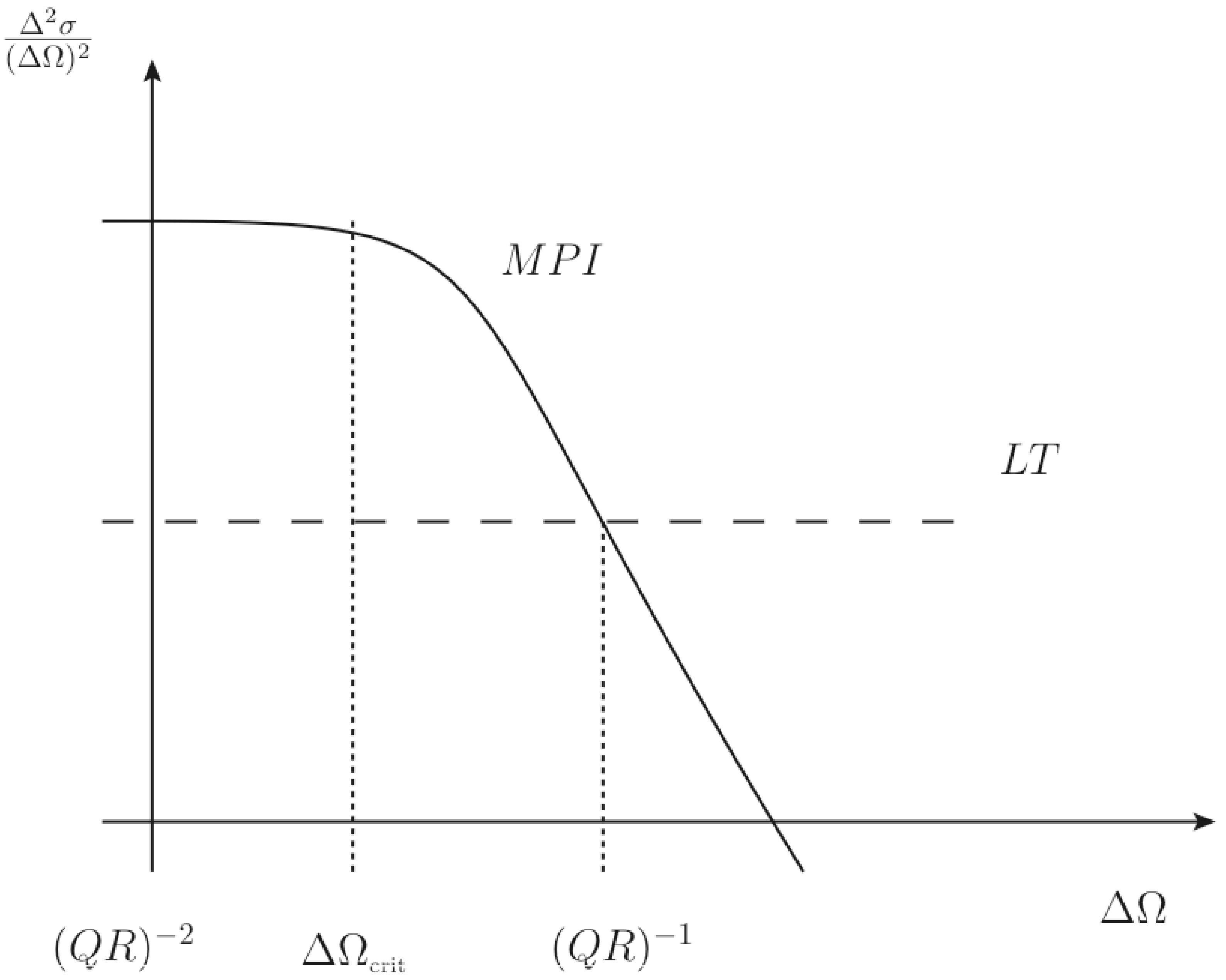

