Scattering Amplitudes at the Integrand Level

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Credits

Recent work:

- "On the Integrand-Reduction Method for Two-Loop Scattering Amplitudes," **arXiv:1107.604**
  P. Mastrolia, G.O.
- "Automated One-Loop Calculations with GoSam," **arXiv:1111.2034**
  G. Cullen, N. Greiner, G. Heinrich, G .Luisoni,
  P. Mastrolia, G.O., T. Reiter, F. Tramontano
- "NLO QCD corrections to the production of W+ W- plus two jets at the LHC," **arXiv:1202.6004**
  N. Greiner, G. Heinrich, P. Mastrolia,
  G.O., T. Reiter, F. Tramontano

Work in progress: E. Mirabella, T. Peraro, Z. Zhang.

Outline

1. Motivation & Introduction
2. The one-loop case
   - Basic features of OPP reduction
   - From 4 to D dimensions (rational term)
   - GoSam: the automated one-loop all-purpose tool
3. What can we say beyond one-loop?
4. Some preliminary results at two-loop
5. conclusions and future outlook

Amplitudes 2012

"We intend to **review progress and discuss promising developments** *in this highly energetic field. Thereby, we hope to inspire further development".*

Intro

Scattering Amplitudes turn our theories into predictions:

- Calculations of Amplitudes are necessary to test our models
- There is a long tradition and huge literature

Several (partially overlapping) Communities:
tree-level, NLO, two-loop, multi-loop, all-loop
QCD, EW, MSSM, SYM, Super-gravity, ...

In this talk:

- Brief review of the one-loop Integrand-Level technique (QCD/EW)
- "Can we extend what we learned at one-loop to study a more general problem?"

One-loop – Notations

Any m-point one-loop amplitude can be written, before integration, as

$$A(\bar{q}) = \frac{N(\bar{q})}{\bar{D}_0 \bar{D}_1 \cdots \bar{D}_{m-1}}$$

where

$$\bar{D}_i = (\bar{q} + p_i)^2 - m_i^2 \quad , \quad \bar{q}^2 = q^2 - \mu^2 \quad , \quad \bar{D}_i = D_i - \mu^2$$

Our task is to calculate, for each phase space point:

$$\mathcal{M} = \int d^n\bar{q}\ A(\bar{q}) = \int d^n\bar{q} \frac{N(\bar{q})}{\bar{D}_0 \bar{D}_1 \ldots \bar{D}_{m-1}}$$

Integrand-Level Approach

Description in terms of Master Integrals I_i → Integral Level

$$\int d^n\bar{q}\ A(\bar{q}) = \int d^n\bar{q}\frac{N(\bar{q})}{\bar{D}_0\bar{D}_1\ldots\bar{D}_{m-1}} = c_0 I_0 + c_1 I_1 + \ldots + C_n I_n$$

Integrand-Level Approach

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$$\int d^n\bar{q}\; A(\bar{q}) = \int d^n\bar{q} \frac{N(\bar{q})}{\bar{D}_0 \bar{D}_1 \ldots \bar{D}_{m-1}} = c_0 I_0 + c_1 I_1 + \ldots + C_n I_n$$

Integrand Level → The $N = N$ identity

$$N(\bar{q}) = ???$$

Challenge: write a complete expression at the l.h.s. for $N(\bar{q})$

- powers of q and μ^2
- scalar products
- reconstructed denominators $\bar{D}_i$

Integrand-Level Approach

Integrand level decomposition:

$$
\begin{aligned}
N(\bar{q}) &= \sum_{i<<m}^{n-1} \Delta_{ijk\ell m}(\bar{q}) \prod_{h\neq i,j,k,\ell,m}^{n-1} \bar{D}_h + \sum_{i<<\ell}^{n-1} \Delta_{ijk\ell}(\bar{q}) \prod_{h\neq i,j,k,\ell}^{n-1} \bar{D}_h + \\
&+ \sum_{i<<k}^{n-1} \Delta_{ijk}(\bar{q}) \prod_{h\neq i,j,k}^{n-1} \bar{D}_h + \sum_{i<j}^{n-1} \Delta_{ij}(\bar{q}) \prod_{h\neq i,j}^{n-1} \bar{D}_h + \sum_{i}^{n-1} \Delta_{i}(\bar{q}) \prod_{h\neq i}^{n-1} \bar{D}_h
\end{aligned}
$$

- both sides are polynomial in q and μ^2
- the functional form is **process-independent**
- the process-dependent coefficients are contained in the Δ's
- **polynomial fitting replaces the integration**
- bonus: we can solve "on the multi-cuts"

Integrand level decomposition:

$$N(\bar{q}) = \sum_{i<<m}^{n-1} \Delta_{ijk\ell m}(\bar{q}) \prod_{h\neq i,j,k,\ell,m}^{n-1} \bar{D}_h + \sum_{i<<\ell}^{n-1} \Delta_{ijk\ell}(\bar{q}) \prod_{h\neq i,j,k,\ell}^{n-1} \bar{D}_h + \sum_{i<<k}^{n-1} \Delta_{ijk}(\bar{q}) \prod_{h\neq i,j,k}^{n-1} \bar{D}_h + \sum_{i<j}^{n-1} \Delta_{ij}(\bar{q}) \prod_{h\neq i,j}^{n-1} \bar{D}_h + \sum_{i}^{n-1} \Delta_{i}(\bar{q}) \prod_{h\neq i}^{n-1} \bar{D}_h$$

Recombining with the denominators:

$$A(\bar{q}) = \sum_{i<<m}^{n-1} \frac{\Delta_{ijk\ell m}(\bar{q})}{\bar{D}_i\bar{D}_j\bar{D}_k\bar{D}_\ell\bar{D}_m} + \sum_{i<<\ell}^{n-1} \frac{\Delta_{ijk\ell}(\bar{q})}{\bar{D}_i\bar{D}_j\bar{D}_k\bar{D}_\ell} + \sum_{i<<k}^{n-1} \frac{\Delta_{ijk}(\bar{q})}{\bar{D}_i\bar{D}_j\bar{D}_k} + \sum_{i<j}^{n-1} \frac{\Delta_{ij}(\bar{q})}{\bar{D}_i\bar{D}_j} + \sum_{i}^{n-1} \frac{\Delta_{i}(\bar{q})}{\bar{D}_i} ,$$

the decomposition exposes **the multi-pole nature of the integrand**

OPP Method

In this approach the numerical evaluation of scattering amplitudes is based on a decomposition at the **integrand level**.

