N=4 SYM and N=8 SUGRA at 4 loops

Lance Dixon (SLAC)

Z. Bern, J.J. Carrasco, L.D., H. Johansson & R. Roiban

1201.5366

Amplitudes 2012

DESY, Hamburg

Full color N=4 SYM and N=8 SUGRA

- For detailed motivation, see talks by Johansson, Bern, O'Connell, Broedel
- In brief, test the ultraviolet behavior of N=8 SUGRA in D=4 by computing amplitudes to high loop order, and inspecting their UV properties.
- First compute N=4 SYM amplitudes for two reasons:

1. Relations between gauge theory and gravity (KLT, BCJ/double copy) help in constructing gravity
2. Assess how N=8 SUGRA is doing by comparing UV behavior in D > 4 to N=4 SYM critical dimension,

$$D_c = 4 + \frac{6}{L}$$

- Need full color N=4 SYM for task 1, but it also provides interesting information for task 2.

Why bother redoing 4 loops?

- N=4 SYM and N=8 SUGRA 4-loop 4-point amplitudes already computed once before

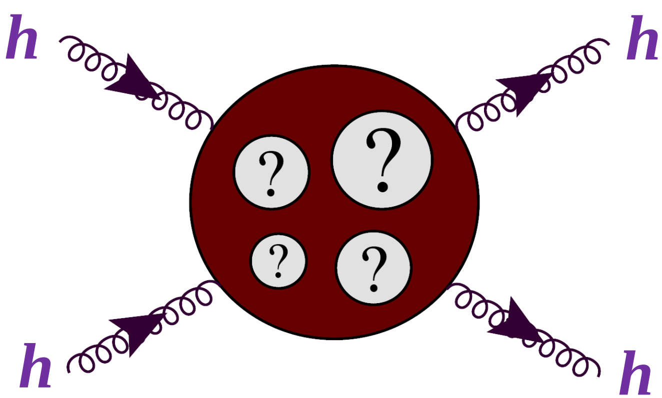
  Bern, Carrasco, LD, Johansson, Roiban, 0905.2326, 1008.3327

- Why do it again?

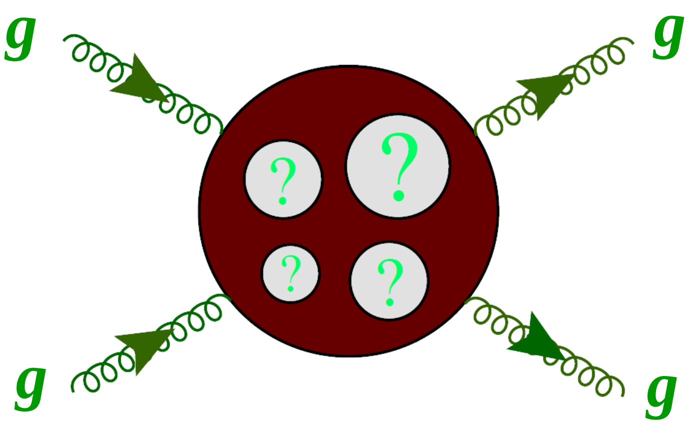
1. **Tradition.** 3-loop amplitudes have been computed 3 times now: BCDJR+Kosower, hep-th/0702112;
   BCDJR, 0808.4112;
   BCJ, 1004.0476
2. **Color-kinematics duality** [Bern, Johansson talks]: simplifies N=4 SYM and especially N=8 SUGRA amplitude construction
3. **Improved UV representation, especially for N=8**
4. **Extract numerical value of N=8 counterterm in D = 4 + 6/L, study relation with N=4 SYM counterterm**

Strategy for Assessing N=8 Supergravity

Johansson talk



vs.



N=8 SUGRA

N=4 Super-Yang-Mills

How does N=8 SUGRA compare to N=4 SYM? What is the critical dimension $D_c(L)$ in which it first diverges?

A "mere" gauge theory. UV finite in $D = 4$.
Strong evidence that it's also finite at L loops for

$$D < 4 + \frac{6}{L}$$

Part I
Amplitude Construction

Color-Kinematic Duality

talks by Johansson, Bern, O'Connell, Broedel

- First realized for 4-point non-Abelian gauge theory amplitudes by Zhu (1980), Goebel, Halzen, Leveille (1981)
- Massless adjoint gauge theory result:

$$\mathcal{A}_4^{\text{tree}} = \frac{n_s C_s}{s} + \frac{n_t C_t}{t} + \frac{n_u C_u}{u}$$

- Group theory → 3 terms are not independent (Jacobi identity):

$$C_t - C_u = C_s$$

- In a suitable "gauge", one finds: $n_t - n_u = n_s$

  Same structure can be extended to an arbitrary number of legs and provides a new "KLT-like" relation to gravity ($n_i = \tilde{n}_i$):

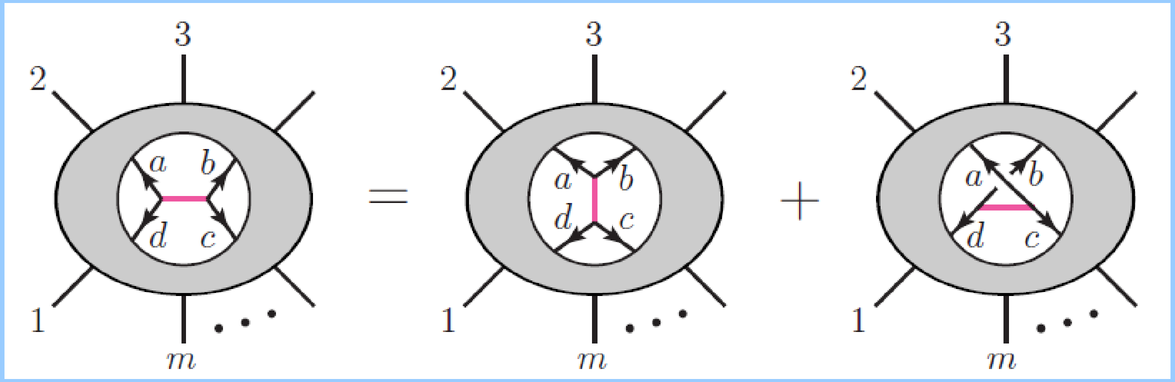
$$M_4^{\text{tree}} = \frac{n_s^2}{s} + \frac{n_t^2}{t} + \frac{n_u^2}{u}$$

Bern, Carrasco, Johansson, 0805.3993

Color-Kinematic Duality at loop level

BCJ, 1004.0476

- Consider any 3 graphs connected by a Jacobi identity



- Color factors obey

$$C_s \quad = \quad C_t \quad - \quad C_u$$

- Duality requires

$$n_s \quad = \quad n_t \quad - \quad n_u$$

- Very strong constraint on structure of integrands; only a handful of independent integral numerators left after imposing it.

Double-copy formula for gravity

BCJ, 1004.0476; Bern, Dennen, Huang, Kiermaier, 1004.0476

- If an all adjoint gauge-theory amplitude is given by a representation in terms of cubic graphs Γ:

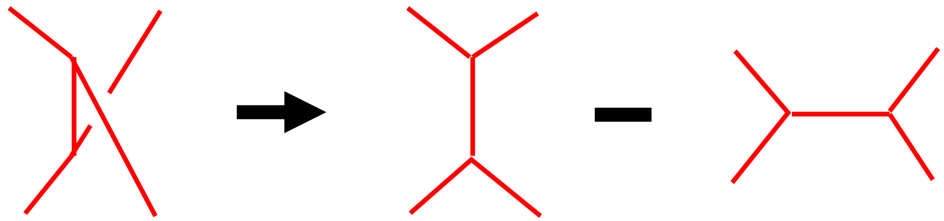
$$\mathcal{A}_4^{(L)} = \sum_{i\in\Gamma} \int \prod_{l=1}^{L} \frac{d^D p_l}{(2\pi)^D} \frac{1}{S_i} \frac{n_i C_i}{\prod_{\alpha_i} p_{\alpha_i}^2}$$

- And the numerator factors n_i obey the color-kinematics duality
- Then the corresponding gravity amplitude is given by

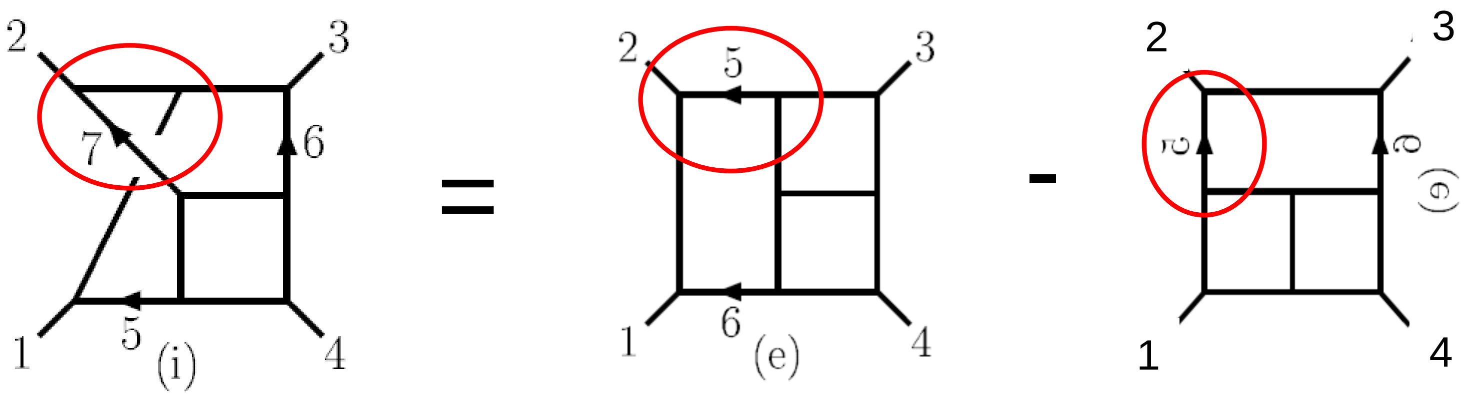
$$\mathcal{M}_4^{(L)} = \sum_{i\in\Gamma} \int \prod_{l=1}^{L} \frac{d^D p_l}{(2\pi)^D} \frac{1}{S_i} \frac{n_i^2}{\prod_{\alpha_i} p_{\alpha_i}^2}$$

- Argument based on a recursion relation on the integrand.

Simple 3 loop example

Using 

we can relate non-planar topologies to planar ones



$$n^{(i)} = n^{(e)} - n^{(e)}|_{1\leftrightarrow 3}$$

In fact all N=4 SYM 3 loop topologies related to (*e*)

(master graph)

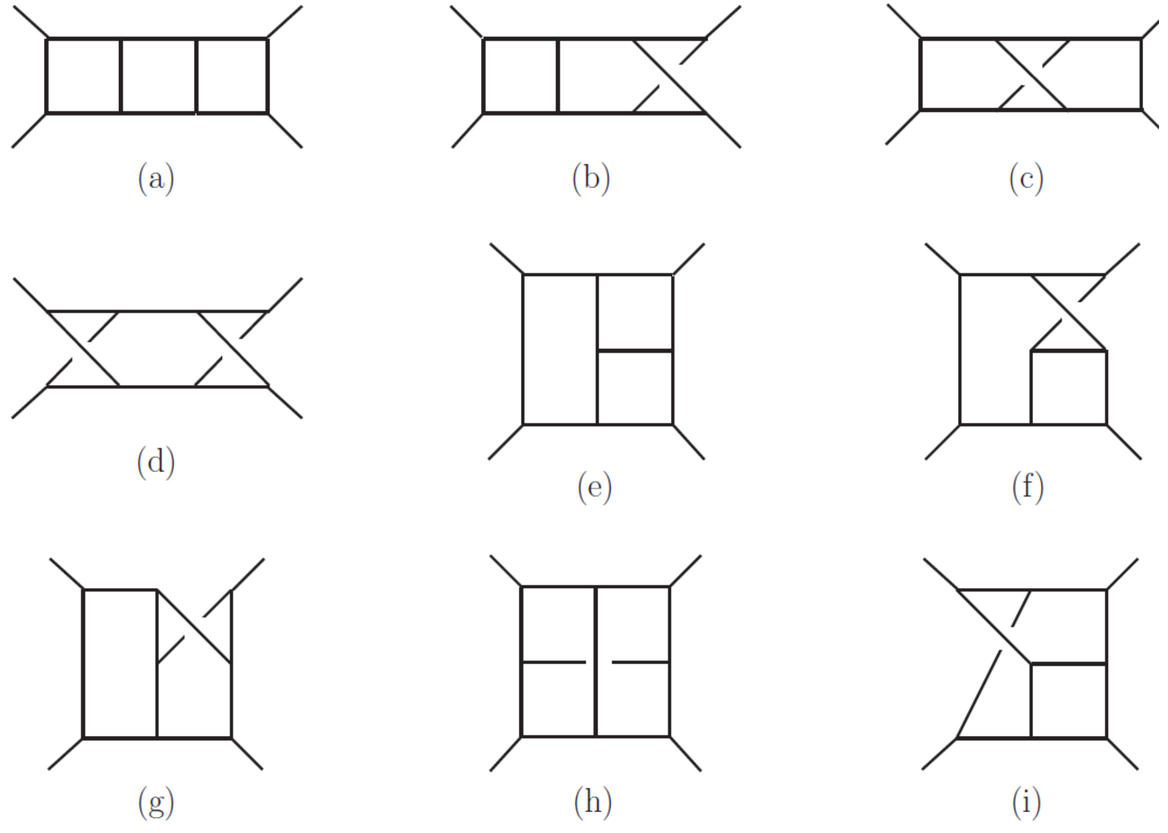
Carrasco, Johansson, 1103.3298; talk by Bern

