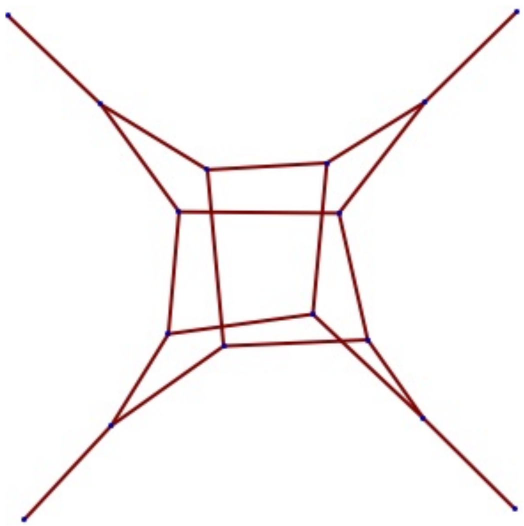
Non-Planar SYM at Five Loops



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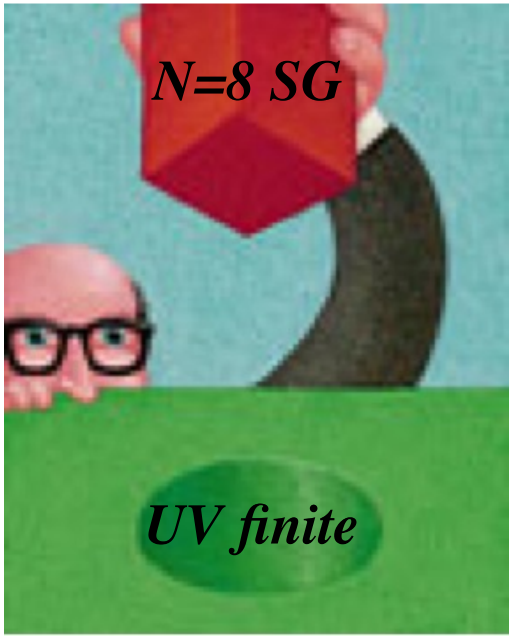
Work in collaboration with:

Z.Bern, J.J.Carrasco, L.Dixon, R.Roiban

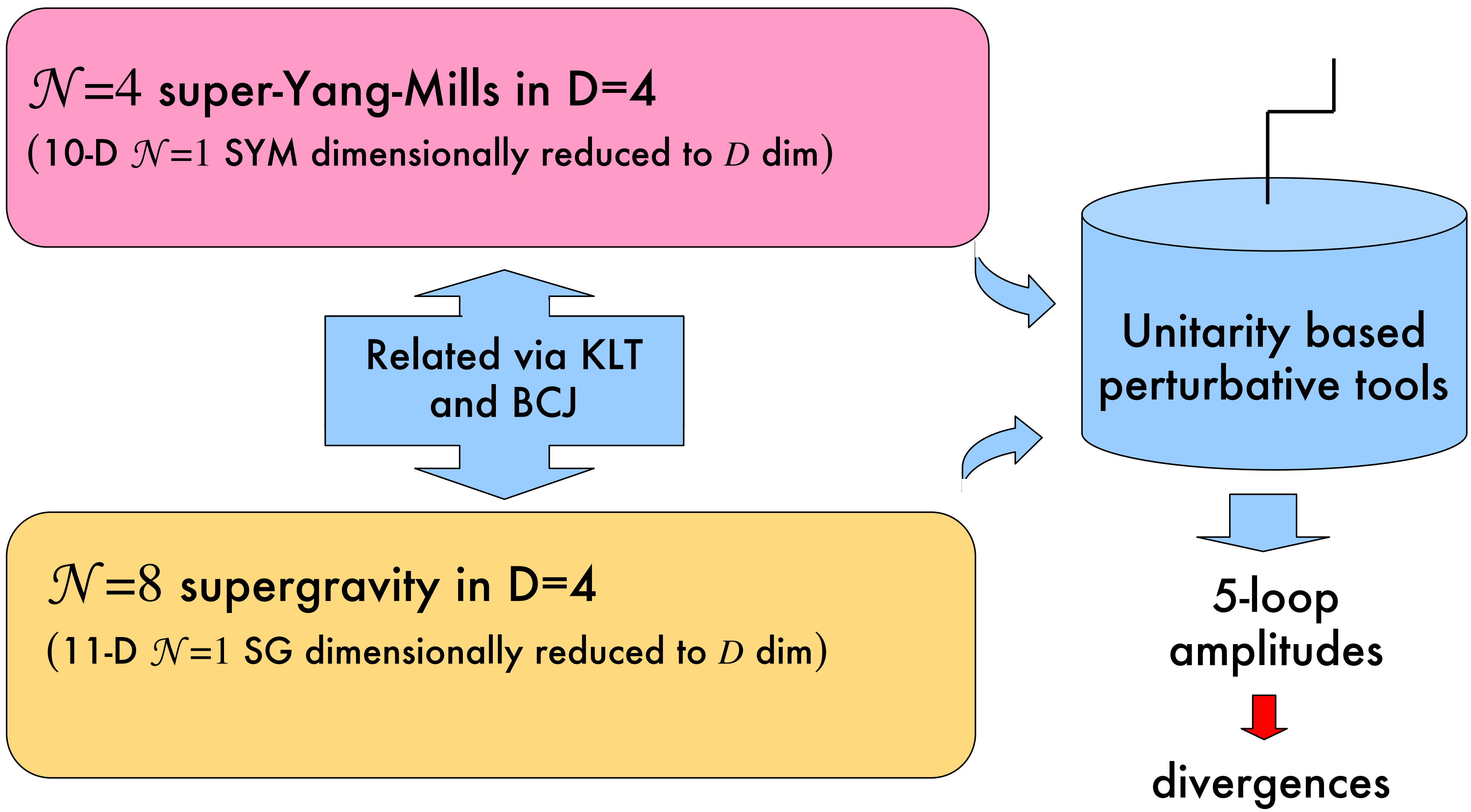
Motivation

- 5-loop N=4 SYM steppingstone to 5-loop N=8 supergravity
- Calculation of 5-loop N=8 UV divergence (in 24/5 dim.) gives prediction on 7-loop divergence in D=4.
- Need UV divergences of N=4 SYM
  - Compare with N=8 SG
  - Double trace structure
  - Single-trace NNLC
- Find BCJ structure at 5-loops



- Most important question: is N=8 SG sufficiently well-behaved to be a UV finite theory ?

Scheme of calculation



Outline

- Motivation
- Review UV status of N=4 SYM and N=8 SG
- How we calculated 5 loops SYM
  - Unitarity, maximal cuts
  - Two-particle cuts, box cuts
  - Duality between color and kinematics at cut level
- 5-loop result and UV properties
- Conclusions

UV properties $\mathcal{N}$=4 SYM and $\mathcal{N}$=8 SG

In D=4 dimensions: see Dixon's talk

- N=4 SYM ultraviolet finite Mandelstam
- N=8 SG: conventional superspace power counting forbids L=1,2 divergences Green, Schwarz, Brink, Howe and Stelle, Marcus and Sagnotti
- Three-loop divergence ruled out by calculation: Bern, Carrasco, Dixon, HJ, Kosower, Roiban, (2007), Bern, Carrasco, Dixon, HJ, Roiban (2008)
- L<7 loop divergences ruled out by counterterm analysis, using $E_{7(7)}$ symmetry and other methods, but a L=7 divergence is still possible

Beisert, Elvang, Freedman, Kiermaier, Morales, Stieberger; Björnsson, Green, Bossard, Howe, Stelle, Vanhove, Kallosh, Ramond, Lindström, Berkovits, Grisaru, Siegel, Russo, and more....

In D>4 dimensions:

Through four loops $\mathcal{N}$=8 SG and $\mathcal{N}$=4 SYM diverge in exactly the same dimension:

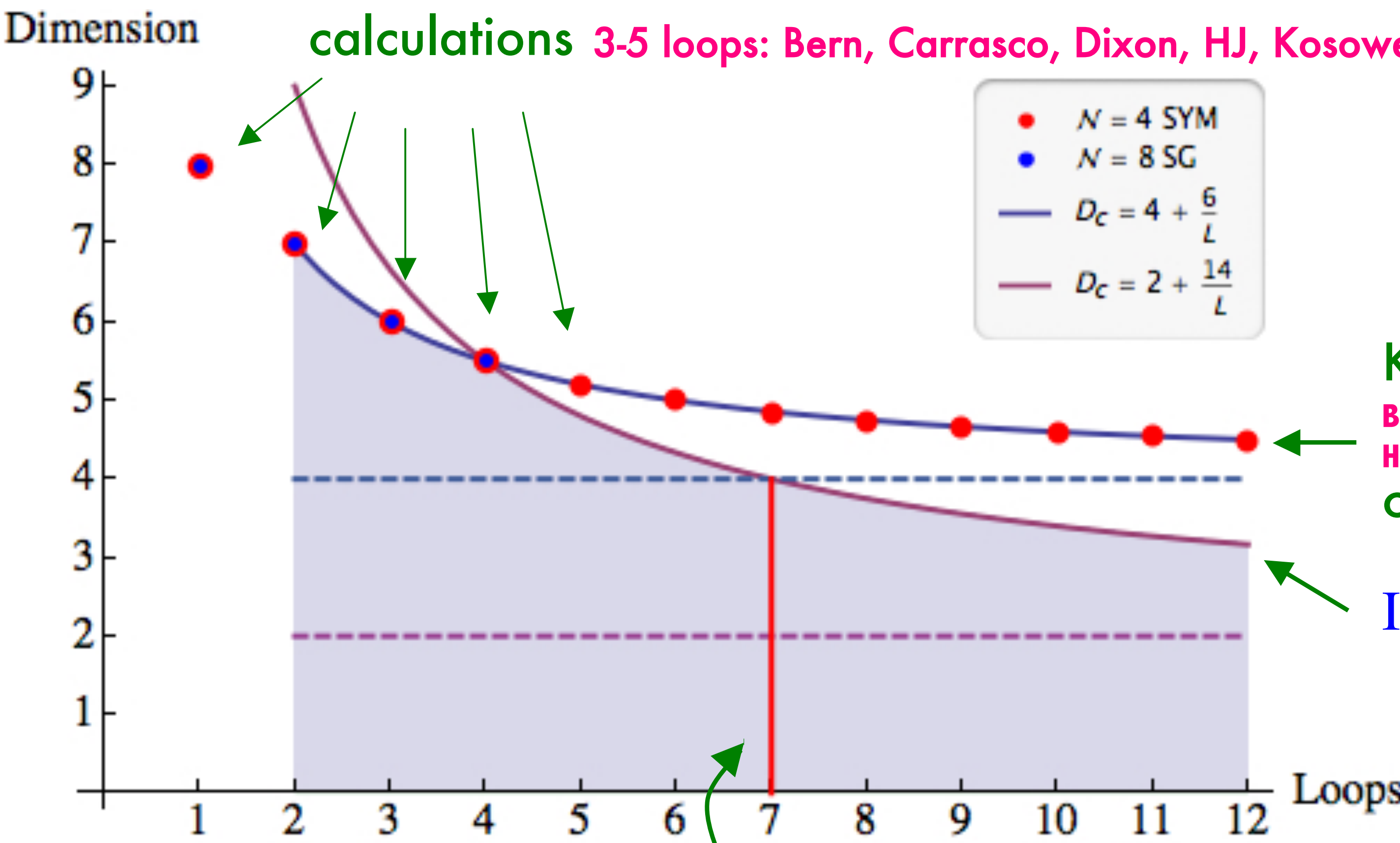
$$D_c = 4 + \frac{6}{L} \qquad (L > 1)$$

Marcus and Sagnotti; Bern, Dixon, Dunbar, Perelstein, Rozowsky; Bern, Carrasco, Dixon, HJ, Kosower, Roiban

UV divergence trend

Plot of critical dimensions of $\mathcal{N}=8$ SUGRA and $\mathcal{N}=4$ SYM

calculations 3-5 loops: Bern, Carrasco, Dixon, HJ, Kosower, Roiban



Known bound for $\mathcal{N}=4$

Bern, Dixon, Dunbar, Rozowsky, Perelstein; Howe, Stelle

current trend for $\mathcal{N}=8$

If $\mathcal{N}=8$ div. at $L=7$

$L=7$ lowest loop order for possible $D=4$ divergence

Beisert, Elvang, Freedman, Kiermaier, Morales, Stieberger;
Björnsson, Green, Bossard, Howe, Stelle, Vanhove Kallosh, Ramond, Lindström, Berkovits, Grisaru, Siegel, Russo, and more….