Some of the advantages:

- Universal - applicable to any process,
  no distinction between massive and massless
- Simple - based on basic algebraic properties
- Automatizable - easy to implement in a computer code

A good starting point to build a fully automated NLO generator

G. O., Papadopoulos and Pittau (2007)

The traditional one-loop "master" formula

At the Integral level in terms of scalar integrals

$$\int \frac{N(\bar{q})}{\bar{D}_{i_0}\bar{D}_{i_1}\ldots\bar{D}_{m-1}} = \sum_{i_0<i_1<i_2<i_3}^{m-1} d(i_0 i_1 i_2 i_3) \int \frac{1}{\bar{D}_{i_0}\bar{D}_{i_1}\bar{D}_{i_2}\bar{D}_{i_3}}$$
$$+ \sum_{i_0<i_1<i_2}^{m-1} c(i_0 i_1 i_2) \int \frac{1}{\bar{D}_{i_0}\bar{D}_{i_1}\bar{D}_{i_2}}$$
$$+ \sum_{i_0<i_1}^{m-1} b(i_0 i_1) \int \frac{1}{\bar{D}_{i_0}\bar{D}_{i_1}}$$
$$+ \sum_{i_0}^{m-1} a(i_0) \int \frac{1}{\bar{D}_{i_0}}$$
$$+ \text{ rational terms}$$

OPP integrand-level "master" formula - I

General expression for the **4-dim** $N(q)$ at the integrand level in terms of D_i

$$
\begin{aligned}
N(q) &= \sum_{i_0<i_1<i_2<i_3}^{m-1} \left[d(i_0 i_1 i_2 i_3) \right] \prod_{i \neq i_0, i_1, i_2, i_3}^{m-1} D_i \\
&+ \sum_{i_0<i_1<i_2}^{m-1} \left[c(i_0 i_1 i_2) \right] \prod_{i \neq i_0, i_1, i_2}^{m-1} D_i \\
&+ \sum_{i_0<i_1}^{m-1} \left[b(i_0 i_1) \right] \prod_{i \neq i_0, i_1}^{m-1} D_i \\
&+ \sum_{i_0}^{m-1} \left[a(i_0) \right] \prod_{i \neq i_0}^{m-1} D_i
\end{aligned}
$$

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\begin{aligned}
N(q) &= \sum_{i_0<i_1<i_2<i_3}^{m-1} \left[d(i_0 i_1 i_2 i_3) + \tilde{d}(q; i_0 i_1 i_2 i_3) \right] \prod_{i \neq i_0, i_1, i_2, i_3}^{m-1} D_i \\
&+ \sum_{i_0<i_1<i_2}^{m-1} \left[c(i_0 i_1 i_2) + \tilde{c}(q; i_0 i_1 i_2) \right] \prod_{i \neq i_0, i_1, i_2}^{m-1} D_i \\
&+ \sum_{i_0<i_1}^{m-1} \left[b(i_0 i_1) + \tilde{b}(q; i_0 i_1) \right] \prod_{i \neq i_0, i_1}^{m-1} D_i \\
&+ \sum_{i_0}^{m-1} \left[a(i_0) + \tilde{a}(q; i_0) \right] \prod_{i \neq i_0}^{m-1} D_i
\end{aligned}
$$

This is 4-dimensional Identity

Spurious Terms - Box

- Each box diagram depends on three independent momenta $\mathbf{p_1}$, $\mathbf{p_2}$, and $\mathbf{p_3}$
- Build ℓ_1 and ℓ_2 ($\ell_{1,2}^2 = 0$) starting from $\mathbf{p_1}$ and $\mathbf{p_2}$

$$\mathbf{p_1} = \ell_1 + \alpha_1 \ell_2 \,, \quad \mathbf{p_2} = \ell_2 + \alpha_2 \ell_1$$

- There is only one direction which is perpendicular to ℓ_1, ℓ_2, and $\mathbf{p_3}$

$$w_\mu = \epsilon_{\alpha\,\beta\,\rho\,\mu}\, {\ell_1}^\alpha {\ell_2}^\beta {\mathbf{p_3}}^\rho$$

- Theorem: since $\mathbf{w} \cdot \ell_{1,2} = 0$ and $\mathbf{w} \cdot \mathbf{p_3} = 0$,

$$\int d^4q \frac{(q \cdot w)}{D_0 D_1 D_2 D_3} = 0$$

- The spurious 4-point function $\tilde{d}(q)$ is

$$\tilde{d}(q) = \tilde{d}\,(q \cdot w)$$

SPURIOUS TERMS - TRIANGLE

- Each triangle diagram depends on two independent momenta $\mathbf{p_1}$ and $\mathbf{p_2}$
- Build a basis of momenta ℓ_i (such that $\ell_i^2 = 0$) starting from $\mathbf{p_1}$ and $\mathbf{p_2}$

$$\mathbf{p_1} = \ell_1 + \alpha_1 \ell_2 \,, \quad \mathbf{p_2} = \ell_2 + \alpha_2 \ell_1$$

$$\ell_3{}^{\mu} = < \ell_1 | \gamma^{\mu} | \ell_2] \,, \; \ell_4{}^{\mu} = < \ell_2 | \gamma^{\mu} | \ell_1]$$

- Theorems: since $\mathbf{p_{1,2}} \cdot \ell_{3,4} = 0$

$$\int d^4 q \frac{q \cdot \ell_3}{D_0 D_1 D_2} = 0 \,, \qquad \int d^4 q \frac{q \cdot \ell_4}{D_0 D_1 D_2} = 0 \,,$$

- The spurious 3-point function $\tilde{c}(q)$ is

$$\tilde{c}(q) = \sum_{j=1}^{j_{max}} \left\{ \tilde{c}_{1j} [q \cdot \ell_3]^j + \tilde{c}_{2j} [q \cdot \ell_4]^j \right\}$$

Spurious Terms - Bubbles and Tadpoles

- Each **bubble** diagram depends only on one momentum $\mathbf{p_1}$
- To build ℓ_1 and ℓ_2, we start from $\mathbf{p_1}$ and an arbitrary massless vector $\mathbf{v}$ (with $\mathbf{v}$ non-parallel to $\mathbf{p_1}$)
- Using the freedom in to the choice of $\mathbf{v}$, we can minimize the non-spurious term (scalar integral only) or introduce additional elements in the basis of MI (to increase numerical stability)
- In the case of the **tadpoles**, there is no momentum dependence
  Any residual (non reducible) dependence on q gives rise to vanishing integrals

ONE-LOOP DICHOTOMY

At the one-loop level, "if it is not reducible, then it is spurious".