3 loop amplitude before color-kinematics duality

Nine basic integral topologies:

BCDJKR th/0702112; BCDJR, 0808.4112

- Cubic 1PI graphs only, no triangle subgraphs

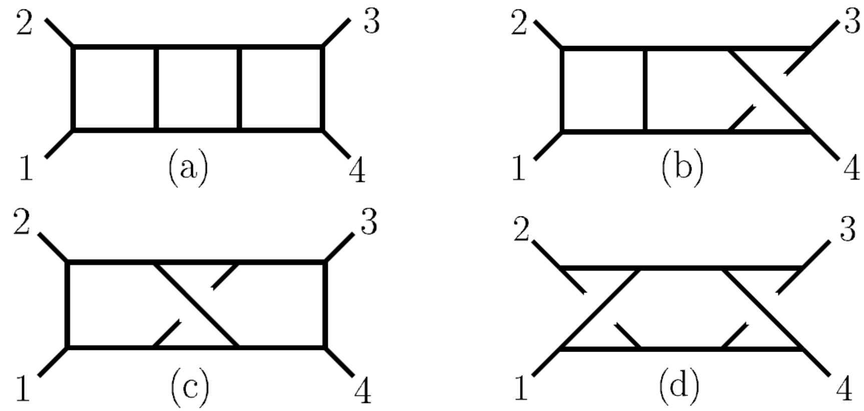


Old N=4 numerators at 3 loops

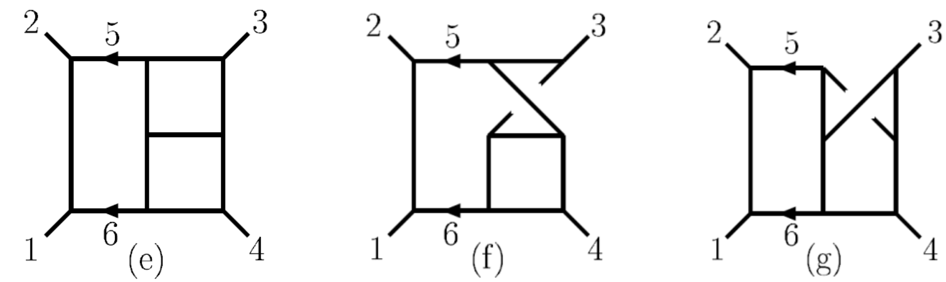
Overall $st\,A_4^{\text{tree}}$

$$s_{iM} = (k_i + \ell_M)^2$$
$$\tau_{iM} = 2k_i \cdot \ell_M$$

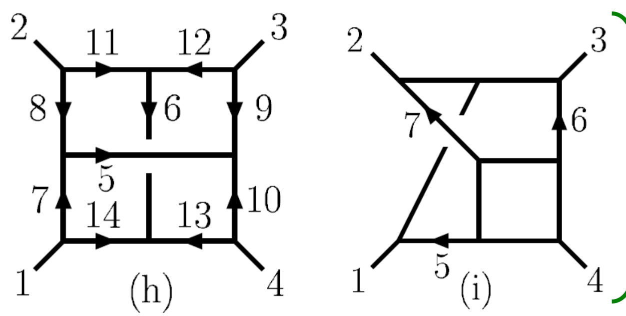
s_{12}^2



$s_{12}\, s_{46}$



$$s_{12}(\tau_{26} + \tau_{36}) + s_{14}(\tau_{15} + \tau_{25}) + s_{12}s_{14}$$



$$s_{12}\, s_{45} - s_{14}\, s_{46} - \tfrac{1}{3}(s_{12} - s_{14})\, \ell_7^2$$

manifestly quadratic in loop momentum ℓ_M

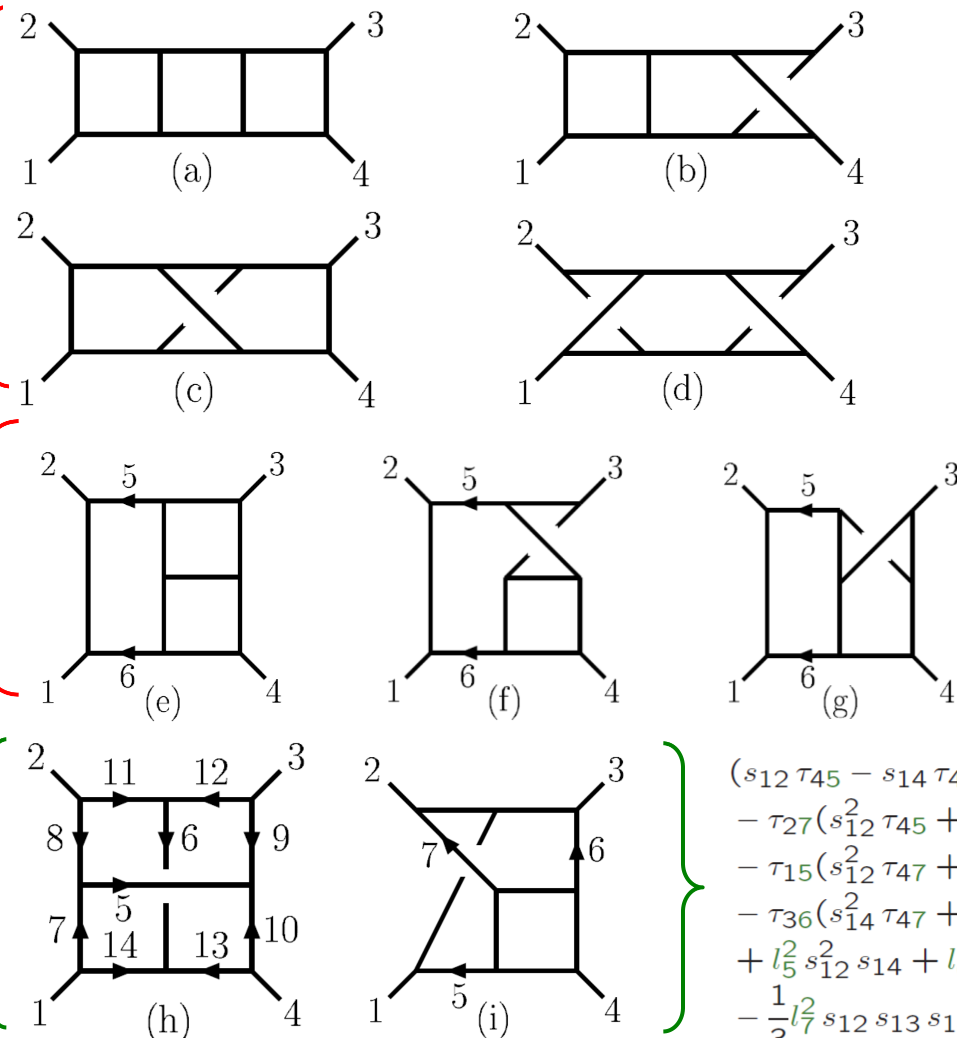
Old N=8 numerators at 3 loops

Overall $(st A_4^{\text{tree}})^2 = stu\, M_4^{\text{tree}}$

$$s_{iM} = (k_i + \ell_M)^2$$
$$\tau_{iM} = 2k_i \cdot \ell_M$$

s_{12}^4 : (a), (b), (c), (d)

$s_{12}^2\, \tau_{35}\, \tau_{46}$: (e), (f), (g)



$$(s_{12}(\tau_{26}+\tau_{36}) + s_{14}(\tau_{15}+\tau_{25}) + s_{12}s_{14})^2$$
$$+ (s_{12}^2(\tau_{26}+\tau_{36}) - s_{14}^2(\tau_{15}+\tau_{25}))$$
$$\times (\tau_{17}+\tau_{28}+\tau_{39}+\tau_{4,10})$$
$$+ s_{12}^2(\tau_{17}\tau_{28}+\tau_{39}\tau_{4,10})$$
$$+ s_{14}^2(\tau_{28}\tau_{39}+\tau_{17}\tau_{4,10})$$
$$+ s_{13}^2(\tau_{17}\tau_{39}+\tau_{28}\tau_{4,10})$$

$$(s_{12}\tau_{45} - s_{14}\tau_{46})^2$$
$$- \tau_{27}(s_{12}^2\tau_{45} + s_{14}^2\tau_{46})$$
$$- \tau_{15}(s_{12}^2\tau_{47} + s_{13}^2\tau_{46})$$
$$- \tau_{36}(s_{14}^2\tau_{47} + s_{13}^2\tau_{45})$$
$$+ l_5^2\, s_{12}^2\, s_{14} + l_6^2\, s_{12}\, s_{14}^2$$
$$- \frac{1}{3} l_7^2\, s_{12}\, s_{13}\, s_{14}$$

Had to work hard to make manifestly quadratic in ℓ_M

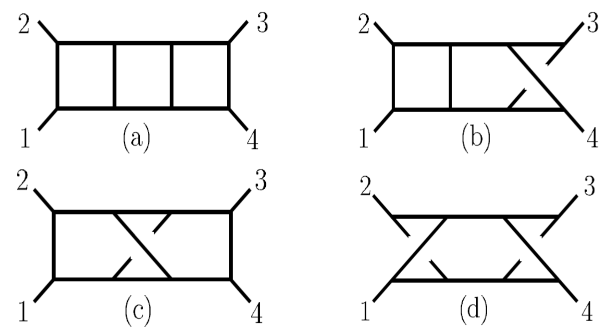
BCDJR (2008)

3 loop amplitude after color-kinematics duality

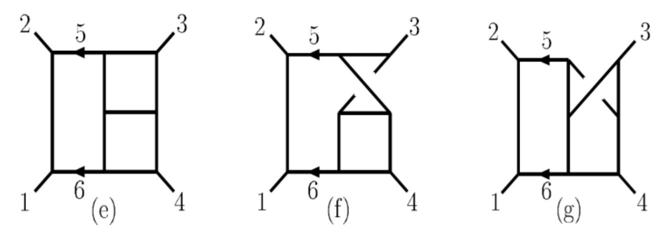
BCJ, 1004.0476

N=8 SUGRA

$$\left[\, s^2 \,\right]^2$$

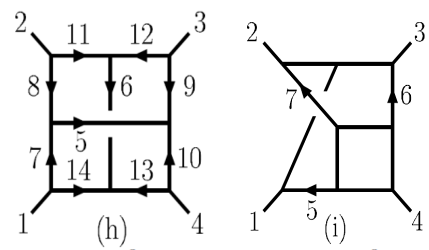


$$\left[\ \tfrac{1}{3}\ \ [s(t-\tau_{36}-\tau_{46}) \\ -\ t(\tau_{26}+\tau_{46}) \\ +\ u(\tau_{26}+\tau_{36})-s^2]\ \right]^2$$



Linear in ℓ_M

$$\left[N^{(h)}(\tau_{ij})\right]^2$$

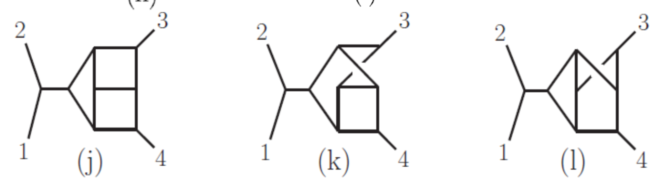


$$\left[N^{(i)}(\tau_{ij})\right]^2$$

Add 3 1PR graphs:

[not UT form – Huber's talk]

$$\left[\frac{1}{3}s(t-u)\right]^2$$



N=8 no worse than N=4 SYM in UV

Manifest quadratic representation at 3 loops – same as N=4 SYM – implies same critical dimension (as for $L = 2$):

$$I_3^{\text{quad.}} \sim \int \frac{(d^6 l_i)^3 \; l_i^2}{[(l_i)^2]^{10}} \sim \ln \Lambda$$

$$D_c = 4 + \frac{6}{L} = 6$$

- Evaluate UV poles in integrals

→ no further cancellation

- At 3 loops, $D_c = 6$ for N=8 SUGRA as well as N=4 SYM:

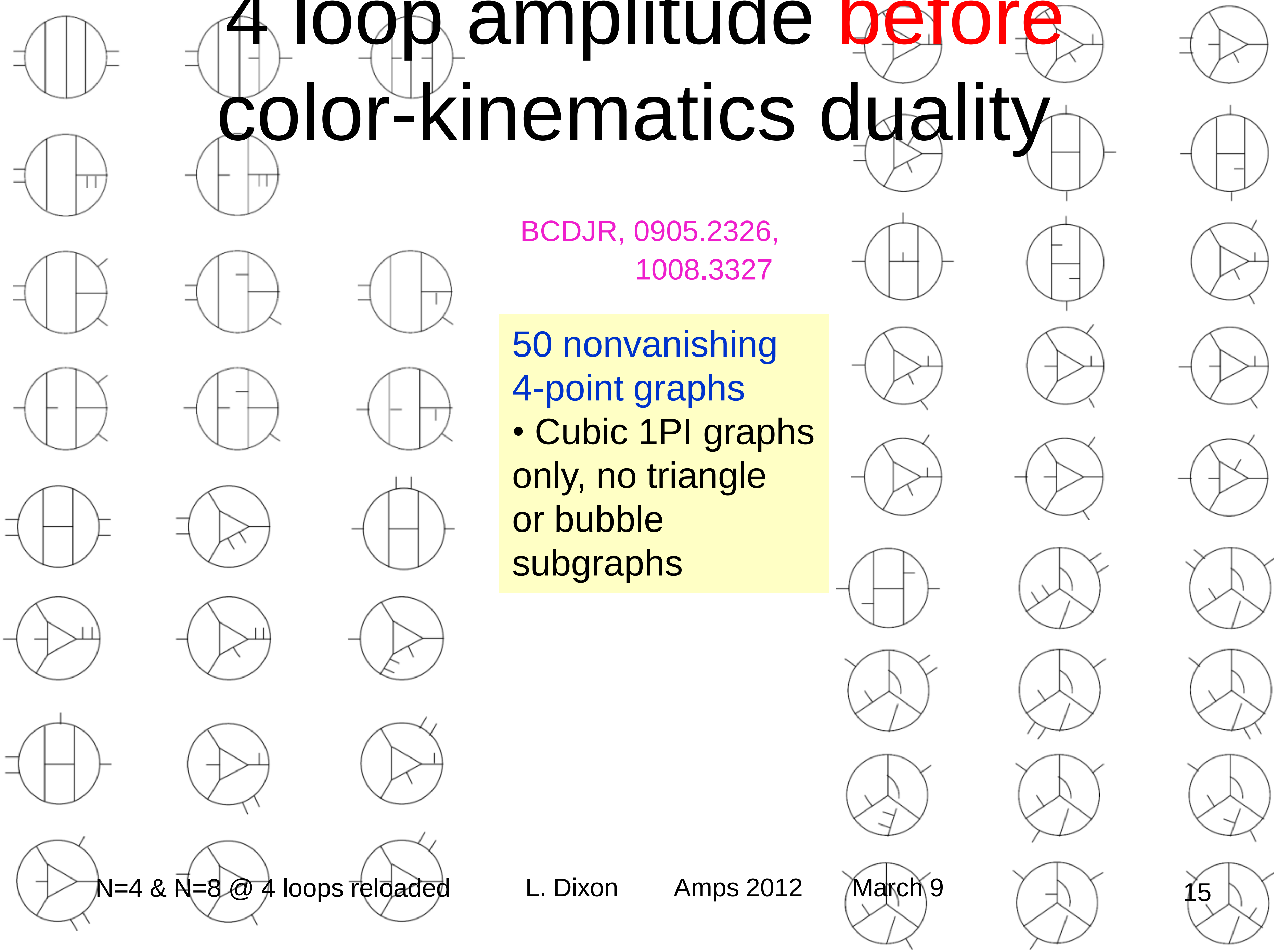
$$M_4^{(3),D=6-2\epsilon}\Big|_{\text{pole}} = \frac{1}{\epsilon}\frac{5\zeta_3}{(4\pi)^9}\left(\frac{\kappa}{2}\right)^8 (s_{12}s_{13}s_{14})^2 M_4^{\text{tree}}$$

$\mathcal{D}^6 R^4$ counterterm

Also recovered via string theory (up to factor of 9?)

Green, Russo, Vanhove,1002.3805

4 loop amplitude before color-kinematics duality



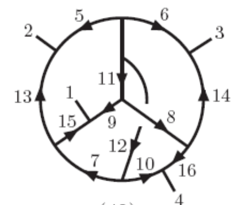
BCDJR, 0905.2326, 1008.3327

50 nonvanishing 4-point graphs

- Cubic 1PI graphs only, no triangle or bubble subgraphs

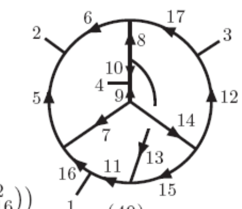
N=4 SYM numerators for most complex graphs [N=8 SUGRA numerators much larger]

$$\begin{aligned}
&s_{12}(s_{2,10}s_{39} - s_{47}s_{18} + s_{2,10}s_{59} + s_{39}s_{6,10} + s_{23}s_{6,11}) - s_{23}s_{57}s_{68} - s_{13}s_{59}s_{6,10} \\
&+l_6^2(s_{12}s_{35} + s_{12}s_{4,\overline{12}} - s_{23}s_{59}) + l_5^2(s_{12}s_{26} + s_{12}s_{1,\overline{11}} - s_{23}s_{6,10}) \\
&+l_9^2(s_{12}s_{12,\overline{13}} - s_{13}s_{10,11}) + l_{10}^2(s_{12}s_{11,\overline{14}} - s_{13}s_{9,12}) \\
&-l_{13}^2 s_{12}s_{11,\overline{14}} - l_{14}^2 s_{12}s_{12,\overline{13}} + (s_{13} - 2s_{12})l_9^2 l_{10}^2 \\
&+s_{23}(l_5^2l_6^2 - l_7^2l_8^2 + l_6^2l_7^2 + l_5^2l_8^2) + s_{12}l_{13}^2l_{14}^2 + s_{12}l_5^2l_6^2 \\
&+s_{12}(-l_5^2l_8^2 + l_5^2l_9^2 - l_5^2l_{11}^2 - l_5^2l_{15}^2 - l_9^2l_{15}^2) \\
&+s_{12}(-l_6^2l_7^2 + l_6^2l_{10}^2 - l_6^2l_{12}^2 - l_6^2l_{16}^2 - l_{10}^2l_{16}^2) \\
&+s_{23}(l_9^2l_{12}^2 + l_{10}^2l_{11}^2 - l_7^2l_9^2 - l_8^2l_{10}^2) + s_{13}(l_9^2l_{11}^2 + l_{10}^2l_{12}^2)
\end{aligned}$$



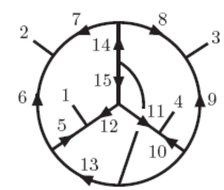
(48)

$$\begin{aligned}
&s_{12}(s_{47}s_{5,12} - s_{19}s_{36} - s_{48}s_{36}) + s_{23}(s_{48}s_{6,11} - s_{15}s_{3,10} - s_{15}s_{47}) - s_{12}s_{23}s_{11,12} \\
&+l_5^2(s_{23}s_{7,12} - s_{23}s_{4,15} - s_{13}s_{10,11}) + l_6^2(s_{12}s_{8,11} - s_{12}s_{4,\overline{15}} - s_{13}s_{9,12}) \\
&+l_9^2(s_{23}s_{3,15} - s_{12}s_{3\overline{8}} + s_{23}s_{6,10}) + l_{10}^2(s_{12}s_{1,\overline{15}} - s_{23}s_{1\overline{7}} + s_{12}s_{59}) \\
&+l_{13}^2(s_{12}s_{23} + s_{12}s_{38} - s_{23}s_{6,11}) + l_{14}^2(s_{23}s_{12} + s_{23}s_{17} - s_{12}s_{5,12}) \\
&+l_{11}^2 s_{23}(s_{4,12} - s_{6,10}) + l_{12}^2 s_{12}(s_{4,11} - s_{59}) \\
&+s_{13}(l_7^2l_8^2 + l_5^2l_8^2 + l_6^2l_7^2 + l_{11}^2l_{12}^2 + l_{10}^2l_{16}^2 + l_9^2l_{17}^2 - l_9^2l_{12}^2 - l_{10}^2l_{11}^2) \\
&+s_{12}(-l_5^2l_{10}^2 + l_6^2(l_{14}^2 + l_{13}^2 - l_{10}^2) + l_{12}^2(l_9^2 - l_5^2 - l_7^2 + l_{14}^2) + l_8^2(l_9^2 + l_{16}^2)) \\
&+s_{23}(-l_6^2l_9^2 + l_5^2(l_{13}^2 + l_{14}^2 - l_9^2) + l_{11}^2(l_{10}^2 - l_6^2 - l_8^2 + l_{13}^2) + l_7^2(l_{10}^2 + l_{17}^2)) \\
&+s_{12}(l_{12}^2l_{13}^2 - l_8^2l_{13}^2 - l_{10}^2l_{13}^2 - l_{10}^2l_{14}^2 - l_{13}^2l_{17}^2) + s_{23}(l_{11}^2l_{14}^2 - l_7^2l_{14}^2 - l_9^2l_{14}^2 - l_9^2l_{13}^2 - l_{14}^2l_{16}^2)
\end{aligned}$$



(49)

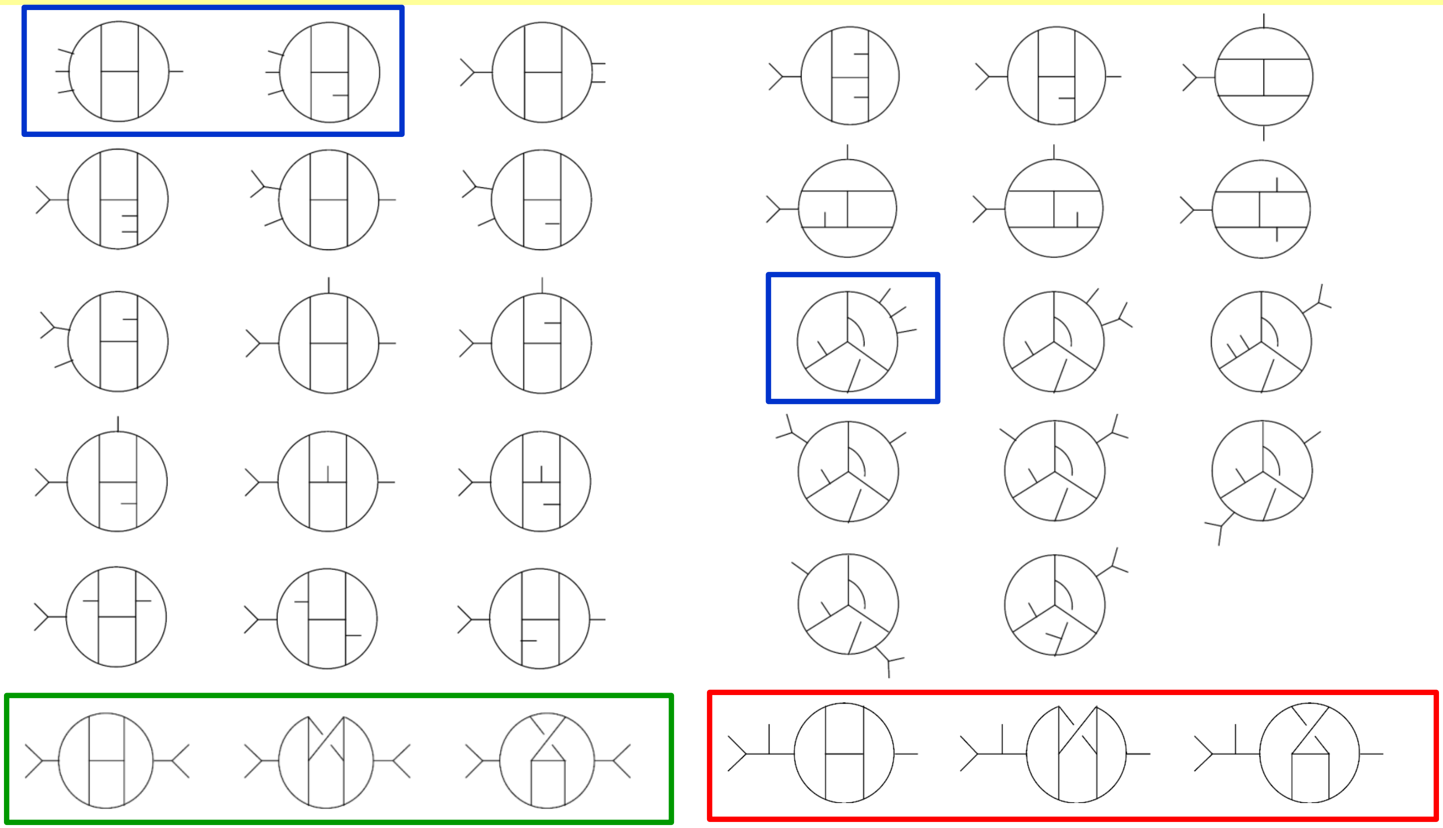
$$\begin{aligned}
&s_{12}s_{28}s_{4,12} - s_{12}s_{37}s_{1,11} - s_{23}s_{16}s_{3,10} \\
&+s_{23}s_{25}s_{49} + \tfrac{1}{2}s_{12}s_{23}(s_{13,15} - s_{13,14}) \\
&+s_{12}(l_6^2l_{10}^2 - l_5^2l_9^2) + s_{23}(l_7^2l_{11}^2 - l_8^2l_{12}^2)
\end{aligned}$$



