Heterotic Geometries

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DESY Hamburg

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Based on:

- M. Fischer, M. Ratz, J. Torrado and P. V.: 1209.3906
- M.-C. Chen, M. Ratz, C. Staudt and P. V.: 1206.5375, NPB 866 (2013) 157-176
- H. P. Nilles, S. Ramos-Sánchez, P. V., A. Wingerter: 1110.5229, CPC 183 (2012) 1363-1380

Ingredients for the Standard Model and Beyond

SM and Beyond

- gauge symmetry
- 3 generations
- Higgs
- Flavour
  $\Rightarrow$ symmetry?
- GUT?
  SU(5) or SO(10)
- SUSY
  $\Rightarrow$ GUT!
- R symmetry
- extra dimensions
- gravity
- strings

Three gauge interactions:

- electromagnetism:
  photon, α_1, $\mathrm{U}(1)_Y$
- weak force:
  $W^{\pm}$ and Z, α_2, $\mathrm{SU}(2)_L$
- strong force:
  8 gluons, α_3, $\mathrm{SU}(3)_C$

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Three generations of

quarks:

- $(\mathbf{3}, \mathbf{2})_{1/6} \leftrightarrow q$
- $(\overline{\mathbf{3}}, \mathbf{1})_{-2/3} \leftrightarrow \bar{u}$
- $(\overline{\mathbf{3}}, \mathbf{1})_{1/3} \leftrightarrow \bar{d}$

leptons:

- $(\mathbf{1}, \mathbf{2})_{-1/2} \leftrightarrow \ell$
- $(\mathbf{1}, \mathbf{1})_{1} \leftrightarrow \bar{e}$
- $(\mathbf{1}, \mathbf{1})_{0} \leftrightarrow \bar{\nu}$

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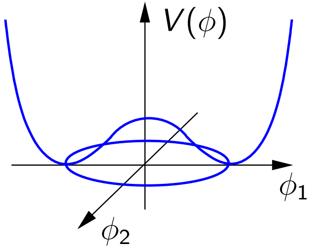
Electroweak symmetry breaking:



- Higgs boson H: $(\mathbf{1},\mathbf{2})_{-1/2}$
- $\mathrm{SU}(2)\times\mathrm{U}(1)_Y \rightarrow \mathrm{U}(1)_{\mathrm{em}}$
  $W^\pm$, Z massive
- Yukawa interactions:
  $$\mathcal{L} \supset \ell\, H\, \bar{e} + \ldots$$
  quarks and leptons massive

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Flavour:

- mass eigenstates
  $\neq$ flavour eigenstates
- CKM matrix, e.g. Cabibbo angle:

$$\begin{pmatrix} d \\ s \end{pmatrix} = \begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix} \begin{pmatrix} d_m \\ s_m \end{pmatrix}$$

  with $\theta_c \approx 13.04^\circ$

- origin?

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Flavour symmetry:

- discrete (finite) symmetry
- quarks and letons in (non-trivial) representations
- e.g. S_3 with 3 representations
  - doublet
  - (non-trivial) singlet
  - (trivial) singlet
- choose:
  1st and 2nd generation S_3 doublet
  3rd generation S_3 singlet
  Higgs pair S_3 doublet
- breaking of discrete symmetry (most) important

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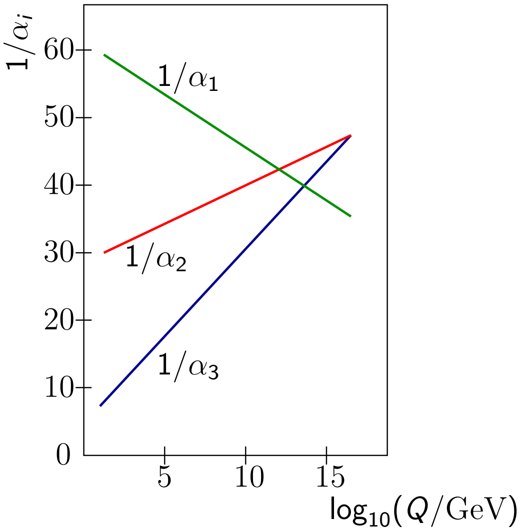

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SU(5) GUT:

- $\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)_Y \subset \mathrm{SU}(5)$
- one generation of matter:

$$\mathbf{10}: \; q, \bar{u}, \bar{e}$$
$$\bar{\mathbf{5}}: \; \bar{d}, \ell$$

SO(10) GUT:

- $\mathrm{SU}(5) \subset \mathrm{SO}(10)$
- one generation of matter:

$$\mathbf{16} \rightarrow \mathbf{10} \oplus \bar{\mathbf{5}} \oplus \bar{\nu}$$

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Higgs and GUTs:

- Higgs boson $(\mathbf{1},\mathbf{2})_{-1/2}$ into rep. of GUT group
- e.g. for SU(5) smallest rep. $\overline{\mathbf{5}}$
- but

$$\overline{\mathbf{5}} \rightarrow (\mathbf{1},\overline{\mathbf{3}})_{1/3} \oplus (\mathbf{1},\mathbf{2})_{-1/2}$$

- extra-triplets $(\mathbf{1},\overline{\mathbf{3}})_{1/3}$ mediate fast proton decay $\Rightarrow$ must be heavy

doublet-triplet splitting problem

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Supersymmetry:

- quantum corrections to Higgs mass

  $$\delta m^2 \sim \Lambda^2$$

- SUSY: fermions $\Leftrightarrow$ bosons
- stabilizes electroweak scale against GUT/Planck scale
- dark matter candidate
- additional Higgs $\bar{H}$: $(\mathbf{1}, \mathbf{2})_{1/2}$
- LHC?

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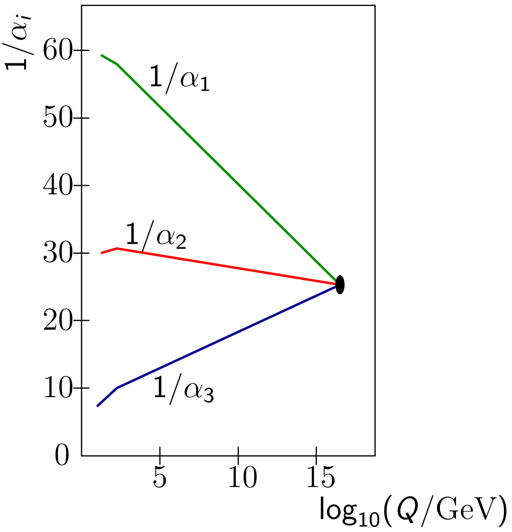

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Problems of SUSY:

- **μ-problem:**

$$\mathcal{W} \supset \mu \bar{H} H$$

  $\mu \sim m_{3/2}$ needed, but
  $\mu \sim M_{\mathrm{GUT}}$ or M_P expected

- **rapid proton decay:**
  operators of dim. 4 and 5, e.g.:

$$\bar{u}\,\bar{d}\,\bar{d} \text{ and } q\,q\,q\,\ell$$

  must be absent/strongly suppressed

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R symmetry:

- does not commute with SUSY

$$q_\theta \neq 0 \text{ and } q_{\mathcal{W}} = 2q_\theta$$

$\Rightarrow$ chiral superfield Φ

$$\Phi = \varphi + \sqrt{2}\theta\Psi + \theta\theta F$$

has R charges

Φ	q_Φ
φ	$q_\varphi = q_\Phi$
Ψ	$q_\Psi = q_\Phi - q_\theta$
F	$q_F = q_\Phi - 2q_\theta$

- consider discrete symmetry $\mathbb{Z}_M^R$
- for $q_\theta = 0$ ordinary $\mathbb{Z}_M$
- discrete anomaly?

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Anomaly of $\mathbb{Z}_{\mathsf{M}}^{\mathsf{R}}$:

- action invariant, but path integral measure maybe not
- gauge group G with rep. $r^{(f)}$

$$A_{G-G-\mathbb{Z}_M^R} = \sum_{r^{(f)}} \ell\left(r^{(f)}\right)\left(q^{(f)} - q_\theta\right) + \ell\,(\mathrm{adj})\, q_\theta$$

- unification + anomaly cancellation

$$A_{G-G-\mathbb{Z}_M^R} = \rho \bmod \eta$$

$$\text{where } \eta = \begin{cases} M & \text{for } M \text{ odd,} \\ M/2 & \text{for } M \text{ even.} \end{cases}$$

for $\rho \neq 0$: discrete Green-Schwarz

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Anomaly of $\mathbb{Z}_{\mathsf{M}}^{\mathsf{R}}$:

- assume SU(5) universal charges for matter $q_{\mathbf{10}_i}$ and $q_{\bar{\mathbf{5}}_i}$, $i = 1, 2, 3$
- $A_{\mathrm{SU}(3)-\mathrm{SU}(3)-\mathbb{Z}_M^R} - A_{\mathrm{SU}(2)-\mathrm{SU}(2)-\mathbb{Z}_M^R}$
  $= 0 \bmod \rho$
  $\Rightarrow \; q_H + q_{\bar{H}} = 4q_\theta \bmod 2\eta$
- μ-term allowed if
  $$q_H + q_{\bar{H}} = 2q_\theta \bmod M$$
- only R symmetry can forbid the μ-term
- and forbids proton decay operators

Lee, Raby, Ratz, Ross, Schieren, Schmidt-Hoberg and P. V. 2010, 2011

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$\mathbb{Z}_\mathbf{M}^\mathbf{R}$ and GUTs:

- assume $\mathrm{SU}(5) \times \mathbb{Z}_M^R$
- break SU(5) but keep R symmetry

$$\mathbf{24} = \underbrace{(\mathbf{8},\mathbf{1})_0}_{S_0} \oplus \underbrace{(\mathbf{1},\mathbf{3})_0}_{T_0} \oplus \underbrace{(\mathbf{1},\mathbf{1})_0}_{\langle H_{24}\rangle \neq 0}$$

$$\oplus \underbrace{(\mathbf{3},\mathbf{2})_{-5/6}}_{X_0,\text{eaten}} \oplus \underbrace{(\overline{\mathbf{3}},\mathbf{2})_{5/6}}_{Y_0,\text{eaten}}$$

- $q_R(\mathbf{24}) = 0 \bmod M \;\Rightarrow\; \mathbb{Z}_M^R$
- $q_R(\mathcal{W}) = 2 \bmod M \;\Rightarrow\; S_0$ and T_0 massless
- add mass partner S_2 and T_2
  $\Rightarrow X_2$ and Y_2 massless

Fallbacher, Ratz and P. V. 2011

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$\mathbb{Z}_\mathsf{M}^\mathsf{R}$ from extra dimensions:

- for example from 6D
- 2D flat: $\mathrm{U}(1)_{56}$ symmetry
- extra components of gauge field A_m:

$$A_5 + \mathrm{i}A_6 \;\to\; \mathrm{e}^{\mathrm{i}\,\zeta}\,(A_5 + \mathrm{i}A_6)$$

- fermions $\Psi = \rho \otimes \chi$:

$$\rho \;\to\; \mathrm{e}^{\mathrm{i}\,\zeta/2}\,\rho$$

- 4D superfield:

$$\Phi \;=\; \frac{1}{\sqrt{2}}\,(A_5 + \mathrm{i}A_6) + \sqrt{2}\,\theta\rho + \theta\theta\,F$$

hence

$$\theta \;\to\; \mathrm{e}^{\mathrm{i}\,\zeta/2}\,\theta$$

$\Rightarrow$ $\mathrm{U}(1)_{56}^{R}$ is R symmetry

e.g. Nilles, Ratz and P. V. 2012

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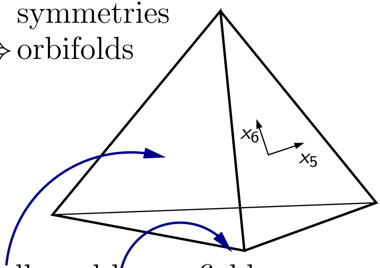
$\mathbb{Z}_M^R$ from extra dimensions:

- generically $U(1)^R_{56}$ broken completely
- want: discrete R symmetry
  $\Rightarrow$ compact space with discrete symmetries
  $\Rightarrow$ orbifolds



- bulk and brane fields
  $\Rightarrow$ flavour & GUT in 6D
- what are the R charges?