Pittau/del Aguila (2004)

OPP "MASTER" FORMULA - II

$$
\begin{aligned}
N(q) &= \sum_{i_0<i_1<i_2<i_3}^{m-1} \left[d(i_0 i_1 i_2 i_3) + \tilde{d}(q; i_0 i_1 i_2 i_3) \right] \prod_{i \neq i_0, i_1, i_2, i_3}^{m-1} D_i + \sum_{i_0<i_1<i_2}^{m-1} \left[c(i_0 i_1 i_2) + \tilde{c}(q; i_0 i_1 i_2) \right] \prod_{i \neq i_0, i_1, i_2}^{m-1} D_i \\
&+ \sum_{i_0<i_1}^{m-1} \left[b(i_0 i_1) + \tilde{b}(q; i_0 i_1) \right] \prod_{i \neq i_0, i_1}^{m-1} D_i + \sum_{i_0}^{m-1} \left[a(i_0) + \tilde{a}(q; i_0) \right] \prod_{i \neq i_0}^{m-1} D_i
\end{aligned}
$$

The quantities d, c, b, a are the coefficients of all possible scalar functions

The quantities $\tilde{d}$, $\tilde{c}$, $\tilde{b}$, $\tilde{a}$ are the "spurious" terms → vanish upon integration

IT IS NOW AN ALGEBRAIC PROBLEM:

Any N(q) just depends on a set of coefficients, to be determined!

OPP "MASTER" FORMULA - II

$$
\begin{aligned}
N(q) = & \sum_{i_0<i_1<i_2<i_3}^{m-1} \left[d(i_0 i_1 i_2 i_3) + \tilde{d}(q; i_0 i_1 i_2 i_3)\right] \prod_{i\neq i_0, i_1, i_2, i_3}^{m-1} D_i + \sum_{i_0<i_1<i_2}^{m-1} \left[c(i_0 i_1 i_2) + \tilde{c}(q; i_0 i_1 i_2)\right] \prod_{i\neq i_0, i_1, i_2}^{m-1} D_i \\
+ & \sum_{i_0<i_1}^{m-1} \left[b(i_0 i_1) + \tilde{b}(q; i_0 i_1)\right] \prod_{i\neq i_0, i_1}^{m-1} D_i + \sum_{i_0}^{m-1} \left[a(i_0) + \tilde{a}(q; i_0)\right] \prod_{i\neq i_0}^{m-1} D_i
\end{aligned}
$$

The quantities d, c, b, a are the coefficients of all possible scalar functions

The quantities $\tilde{d}$, $\tilde{c}$, $\tilde{b}$, $\tilde{a}$ are the "spurious" terms → vanish upon integration

It is now an algebraic problem:

Any N(q) just depends on a set of coefficients, to be determined!

Choose $\{q_i\}$ wisely

by evaluating N(q) for a set of values of the integration momentum $\{q_i\}$ such that some denominators D_i vanish ("cuts")

Example: 4-particles process

$$
\begin{aligned}
N(q) &= d + \tilde{d}(q) + \sum_{i=0}^{3} \left[c(i) + \tilde{c}(q;i) \right] D_i + \sum_{i_0 < i_1}^{3} \left[b(i_0 i_1) + \tilde{b}(q; i_0 i_1) \right] D_{i_0} D_{i_1} \\
&+ \sum_{i_0 = 0}^{3} \left[a(i_0) + \tilde{a}(q; i_0) \right] D_{i \neq i_0} D_{j \neq i_0} D_{k \neq i_0}
\end{aligned}
$$

We look for a q such that

$$D_0 = D_1 = D_2 = D_3 = 0$$

$\rightarrow$ there are two solutions $q_0^{\pm}$

Example: 4-particles process

$$N(q) \;=\; d + \tilde{d}(q)$$

Our "master formula" for $q = q_0^{\pm}$ is:

$$N(q_0^{\pm}) = [d + \tilde{d}\,(w \cdot q_0^{\pm})]$$

$\rightarrow$ solve to extract the coefficients d and $\tilde{d}$

Example: 4-particles process

$$N(q) - d - \tilde{d}(q) = \sum_{i=0}^{3} [c(i) + \tilde{c}(q;i)] D_i + \sum_{i_0<i_1}^{3} \left[b(i_0 i_1) + \tilde{b}(q; i_0 i_1) \right] D_{i_0} D_{i_1}$$

$$+ \sum_{i_0=0}^{3} [a(i_0) + \tilde{a}(q; i_0)] D_{i \neq i_0} D_{j \neq i_0} D_{k \neq i_0}$$

Then we can move to the extraction of c coefficients using

$$N'(q) = N(q) - d - \tilde{d}(w \cdot q)$$

and setting to zero three denominators (ex: $D_1 = 0$, $D_2 = 0$, $D_3 = 0$)

Example: 4-particles process

$$N(q) - d - \tilde{d}(q) \quad = \quad [c(0) + \tilde{c}(q;0)]\, D_0$$

We have infinite values of q for which

$$D_1 = D_2 = D_3 = 0 \quad \text{and} \quad D_0 \neq 0$$

$\rightarrow$ Here we need 7 of them to determine $c(0)$ and $\tilde{c}(q;0)$

The $N \equiv N$ test

Our "master" formula again!

$$
\begin{aligned}
N(q) = & \sum_{i_0<i_1<i_2<i_3}^{m-1} \left[d(i_0 i_1 i_2 i_3) + \tilde{d}(q; i_0 i_1 i_2 i_3) \right] \prod_{i \neq i_0, i_1, i_2, i_3}^{m-1} D_i \\
+ & \sum_{i_0<i_1<i_2}^{m-1} \left[c(i_0 i_1 i_2) + \tilde{c}(q; i_0 i_1 i_2) \right] \prod_{i \neq i_0, i_1, i_2}^{m-1} D_i \\
+ & \sum_{i_0<i_1}^{m-1} \left[b(i_0 i_1) + \tilde{b}(q; i_0 i_1) \right] \prod_{i \neq i_0, i_1}^{m-1} D_i \\
+ & \sum_{i_0}^{m-1} \left[a(i_0) + \tilde{a}(q; i_0) \right] \prod_{i \neq i_0}^{m-1} D_i
\end{aligned}
$$