(50)

4 loop amplitude after color-kinematics duality

To the 50 nonvanishing 1PI cubic 4-point graphs BCDJR, 1201.5366
we must add 3 more 1PI graphs (0 in previous representation)
and 32 1PR graphs (6 of which are 2PR) → 85 in all



Minimal set of (simplified) duality relations

$$n_i \equiv st A_4^{\text{tree}} N_i$$

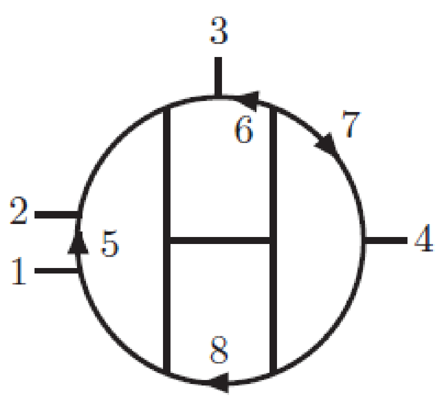
2 terms only, due to generation of vanishing Triangle subgraph:

$$
\begin{aligned}
N_5 &= N_4 = N_3 = N_2 = N_1 \,, \\
N_{11} &= N_{10} = N_9 = N_8 = N_7 = N_6 \,, \\
N_{40} &= N_{13} = -N_{12} \,, \\
N_{41} &= -N_{17} = -N_{16} = -N_{15} = N_{14} \,, \\
N_{42} &= N_{20} = -N_{19} = N_{18} \,, \\
N_{43} &= -N_{23} = -N_{22} = -N_{21} \,, \\
N_{25} &= N_{24} \,, \\
N_{44} &= -N_{26} \,, \\
N_{31} &= -N_{30} = N_{29} = N_{28} \,, \\
N_{46} &= N_{34} = N_{32} \,, \\
N_{36} &= -N_{35} = -N_{33} \,, \\
N_{47} &= N_{38} \,, \\
N_{72} &= N_{52} = N_{51} \,, \\
N_{74} &= -N_{54} = -N_{53} \,, \\
N_{73} &= N_{57} = N_{56} = N_{55} \,, \\
N_{76} &= -N_{62} = -N_{61} = -N_{60} = -N_{59} = -N_{58} \\
N_{77} &= -N_{65} = N_{64} = N_{63} \,, \\
N_{78} &= -N_{67} = N_{66} \,, \\
N_{75} &= N_{71} = N_{70} = N_{69} = N_{68} \,, \\
N_{82} &= N_{81} = N_{80} \,, \\
N_{85} &= N_{84} = N_{83} \,.
\end{aligned}
$$

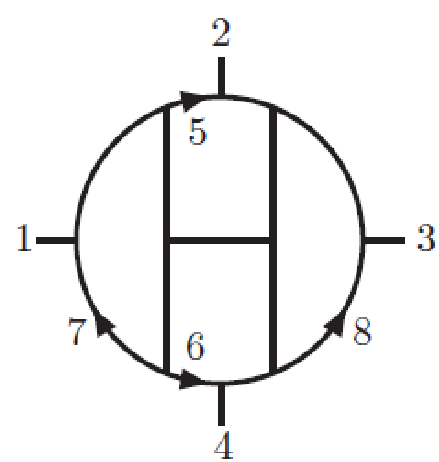
$$
\begin{aligned}
N_{58} &= N_{18}(k_1,k_2,k_3,k_2-l_6,l_5,l_7,l_8) - N_{18}(k_2,k_1,k_3,k_1-l_6,l_5,l_7,l_8) \,, \\
N_{33} &= N_{28}(k_4,k_3,k_2,k_3-l_5,k_2-l_6+l_7,l_7,l_8) - N_{18}(k_1,k_2,k_3,k_2-l_6,k_3-l_5,l_7,l_8) \,, \\
N_{50} &= N_{28}(k_2,k_1,k_4,l_5,k_3-l_7,l_7,l_8) - N_{28}(k_1,k_2,k_3,l_6,k_4-l_8,l_7,l_8) \,, \\
N_6 &= -N_{33}(k_1,k_2,k_4,l_7,l_5-l_6,k_1-l_6,l_8) - N_{33}(k_2,k_1,k_4,l_7,l_6,k_2-l_5+l_6,l_8) \,, \\
N_{14} &= -N_{33}(k_3,k_2,k_1,l_5,-l_5-l_7,k_3-l_7+l_8,l_6) - N_{33}(k_3,k_2,k_1,l_5,k_2+l_7,l_7-l_8,l_8) \,, \\
N_{24} &= -N_{28}(k_1,k_2,k_3,l_5-l_7,-l_6,l_7,l_8) - N_{33}(k_1,k_2,k_4,-l_6,-l_7,-l_5,l_8) \,, \\
N_{32} &= -N_{28}(k_4,k_2,k_1,l_7,k_3-l_5,l_7,l_8) - N_{33}(k_2,k_1,k_3,l_5,l_6,k_2+l_5+l_6-l_7,l_8) \,, \\
N_{48} &= N_{28}(k_3,k_4,k_1,l_8,k_2-l_5,l_7,l_8) - N_{33}(k_1,k_2,k_3,k_3-l_6,k_2-l_5,l_7,l_8) \,, \\
N_{49} &= -N_{33}(k_1,k_2,k_3,k_3-l_8,k_2-l_5,-l_7,l_8) - N_{33}(k_4,k_1,k_2,l_5,-l_7,l_6,l_8) \,, \\
N_{66} &= N_{58}(k_1,k_2,k_4,l_5-k_3-l_6,l_6,l_7,l_8) - N_{58}(k_1,k_2,k_3,k_3+l_6,l_6,l_7,l_8) \,, \\
N_1 &= -N_6(k_1,k_2,k_3,l_6,l_5,l_7,l_8) - N_6(k_1,k_2,k_4,l_6,l_5,l_7,l_8) \,, \\
N_{68} &= N_{14}(k_1,k_2,k_3,k_1-l_5,-l_6,-l_7,-l_8) - N_{14}(k_1,k_2,k_4,l_5-k_2,-l_7,-l_6,l_8) \,, \\
N_{21} &= -N_{14}(k_2,k_1,k_3,l_5,l_6,l_7,l_8) - N_{18}(k_2,k_1,k_3,-l_5,k_1+k_3+l_5-l_6,l_7,l_8) \,, \\
N_{26} &= N_{24}(k_2,k_1,k_3,-l_5,-k_4-l_6-l_7,l_8,l_6) - N_{24}(k_2,k_1,k_4,-l_5,l_7-k_3,l_6-k_1-l_5-l_8,l_6) \,, \\
N_{27} &= -N_{18}(k_2,k_1,k_4,-l_5,l_7,l_7,l_8) - N_{24}(k_1,k_2,k_4,l_5,-k_3-l_7-l_8,k_3-l_6+l_7+l_8,l_8) \,, \\
N_{37} &= -N_{28}(k_2,k_1,k_3,k_1-l_5,k_4+l_8,l_7,l_6) - N_{49}(k_2,k_1,k_3,k_1-l_5,-l_8,l_7-k_2,l_6) \,, \\
N_{39} &= N_{28}(k_2,k_1,k_3,-l_5-l_7,k_4+l_6+l_8,l_5,l_6) - N_{48}(k_1,k_2,k_3,l_7,l_8,-l_5-l_7,-l_6-l_8) \,, \\
N_{45} &= N_{49}(k_1,k_2,k_3,l_5-l_6-l_7-l_8,k_4-l_6,l_5,l_7) \\
&\quad + N_{49}(k_1,k_2,k_4,k_2+l_6+l_7+l_8,l_7,l_5,k_4-l_6) \,, \\
N_{38} &= N_{49}(k_2,k_1,k_4,l_6,k_3+l_5+l_7,-l_5+l_6,k_4-l_8) \\
&\quad - N_{49}(k_1,k_2,k_4,l_5-l_6,k_3+l_5+l_7,-l_6,l_7+l_8) \,, \\
N_{53} &= N_{58}(k_1,k_2,k_3,k_3-l_8,l_6,l_7,l_8) + N_{66}(k_1,k_2,k_4,l_8,-k_4-l_5,l_7,l_8) \,, \\
N_{12} &= N_{18}(k_4,k_3,k_2,l_6,k_2+l_8,l_5,l_7) + N_{26}(k_3,k_4,k_1,-l_6,l_8,-l_5,l_8) \,, \\
N_{51} &= N_{18}(k_3,k_2,k_1,k_1+k_2-l_5,-l_6,l_7,l_8) - N_{21}(k_2,k_3,k_1,l_5-k_1-k_2,-l_6,l_7,l_8) \,, \\
N_{63} &= N_{21}(k_1,k_2,k_3,k_2-l_5,k_1+k_2-l_5-l_6,l_7,l_8) \\
&\quad - N_{21}(k_2,k_1,k_3,k_1-l_5,k_1+k_2-l_5-l_6,l_7,l_8) \,, \\
N_{79} &= N_{45}(k_1,k_2,k_3,k_2-l_5,k_4-l_7,l_6,-l_6-l_8) \\
&\quad - N_{45}(k_1,k_2,k_3,l_5-k_1,l_7,k_3-l_6,k_4+l_5-l_7-l_8) \,, \\
N_{80} &= N_{53}(k_1,k_2,k_3,k_3-l_7,l_6,l_7,l_8) + N_{53}(k_1,k_2,k_3,l_7-k_4,l_5,l_6,l_8) \,, \\
N_{55} &= N_{51}(k_1,k_2,k_3,k_1+l_5,l_6,l_7,l_8) - N_{51}(k_1,k_3,k_2,k_1+l_5,l_6,l_7,l_8) \,, \\
N_{83} &= -N_{55}(k_3,k_1,k_2,k_1+k_2-l_5,l_8,l_6,l_7) - N_{55}(k_3,k_1,k_2,l_5-k_3,l_6,l_7,l_8) \,.
\end{aligned}
$$

Duality relations and master graphs

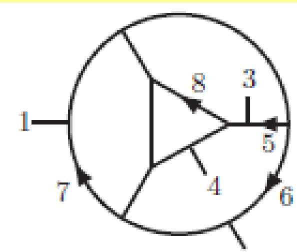
- System of linear relations between numerators.
- Solve by Gaussian elimination in terms of “master” numerators (reminiscent of Laporta IBP method).
- Ambiguity in which integral(s) to choose as masters.
- Convenient to choose 2 planar integrals, (18) and (28).
- Could have used 1 nonplanar integral, (33) instead.