UV divergence trend

Plot of critical dimensions of $\mathcal{N}=8$ SUGRA and $\mathcal{N}=4$ SYM

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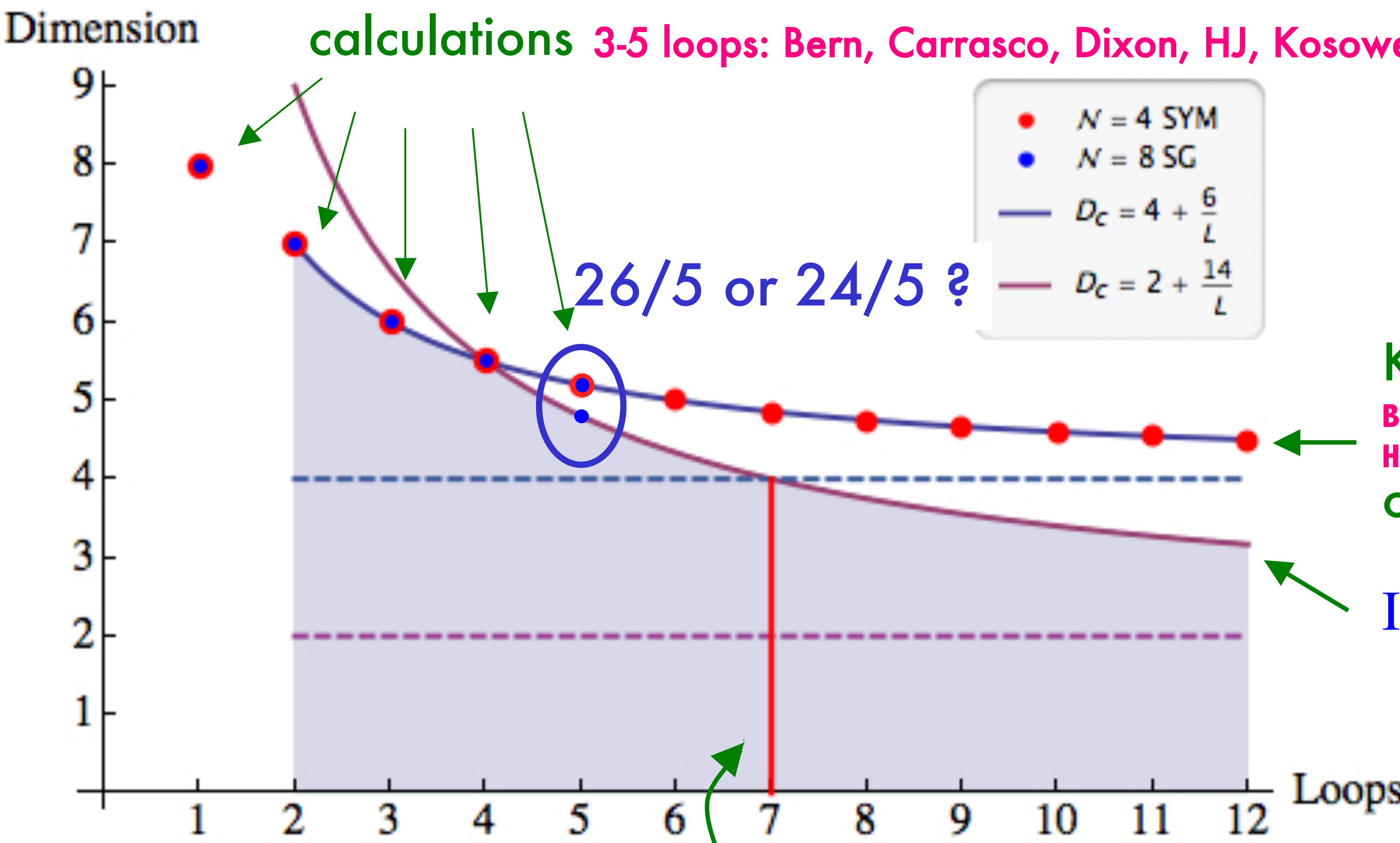

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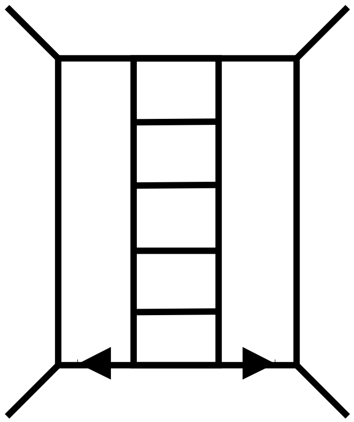
7-loop feature accessible at 5 loops



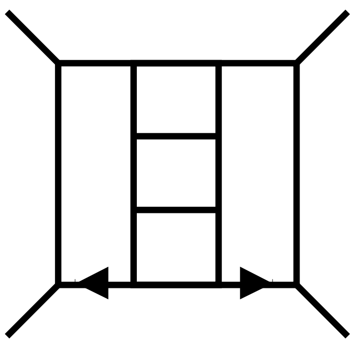
rung rule numerator $\sim ((l_1+l_2)^2)^4$

gravity numerator $\sim \left[((l_1+l_2)^2)^4\right]^2$

log div. integral $\sim \int d^{4\times 7}l \, \frac{\left[((l_1+l_2)^2)^4\right]^2}{(l^2)^{22}} \sim \int \frac{d^{28}l}{l^{28}}$

Can a local gauge transformation remove the offending numerator ?

No, maximal cut is gauge invariant, shows $\left[(2l_1 \cdot l_2)^4\right]^2$ is there in any rep.



rung rule numerator $\sim ((l_1+l_2)^2)^2$

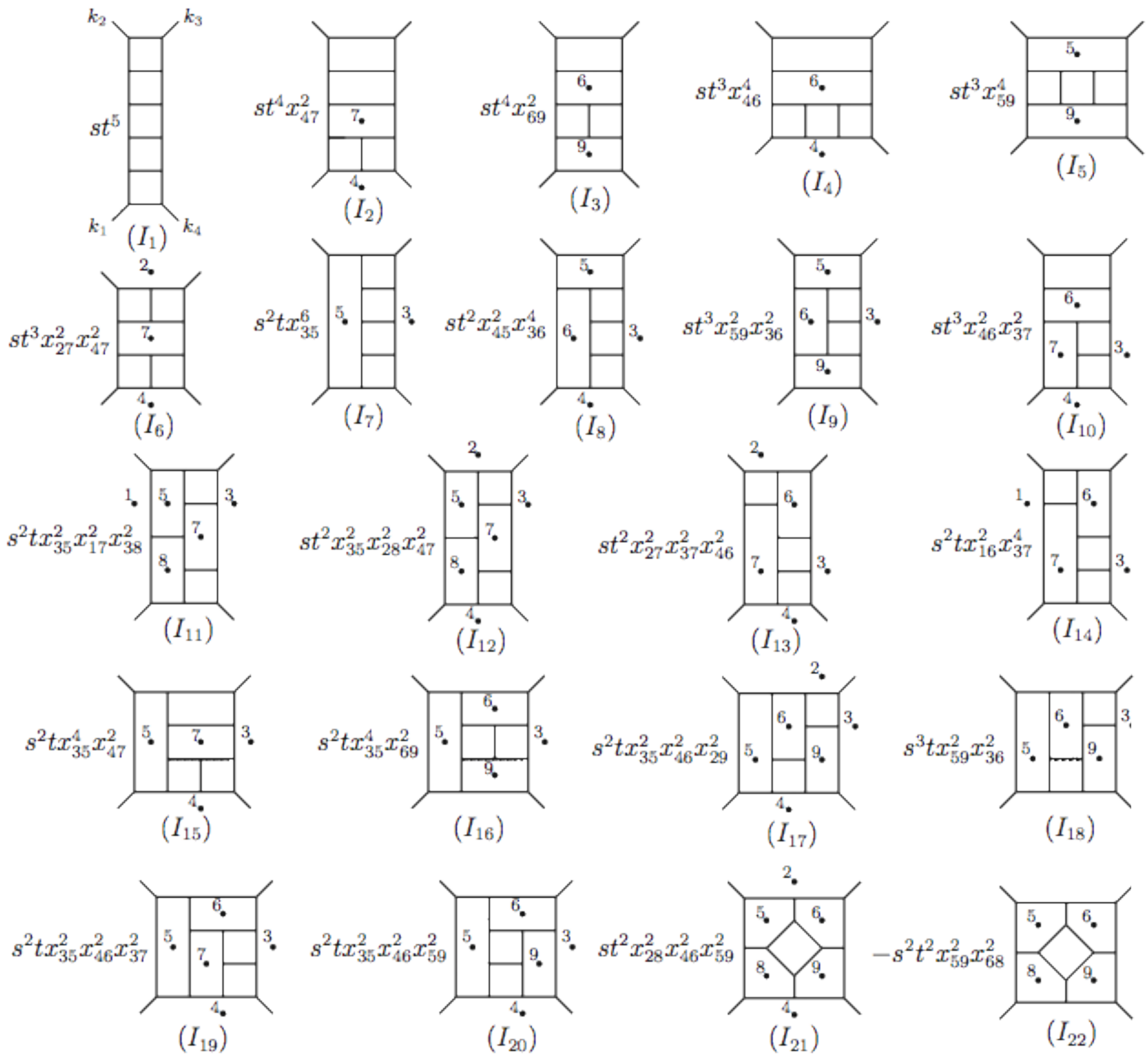
gravity numerator $\sim \left[((l_1+l_2)^2)^2\right]^2$