After determining all coefficients $\rightarrow$ this should hold for any q

One-loop OPP summary

1. The **functional form** of the **OPP-master formula** is **universal** (**process independent**)
2. There are **no limitations to the presence of masses** (internal or external)
3. To extract **all coefficients** d, c, b, and a we ONLY need to evaluate numerator N(q) **numerically** at **fixed given values of** q.
4. Self-consistency **test** on the 4-dimensional reduction $\rightarrow$ $N = N$ (no previous or external information required)

$$\begin{aligned}\mathcal{M} &= \sum_i d_i \,\mathrm{Box}_i + \sum_i c_i \,\mathrm{Triangle}_i \\ &+ \sum_i b_i \,\mathrm{Bubble}_i + \sum_i a_i \,\mathrm{Tadpole}_i\end{aligned}$$

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$$
\begin{aligned}
\mathcal{M} &= \sum_i d_i \, \mathrm{Box}_i + \sum_i c_i \, \mathrm{Triangle}_i \\
&+ \sum_i b_i \, \mathrm{Bubble}_i + \sum_i a_i \, \mathrm{Tadpole}_i + \mathrm{R}
\end{aligned}
$$

We need to reconstruct **n-dimensional objects**, not 4-dim!
This generates the **rational terms**

Overview Rational Terms

Within the original **4-dimensional** reduction:

$$R = R_1 + R_2$$

R_1 – The OPP expansion is written in terms of 4-dim D_i, while n-dim $\bar{D}_i$ appear in scalar integrals.

$$A(\bar{q}) = \frac{N(q)}{\bar{D}_0 \bar{D}_1 \cdots \bar{D}_{m-1}}$$

R_1 can be calculated in **two different ways**, both **fully automatized**.

R_2 – The numerator $\bar{N}(\bar{q})$ can be also split into a 4-dim plus a ϵ-dim part

$$\bar{N}(\bar{q}) = N(q) + \tilde{N}(\tilde{q}^2, q, \epsilon)\,.$$

Compute R_2 using **tree-level like Feynman Rules**.

Identity in d-dimensions: $q \to \bar{q}$

Ellis, Giele, Kunszt, Melnikov (2008), Melnikov, Schulze (2010)
Mastrolia, G.O., Reiter, Tramontano (2010)

Reconstruct directly d-dimensional denominators

$$N(\bar{q}) = \sum_{i<<m}^{n-1} \Delta_{ijk\ell m}(\bar{q}) \prod_{h\neq i,j,k,\ell,m}^{n-1} \bar{D}_h + \sum_{i<<\ell}^{n-1} \Delta_{ijk\ell}(\bar{q}) \prod_{h\neq i,j,k,\ell}^{n-1} \bar{D}_h + \sum_{i<<k}^{n-1} \Delta_{ijk}(\bar{q}) \prod_{h\neq i,j,k}^{n-1} \bar{D}_h + \sum_{i<j}^{n-1} \Delta_{ij}(\bar{q}) \prod_{h\neq i,j}^{n-1} \bar{D}_h + \sum_{i}^{n-1} \Delta_{i}(\bar{q}) \prod_{h\neq i}^{n-1} \bar{D}_h$$

1. Denominators in d-dimensions: $D_h \to \bar{D}_h$
2. $N(\bar{q})$ is d-dimensions: $N(q) \to N(q, \mu^2)$
3. The polynominals in the coefficients have a **more complicated structure**

Identity in d-dimensions: Coefficients

- Add a spurious pentagon term in μ^2

$$\Delta_{ijk\ell m}(\bar{q}) = c_{5,0}^{(ijk\ell m)}\, \mu^2$$

- The coefficients have a more complicated structure

Box in 4-dimensions

$$\Delta_{ijk\ell}(q) = c_{4,0} + c_{4,1}\, \tilde{F}(q)$$

Box in d-dimensions

$$\Delta_{ijk\ell}(\bar{q}) = c_{4,0} + c_{4,2}\, \mu^2 + c_{4,4}\, \mu^4 + \left(c_{4,1} + c_{4,3}\, \mu^2\right) \tilde{F}(q)$$

Master Integrals

The blue coefficients, such as $c_{4,0}$ or $c_{4,4}$ multiply non-vanishing integrals:

$$\int d^d\bar{q}\ A(\bar{q}) = c_{4,0} \int d^d\bar{q} \frac{1}{\bar{D}_i\bar{D}_j\bar{D}_k\bar{D}_\ell} + c_{4,4} \int d^d\bar{q} \frac{\mu^4}{\bar{D}_i\bar{D}_j\bar{D}_k\bar{D}_\ell} + \dots$$

In addition to the standard **scalar integrals**

$$\int d^d\bar{q} \frac{1}{\bar{D}_i\bar{D}_j} \quad , \quad \int d^d\bar{q} \frac{1}{\bar{D}_i\bar{D}_j\bar{D}_k} \quad , \quad \int d^d\bar{q} \frac{1}{\bar{D}_i\bar{D}_j\bar{D}_k\bar{D}_l}$$

we need the following **additional** well-known integrals → **Rational Term**

$$\int d^d\bar{q} \frac{\mu^2}{\bar{D}_i\bar{D}_j} \quad , \quad \int d^d\bar{q} \frac{\mu^2}{\bar{D}_i\bar{D}_j\bar{D}_k} \quad , \quad \int d^d\bar{q} \frac{\mu^4}{\bar{D}_i\bar{D}_j\bar{D}_k\bar{D}_l}$$

One-Loop as a 3 step process

1) Compute the numerator $N(\bar{q})$ numerically at given q, μ^2

2) Extract coefficients/rats with D-dimensional integrand reduction

3) Combine with one-loop Master Integrals (scalar integrals)

$$\begin{aligned}\mathcal{M} &= \sum_i d_i \,\mathrm{Box}_i + \sum_i c_i \,\mathrm{Triangle}_i \\ &+ \sum_i b_i \,\mathrm{Bubble}_i + \sum_i a_i \,\mathrm{Tadpole}_i + \mathrm{R}\,,\end{aligned}$$

One-Loop as a 3 step process

1) Compute the numerator $N(\bar{q})$ numerically at given q, μ^2

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Samurai

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OneLOop, **QCDLoop**, **Golem95c**, **LoopTools**

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There are **publicly available tools** for reduction and scalar integrals

One-Loop as a 3 step process

1) Compute the numerator $N(\bar{q})$ numerically at given q, μ^2

How can we provide this?