(18)



(28)



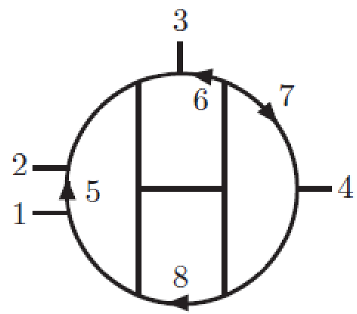
(33)

Ansatz for master graphs

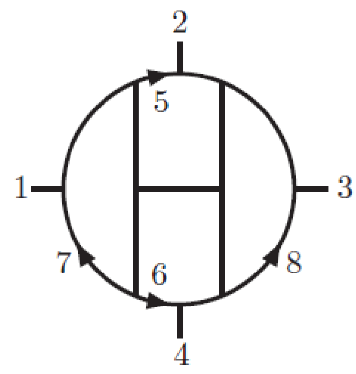
- To solve the duality relations, insert ansatz for the numerators of (18) and (28) based on an assumption of loop-momentum independent boxes and linear pentagons:

$$M = \{s^3, st^2, s^2t, t^3, \tau_{i5}s^2, \tau_{i5}t^2, \tau_{i5}st, \tau_{i6}s^2, \tau_{i6}t^2, \tau_{i6}st, \tau_{i5}\tau_{j6}s, \tau_{i5}\tau_{j6}t, \tau_{56}s^2, \tau_{56}t^2, \tau_{56}st\}$$

$$N_{18} = \sum_{j=1}^{43} a_j M_j \,, \qquad N_{28} = \sum_{j=1}^{43} b_j M_j$$



(18)



(28)

Determining Ansatz parameters

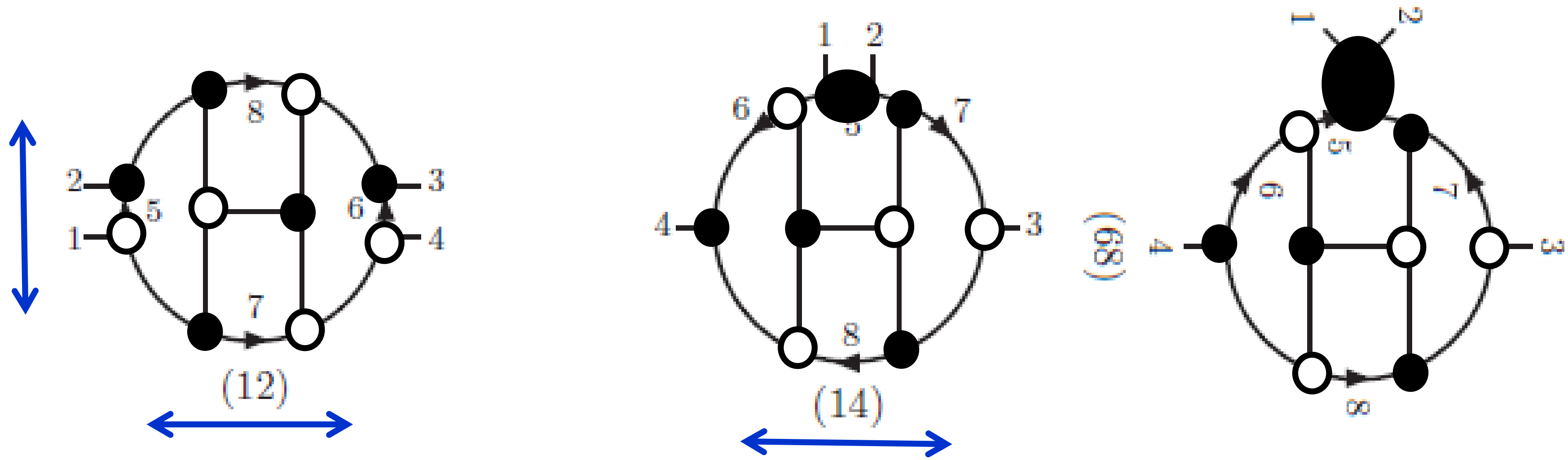
Sufficient to enforce:

86 1. Automorphism symmetries for N_{12}, N_{14}, N_{28}

17 2. Maximal cut of graph 12

8 3. The next-to maximal cut of graphs 14, with l_5 off-shell

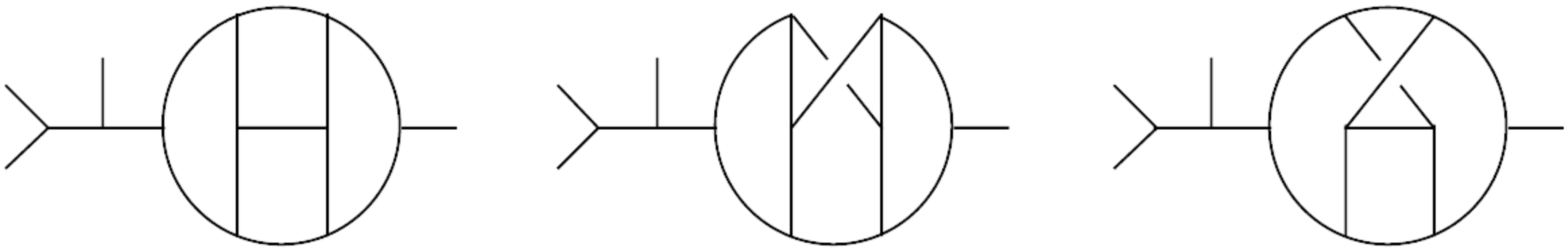
0



$$N_{12}^{\mathrm{rr}} = s^2(l_5 + l_6 + k_1 + k_4)^2 \Rightarrow N_{12}^{\mathrm{max.\,cut}} = s^2(t - \tau_{26} - \tau_{35} + \tau_{56})$$

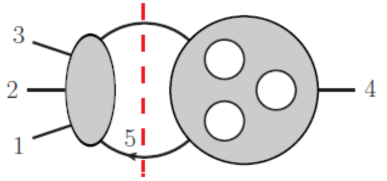
Step 2, matching this cut, reduces 17 parameters → 8 parameters

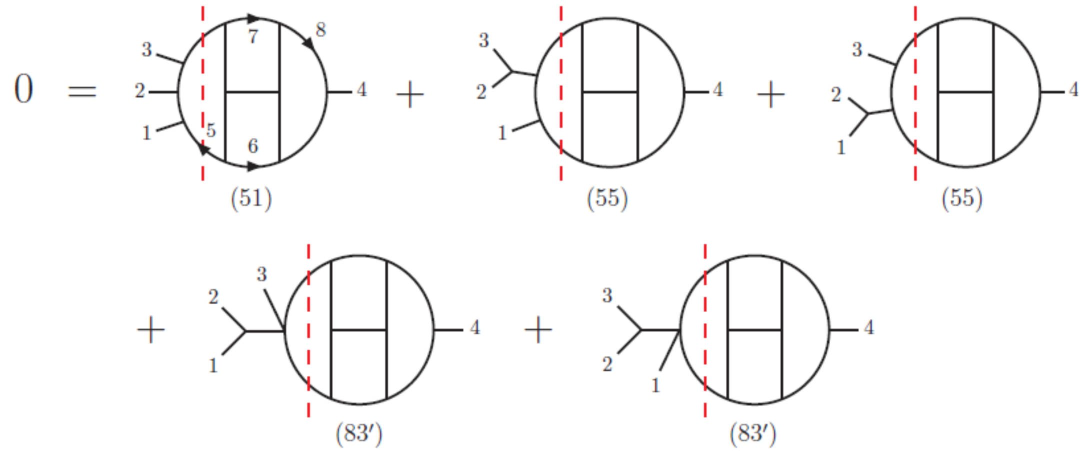
Resolving the Snails



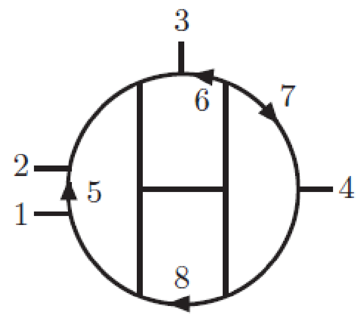
- External leg graphs a fake (color bookkeeping trick)
- They all contain a factor of $k_4{}^2$ in N_i to cancel a singular propagator factor of $1/k_4{}^2$
- To determine their N_i use another cut: → $N_{83} = -\frac{9}{2} k_4^2 s(u-t)$

due to N=4 SYM nonrenormalization of 3-point amplitude

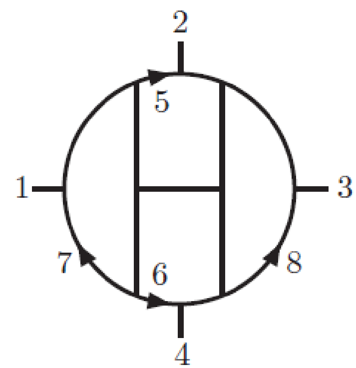




The Answer



(18)



(28)

$$N_{18} = \tfrac{1}{4}(6u^2\tau_{25} + u(2s(5\tau_{25} + 2\tau_{26}) - \tau_{15}(7\tau_{16} + 6t)) + t(\tau_{15}\tau_{26} - \tau_{25}(\tau_{16} + 7\tau_{26})) + s(4\tau_{15}(t - \tau_{26}) + 6\tau_{36}(\tau_{35} - \tau_{45}) - \tau_{16}(4t + 5\tau_{25}) - \tau_{46}(5\tau_{35} + \tau_{45})) + 2s^2(t + \tau_{26} - \tau_{35} + \tau_{36} + \tau_{56}))$$

$$N_{28} = \tfrac{1}{4}(s(2\tau_{15}t + \tau_{16}(2t - 5\tau_{25} + \tau_{35}) + 5\tau_{35}(\tau_{26} + \tau_{36}) + 2t(2\tau_{46} - \tau_{56}) - 10u\tau_{25}) - 4s^2\tau_{25} - 6u(\tau_{46}(t - \tau_{25} + \tau_{45}) + \tau_{25}\tau_{26}) - t(\tau_{15}(4\tau_{36} + 5\tau_{46}) + 5\tau_{25}\tau_{36}))$$

plus the duality relations for the rest

Checks and gravity amplitude

- Since we computed the N=4 SYM 4-loop 4-point amplitude once before [1008.3327], we can just check that the cuts of the new integrands agree with the cuts of the old answer.
- To get N=8 SUGRA, we use double copying:

$$\mathcal{A}_4^{(L)} = \sum_{i\in\Gamma} \int \prod_{l=1}^{L} \frac{d^D p_l}{(2\pi)^D} \frac{1}{S_i} \frac{n_i C_i}{\prod_{\alpha_i} p_{\alpha_i}^2}$$

$$\mathcal{M}_4^{(L)} = \sum_{i\in\Gamma} \int \prod_{l=1}^{L} \frac{d^D p_l}{(2\pi)^D} \frac{1}{S_i} \frac{n_i^2}{\prod_{\alpha_i} p_{\alpha_i}^2}$$

- And then we check the cuts of the new gravity amplitude against the previous (KLT driven) construction [0905.2326]

Part II
Ultraviolet Behavior

UV divergences at 3 loops in $D = 4 + 6/3 = 6$

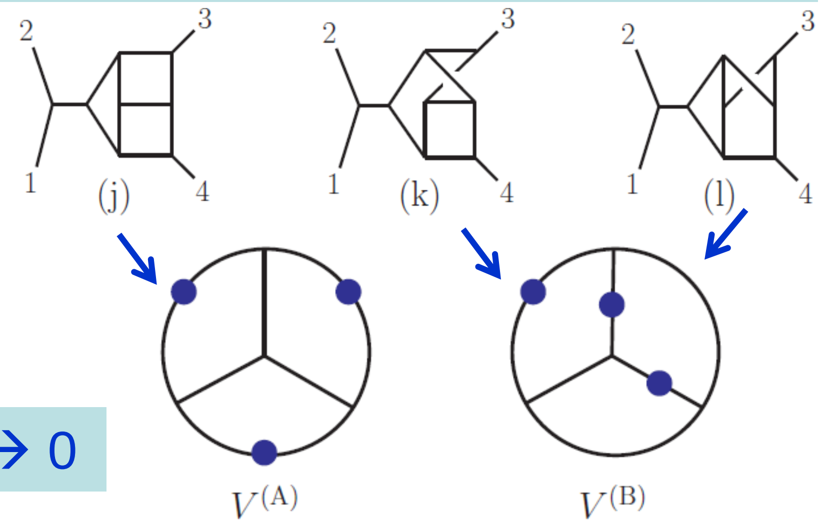
- N=4 SYM: 1PI graphs (x) = (a), (b), … , (i) all have 10 propagators, and numerators $N^{(x)}(l_i)$ that are at most linear in loop momenta l_i.

$$\Rightarrow \quad I^{(x)} \sim \int \frac{(d^6 l_i)^3 \; l_i^\mu}{[(l_i)^2]^{10}}$$ is finite

Only divergences come from 1PR 9 propagator graphs (y) = (j), (k), (l)

$$N^{(y)} = \frac{1}{3} s(t-u)$$

$$\Rightarrow \quad I^{(y)} \sim \int \frac{(d^{6-2\epsilon} l_i)^3}{[(l_i)^2]^9}$$