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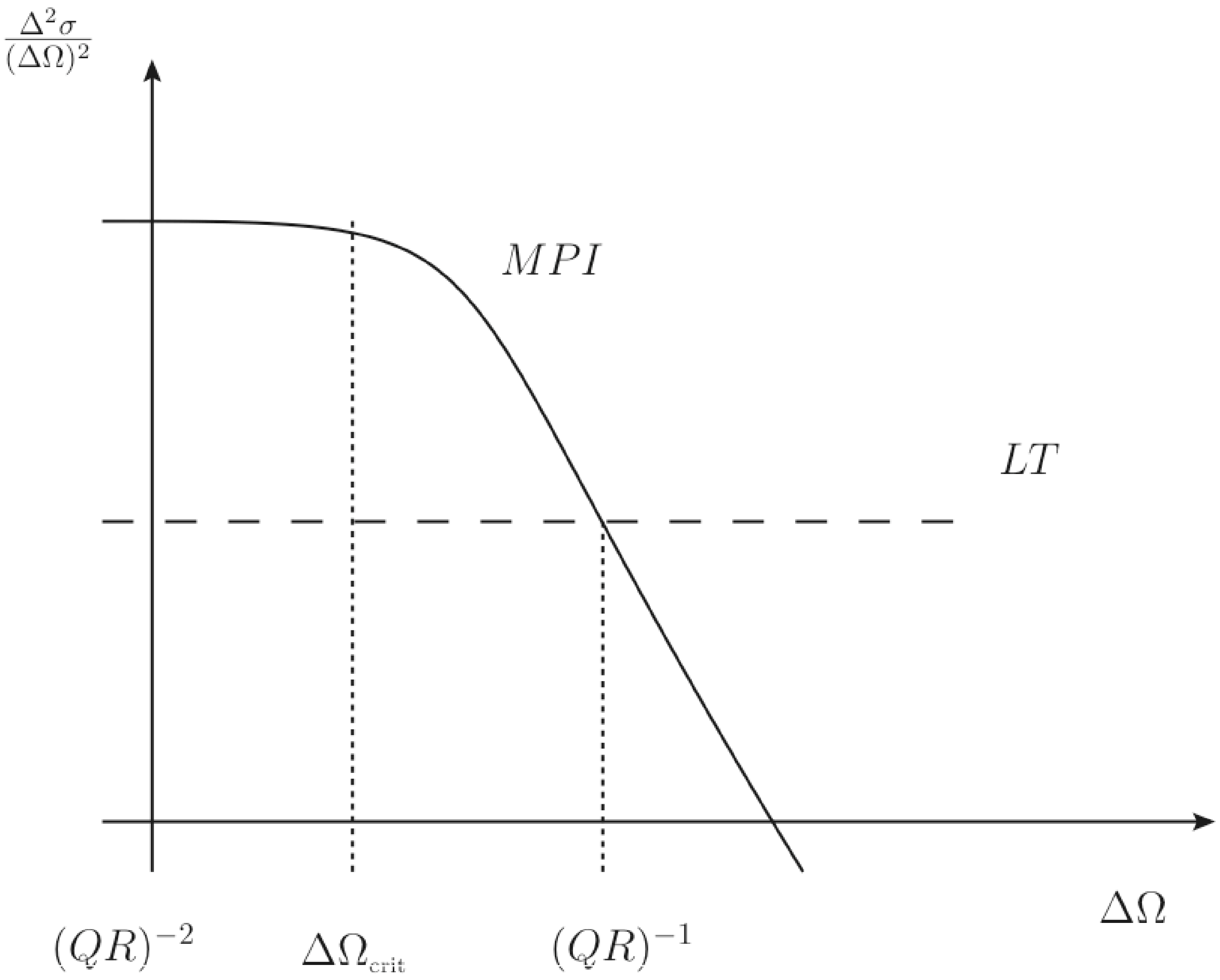