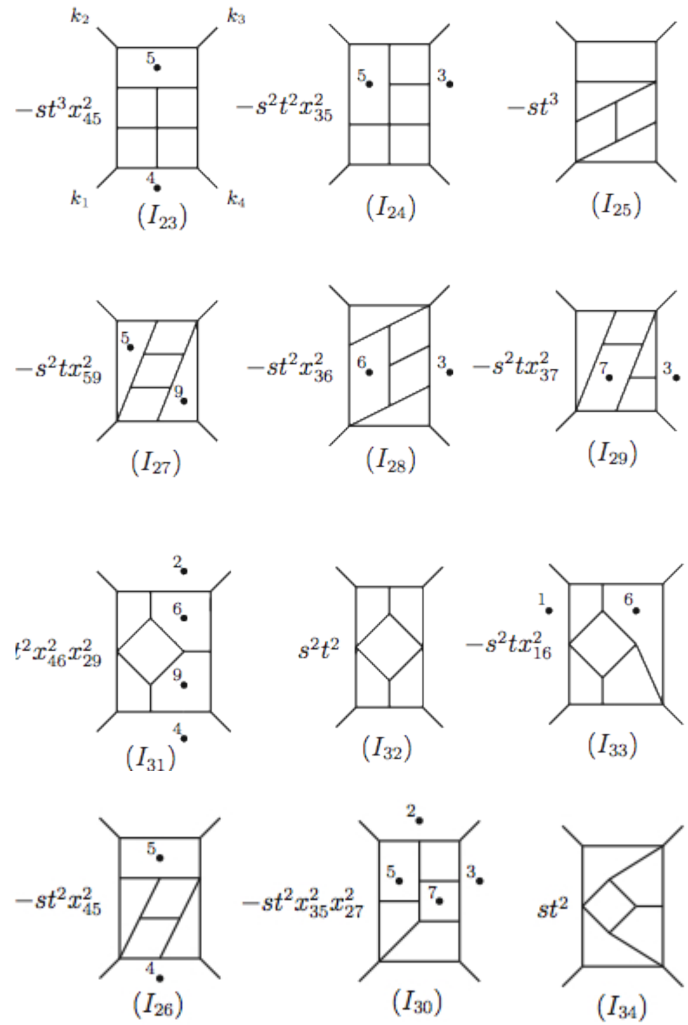
log div. integral $\sim \int d^{D\times 5}l \, \frac{\left[((l_1+l_2)^2)^2\right]^2}{(l^2)^{16}} \sim \int \frac{d^{5D}l}{l^{24}}$

Similar analysis: Björnsson, Green; Vanhove

Conclusion: when in doubt, calculate! *see Bern's talk on N=4 SG*

Planar $\mathcal{N}$=4 SYM at 5 loops





Bern, Carrasco, HJ, Kosower (2007)

34 dual-conformal integrals

Computed via maximal-cut method

Planar part well understood

How nonplanar 5-loops is calculated

Overcoming 5 loop complexity

At 5 loops factorial growth start to become an issue

- cubic diagram topologies: ~ 400-900 (not counting triangles, bubbles)
- possible numerator terms: ~10 000 (conservative)
- maximal cuts ~500 000 (conservative)

Divide and conquer approach

- Use maximal cut method in *D* dimensions
  - Exploit known power counting
- Obtain compact analytical cut expressions through
  - Two-particle cuts
  - Box cuts
  - Color-kinematics duality: non-planar cuts from planar ones
- Automate calculation

With the N=4 SYM amplitude in our hands

- analyze its structure, UV behavior
- find representation where color-kinematics duality is manifest
- get N=8 supergravity

see Dixon's talk

Maximal Cuts

Idea: reconstruct amplitude piece by piece Bern, Carrasco, HJ, Kosower (2007)

similar to one-loop technology: BCF, Forde, OPP, Badger, etc.
and multi-loop leading singularity: Cachazo, Skinner; Arkani-Hamed et al.
and current two-loop progress: Mastrolia,Ossola; Kosower, Larsen; Badger

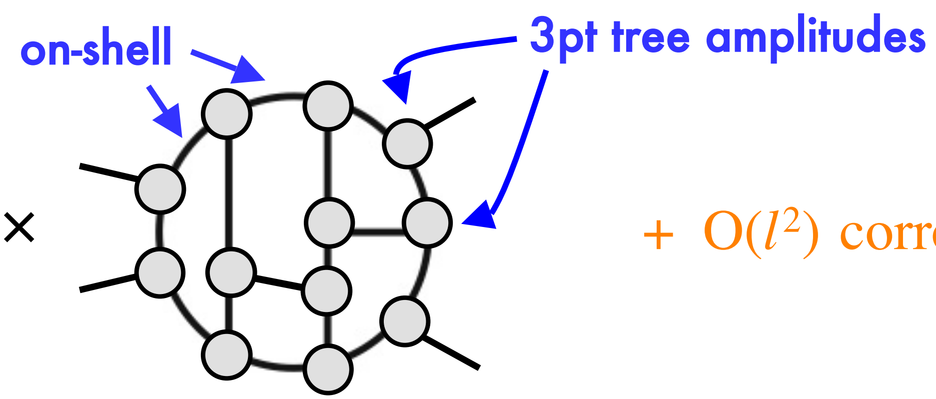
see talks by Kosower, Ossola

Used to construct 3,4-loop N=4 SYM and N=8 SG previously (08-09)

Bern,Carrasco, Dixon,HJ, Roiban

put maximum number of propagator on-shell → simplifies calculation

$$N_i = \frac{1}{st A_4^{\text{tree}}} \times$$



$+ \; O(l^2)$ corrections

Detects terms: $l_i \cdot l_j \,, \quad (l_i + l_j)^2 \,, \quad (l_i + l_j + l_k + \ldots)^2 \,, \ldots$

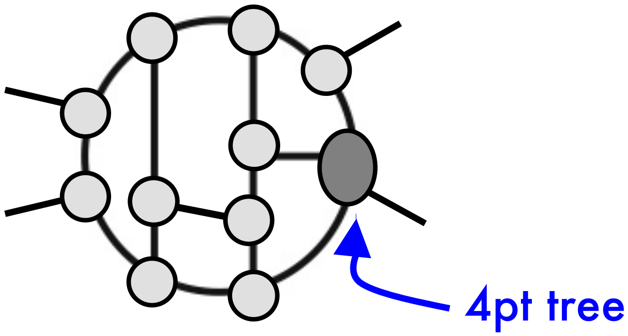
Blind to: $l_i^2 \,, \quad l_i^2\,(l_j \cdot l_k) \,, \quad (l_i + l_j)^2 - 2 l_i \cdot l_j \,, \ldots$

Near-Maximal Cuts

Bern, Carrasco, HJ and Kosower

- systematically release cut conditions → get missing contact terms

remove 1 cut line: next-to-maximal cuts

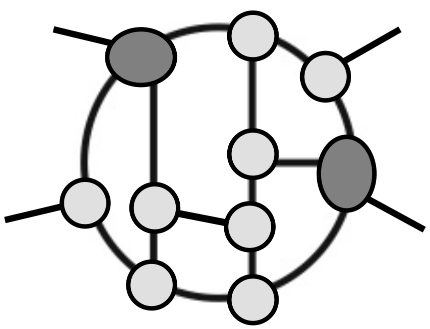


4⊗3⊗3⊗3⊗…

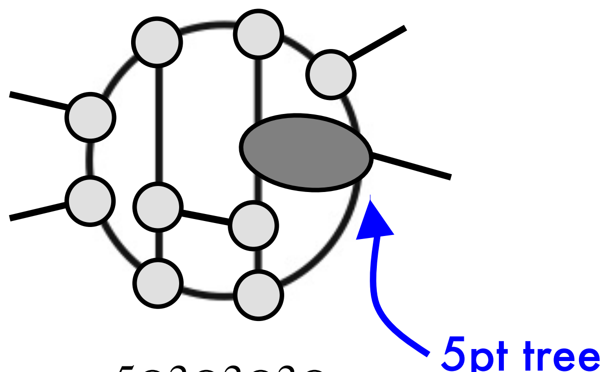
Detects: $l_i^2\,, \quad l_i^2\,(l_j \cdot l_k)\,, \ldots$

Blind to: $l_i^2 l_j^2\,, \ldots$

remove 2 cut lines: next-to-next-to-maximal cuts



4⊗4⊗3⊗3⊗…



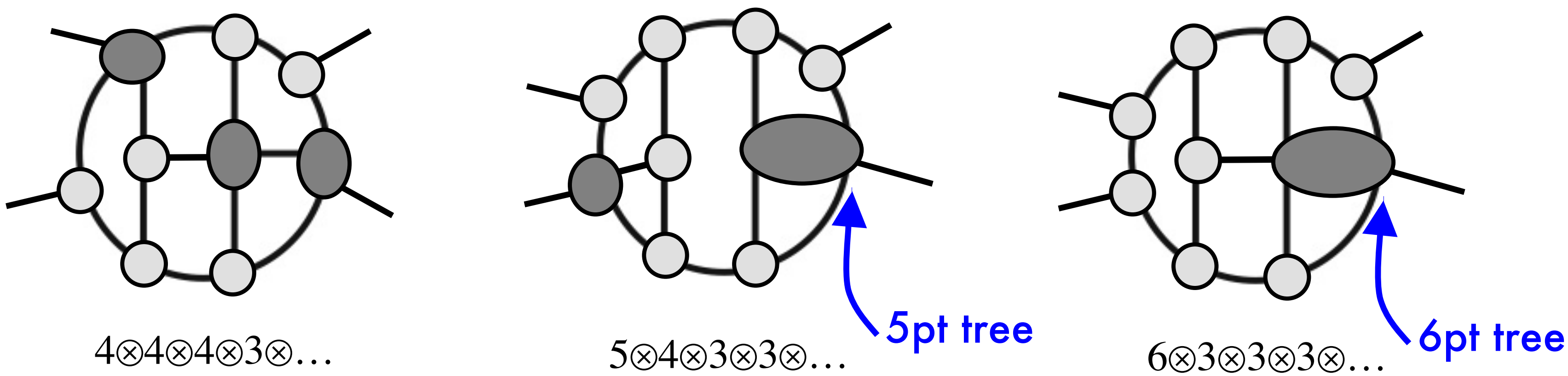
5⊗3⊗3⊗3⊗…

Detects: $l_i^2 l_j^2\,, \ldots$

Blind to: $l_i^2 l_j^2 l_k^2\,, \ldots$

Near-Maximal Cuts cont.

remove 3 cut lines: (next-to)^3-maximal cut



Detects: $l_i^2 l_j^2 l_k^2 , \ldots$ Blind to: $l_i^2 l_j^2 l_k^2 l_m^2 , \ldots$