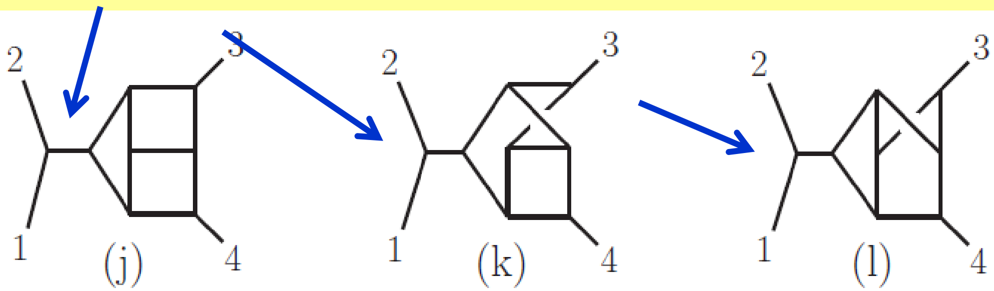


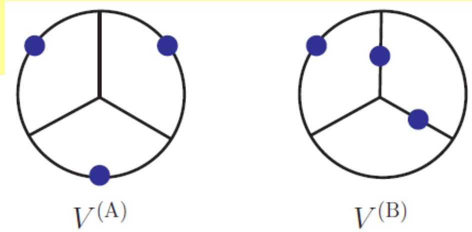
Log divergence → just set external $k_i \to 0$

3 loop N=4 SYM UV color structure

- BCJ form makes manifest that there are no double trace terms in critical dimension $D_c = 6$:
- Color factors for only divergent graphs contain explicit

$$f^{a_1 a_2 b} f^{b a_3 a_4} = \mathrm{Tr}(T^{a_1} T^{a_2} T^{a_3} T^{a_4}) \pm \cdots$$





$$\mathrm{Tr}_{ijkl} \equiv \mathrm{Tr}(T^{a_i} T^{a_j} T^{a_k} T^{a_l})$$

$$\mathcal{A}_4^{(3)}(1,2,3,4)\Big|_{\text{pole}}^{SU(N_c)} = 2\, g^8\, \mathcal{K} \left(N_c^3\, V^{(A)} + 12\, N_c\, (V^{(A)} + 3\, V^{(B)}) \right) \times \left(s\, (\mathrm{Tr}_{1324} + \mathrm{Tr}_{1423}) + t\, (\mathrm{Tr}_{1243} + \mathrm{Tr}_{1342}) + u\, (\mathrm{Tr}_{1234} + \mathrm{Tr}_{1432}) \right)$$

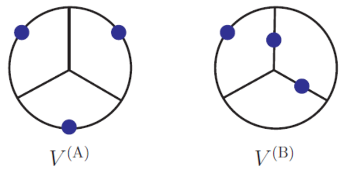
LD @ Amps 2009, BCDJR, 1008.3327

String-theory argument for double-trace absence via collision of 2 vertex operators: Berkovits, Green, Russo, Vanhove, 0908.1923

3 loop N=8 SUGRA UV structure

• 1PI graphs (x) = (a), (b), (c), (d) have loop-momentum independent (scalar) numerators, also after squaring
→ finite in $D = 6$.

• 1PI graphs (x) = (e), (f), (g), (h), (i) were linear in l_i in SYM, become quadratic in SUGRA, so they do contribute to the UV pole



• As do 1PR scalar graphs (y) = (i), (j), (k).

• Total:

$$\mathcal{M}_4^{(3)}\Big|_{\rm pole} = -\left(\frac{\kappa}{2}\right)^8 (stu)^2 M_4^{\rm tree}\left[10\left(V^{(A)} + 3\,V^{(B)}\right)\right]$$

Curiously, this is the **same linear combination** of $V^{(A)}$ and $V^{(B)}$ as in the subleading-color part of the N=4 SYM divergence!
Can understand this for the (y) graphs, but why for the 1PI ones?

UV divergences at 4 loops in $D = 4 + 6/4 = 11/2 = 5.5$

• N=4 SYM: Master numerators N_{18} and N_{28} are quadratic in l_i. Duality relations preserve this for all numerators. Therefore the 1PI, 13-propagator graphs (1)-(52) and (72)

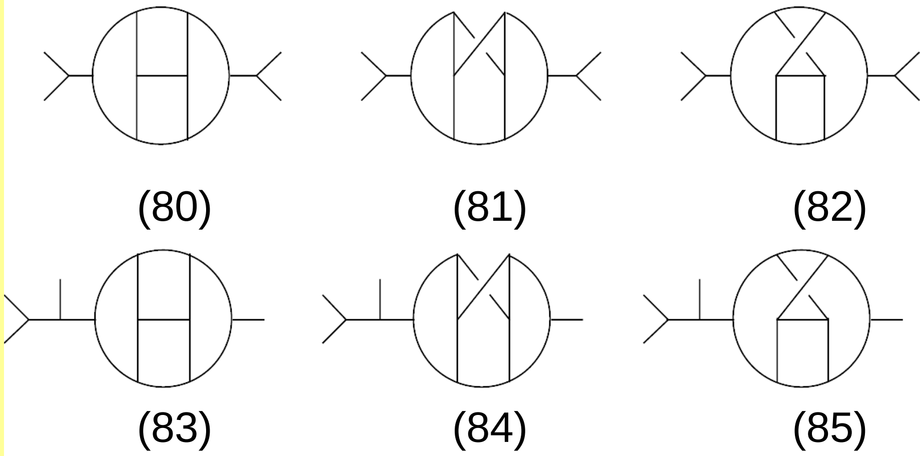
$$\Rightarrow \ I \sim \int \frac{(d^{11/2} l_i)^4 \ l_i^2}{[(l_i)^2]^{13}}$$

are finite in $D = 11/2$

• The 1PR but 2PI 12-propagator graphs are linear → they are also finite in $D = 11/2$

• Only divergences are again from most reducible graphs: scalar 2PR 11-propagators

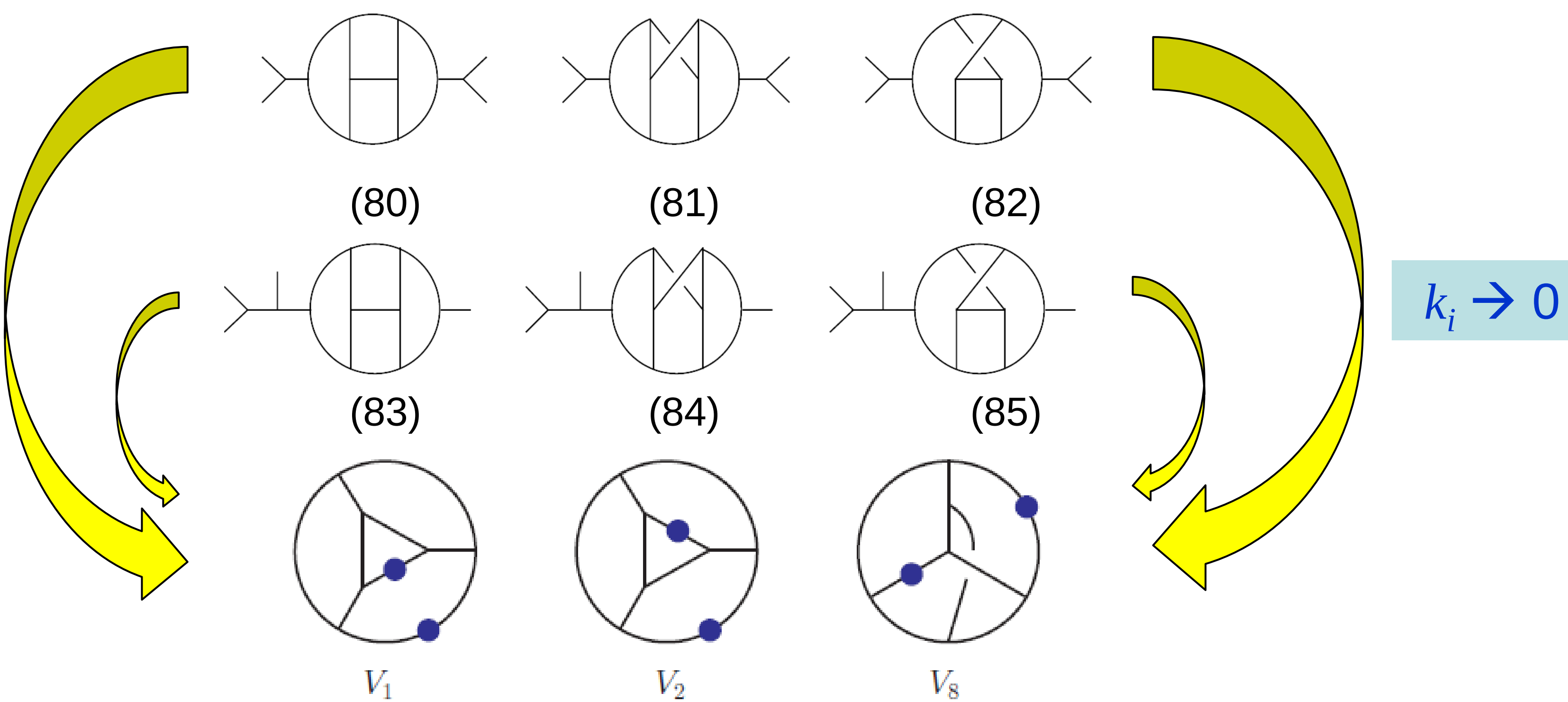
$$I \sim \int \frac{(d^{11/2-2\epsilon} l_i)^4}{[(l_i)^2]^{11}}$$



4 loop N=4 SYM UV color structure

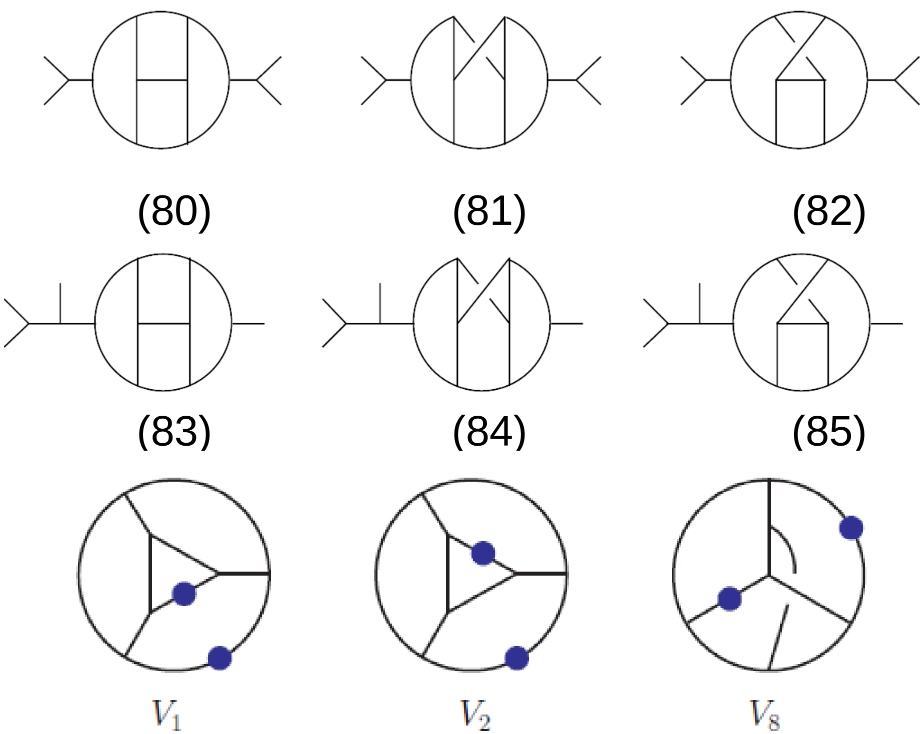
- BCJ form again makes manifest that there are no double trace terms in critical dimension $D_c = 11/2$:
- Color factors for only divergent graphs contain explicit

$$f^{a_1 a_2 b} f^{b a_3 a_4} = \mathrm{Tr}(T^{a_1} T^{a_2} T^{a_3} T^{a_4}) \pm \cdots$$



4 loop N=4 SYM UV pole

$$\mathcal{A}_4^{(4)}(1,2,3,4)\Big|_{\text{pole}}^{SU(N_c)} = -6\,g^{10}\,\mathcal{K}\,N_c^2\Big(N_c^2\,V_1 + 12\,(\underline{V_1 + 2\,V_2 + V_8})\Big)$$
$$\times\Big(s\,(\mathrm{Tr}_{1324} + \mathrm{Tr}_{1423}) + t\,(\mathrm{Tr}_{1243} + \mathrm{Tr}_{1342}) + u\,(\mathrm{Tr}_{1234} + \mathrm{Tr}_{1432})\Big)$$



Note:

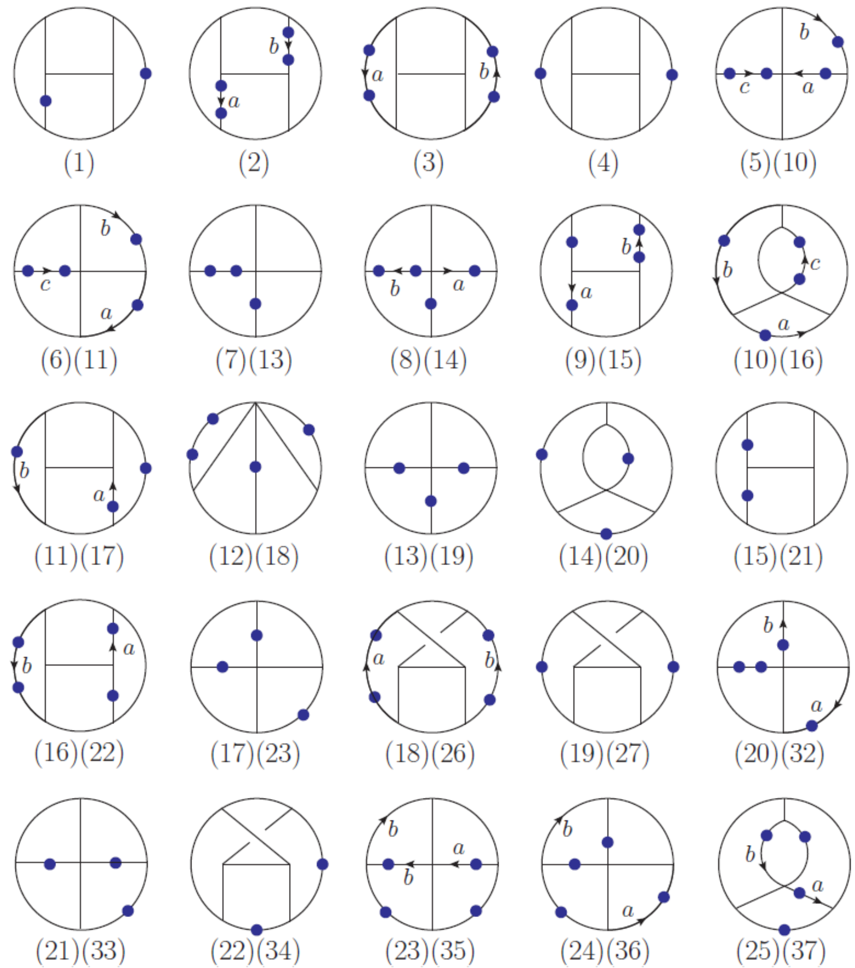
1. No $N_c^{\,0}$ term
2. As at 3 loops, relative factors in $N_c^{\,2}$ are purely from graph symmetry factors S_i: Kinematic (color) Jacobi equates N_i ($N_c^{\,2}$ part of C_i)

4 loop N=8 SUGRA UV pole

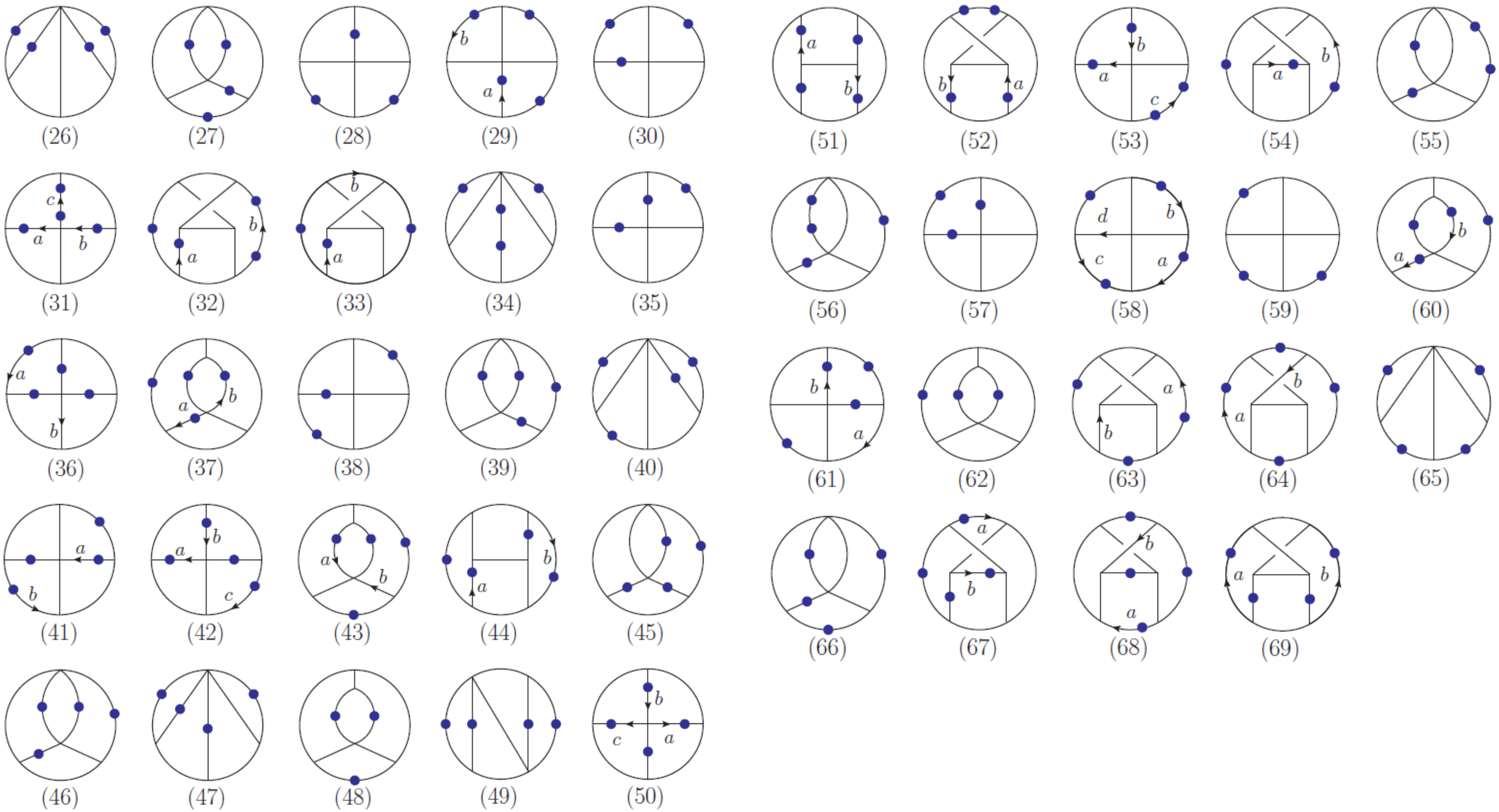
- In new form of amplitude, all integrals are at worst log-divergent in $D = 11/2$.
- After doing standard tensor reductions like

$$l_i^{\mu_i} l_j^{\mu_j} \mapsto \frac{1}{D} \eta^{\mu_i \mu_j} \, l_i \cdot l_j$$

we can set $k_i \to 0$ inside the integrals, resulting in 69 different 4 loop vacuum integrals.
- 25 are shown here



The other 44


(26) (27) (28) (29) (30) (51) (52) (53) (54) (55)
(31) (32) (33) (34) (35) (56) (57) (58) (59) (60)
(36) (37) (38) (39) (40) (61) (62) (63) (64) (65)
(41) (42) (43) (44) (45) (66) (67) (68) (69)
(46) (47) (48) (49) (50)

Evaluating the vacuum integrals

We did it 2 different ways:

1. Consistency relations from expanding a large set of integrals with different loop momentum labelings as $k_i \to 0$, and requiring equality (Johansson's talk)
2. Inject and remove off-shell momenta in 2 places (IRR), to make a 4-loop propagator integral that factorizes as

   [1-loop UV divergent outer bubble]

   x [inner finite 3-loop propagator].

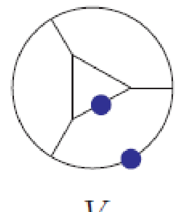
   Use IBP/MINCER/AIR to reduce 3-loop ones to master integrals. Easy problem compared to Smirnov's talk.

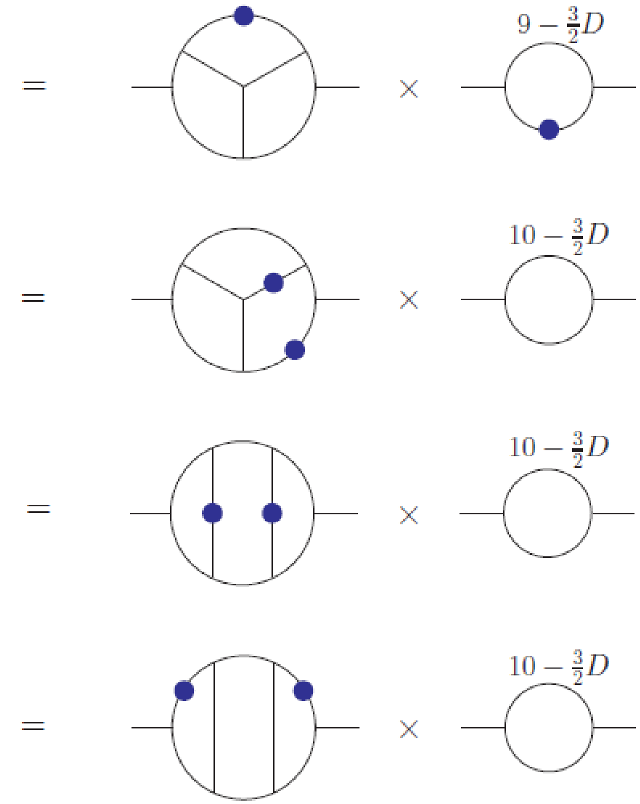
Vladimirov (1980); Chetyrkin, Kataev, Tkachov (1980); Chetyrkin, Tkachov (1982); Chetyrkin, Smirnov (1984); Gorishny, Larin, Surguladze, Tkachov (1989); Larin, …,Vermaseren (1991);

Laporta, hep-ph/0102033; Anastasiou, Lazopoulos, hep-ph/0404258

Evaluating the master integrals

Most can be determined by gluing relations:
Chetyrkin, Tkachov (1982)





$$V_1 = \frac{1}{(4\pi)^{11}\,\epsilon}\left[\frac{512}{5}\,\Gamma^4(\tfrac{3}{4}) - \frac{2048}{105}\,\Gamma^3(\tfrac{3}{4})\Gamma(\tfrac{1}{2})\Gamma(\tfrac{1}{4})\right]$$

$$V_2 = \frac{1}{(4\pi)^{11}\,\epsilon}\left[-\frac{4352}{105}\,\Gamma^4(\tfrac{3}{4}) + \frac{832}{105}\,\Gamma^3(\tfrac{3}{4})\Gamma(\tfrac{1}{2})\Gamma(\tfrac{1}{4})\right]$$

but

$$V_8 = \frac{1}{(4\pi)^{11}\,\epsilon}\left[-\frac{20992}{2625}\,\Gamma^4(\tfrac{3}{4}) + \frac{128}{75}\,\Gamma^3(\tfrac{3}{4})\Gamma(\tfrac{1}{2})\Gamma(\tfrac{1}{4}) + \frac{8}{21\,\Gamma(\frac{3}{4})}\,\mathrm{NO}_m\right]$$

$$\mathrm{NO}_m = -6.1983992267\ldots$$

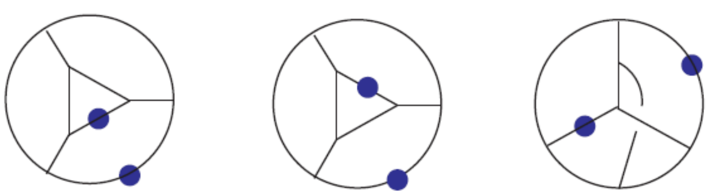
4 loop UV pole in $D = 11/2$

- Best to use V_1, V_2, V_8 as the basis anyway
- Remarkably, final answer is simply:

$$\mathcal{M}_4^{(4)}\Big|_{\rm pole} = -\frac{23}{8}\left(\frac{\kappa}{2}\right)^{10} stu\,(s^2+t^2+u^2)^2\, M_4^{\rm tree}\,\underline{(V_1+2V_2+V_8)}$$

- Again, same linear combination as in N_c^2 part of N=4 SYM pole!