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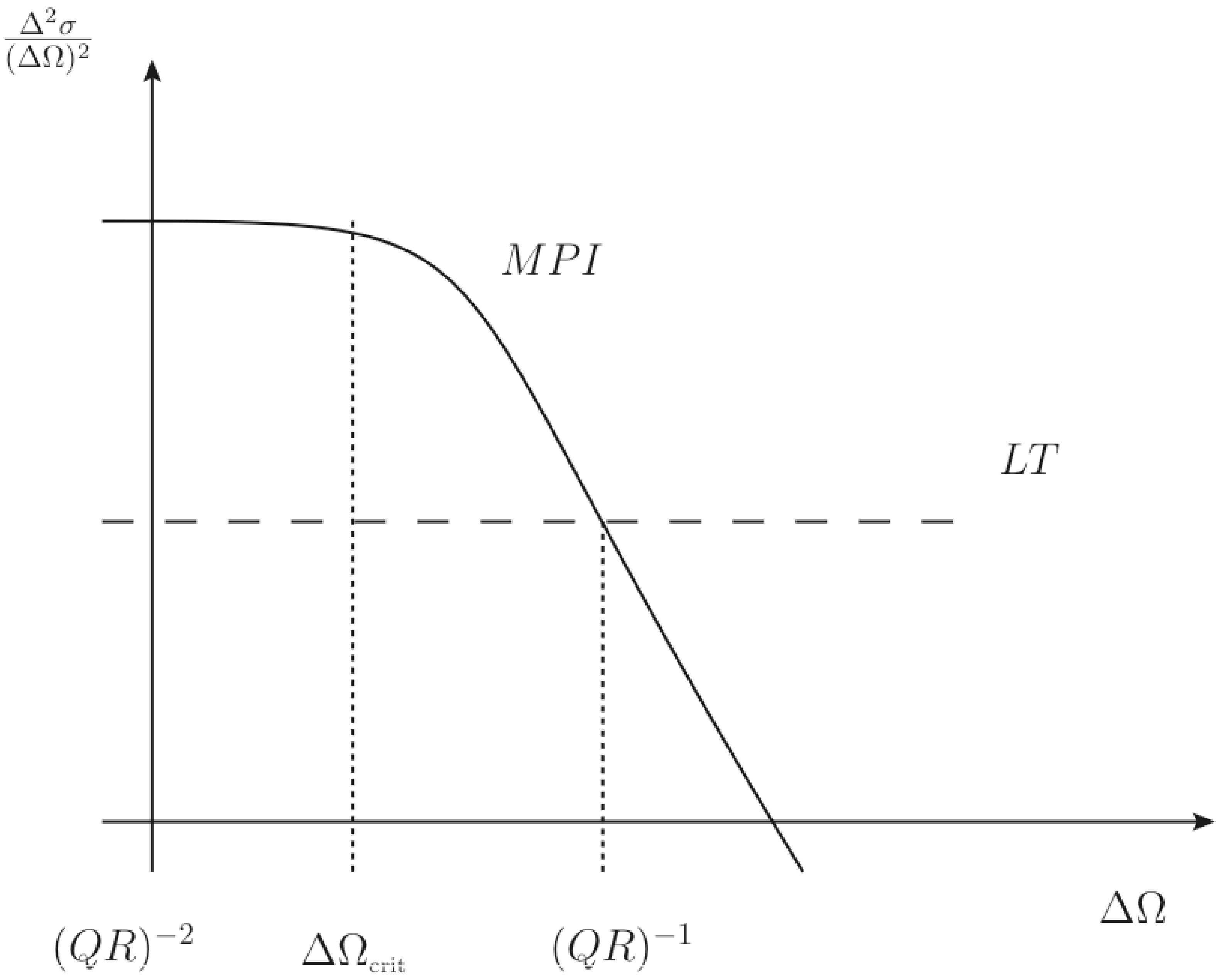