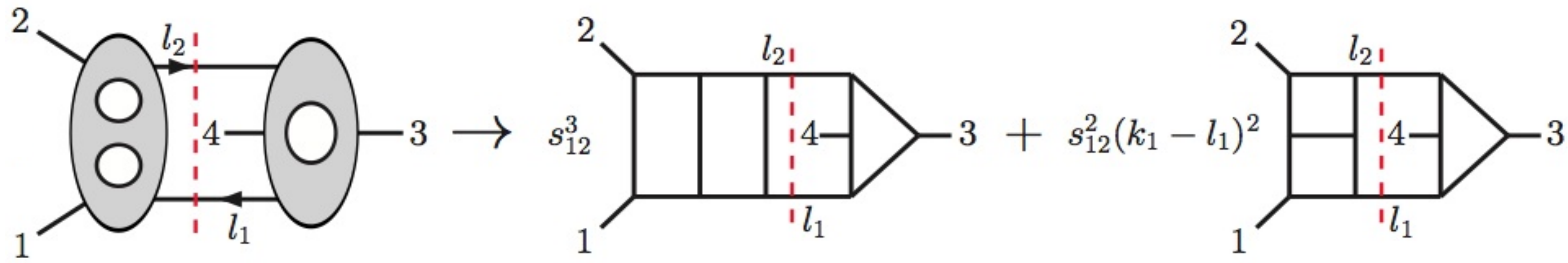
- By power counting $D_c = 4 + \dfrac{6}{L}$ numerators should take the form

$$N = st A^{\text{tree}} \left(s P_1^{(6)}(l_i) + t P_2^{(6)}(l_i) \right)$$

- All such terms are detected on the *(next-to)^3-maximal cuts*

Two-particle cut

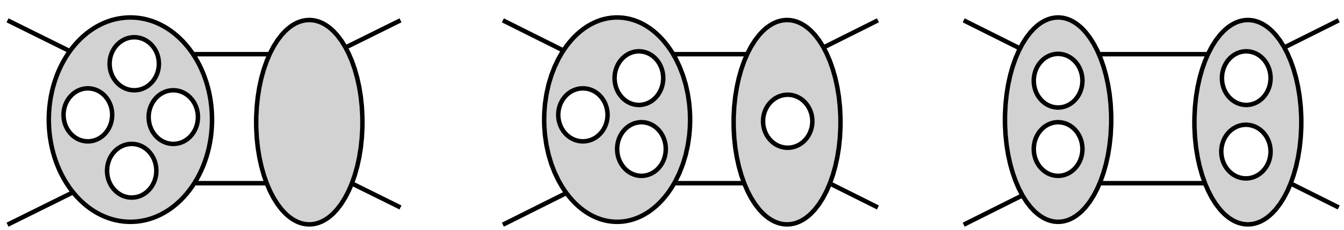
In $\mathcal{N}$=4 SYM, 2-particle cuts of the form 2→2→2 are trivial, e.g.



- Works for non-planar cuts and in D-dimensions
- Cut expressions are recycled from lower loops, thus very compact

(closely related to the rung rule in the planar case Bern, Rozowsky, Yan)

At 5 loops one obtains...

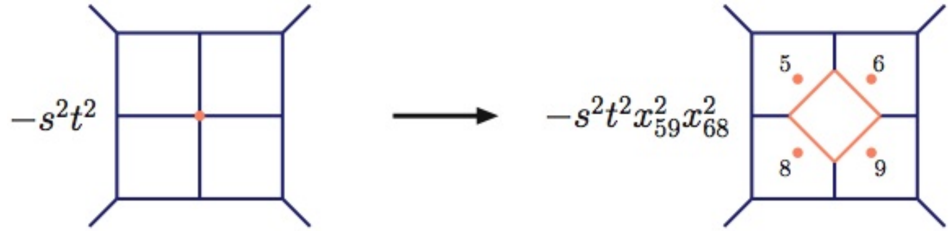


...cutting these further gives maximal and near-maximal cuts

Box cut (box substitution rule)

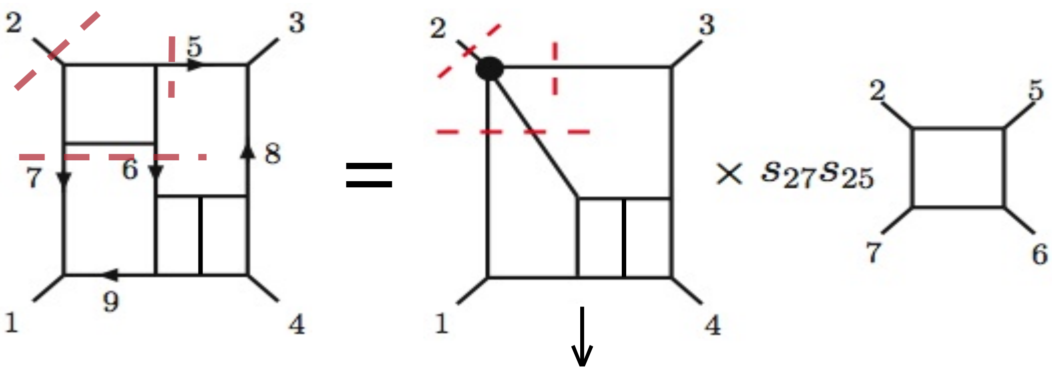
Observation in 0705.1864 [hep-th] inspired a "box-substitution rule"

Bern, Carrasco, HJ and Kosower



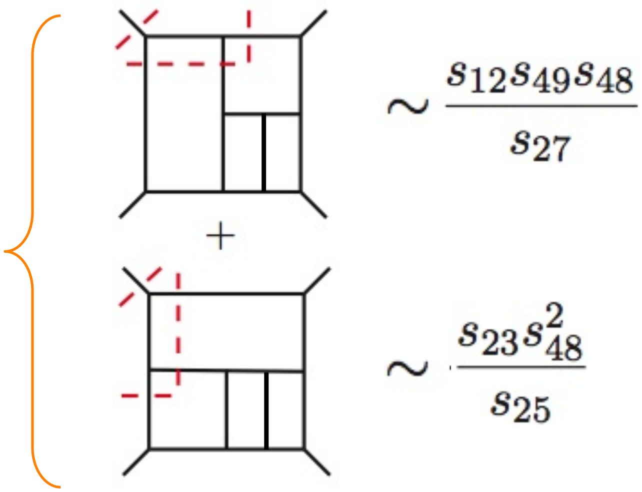
- Can be applied to any diagram containing a 4-pt "box" subdiagram

Bern,Carrasco, Dixon,HJ, Roiban



- Works in D dim, non-planar
- Recycles lower loop cuts
- Back-of-the-envelope calculations of non-trivial N's

Known 4-loop diagrams

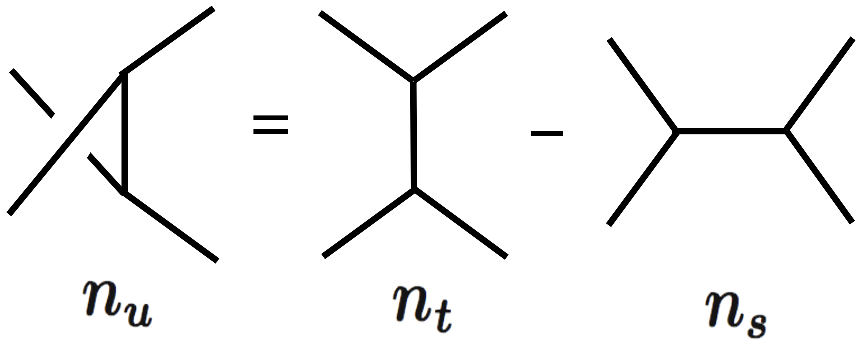


$$N = s_{25}s_{27}\left(\frac{s_{12}s_{49}s_{48}}{s_{27}} + \frac{s_{23}s_{48}^2}{s_{25}}\right)$$
$$= \left(s_{12}s_{25}s_{49}s_{48} + s_{23}s_{27}s_{48}^2\right)$$

similar rules: Cachazo, Skinner

Color-kinematics relations for cuts

We use the Jacobi-like numerator identity at the level of the cuts

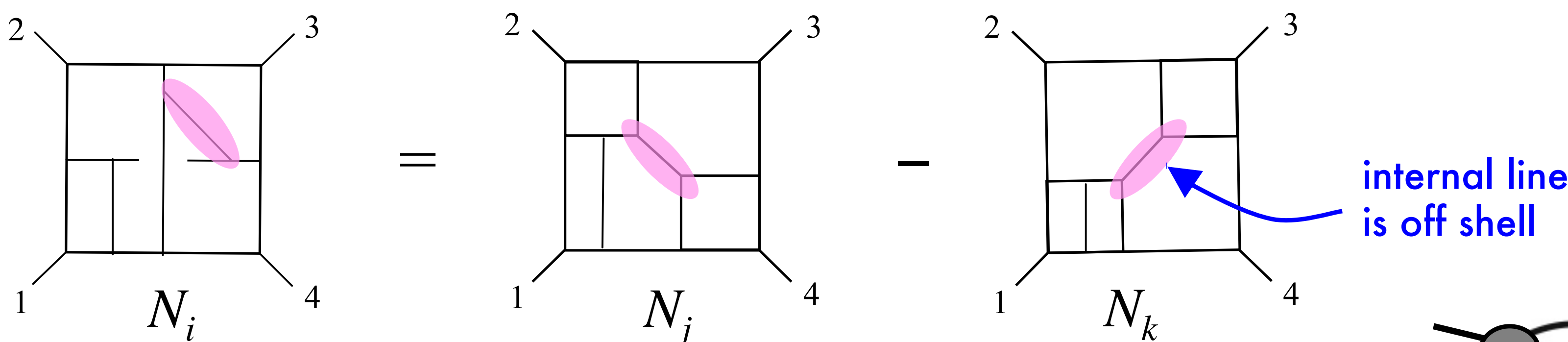


Bern, Carrasco, HJ

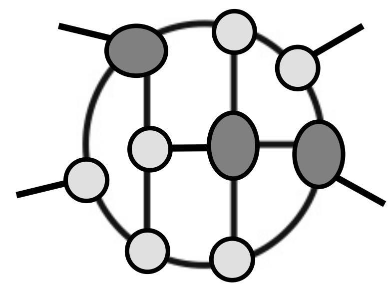
$$\mathcal{A}_4^{\text{tree}}(1,2,3,4) = g^2\Big(\frac{n_s c_s}{s} + \frac{n_t c_t}{t} + \frac{n_u c_u}{u}\Big)$$

see talks by Broedel, Dixon, O'Connell

Diagram numerators are dependent, so are the color ordered cuts



- Obtain non-planar cuts from planar ones
- Valid in *D* dimensions (for generic gauge theories)
- Input is planar ⇒ compact expressions

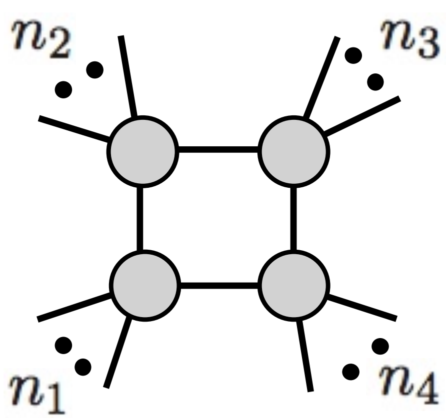


Color-kinematics cut basis

Good bookkeeping saves a lot of work:

- half a million color-ordered cuts → stored as 25000 (5%) cuts via Kleiss-Kujf and BCJ relations.