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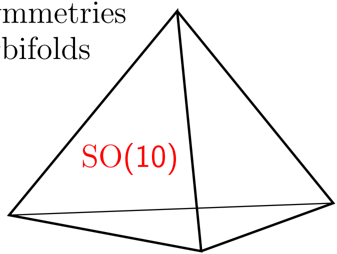

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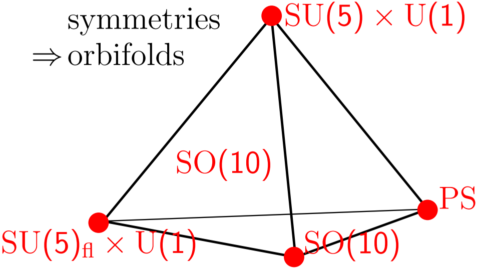

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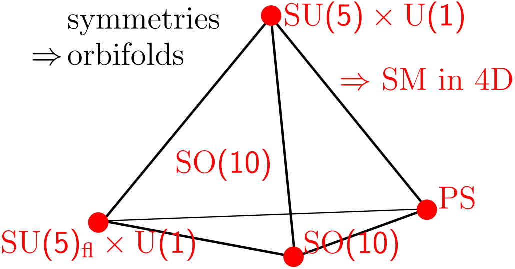

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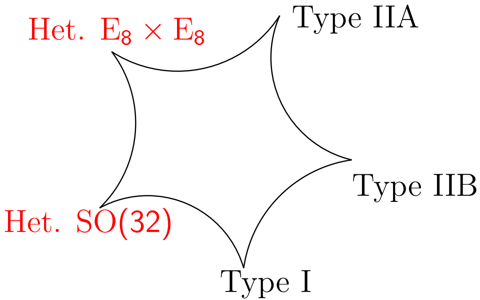
Include gravity:

- unification of GR and QFT in higher dimensions
  ⇒ string theory in 10D



⇒ compactify to 4D

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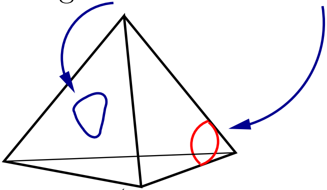
$E_8 \times E_8$ heterotic string theory:

- closed strings: untwisted or twisted



- twisted strings (not from $E_8 \times E_8$):
  - rep. of local GUT
  - rep. of flavour group
  - either $\mathcal{N} = 1$ or $\mathcal{N} = 2$
- untwisted strings (from $E_8 \times E_8$):
  - rep. of 4D gauge symmetry
  - invariant under flavour group
  - $\mathcal{N} = 4$

Classification of Toroidal Orbifolds

Outline

- space group S
- lattice Λ
- point group P
- orbifolding group G (includes roto-translations)
- equivalences and SUSY
- results of classification, summary:

60 inequivalent point groups with $\mathcal{N} \geq 1$
186 inequivalent lattices
520 inequivalent toroidal orbifolds with $\mathcal{N} \geq 1$
- → 162 with Abelian point group
  - → 138 with $\mathcal{N} = 1$
- → 358 with non-Abelian point group
  - → 331 with $\mathcal{N} = 1$

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22 Abelian (known)
like $\mathbb{Z}_3$, $\mathbb{Z}_4$, $\mathbb{Z}_6 - I$, ...
38 non-Abelian (mostly unknown)
like S_3, D_4, A_4, ...

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e.g. many non-factorizable ones like Lie lattice $F_4 \times SU(3)$ or E_6, but many unknown more, i.e. not all lattices are Lie lattices

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only classified for $\mathbb{Z}_2 \times \mathbb{Z}_2$ before

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non-local GUT breaking:
21 space groups from $\mathbb{Z}_2 \times \mathbb{Z}_2$
6 space groups from $\mathbb{Z}_2 \times \mathbb{Z}_4$
4 space groups from $\mathbb{Z}_3 \times \mathbb{Z}_3$

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Classification of Toroidal Orbifolds

Space group S

- def. S: discrete subgroup of the group of motions in R^6 with 6 linearly independent translations
- space group elements:

$$g = (\vartheta, \lambda) \text{ for } g \in S$$

acts on $x \in \mathrm{R}^6$ as

$$x \;\mapsto\; g\,x = \vartheta x + \lambda$$

- define orbifold

$$O = \mathrm{R}^6/S \text{ where } x \;\sim\; \vartheta x + \lambda \text{ for all } g \in S$$

Classification of Toroidal Orbifolds

Lattice Λ

- def. Λ: set of all translations in S
  $\Rightarrow$ e_i basis and B_e matrix whose columns are e_i
- for general $g = (\vartheta, \lambda) \in S$, $\lambda \notin \Lambda$ possible, e.g.