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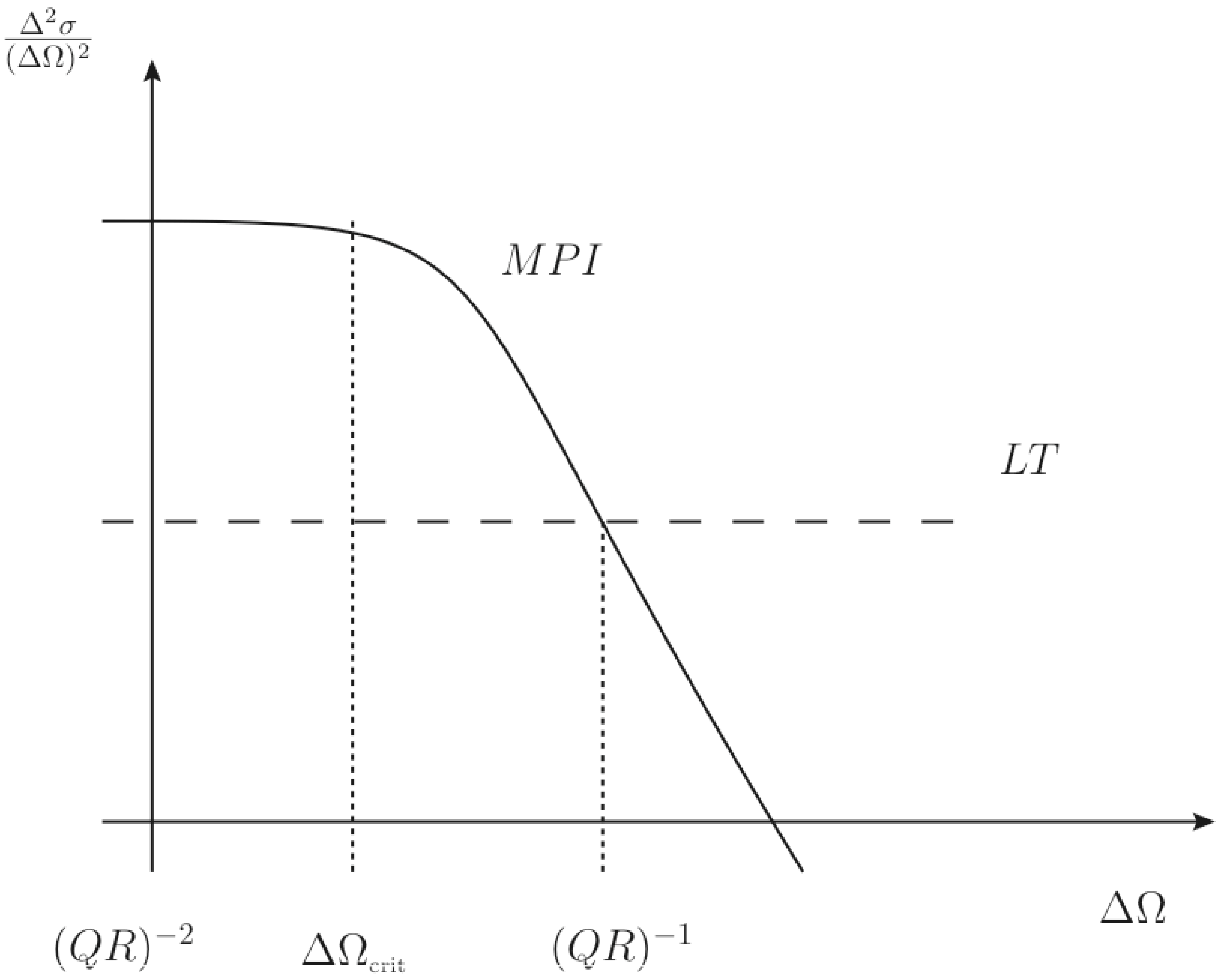

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for collision of two *gluons*

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- the quest for understanding the nature of ${}_2$GPDs calls for new ideas


ИДЕЯ

a missing piece

EXTRAS

Soft Gluon

Soft Gluon

PUZZLE

Hidden message from ***QCD Radiophysics***

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An additional look at the problem (G.Marchesini & YLD, 2005)

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*interchanging internal **(group rank)** and external **(scattering angle)** variables of the problem . . .*

Soft Photon

Soft Photon

Puzzle

Study of the Dependence of Direct Soft Photon Production on the Jet Characteristics in Hadronic Z^0 Decays

DELPHI Collaboration

$$\frac{dN_\gamma}{d^3\vec{k}} = \frac{\alpha}{(2\pi)^2}\frac{1}{E_\gamma}\int d^3\vec{p}_1 ... d^3\vec{p}_N \sum_{i,j} \eta_i\eta_j \frac{(\vec{p}_{i\perp}\cdot\vec{p}_{j\perp})}{(P_iK)(P_jK)} \frac{dN_{hadrons}}{d^3\vec{p}_1 ... d^3\vec{p}_N}$$