One-loop example:



Color orders: $\frac{1}{2^4}(n_1-1)!(n_2-1)!(n_3-1)!(n_4-1)!$

KK basis: $(n_1-2)!(n_2-2)!(n_3-2)!(n_4-2)!$

BCJ basis: $(n_1-3)!(n_2-3)!(n_3-3)!(n_4-3)!$

Bjerrum-Bohr, Damgaard, Sondergaard, Vanhove

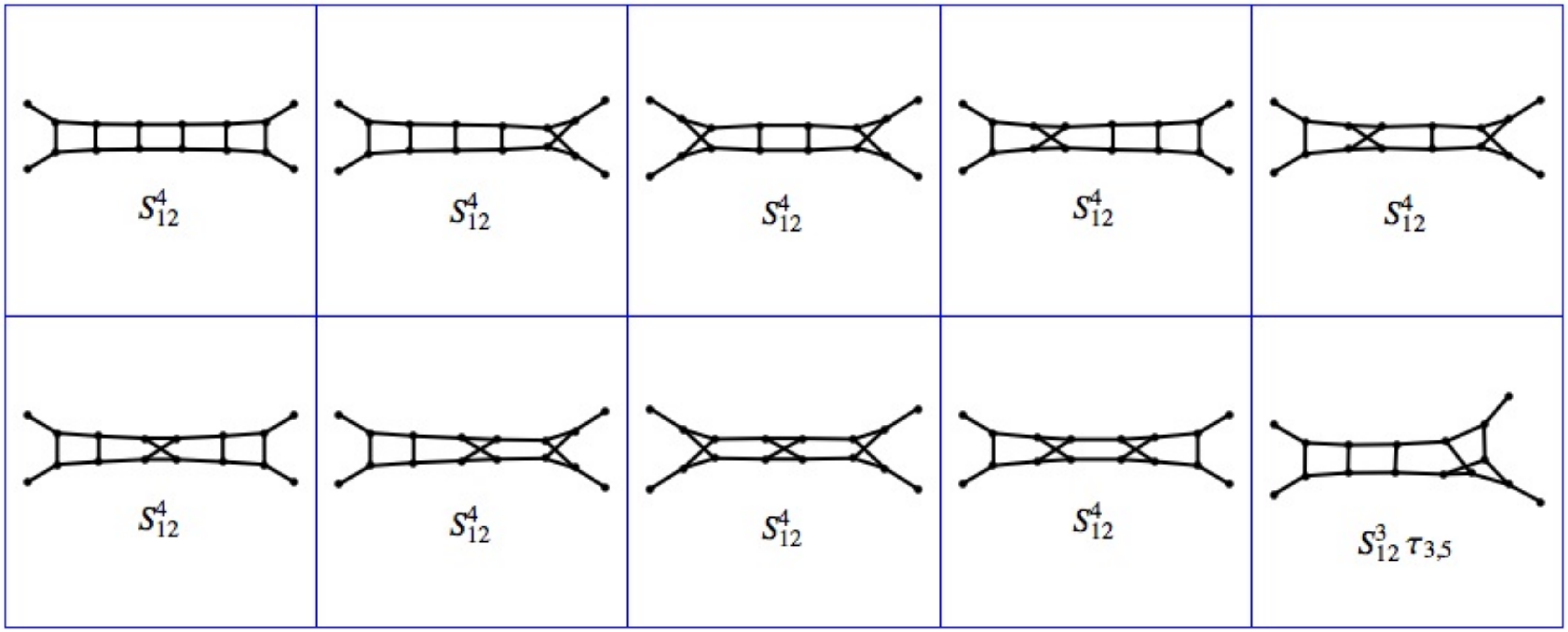
similar to: Boels, Isermann

The 5-loop amplitude

Amplitude diagrams I

Some examples of the 435 diagrams...

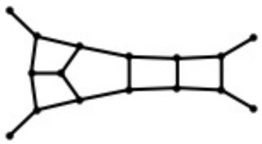
The simplest ones: ladder diagrams



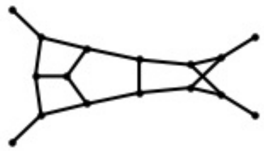
Color structures f^{abc}s are obtained from diagram topology

$$\tau_{i,j} = 2l_i \cdot l_j$$

Amplitude diagrams II



$\frac{1}{2} s_{12}^3 \left(\tau_{3,5} + \tau_{4,15}\right)$



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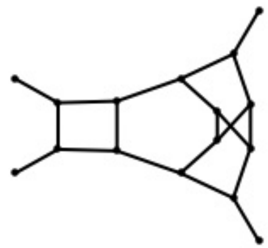
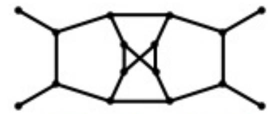
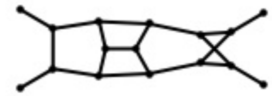


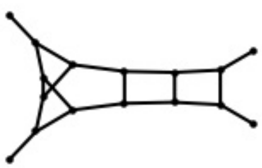
$\frac{1}{2} s_{12}^2 \left(\tau_{3,5}^2 + \tau_{4,13}^2\right)$



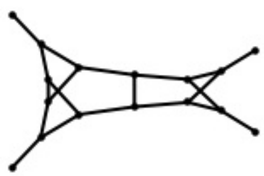
$\frac{1}{2} s_{12}^2 \left(\tau_{10,20}^2 + \tau_{13,18}^2\right)$



$\frac{1}{2} s_{12}^3 \left(\tau_{6,19} + \tau_{13,20} - 2\, s_{12}\right)$



$\frac{1}{2} s_{12}^3 \left(\tau_{3,5} + \tau_{4,15} + l_5^2 + l_{15}^2\right)$



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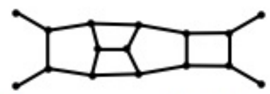
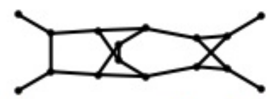
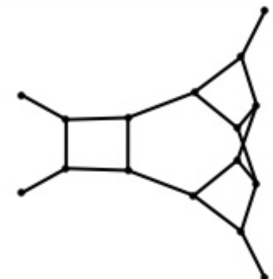


$\frac{1}{2} s_{12}^3 \left(\tau_{6,19} + \tau_{13,20} + l_6^2 + l_{13}^2 - 2\, s_{12}\right)$

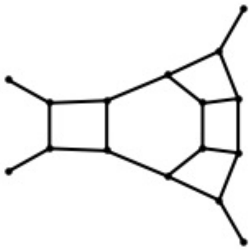


$\frac{1}{2} s_{12}^3 \left(\tau_{6,19} + \tau_{13,20} + l_{19}^2 + l_{20}^2 - 2\, s_{12}\right)$

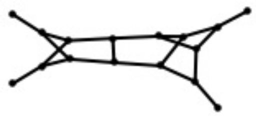


$\frac{1}{2} s_{12}^2 \left(\tau_{3,5}^2 + \tau_{4,13}^2 + 2\, l_5^2\, l_{13}^2\right)$

Amplitude diagrams III

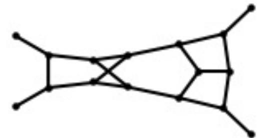


$$\frac{1}{2} S_{12}^2 \left(\tau_{3,5}^2 + \tau_{4,13}^2 + 2\, l_5^2\, l_{13}^2\right)$$

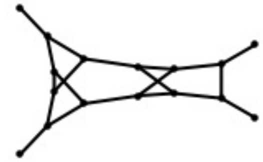


$$S_{12}^3 \left(\tau_{3,5} + l_5^2\right) - \frac{5}{2}\, l_5^2\, l_{13}^2\, l_{15}^2\, S_{12}$$

$$-\frac{1}{2} S_{12}\, \tau_{1,20} \left(2\, l_6^2 \left(\tau_{1,5} - \tau_{1,20} + S_{12}\right) + 2\, \tau_{1,6}\, \tau_{1,18}\right)$$

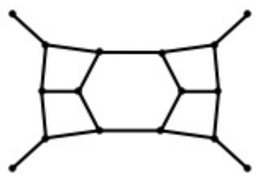


$$-\frac{1}{2} S_{12}^2 \left(S_{12}\left(\tau_{3,15} + \tau_{4,5} + 2\, S_{12}\right) + 2\left(l_8^2 + l_9^2\right)\left(l_7^2 + l_{10}^2\right)\right)$$

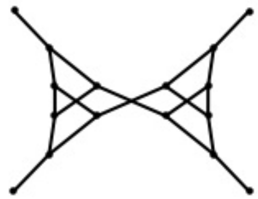


$$-\frac{1}{2} S_{12}^2 \left(S_{12}\left(\tau_{3,15} + \tau_{4,5} + 2\, S_{12}\right) + 2\left(l_8^2 + l_9^2\right)\left(l_7^2 + l_{10}^2\right)\right)$$

$$-\frac{1}{4} S_{12}^2 \left(4\, l_{13}^2 \left(-\tau_{1,5} - \tau_{2,5} - 2\, \tau_{4,5} + l_5^2 + S_{12}\right) - 4\, \tau_{4,13}^2\right)$$

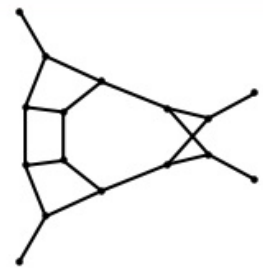


$$\frac{1}{2} S_{12}^2 \left(S_{12}\left(\tau_{1,17} + \tau_{2,5} - \tau_{3,5} - \tau_{4,17}\right) - \tau_{2,5}\, \tau_{3,5} - \tau_{1,17}\, \tau_{4,17} + 2\, S_{12}^2\right)$$



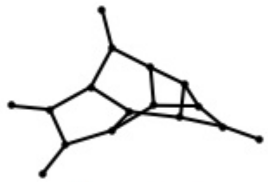
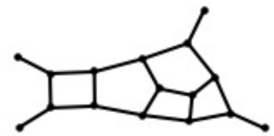
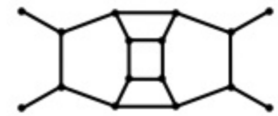
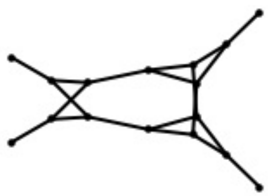
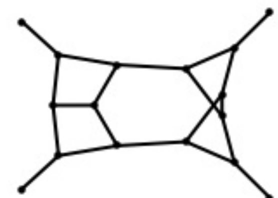
$$\frac{1}{2} S_{12}^2 \left(S_{12}\left(\tau_{1,17} + \tau_{2,5} - \tau_{3,5} - \tau_{4,17}\right) - \tau_{2,5}\, \tau_{3,5} - \tau_{1,17}\, \tau_{4,17} + 2\, S_{12}^2\right)$$

$$\frac{1}{64} S_{12} \left(64\, S_{12}\left(\tau_{1,6} + \tau_{2,6} + l_{19}^2\right)\left(l_5^2 - \tau_{3,5}\right) - 8\, l_7^2 \left(l_5^2 - 20\, l_{12}^2\right) l_{13}^2\right)$$