I^v	Effective numerator	V_1	V_2	V_8
I_1^v	$-\frac{117674}{1485}$	0	$-\frac{117674}{1485}$	0
I_2^v	$\frac{19112}{1485}\,\tau_{a,b}^2$	$\frac{8798687}{5346000}$	$\frac{212621}{27000}$	0
I_3^v	$\frac{9556}{1485}\,\tau_{a,b}^2$	$\frac{15937019}{1782000}$	$-\frac{140951}{33000}$	0
I_4^v	$-\frac{16427}{495}$	$-\frac{16427}{495}$	0	0
I_5^v	$\frac{19112}{1485}\,\tau_{a,c} - \frac{19112}{1485}\,\tau_{b,c}$	$-\frac{2389}{2970}$	$-\frac{2389}{1485}$	0
I_6^v	$-\frac{4778}{495}\,\tau_{a,c} + \frac{4778}{1485}\,\tau_{b,c}$	$\frac{16723}{2970}$	$-\frac{4778}{1485}$	0
I_7^v	$-\frac{9556}{1485}$	$\frac{109894}{7425}$	$\frac{90782}{2475}$	0
I_8^v	$\frac{38224}{1485}\,\tau_{a,b}$	$-\frac{2389}{675}$	$-\frac{19112}{2475}$	0
I_9^v	$\frac{38224}{1485}\,\tau_{a,b}^2$	$-\frac{1617353}{148500}$	$\frac{2606399}{74250}$	0
I_{10}^v	$-\frac{19112}{1485}\,\tau_{a,c} - \frac{19112}{1485}\,\tau_{b,c}$	$-\frac{2389}{2970}$	$-\frac{2389}{1485}$	0
I_{11}^v	$-\frac{38224}{1485}\,\tau_{a,b}$	$\frac{90782}{22275}$	$-\frac{9556}{825}$	0
I_{12}^v	$-\frac{19112}{1485}$	$\frac{31057}{990}$	$\frac{38224}{495}$	0
I_{13}^v	$\frac{10048}{99}$	$\frac{2512}{99}$	$\frac{10048}{99}$	0
I_{14}^v	$-\frac{19112}{1485}$	$-\frac{4778}{275}$	$-\frac{324904}{7425}$	0
I_{15}^v	$\frac{19112}{1485}$	$\frac{66892}{4455}$	$\frac{19112}{495}$	0
I_{16}^v	$\frac{19112}{1485}\,\tau_{a,b}^2$	$\frac{977101}{267300}$	$\frac{88393}{14850}$	0
I_{17}^v	$\frac{39676}{1485}$	$\frac{9919}{495}$	$\frac{19838}{1485}$	0
I_{18}^v	$\frac{9556}{1485}\,\tau_{a,b}^2$	$-\frac{1478791}{297000}$	$\frac{661753}{148500}$	$\frac{2389}{396}$
I_{19}^v	$-\frac{64441}{1485}$	0	0	$-\frac{64441}{1485}$
I_{20}^v	$\frac{38224}{1485}\,\tau_{a,b}$	$-\frac{102727}{14850}$	$-\frac{74059}{7425}$	0
I_{21}^v	$\frac{5284}{1485}$	$\frac{18494}{7425}$	$\frac{34346}{7425}$	0
I_{22}^v	$\frac{934}{165}$	$\frac{467}{165}$	$\frac{1868}{165}$	$-\frac{934}{165}$
I_{23}^v	$\frac{526}{135}\,\tau_{a,b} - \frac{91}{1485}\,\tau_{a,c}$	$\frac{279199}{297000}$	$\frac{72052}{37125}$	0
I_{24}^v	$\frac{3736}{495}\,\tau_{a,b}$	$\frac{26152}{12375}$	$-\frac{91532}{12375}$	0
I_{25}^v	$-\frac{9556}{1485}\,\tau_{a,b}$	$-\frac{2389}{2475}$	$-\frac{16723}{7425}$	0
I_{26}^v	$-\frac{11048}{1485}$	$-\frac{17953}{2475}$	$-\frac{44192}{2475}$	0
I_{27}^v	$-\frac{1228}{135}$	$-\frac{307}{50}$	$-\frac{10438}{675}$	0
I_{28}^v	$-\frac{3736}{495}$	$\frac{934}{825}$	$-\frac{14944}{825}$	0
I_{29}^v	$\frac{3736}{495}\,\tau_{a,b}$	$-\frac{48568}{4125}$	$\frac{76588}{4125}$	0
I_{30}^v	$\frac{934}{495}$	$\frac{90131}{24750}$	$\frac{119552}{12375}$	0
I_{31}^v	$\frac{4778}{1485}\,\tau_{a,b} + \frac{9556}{1485}\,\tau_{a,c}$	$-\frac{45391}{19800}$	$-\frac{112283}{14850}$	0
I_{32}^v	$\frac{19112}{1485}\,\tau_{a,b}^2$	$\frac{2721071}{148500}$	$-\frac{327293}{74250}$	$-\frac{2389}{495}$
I_{33}^v	$-\frac{3736}{495}\,\tau_{a,b}$	$-\frac{1868}{2475}$	$-\frac{1868}{275}$	$\frac{1868}{165}$
I_{34}^v	$\frac{4778}{297}$	$-\frac{155285}{2376}$	$-\frac{47780}{297}$	0
I_{35}^v	$-\frac{7904}{1485}$	$-\frac{3952}{495}$	$-\frac{27664}{1485}$	0
		. . .		
Total	–	$\frac{23}{2}$	23	$\frac{23}{2}$



V_1 V_2 V_8

4 loop UV pole in $D = 11/2$ (cont.)

- Again, we understand that it is the same linear combination as the 2PR contributions (80), (81), (82), due to double-copy + group theory (same S_i in both cases):

$$\mathcal{M}_4^{(4)}\Big|_{\text{pole}}^{I_{80,81,82}} = -64\left(\frac{\kappa}{2}\right)^{10} stu\,(s^2+t^2+u^2)^2\, M_4^{\text{tree}}\,(V_1+2V_2+V_8)$$

- But we don't understand why all the other, much more complicated contributions arrange themselves in this way.

One more piece of numerology

- Numerators are squared in the double-copy formulat, but not propagators. Sign cancellations might happen because of different numbers of internal and external propagators.
- Motivated by this, we broke up the answer further, into 11-, 12- and 13-internal-propagator contributions, then considered if the odd (11-,13-) terms cancel somewhat against the even (12-).

$$\mathcal{M}_4^{(4)}\Big|_{\rm pole}^{p-{\rm prop's}} = X_p \left(\frac{\kappa}{2}\right)^{10} stu\,(s^2+t^2+u^2)^2\,M_4^{\rm tree}\,(V_1+2V_2+V_8)$$

Numerology (cont.)

$$\mathcal{M}_4^{(4)}\Big|_{\text{pole}}^{p-\text{prop's}} = X_p \left(\frac{\kappa}{2}\right)^{10} stu\,(s^2+t^2+u^2)^2\, M_4^{\text{tree}}\,(V_1 + 2V_2 + V_8)$$

$$X_{11} = -64$$

$$X_{12} = +142$$

$$X_{13} = -\frac{647}{8}$$

$$X_{\text{tot}} = -\frac{23}{8}$$

- Cancellation between odd (11-,13-) and even (12-) terms is quite strong, at the level of (23/8)/142 = 0.020…

What about $L = 5$?

Talk by H. Johansson

- Motivation: Various arguments point to 7 loops as the possible first divergence for N=8 SUGRA in D=4, associated with a D^8R^4 counterterm:

Howe, Lindstrom, NPB181, 487 (1981); Bossard, Howe, Stelle, 0908.3883; Kallosh, 0903.4630; Green, Russo, Vanhove, 1002.3805; Bjornsson, Green, 1004.2692; Bossard, Howe, Stelle, 1009.0743; Beisert, Elvang, Freedman, Kiermaier, Morales, Stieberger,1009.1643

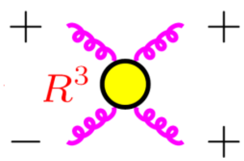
- Same D^8R^4 counterterm shows up at $L = 4$ in $D = 5.5$
- Does 5 loops → $D^{10}R^4$ (same UV as N=4 SYM)?
  or → D^8R^4 (worse UV as N=4 SYM)?
- 5 loops would be a very strong indicator for 7 loops
- Now 100s of nonvanishing cubic 4-point graphs!

Outlook

- Through 4 loops, the 4-graviton scattering amplitude of N=8 supergravity has UV behavior no worse than the corresponding 4-gluon amplitude of N=4 SYM.
- The precise pole for N=8 supergravity bears a remarkable relation with the subleading-color single trace pole in N=4 SYM in the same critical dimension, not only at 4 loops, but also at 2 and 3 loops.
- Is this an accident, or does it portend something at higher loops?
- In particular, could it be the harbinger of equal critical dimensions $D_c = 26/5$ at 5 loops? Which in turn would suggest 7 loops is not where N=8 supergravity first diverges (contrary to much speculation).
- Stay tuned!

Extra Slides

Pure supergravity ($\mathcal{N} \geq 1$):
Divergences deferred to at least three loops



$R^3 \equiv R^{\lambda\rho}{}_{\mu\nu} R^{\mu\nu}{}_{\sigma\tau} R^{\sigma\tau}{}_{\lambda\rho}$ cannot be supersymmetrized

produces helicity amplitude (-+++) incompatible with SUSY Ward identitites

Grisaru (1977); Deser, Kay, Stelle (1977); Tomboulis (1977)

However, at three loops, there is an N=8 supersymmetric counterterm, abbreviated R^4,
plus (many) other terms containing other fields in N=8 multiplet.

Deser, Kay, Stelle (1977); Howe, Lindstrom (1981); Kallosh (1981); Howe, Stelle, Townsend (1981)

R^4 produces first subleading term in low-energy limit of 4-graviton scattering in type II string theory:

$$\alpha'^3 R^4 \;\Rightarrow\; \alpha'^3 \underbrace{stu}_{\text{Bose symmetric polynomial}}\; \underbrace{M_4^{\text{tree}}(1,2,3,4)}_{\text{4-graviton amplitude in (super)gravity}}$$

Gross, Witten (1986)

Bose symmetric polynomial

4-graviton amplitude in (super)gravity

N=8 allowed

Chart of potential counterterms

Elvang, Freedman, Kiermaier (2010)

Analytic proofs:

- $D^{2k}R^n$ MHV ∄ for $n > 4$ and $k < 4$.
- $D^{2k}R^n$ NMHV ∄ for $n > 5$ and $k < 2$.

Drummond, Heslop, Howe, Kerstan, th/0305202; Kallosh, 0906.3495

Until 7 loops, any divergences show up in 4-point amplitude!

L							
3	R^4 MHV ∃!						
4	D^2R^4 MHV ∄	R^5 MHV ∄					
5	D^4R^4 MHV ∃!	D^2R^5 MHV ∄	R^6 (N)MHV ∄				
6	D^6R^4 MHV ∃!	D^4R^5 MHV ∄	D^2R^6 (N)MHV ∄	R^7 (N)MHV ∄			
7	D^8R^4 MHV ∃!	D^6R^5 MHV ∄	D^4R^6 MHV ∄ NMHV	D^2R^7 (N)MHV ∄	R^8 (N)MHV ∄ N^2MHV?		
8	$D^{10}R^4$ MHV ∃!	D^8R^5 MHV ∃!	D^6R^6 MHV ∄ NMHV?	D^4R^7 MHV ∄ NMHV?	D^2R^8 (N)MHV ∄ N^2MHV?	R^9 (N)MHV ∄ N^2MHV?	
9	$D^{12}R^4$ 2×MHV	$D^{10}R^5$?×MHV	D^8R^6 2×MHV NMHV?	D^6R^7 MHV ∄ NMHV?	D^4R^8 MHV ∄ N or N^2MHV?	D^2R^9 (N)MHV ∄ N^2MHV?	R^{10} (N)MHV ∄ N^2 or N^3MHV?

- red: not excluded
- green: ?
- gray: excluded

$E_{7(7)}$ Constraints on Counterterms

- N=8 SUGRA has continuous symmetries: noncompact form of E_7.
- 70 scalars → coset $E_{7(7)}/\mathrm{SU}(8)$. Non-$\mathrm{SU}(8)$ part realized nonlinearly.

Cremmer, Julia (1978,1979) quantum level: Bossard, Hillmann, Nicolai, 1007.5472

- $E_{7(7)}$ also implies amplitude Ward identities, associated with limits as one or two scalars become soft Bianchi, Elvang, Freedman, 0805.0757; Arkani-Hamed, Cachazo, Kaplan, 0808.1446; Kallosh, Kugo, 0811.3414
- Single-soft limit of NMHV 6-point matrix element of R^4 doesn't vanish; violates $E_{7(7)}$ Elvang, Kiermaier, 1007.4813
- Similar arguments also rule out $\mathcal{D}^4R^4$ and $\mathcal{D}^6R^4$
- However, $\mathcal{D}^8R^4$ is allowed (L=7 for D=4) Beisert et al., 1009.1643
- Same conclusions reached by other methods Bossard, Howe, Stelle, 1009.0743
- Volume of full N=8 superspace is same dimension as $\mathcal{D}^8R^4$ – but it vanishes! Invariant candidate $\mathcal{D}^8R^4$ counterterm exists, but not full superspace integral. Bossard, Howe, Stelle, Vanhove, 1105.6087