- ***calculate***
- ***compare with the data***
- ***say: "oh-la-la..."***

- $200 \text{ MeV} \leq E_\gamma \leq 1 \text{ GeV}$

DELPHI photons vs. hadron multiplicity

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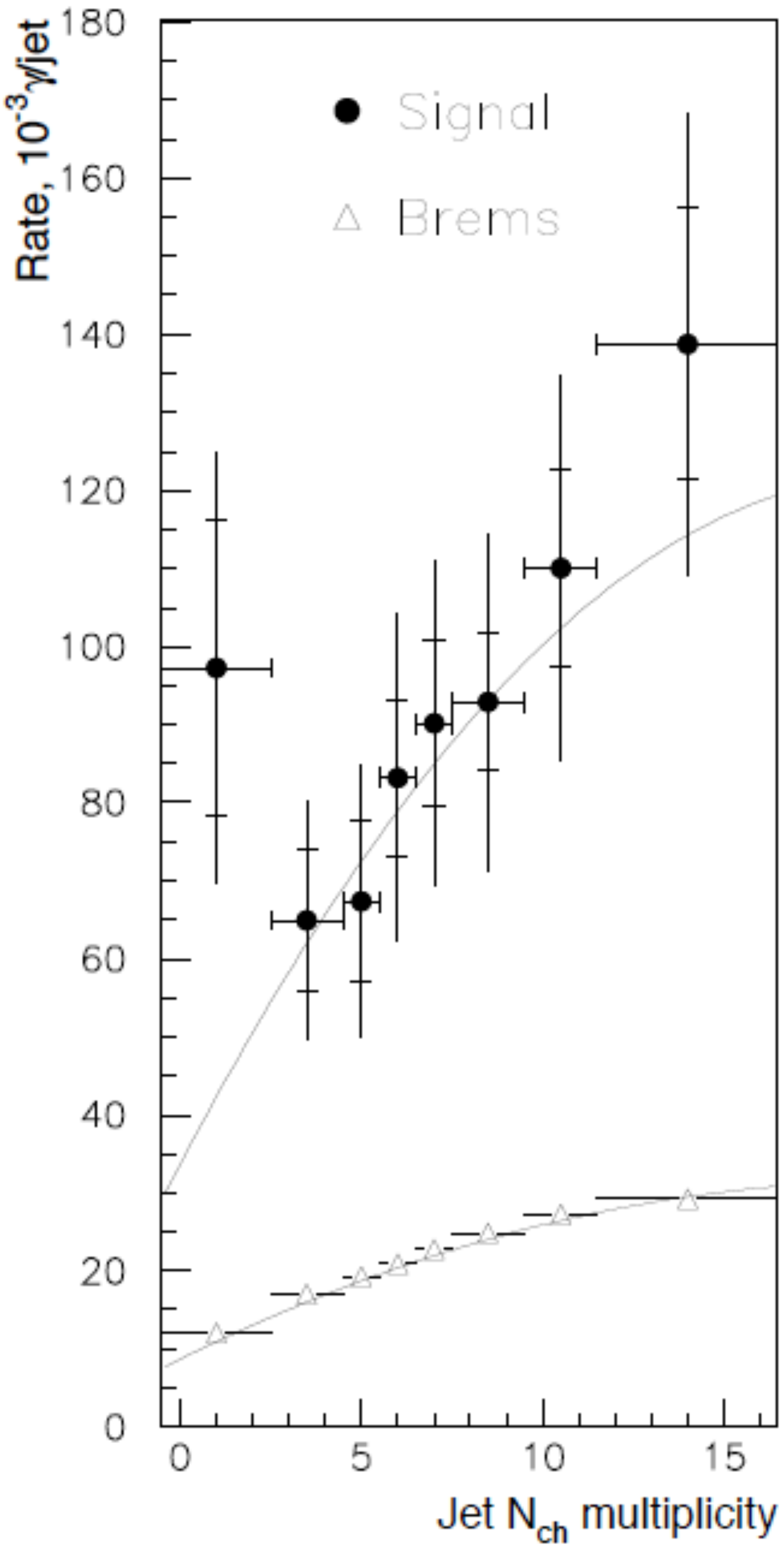

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