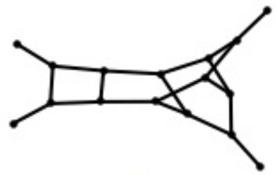
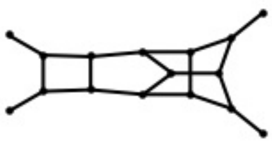
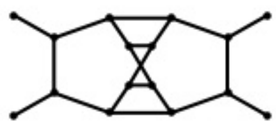
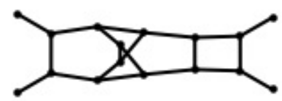


$$\frac{1}{2} S_{12}^2 \left(l_5^2 \left(\tau_{1,13} + \tau_{2,13} + 2\, l_{13}^2\right) + l_{13}^2 \left(\tau_{1,5} + \tau_{2,5}\right) + \tau_{3,5}^2 + \tau_{4,13}^2\right)$$

Amplitude diagrams IV

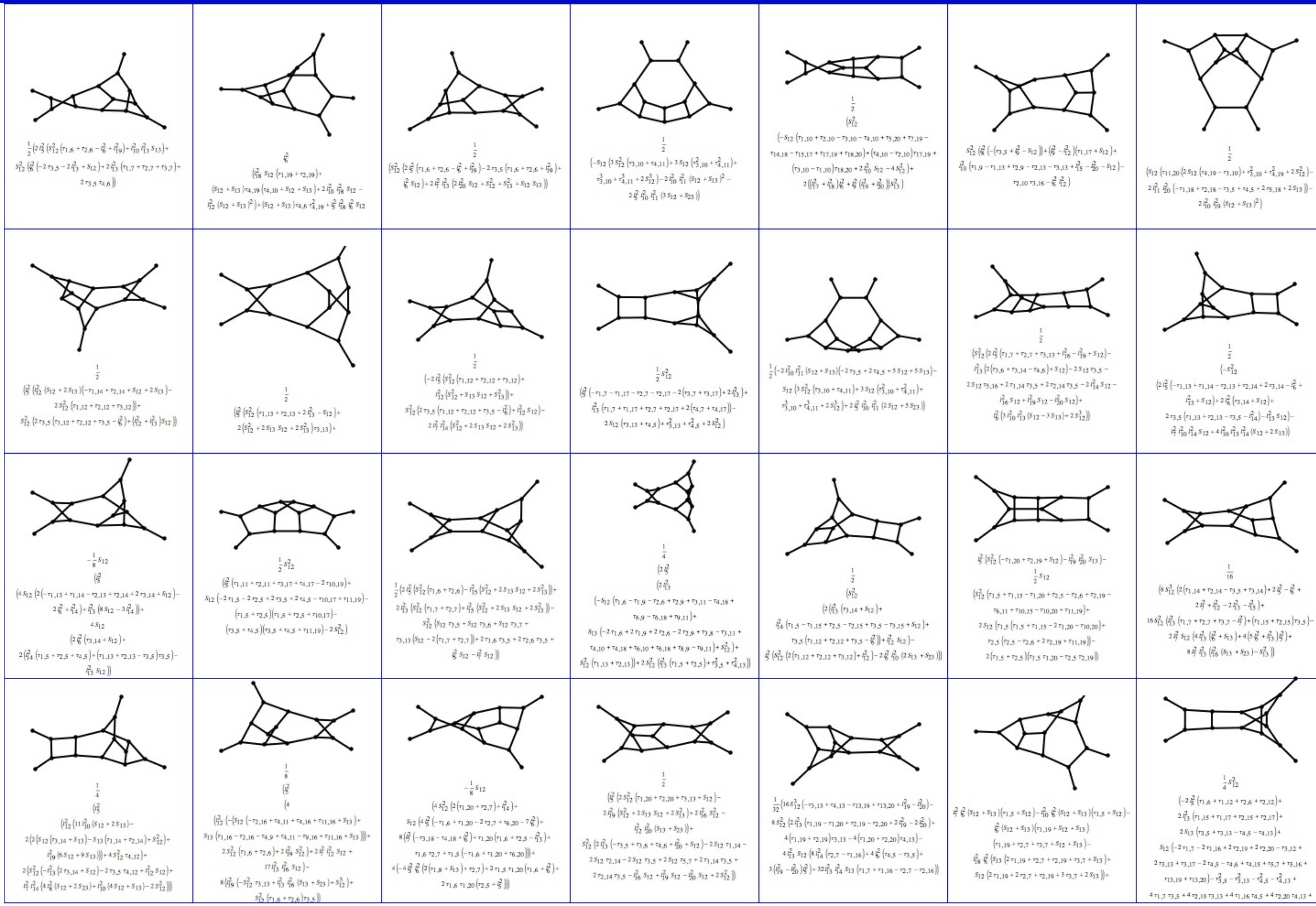
 $-\frac{1}{2}S_{12}^2\left(l_{13}^2\left(\tau_{3,8}+\tau_{4,8}-3S_{12}\right)+S_{12}\left(2\tau_{13,19}-2l_{19}^2+l_{20}^2\right)+l_6^2\left(2l_{19}^2-2l_{20}^2+S_{12}\right)\right)$	 $\frac{1}{4}S_{12}\left(-4l_6^2\left(l_5^2\left(-\tau_{2,7}+S_{12}+S_{13}\right)-\left(\tau_{3,18}+\tau_{4,18}\right)\left(S_{12}-\tau_{2,20}\right)\right)-4\tau_{1,6}\,\tau_{1,18}\,\tau_{1,20}\right)$	 $\frac{1}{8}\left(8S_{12}^2\left(l_5^2\left(-\left(\tau_{1,12}+\tau_{2,12}+\tau_{3,12}\right)\right)-\tau_{3,5}\,\tau_{4,12}+l_{12}^2\,S_{12}\right)+8l_5^2\,l_{10}^2\,l_{14}^2\left(S_{12}+S_{13}\right)\right)$	 $\frac{1}{2}S_{12}^2\left(l_6^2\left(-2\tau_{3,5}-2l_{13}^2+S_{12}\right)+2l_5^2\left(\tau_{1,6}+\tau_{2,6}+l_{19}^2\right)+2\tau_{3,5}\,\tau_{4,6}+l_{12}^2\,S_{12}\right)$	 $\frac{1}{2}\left(S_{12}^2\left(2S_{12}\left(\tau_{10,13}+\tau_{18,20}\right)+\tau_{10,13}^2+\tau_{18,20}^2+2S_{12}^2\right)+4\left(l_5^2+l_6^2\right)\left(l_{13}^2\,l_{18}^2+l_{10}^2\,l_{20}^2\right)S_{23}\right)$
 $\frac{1}{2}S_{12}\left(S_{12}\left(S_{12}\left(2\tau_{6,19}+l_{20}^2-2S_{12}\right)+2l_{13}^2\left(S_{12}-l_{20}^2\right)\right)-l_6^2\left(2\left(l_{19}^2-l_{20}^2\right)S_{12}+l_7^2\,l_{19}^2\right)\right)$	 $\frac{1}{16}S_{12}\left(8S_{12}\left(-2S_{12}\left(\tau_{3,13}+\tau_{4,5}\right)+\tau_{3,13}^2+\tau_{4,5}^2+2S_{12}^2\right)+l_5^2\,l_{13}^2\left(19\left(l_7^2+l_{17}^2\right)+16S_{12}\right)\right)$	 $-\frac{1}{2}S_{12}\left(3l_7^2\,l_{10}^2\left(\tau_{1,15}+\tau_{2,15}+\tau_{4,15}+l_9^2-l_{14}^2-l_{17}^2+l_{20}^2-S_{12}\right)+2S_{12}^2\left(\tau_{3,15}+S_{12}\right)\right)$	 $\frac{1}{2}S_{12}^2\left(2\left(\tau_{1,6}+\tau_{2,6}+l_{19}^2\right)\left(l_5^2-\tau_{3,5}\right)+l_6^2\left(S_{12}-2l_{13}^2\right)+l_{12}^2\,S_{12}\right)$	 $\frac{1}{2}\left(l_5^2\,l_{17}^2\,S_{13}\left(\tau_{2,13}-\tau_{1,12}\right)+S_{12}^3\left(\tau_{1,17}+\tau_{2,5}-\tau_{3,5}-\tau_{4,17}\right)-S_{12}^2\left(\tau_{2,5}\,\tau_{3,5}+\tau_{1,17}\,\tau_{4,17}\right)+2S_{12}^4\right)$