$$g = (\vartheta, \tfrac{1}{2}\lambda) \in S$$

  is called roto-translation
- basis change

$$B_e\, M \;=\; B_f \text{ for } M \in \mathrm{GL}(6, \mathbb{Z})$$

then, B_e and B_f define same lattice

$\Rightarrow$ equivalence of lattices

Classification of Toroidal Orbifolds

Point group P

- def. P: set of all twists ϑ for $(\vartheta, \lambda) \in S$
- in general $P \not\subset S$ possible
- P maps Λ of S to itself $\Rightarrow$ can be represented by $\mathrm{GL}(6, \mathbb{Z})$ matrix

$$\vartheta_e = B_e^{-1} \vartheta B_e \text{ and } \vartheta_e \in \mathrm{GL}(6, \mathbb{Z})$$

- under basis change with $M \in \mathrm{GL}(6, \mathbb{Z})$

$$\vartheta_f = M^{-1} \vartheta_e M$$

- P either Abelian (e.g. $\mathbb{Z}_3$) or non-Abelian (e.g. S_3)
- equivalence:

ϑ_f and ϑ_e generate equivalent point groups if $M \in \mathrm{GL}(6, \mathbb{Z})$ exists with

$$\vartheta_f = M^{-1} \vartheta_e M \quad \Leftrightarrow \quad \mathbb{Z}\textbf{-class}$$

Classification of Toroidal Orbifolds

Orbifolding group G

- def. G: generated by those elements $g \in S$ with non-trivial twist, identifying elements whose shift parts differ by a lattice vector
- roto-translations $\Leftrightarrow$ $G \neq P$
- orbifold

$$O \;=\; \mathrm{R}^6/S \;=\; T^6/G$$

- examples (in 2D) ...

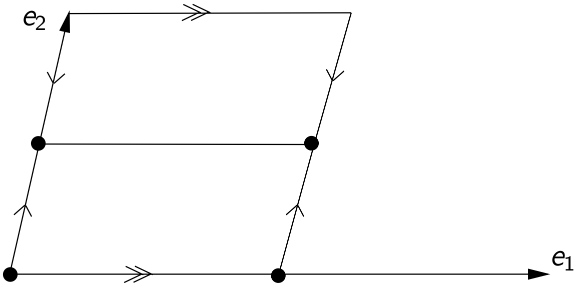
Examples in 2D

- space group generated by

  $(\mathbb{1}, e_1)$, $(\mathbb{1}, e_2)$ and $(\vartheta, 0)$

  where $\vartheta\, e_i = -e_i$

- orbifold: **pillow**



Classification of Toroidal Orbifolds

Examples in 2D

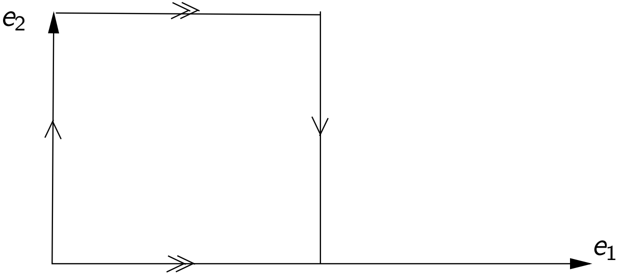
roto-translation

- space group generated by

  $(\mathbb{1}, e_1)$, $(\mathbb{1}, e_2)$ and $(\vartheta, \frac{1}{2} e_1)$

  where $\vartheta\, e_1 = e_1$ and $\vartheta\, e_2 = -e_2$

- orbifold: **Klein bottle**



Classification of Toroidal Orbifolds

Equivalences

- $\mathbb{Q}$-class: point group P, # of SUSY, # of moduli
- $\mathbb{Z}$-class: lattice Λ, nature of moduli
- affine-class: flavour group, gauge symmetry breaking

- **affine classes**

  two space groups $S_1 \sim S_2$ if: $f^{-1} S_1 f = S_2$
  for affine mapping f, i.e.