2) Extract coefficients/rats with D-dimensional integrand reduction
Samurai

3) Combine with one-loop Master Integrals (scalar integrals)
OneLOop, **QCDLoop**, **Golem95c**, **LoopTools**

$$\mathcal{M} = \sum_i d_i \, \mathrm{Box}_i + \sum_i c_i \, \mathrm{Triangle}_i + \sum_i b_i \, \mathrm{Bubble}_i + \sum_i a_i \, \mathrm{Tadpole}_i + \mathrm{R} \,,$$

There are **publicly available tools** for reduction and scalar integrals
What about **the numerator N(q)** ?

GoSam: automated one-loop Calculations

GoSam

Algebraic generation of **d-dimensional integrands** via **Feynman diagrams**

Numerical reduction at the Integrand Level: **d-dimensional integrand-level reduction**

G. Cullen, N. Greiner, G. Heinrich, G. Luisoni, P. Mastrolia, G. O., T. Reiter, F. Tramontano (2011)

Target: provide an **automated tool** for stable evaluation of one-loop matrix elements

- **be generic** (QCD, EW, BSM)
- **interface with other existing tools** like Sherpa, PowHEG, ...
- build upon **open source tools only** (i.e. Samurai, Golem95, QGraf, Form, Spinney, Haggies, QCDLoop, OneLOop)
- **support open standards** (BLHA)

Automation at One-Loop: "Algebraic Way"

Main features of the "Algebraic Way":

- Amplitudes generated with **Feynman diagrams**
- **Algebraic manipulations are allowed** before starting the numerical integration
- The **generation** of numerators is executed **separately from the numerical reduction**
- Optimization: grouping of diagrams, smart caching
- Control over sub-parts of the computation (move in/out subsets of diagrams)
- Algebra in dimension d, different schemes

Great flexibility in the reduction
Choice between different algorithms at runtime

♢ **Diagram Generation** → automated, based on Feynman diagrams

- FORM Vermaseren (1991)
- QGRAF Nogueira (1993)
- Haggies Reiter (2009)
- Spinney Cullen, Koch-Janusz, Reiter (2010)

♢ **Diagram Reduction**

- Default Option: **Samurai** Mastrolia, G.O., Reiter, Tramontano (2010)
- OPP Reduction Algorithm G.O., Papadopoulos, Pittau (2007)
- d-dimensional extension Ellis, Giele, Kunszt, Melnikov (2008)
- Coefficients of Polynomials via DFT Mastrolia et al. (2008)
- Model-independent Computation of the full Rational Term

♢ Other options available at runtime:

- **Golem95**

Binoth, Cullen, Guillet, Heinrich, Kleinschmidt, Pilon, Reiter, Rodgers (2008)

- Tensorial Integrand-level Reduction

Heinrich, G.O., Reiter, Tramontano (2010)

Calculations tested with GoSam

process
$\gamma\gamma \to \gamma\gamma$ (W loop)
$\gamma\gamma \to \gamma\gamma\gamma\gamma$ (fermion loop)
$pp \to t\bar{t}$
$pp \to W^{\pm}\,jj$
$pp \to W^{\pm}\,b\bar{b}$ (massive b)
$e^+e^- \to e^+e^-\gamma$ (QED, massive)
$e^+e^- \to \mu^+\mu^-\gamma$ (QED, two masses)
$pp \to H\,t\bar{t}$
$pp \to Z\,t\bar{t}$
$pp \to W^+W^+jj$
$pp \to b\bar{b}b\bar{b}$
$pp \to W^+W^-\,b\bar{b}$
$pp \to t\bar{t}b\bar{b}$
$u\bar{d} \to W^+ggg$
$pp \to W^+W^-jj$ see next slide

$pp \to W^+W^-jj$

Melia, Melnikov, Rontsch, Zanderighi (2011)
Greiner, Heinrich, Mastrolia, G.O., Reiter, Tramontano (2012)

Virtual Part: **GoSam**
Eight basic partonic sub-processes:

$$
\begin{aligned}
d\,\bar{u} &\to c\,\bar{s}\,W^+\,W^- \\
u\,\bar{u} &\to c\,\bar{c}\,W^+\,W^- \\
d\,\bar{d} &\to s\,\bar{s}\,W^+\,W^- \\
u\,\bar{u} &\to d\,\bar{d}\,W^+\,W^- \\
u\,\bar{u} &\to u\,\bar{u}\,W^+\,W^- \\
d\,\bar{d} &\to d\,\bar{d}\,W^+\,W^- \\
u\,\bar{u} &\to g\,g\,W^+\,W^- \\
d\,\bar{d} &\to g\,g\,W^+\,W^-
\end{aligned}
$$

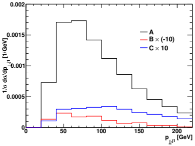
Tree-level + real emission: **MadGraph**
Maltoni, Steltzer
Subtraction terms: **MadDipole**
Frederix, Gehrmann, Greiner
Phase space integration: **MadEvent**

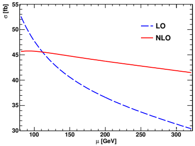
Virtual contributions divided in 3 parts:

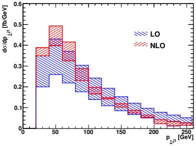
- A same setup as Melia et al. (2011)
- B W's attached to a fermion loop (2%)
- C third generation in the loops (4%)
- Reduction of scale uncertainty at NLO

$$\sigma_{\rm lo} = 39.57\,^{+34\%}_{-23\%}\ \text{fb}$$

$$\sigma_{\rm nlo} = 44.51\,^{+2.5\%}_{-7.4\%}\ \text{fb}$$







Integrand reduction via Laurent expansion

Mastrolia, Mirabella, Peraro, arXiv:1203.0291

Advantages of analytic techniques + integrand decomposition.

- ♢ 5-point coefficients and 4-point spurious coefficients: eliminated.
- ♢ The computation of 3-, 2-, and 1-point coefficients is separated from the 4-point residues.
- ♢ 3-point residues reconstructed from 3ple-cut, without any subtraction.

Forde (2007), Badger (2009)

Integrand reduction via Laurent expansion

Mastrolia, Mirabella, Peraro, arXiv:1203.0291

Advantages of analytic techniques + integrand decomposition.

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- ♢ The computation of 3-, 2-, and 1-point coefficients is separated from the 4-point residues.
- ♢ 3-point residues reconstructed from 3ple-cut, without any subtraction.