Amplitude diagrams V

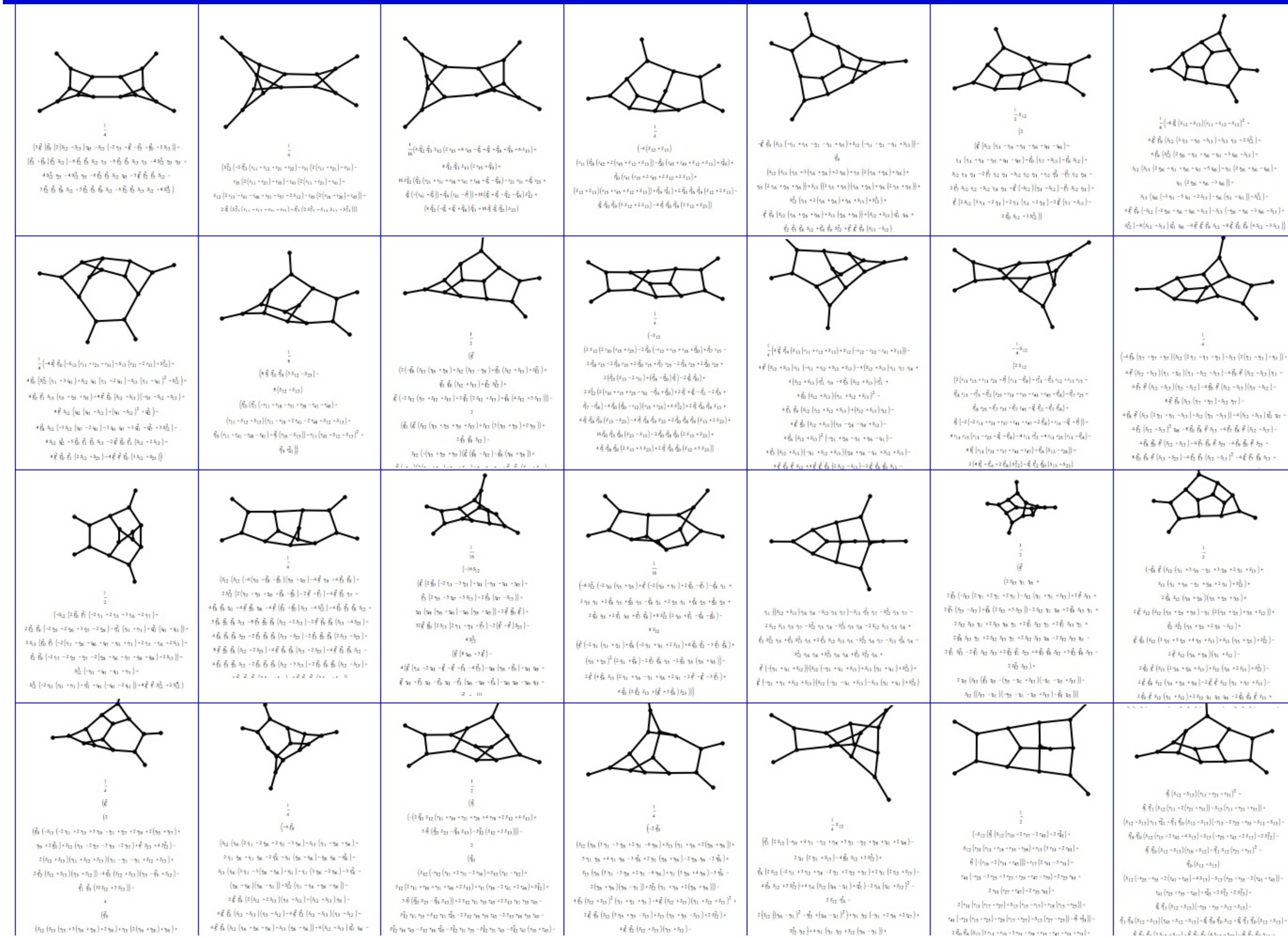
 $\frac{1}{4}\left(4 S_{12}^2 \left(\ell_{13}^2 \left(\tau_{1,5}+\tau_{2,5}+2\tau_{4,5}-S_{12}\right)+\tau_{4,13}^2\right)+ \ell_5^2\, \ell_{13}^2 \left(3\, \ell_{16}^2\, S_{12}-4 S_{13}\left(S_{12}+S_{13}\right)\right)\right)$	 $-\frac{1}{4} S_{12}^2 \left(2 S_{12}\left(2\tau_{1,7}+2\tau_{2,6}+\tau_{3,5}+\tau_{3,6}+\tau_{3,7}+ \tau_{3,13}+\ell_6^2-\ell_7^2\right)+ 4\left(\tau_{1,6}-\tau_{1,7}\right)\tau_{3,5}+4\left(\tau_{2,6}-\tau_{2,7}\right)\tau_{3,13}\right)$	 $\frac{1}{2}\left(2 S_{12}^2\left(\tau_{3,5}\left(\tau_{4,6}-\ell_6^2\right)+ \ell_{12}^2\, S_{12}+\left(\ell_6^2+\ell_8^2-\ell_{20}^2\right)\ell_5^2\right)+ \ell_7^2\, \ell_{13}^2\left(2\,\ell_9^2\left(S_{12}-S_{13}\right)+\ell_{10}^2\, S_{12}\right)\right)$	 $\ell_5^2\left(S_{12}^2\left(\tau_{1,6}+\tau_{2,6}+\ell_{19}^2\right)- 2\,\ell_6^2\,\ell_9^2\left(S_{12}+2 S_{13}\right)\right)+ S_{12}^2\,\tau_{3,5}\left(\tau_{4,6}-\ell_6^2\right)+\ell_9^2\,\ell_{13}^2\,\ell_{16}^2\left(2 S_{12}+S_{13}\right)$	 $S_{12}^2\left(\ell_5^2\left(-\left(\tau_{1,12}+\tau_{2,12}+\tau_{3,12}+\ell_{12}^2\right)\right)+\tau_{3,5} \left(\tau_{1,12}+\tau_{2,12}+\tau_{3,5}-\ell_6^2\right)+\left(\ell_{12}^2+\ell_{13}^2\right)S_{12}\right)$
 $\frac{1}{2} S_{12}^2\left(S_{12}\left(\tau_{1,6}-\tau_{1,12}+\tau_{2,6}-\tau_{2,12}+\tau_{3,5}+ \tau_{3,12}-\tau_{4,6}-\tau_{4,13}+\tau_{5,17}+\tau_{13,17}\right)+ 2\left(\tau_{1,12}-\tau_{1,6}\right)\tau_{3,5}+2\left(\tau_{2,6}-\tau_{2,12}\right)\tau_{4,13}- 2 S_{12}^2\right)$	 $\frac{1}{2} \left(S_{12}^2\left(2 S_{12}\left(\tau_{10,13}+\tau_{18,20}\right)+\tau_{10,13}^2+\tau_{18,20}^2+ 2 S_{12}^2\right)-\left(\ell_{13}^2\,\ell_{18}^2+\ell_{10}^2\,\ell_{20}^2\right)\ell_5^2\left(4 S_{12}+5 S_{23}\right)- \ell_6^2\left(\ell_{13}^2\,\ell_{18}^2+\ell_{10}^2\,\ell_{20}^2\right)\left(4 S_{12}+5 S_{23}\right)\right)$	 $\frac{1}{2} \left(S_{12}^2\left(S_{12}\left(\tau_{6,19}+\tau_{13,20}+\ell_{19}^2+\ell_{20}^2-2 S_{12}\right)+ \ell_{13}^2\left(S_{12}-2\,\ell_{20}^2\right)\right)+ \ell_6^2\left(4\,\ell_{10}^2\,\ell_{20}^2\left(S_{12}+2 S_{13}\right)+S_{12}^2\left(S_{12}-2\,\ell_{19}^2\right)\right)+ 4\,\ell_9^2\,\ell_{13}^2\,\ell_{19}^2\left(S_{12}+2 S_{13}\right)\right)$	 $\frac{1}{8}\left(2 S_{12}^2 \left(2 S_{12}\left(-2\left(\tau_{3,5}-\tau_{3,14}+\tau_{5,14}\right)+2\,\ell_5^2-\ell_6^2+ \ell_{12}^2+\ell_{15}^2\right)+4\,\ell_{15}^2\left(\tau_{3,13}+\ell_{13}^2\right)-4\,\tau_{3,5}\,\tau_{4,15}\right)+ 8\,\ell_5^2\,\ell_6^2\,\ell_{10}^2\, S_{13}+40\,\ell_5^2\,\ell_6^2\,\ell_{18}^2\left(S_{12}+2 S_{13}\right)\right)$	 $\frac{1}{2} S_{12}\left(S_{12} \left(S_{12}\left(-\tau_{1,5}-\tau_{2,5}+\tau_{3,5}+\tau_{4,5}+\tau_{10,20}- \tau_{18,19}\right)+\left(\tau_{1,5}+\tau_{2,5}\right)\tau_{10,20}+ \left(\tau_{3,5}+\tau_{4,5}\right)\tau_{18,19}-2\,\ell_{18}^2\,\ell_{20}^2-2 S_{12}^2\right)- 2\,\ell_5^2\left(S_{12}\,\tau_{10,19}+\ell_{18}^2\,\ell_{20}^2\right)\right)$

So far: the 50 simplest diagrams. Here are some more...