- Subtraction at the **coefficient level**: the Laurent expansion makes each function in the subtraction separately polynomial.
- The Laurent expansion for the determination of 3-, 2-, and 1-point coefficients implemented via *polynomial division*: the Laurent series is *quotient* of the division between the numerator and the product of the uncut denominators, *neglecting the remainder*.
- The 2- and 1-point coefficients are obtained directly as trivial combination of the coefficients coming out of the polynomial division and the corrective coefficients.
- Correction terms of 2-, and 1-point functions have parametric expressions known a priori.

Two-loops (more loops)

Mastrolia, G.O. (2011)
Badger, Frellesvig, Zhang (2012)
Maximal Unitarity → see Kosower's talk

Extension to higher loops:

1. The discussion on orthogonal directions in momentum space is true at all loops
   - 5-point or more: 4 independent momenta → NO orthogonal directions
   - 4-point: 3 independent momenta → 1 orthogonal direction
   - 3-point: 2 independent momenta → 2 orthogonal directions
   - 2-point: 1 independent momentum → 3 orthogonal directions

Two-loops (more loops)

Mastrolia, G.O. (2011)
Badger, Frellesvig, Zhang (2012)
Maximal Unitarity → see Kosower's talk

Extension to higher loops:

1. The discussion on orthogonal directions in momentum space is true at all loops
2. Build an "$N = N$" identity that takes into account **more than one virtual momentum** (say q and k)
   - The dichotomy "reducible" or "spurious" doesn't hold any longer.
   - We should generalize and introduce Irreducible Scalar Products (ISP)
   - ISP can be determined in a process independent way
   - direct inspection (i.e. reconstruction of numerators)
   - relations between scalar products via Gram determinants → BFZ

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   - ISP can be determined in a process independent way
   - direct inspection (i.e. reconstruction of numerators)
   - relations between scalar products via Gram determinants → BFZ
3. Fit all coefficients by sampling on the multiple cuts

Two-loop integrand reduction

For each diagram (or topology), we can define:
m-fold cut → set of on-shell conditions corresponding to the vanishing of m denominators present in that topology.

$$D_1 = D_2 = \ldots = D_m = 0$$

m-residue $\Delta_{i_1,\ldots,i_m}$ → most general polynomial residue, written in the Irreducible Scalar Products (ISP)

As in the one-loop case:

$$N(q,k) = \sum_{i_1<<i_8}^{n} \Delta_{i_1,\ldots,i_8}(q,k) \prod_{h\neq i_1,\ldots,i_8}^{n} \bar{D}_h + \ldots + \sum_{i_1<<i_2}^{n} \Delta_{i_1,i_2}(q,k) \prod_{h\neq i_1,i_2}^{n} \bar{D}_h$$

$$A(q,k) = \sum_{i_1<<i_8}^{n} \frac{\Delta_{i_1,\ldots,i_8}(q,k)}{\bar{D}_{i_1}\bar{D}_{i_2}\ldots\bar{D}_{i_8}} + \sum_{i_1<<i_7}^{n} \frac{\Delta_{i_1,\ldots,i_7}(q,k)}{\bar{D}_{i_1}\bar{D}_{i_2}\ldots\bar{D}_{i_7}} + \ldots + \sum_{i_1<<i_2}^{n} \frac{\Delta_{i_1,i_2}(q,k)}{\bar{D}_{i_1}\bar{D}_{i_2}}$$

which hold for **all values of k and q (also outside the cuts)**

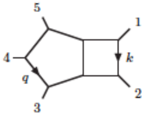
Two-loop rationale

- In the case of **5-point** (or higher) diagrams, four external momenta can be chosen to form a real four-dimensional vector basis.
- Starting with the **4-point** diagrams, we can choose only up to three independent external momenta. But they are not sufficient, and additional elements, orthogonal to them, have to be taken into account to complete the four dimensional basis.
- Each m-fold cut will be characterized by one of these two kinds of basis, and the loop momenta will be decomposed along the vectors forming it.
- The residue of an m-fold cut $\Delta_{i_1,\ldots,i_m}$ is polynomial in the components of the loop momenta, and can be expressed as a linear combination of ISPs constructed from the loop momenta and external momenta or the elements of the basis.
- The **polynomial form of the residue determines the MIs potentially appearing in the final decomposition**.

The PentaBox diagram in N=4 SYM

Mastrolia & G.O. (2011)

Mastrolia, Mirabella, G.O., Peraro, & Zhang (in progress)

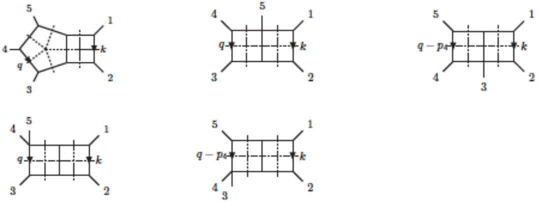


$$
\begin{aligned}
D_1 &= k^2\\
D_2 &= (k+p_2)^2\\
D_3 &= (k-p_1)^2\\
D_4 &= q^2\\
D_5 &= (q+p_3)^2\\
D_6 &= (q-p_4)^2\\
D_7 &= (q-p_4-p_5)^2\\
D_8 &= (q+k+p_2+p_3)^2\,.
\end{aligned}
$$

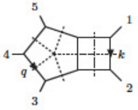
$N(q,k) = 2\,q\cdot v + \alpha$ Carrasco & Johansson (2011)

$$v^\mu = \frac{1}{4}\Big(\gamma_{12}(p_1^\mu - p_2^\mu) + \gamma_{23}(p_2^\mu - p_3^\mu) + 2\,\gamma_{45}(p_4^\mu - p_5^\mu) + \gamma_{13}(p_1^\mu - p_3^\mu)\Big)$$

$$\alpha = \frac{1}{4}\Big(2\,\gamma_{12}(s_{45} - s_{12}) + \gamma_{23}(s_{45} + 3s_{12} - s_{13}) + 2\,\gamma_{45}(s_{14} - s_{15}) + \gamma_{13}(s_{12} + s_{45} - s_{13})\Big)$$



5-POINT 8FOLD-CUT



$$D_1 = \ldots = D_8 = 0$$

$$q = x_1\, p_3 + x_2\, p_4 + x_3 \frac{\langle p_3|\gamma^\mu|p_4]}{2} + x_4 \frac{\langle p_4$$
$$k = y_1\, p_1 + y_2\, p_2 + y_3 \frac{\langle p_1|\gamma^\mu|p_2]}{2} + y_4 \frac{\langle p_2|$$