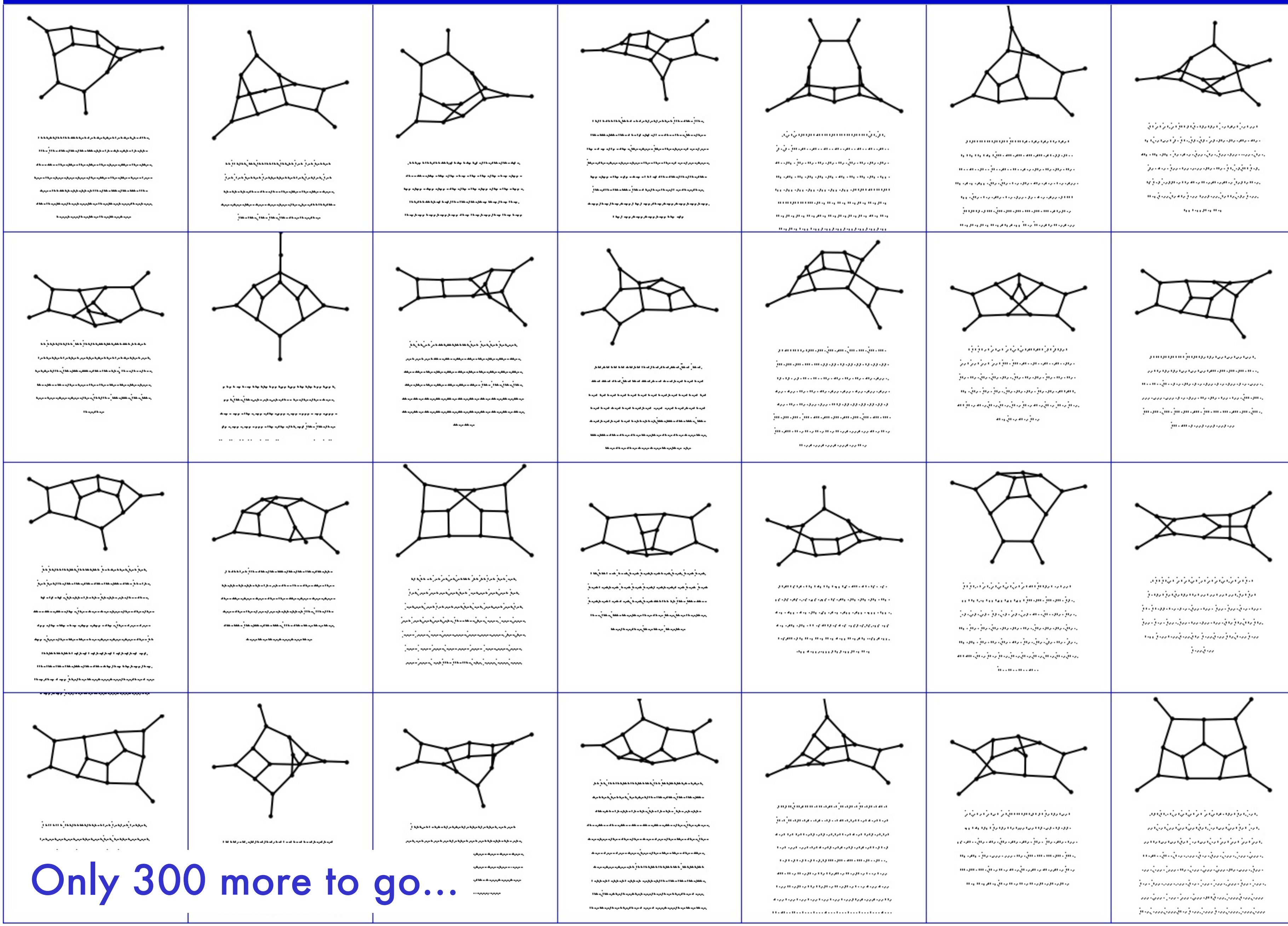
Amplitude diagrams VI



Amplitude diagrams VII

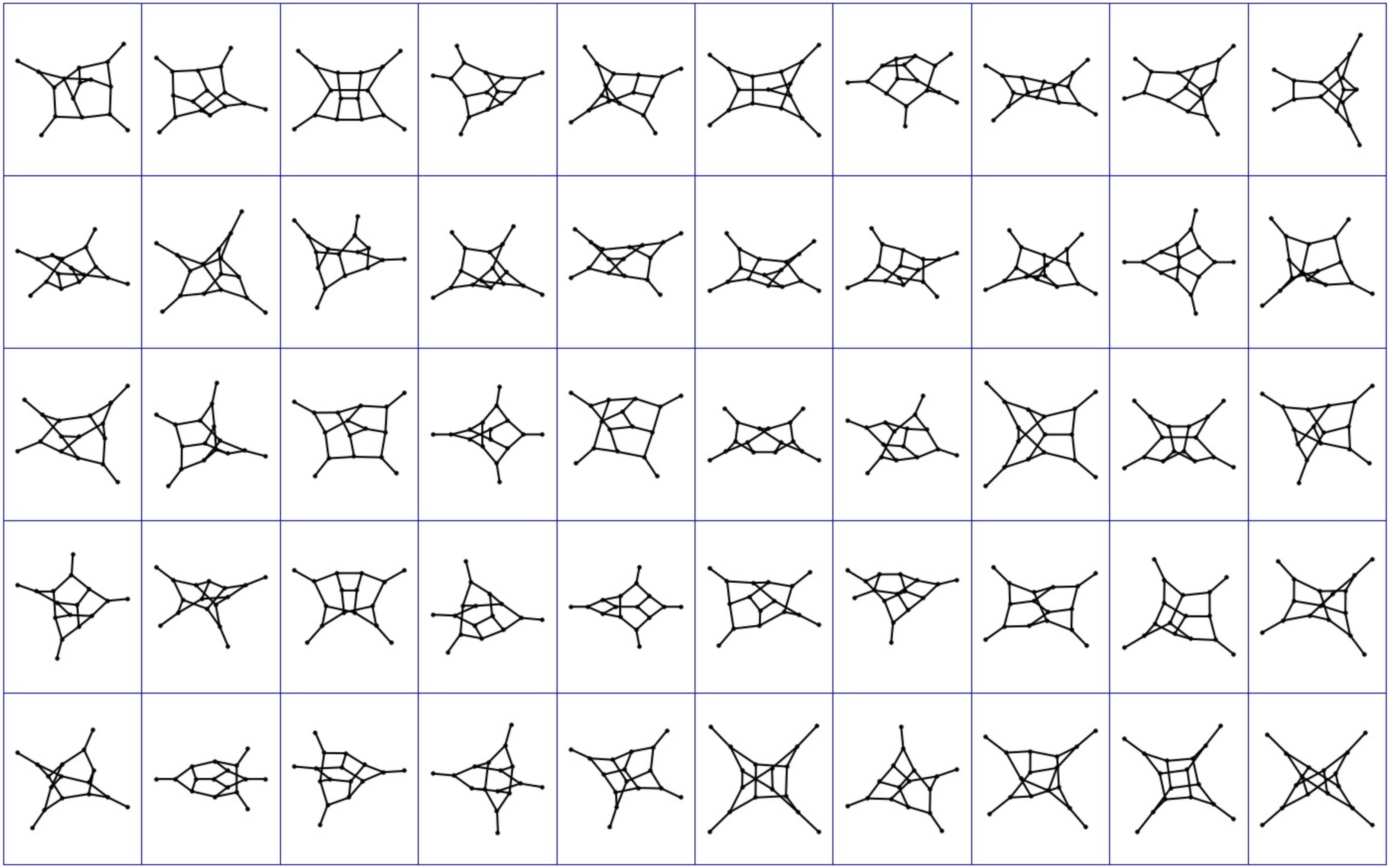


Amplitude diagrams VIII



Only 300 more to go...

Amplitude diagrams IX



....here are 50 of the most complicated remaining diagrams

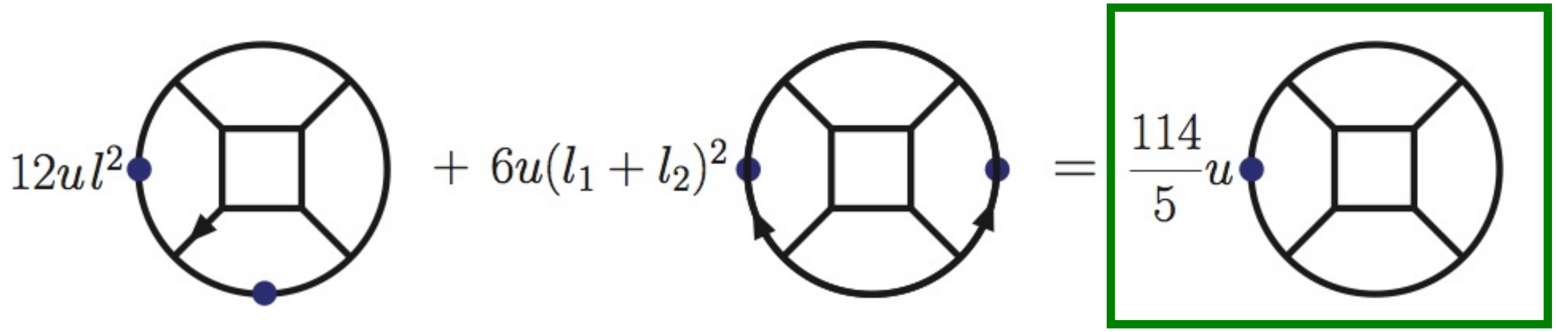
UV properties

Planar $\mathcal{N}$=4 SYM 5-loop UV divergence

Planar UV divergence in D=26/5,

Bern, Carrasco, Dixon, HJ, Roiban (2009)

divergence $\propto N_c^5 \,\mathrm{Tr}[T^{a_1}T^{a_2}T^{a_3}T^{a_4}]$



With IBP relations expression collapses to a single scalar integral

One integral: by positivity non-zero, D_c = 26/5 as expected

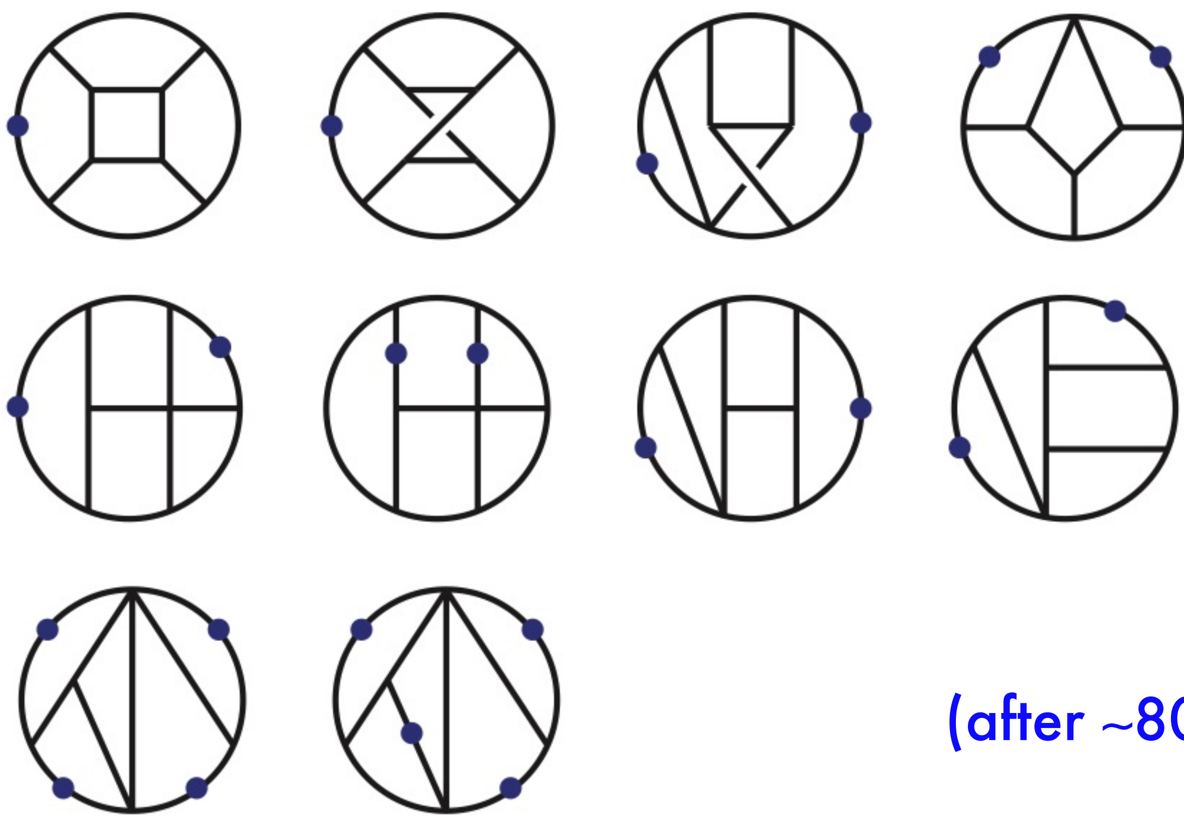
Non-planar $\mathcal{N}$=4 SYM 5-loop UV divergence I

Non-Planar UV divergence in D=26/5:

Preliminary!

divergence $\propto N_c^3 \, \mathrm{Tr}[T^{a_1} T^{a_2} T^{a_3} T^{a_4}]$

expressible in terms of 10 vacuum integrals (not independent, ongoing work)



(after ~8000 IBP relations)

Non-planar $\mathcal{N}$=4 SYM 5-loop UV divergence II

Non-Planar UV divergence in D=26/5:

Components: $N_c^1\,\mathrm{Tr}[T^{a_1}T^{a_2}T^{a_3}T^{a_4}]$

$N_c^4\,\mathrm{Tr}[T^{a_1}T^{a_2}]\,\mathrm{Tr}[T^{a_3}T^{a_4}]$

$N_c^2\,\mathrm{Tr}[T^{a_1}T^{a_2}]\,\mathrm{Tr}[T^{a_3}T^{a_4}]$

$N_c^0\,\mathrm{Tr}[T^{a_1}T^{a_2}]\,\mathrm{Tr}[T^{a_3}T^{a_4}]$ Vanish!

Double traces and single-trace NNLC finite in D=26/5, only single-trace LC and NLC are divergent

similar to 4 loops: *see Dixon's talk*

Summary / Outlook

- The 5-loop divergence is the most important tangible calculation on the question of UV finiteness of N=8 SG.

- This calculation will provide critical information on the amplitude structure: 24/5 vs. 26/5 will correspond to

$$A_4^{(L=5)} \sim \partial^8 R^4 \times (\text{intgrl.}) \qquad \to L=7 \text{ divergent}$$
$$A_4^{(L=5)} \sim \partial^{10} R^4 \times (\text{intgrl.}) \qquad \to L=7 \text{ finite}$$

- As a first step, we have obtained the 5-loop N=4 SYM amplitude and UV divergences

- Next step is to use the duality between color and kinematics to obtain the gravity amplitude. This task is greatly simplified with a representation of the N=4 SYM amplitude in our hands.

- Stay tuned...