- case $x_4 = y_4 = 0$

$$q = \frac{\langle p_4|p_5\rangle}{\langle p_3|p_5\rangle}\frac{\langle p_3|\gamma^\mu|p_4]}{2}, \qquad k = \frac{\langle p_3|p_2\rangle}{\langle p_1|p_3\rangle}\frac{\langle p_1|\gamma^\mu|p_2]}{2}$$

- case $x_4 = y_3 = 0$

$$q = \frac{\langle p_4|p_5\rangle}{\langle p_3|p_5\rangle}\frac{\langle p_3|\gamma^\mu|p_4]}{2}, \qquad k = \frac{\langle p_1|p_5\rangle}{\langle p_2|p_5\rangle}\frac{\langle p_2|\gamma^\mu|p_1]}{2}$$

- case $x_3 = y_4 = 0$

$$q = \frac{[p_4|p_5]}{[p_3|p_5]}\frac{\langle p_4|\gamma^\mu|p_3]}{2}, \qquad k = \frac{[p_1|p_5]}{[p_2|p_5]}$$

- case $x_3 = y_3 = 0$

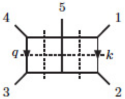
$$q = \frac{[p_4|p_5]}{[p_3|p_5]}\frac{\langle p_4|\gamma^\mu|p_3]}{2}, \qquad k = \frac{[p_3|p_2]}{[p_1|p_3]}$$

$$\Delta_{12345678}(q,k) = c_0^{(12345678)} + c_{q1,1}^{(12345678)}\,(q \cdot p_1) + c_{k4,1}^{(12345678)}\,(k \cdot p_4) + c_{k5,1}^{(12345678)}\,(k \cdot p_5).$$

$$c_0^{(12345678)} = -\frac{1}{\langle p_5|p_4\rangle\langle p_3|p_1\rangle[p_5|p_3][p_4|p_1] - \langle p_5|p_3\rangle\langle p_4|p_1\rangle[p_5|p_4][p_3|p_1]}$$
$$\times\Big(2\langle p_5|p_4\rangle\langle p_4|p_1\rangle(\tau_3\cdot v)[p_5|p_4][p_3|p_1] - 2\langle p_5|p_4\rangle\langle p_3|p_1\rangle(\tau_4\cdot v)[p_5|p_4][p_4|p_1]$$
$$- \alpha\langle p_5|p_4\rangle\langle p_3|p_1\rangle[p_5|p_3][p_4|p_1] + \alpha\langle p_5|p_3\rangle\langle p_4|p_1\rangle[p_5|p_4][p_3|p_1]\Big)$$

$$c_{q1,1}^{(12345678)} = -4\frac{\langle p_5|p_4\rangle(\tau_3\cdot v)[p_5|p_3] - \langle p_5|p_3\rangle(\tau_4\cdot v)[p_5|p_4]}{\langle p_5|p_4\rangle\langle p_3|p_1\rangle[p_5|p_3][p_4|p_1] - \langle p_5|p_3\rangle\langle p_4|p_1\rangle[p_5|p_4][p_3|p_1]}.$$

5-POINT 7FOLD-CUT



$$D_1 = \ldots = D_6 = D_8 = 0$$

$$q = x_1\, p_3 + x_2\, p_4 + x_3 \frac{\langle p_3|\gamma^\mu|p_4]}{2} + x_4 \frac{\langle p_4|\gamma^\mu|p_3]}{2}$$
$$k = y_1\, p_1 + y_2\, p_2 + y_3 \frac{\langle p_1|\gamma^\mu|p_2]}{2} + y_4 \frac{\langle p_2|\gamma^\mu|p_1]}{2}.$$

- case $x_4 = y_4 = 0$

$$q = x_3\, \tau_3 \qquad k = -\frac{\langle p_2|p_3\rangle}{\langle p_1|p_3\rangle} e_3$$
$$q = -\frac{[p_3|p_2]}{[p_4|p_2]} \tau_3 \qquad k = y_3\, e_3$$

- case $x_3 = y_4 = 0$

$$q = x_4\, \tau_4 \qquad k = -\frac{\langle p_4|p_2\rangle\, x_4 + \langle p_3|p_2\rangle}{\langle p_4|p_1\rangle\, x_4 + \langle p_3|p_1\rangle} e_3$$
$$q = -\frac{\langle p_3|p_1\rangle\, y_3 + \langle p_3|p_2\rangle}{\langle p_4|p_1\rangle\, y_3 + \langle p_4|p_2\rangle} \tau_4 \qquad k = y_3\, e_3$$

- case $x_4 = y_3 = 0$

$$q = x_3\, \tau_3 \qquad k = -\frac{[p_4|p_2]\, x_3 + [p_3|p_2]}{[p_4|p_1]\, x_3 + [p_3|p_1]} e_4$$
$$q = -\frac{[p_3|p_1]\, y_4 + [p_3|p_2]}{[p_4|p_1]\, y_4 + [p_4|p_2]} \tau_3 \qquad k = y_4\, e_4$$

- case $x_3 = y_3 = 0$

$$q = x_4\, \tau_4 \qquad k = -\frac{[p_2|p_3]}{[p_1|p_3]} e_4$$
$$q = -\frac{\langle p_3|p_2\rangle}{\langle p_4|p_2\rangle} \tau_4 \qquad k = y_4\, e_4.$$

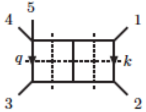
ISPs $(q \cdot p_1), (q \cdot p_1), (k \cdot p_3), (k \cdot p_4)$.

$$\Delta_{1234568}(q, k) = c_0^{(1234568)}.$$

$$\left.\frac{N(q, , k)}{D_7}\right|_{\rm cut} - \left.\frac{\Delta_{12345678}(q, k)}{D_7}\right|_{\rm cut} = \Delta_{1234568}(q, k)|_{\rm cut}$$

$$c_0^{(1234568)} = \frac{2(\tau_3 \cdot v) - c_{q1,1}^{(12345678)}\, (\tau_3 \cdot p_1)}{-2(\tau_3 \cdot p_5)}.$$

4-POINT 7FOLD-CUT



$$D_1 = \ldots = D_5 = D_7 = D_8 = 0.$$

$$q = -p_q + x_1\,\tau_1 + x_2\,\tau_2 + x_3\,\tau_3 + x_4\,\tau_4$$
$$k = -p_k + y_1\,e_1 + y_2\,e_2 + y_3\,e_3 + y_4\,e_4$$

$$e_1 = p_1, \qquad e_2 = p_2 \qquad \tau_1 = p_3, \qquad \tau_2 = p_4 + p_5 - \frac{s_{45}}{2(p_4+p_5)\cdot p_3}\,p_3.$$

4×2 parametrizations.

ISPs $(k \cdot p_3), (k \cdot \omega), (q \cdot \omega), (q \cdot p_1)$

$$\Delta_{1234578}(q,k) = c_0^{(1234578)}.$$

$$\left.\frac{N(q,,k)}{D_6}\right|_{\text{cut}} - \left.\frac{\Delta_{12345678}(q,k)}{D_6}\right|_{\text{cut}} = \Delta_{1234578}(q,k)|_{\text{cut}}$$

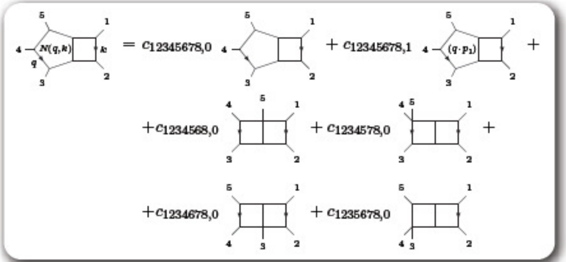
$$c_0^{(1234568)} = \frac{2(\tau_3 \cdot v) - c_{q1,1}^{(12345678)}\,(\tau_3 \cdot p_1)}{-2(\tau_3 \cdot p_5)}.$$

A systematic treatment of higher-rank 2-Loop 4-point integrand reduction has been presented by Badger, Frellesvig & Zhang (2012)

classification via Gram Id's ~ sampling on all classes of solutions required

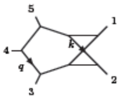
PENTABOX INTEGRAND DECOMPOSITION

$$
\begin{aligned}
N(q,k) &= \Delta_{12345678}(q,k) + \\
&\quad + \Delta_{1234568}(q,k) D_7 + \Delta_{1234578}(q,k) D_6 + \\
&\quad + \Delta_{1234678}(q,k) D_5 + \Delta_{1235678}(q,k) D_4 = \\
&= c_{12345678,0} + c_{12345678,1}\ (q \cdot p_1) + \\
&\quad + c_{1234568,0} D_7 + c_{1234578,0} D_6 + \\
&\quad + c_{1234678,0} D_5 + c_{1235678,0} D_4 \ ,
\end{aligned}
$$



Global (N=N)-test: OK

PentaCross Integrand Decomposition



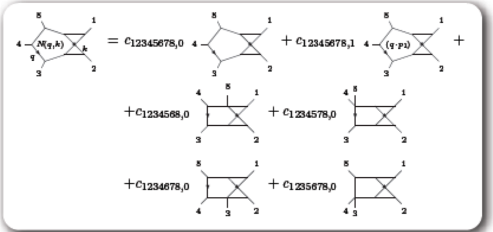
$$
\begin{aligned}
D_1 &= k^2 \\
D_2 &= (k+p_2)^2 \\
D_3 &= (k+q-p_4-p_5)^2 \\
D_4 &= q^2 \\
D_5 &= (q+p_1)^2 \\
D_6 &= (q-p_4)^2 \\
D_7 &= (q-p_4-p_5)^2 \\
D_8 &= (q+k+p_2+p_1)^2 .
\end{aligned}
$$

$$N(q,k) = 2\, q \cdot v + \alpha$$ Carrasco & Johansson (2011)

$$v^\mu = \frac{1}{4}\Big(\gamma_{12}(p_1^\mu - p_2^\mu) + \gamma_{23}(p_2^\mu - p_3^\mu) + 2\,\gamma_{45}(p_4^\mu - p_5^\mu) + \gamma_{13}(p_1^\mu - p_3^\mu)\Big)$$

$$\alpha = \frac{1}{4}\Big(2\,\gamma_{12}(s_{45} - s_{12}) + \gamma_{23}(s_{45} + 3s_{12} - s_{13}) + 2\,\gamma_{45}(s_{14} - s_{15}) + \gamma_{13}(s_{12} + s_{45} - s_{13})\Big)$$

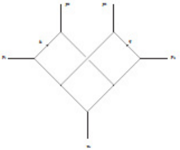
$$
\begin{aligned}
N(q,k) = {} & \Delta_{12345678}(q,k) + \\
& + \Delta_{1234568}(q,k) D_7 + \Delta_{1234578}(q,k) D_6 + \\
& + \Delta_{1234678}(q,k) D_5 + \Delta_{1235678}(q,k) D_4 = \\
= {} & c_{12345678,0} + c_{12345678,1}\ (q \cdot p_1) + \\
& + c_{1234568,0} D_7 + c_{1234578,0} D_6 + \\
& + c_{1234678,0} D_5 + c_{1235678,0} D_4 \ ,
\end{aligned}
$$



The coefficients are the same of the planar case.

Global (N=N)-test: OK

The Last Contribution to the 5-point N=4 SYM

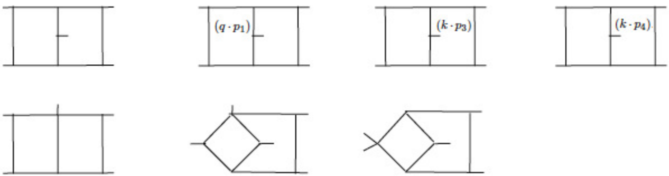


$D_1 = k^2$
$D_2 = (k - p_1)^2$
$D_3 = (k + p_2)^2$
$D_4 = q^2$
$D_5 = (q + p_3)^2$
$D_6 = (q - p_4)^2$
$D_7 = (q - k + p_1 + p_3)^2$
$D_8 = (q - k - p_2 - p_4)^2.$

$N(q,k)$ Carrasco & Johansson (2011)

$D_1 = \ldots = D_8 = 0$ 8 solutions

$$\Delta_{12345678}(q,k) = c_0^{(12345678)} + c_{q1,1}^{(12345678)}\,(q\cdot p_1) + c_{k3,1}^{(12345678)}\,(k\cdot p_3) + c_{k4,1}^{(12345678)}\,(k\cdot p_4).$$



Global (N=N)-test fulfilled!

Scattering Amplitudes at the Integrand Level

1. **The one-loop is in production**
   - GoSam is a flexible (public) multi-purpose tool
   - Recently competed $pp \to W^+W^-jj$
2. **The two-loop in the making**
   - First steps towards an integrand-reduction algorithm
   - ISP's determine the MI-basis
   - Work is in progress
3. **Three-loop or more...**
   - Basic Criteria for determining the residues should hold to higher loops
   - It would be interesting to test the algorithm on real numerators