  $$f = (A, t) \text{ with } A \text{ linear map and } t \text{ translation}$$

- example: $T^2/\mathbb{Z}_2$ see blackboard

Classification of Toroidal Orbifolds

$\mathcal{N} \geq 1$ SUSY

- point group $P \subset \mathrm{O}(6)$ (can be non-Abelian)
- check: $P \in \mathrm{SU}(3)$
- take generators $\vartheta_i \in P$
- ϑ_i in (reducible) $\mathbf{6}$ of P
- decompose $\mathbf{6}$ into irreps $\mathbf{6} \;\rightarrow\; \boldsymbol{\rho_1} \oplus \boldsymbol{\rho_2} \oplus \ldots$ (using the characters of finite groups P)
- check: $\mathbf{6} \;\rightarrow\; \mathbf{a} \oplus \overline{\mathbf{a}}$ because $\mathbf{6} \;\rightarrow\; \mathbf{3} \oplus \overline{\mathbf{3}}$ of $\mathrm{SU}(3)$
- create $\mathbf{a}$ and check det=+1
- example: S_3 see blackboard

Classification of Toroidal Orbifolds

Classification

- CARAT: all space groups in up to 6 dim.
- get $\mathbb{Q}$ classes (point groups) $\Rightarrow$ 7103 point groups in 6D
- check $\mathcal{N} \geq 1$ using GAP
- create $\mathbb{Z}$-classes (lattices)
- create affine classes (inequivalent space groups)

Classification of Toroidal Orbifolds

Result

60 inequivalent point groups with $\mathcal{N} \geq 1$
186 inequivalent lattices
520 inequivalent toroidal orbifolds with $\mathcal{N} \geq 1$

- → 162 with Abelian point group
  - → 138 with $\mathcal{N} = 1$
- → 358 with non-Abelian point group
  - → 331 with $\mathcal{N} = 1$

Fischer, Ratz, Torrado and P. V. 2012

Classification of Toroidal Orbifolds

Conclusion

- New paths of the Heterotic Road
- Classification of all (toroidal) orbifold geometries
- 520 space groups with $\mathcal{N} \geq 1$
- most of them unknown
- gauge embedding? modular invariance?
- pheno? e.g. of non-Abelian orbifolds

Backup

Green-Schwarz mechanism

- Symmetry generator α is anomalous if action invariant but path integral measure not, i.e.

$$D\Psi D\bar{\Psi} \mapsto e^{i\int \mathrm{d}^4 x A(\alpha)} D\Psi D\bar{\Psi}$$

where

$$A(\alpha) = \frac{1}{32\pi^2} \mathrm{Tr}\left(\alpha F \tilde{F}\right) - \frac{1}{384\pi^2} R\tilde{R}\,\mathrm{Tr}\left(\alpha\right)$$

and $A(\alpha) \neq 0$. Álvarez-Gaumé, Witten/ Álvarez-Gaumé, Ginsparg

- Note: $\int \mathrm{d}^4 x A(\alpha) = 2\pi \frac{\text{integer}}{N}$ for $\mathbb{Z}_N$,
i.e. phase can be non-trivial for discrete transformation α

Green-Schwarz mechanism

- For example: anomalous $\mathrm{U}(1)$, gauge transformation Λ

$$V \mapsto V + \frac{i}{2}\left(\Lambda - \Lambda^\dagger\right) \qquad \text{vector superfield}$$
$$\Phi^{(f)} \mapsto e^{-iq^{(f)}\Lambda}\Phi^{(f)} \qquad \text{chiral superfields}$$

- Action invariant but path integral measure not
- From $A(\alpha) \supset \mathrm{Tr}\left(\alpha F\tilde{F}\right) \neq 0 \quad \Rightarrow$

$$\sum_f {q^{(f)}}^3 \neq 0 \quad \text{and} \quad \sum_{r^{(f)}} \ell\left(r^{(f)}\right) q^{(f)} \neq 0$$

$\Rightarrow$ Anomaly!

Green-Schwarz mechanism

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$\Rightarrow$ Anomaly!

(second term also for discrete symmetry)

Green-Schwarz mechanism

- Add Dilaton superfield S

$$L \supset -\int \mathrm{d}^4\theta \ln\left(S + S^\dagger - \delta_{\mathrm{GS}} V\right) + \int \mathrm{d}^2\theta \frac{S}{4} W_\alpha W^\alpha + \text{h.c.}$$

with transformation property

$$S \mapsto S + \frac{i}{2}\delta_{\mathrm{GS}}\Lambda \quad \text{with} \quad \delta_{\mathrm{GS}} = \frac{1}{6\pi^2}\sum_f {q^{(f)}}^3$$

- Second term compensates non-invariance of path integral measure

$\Rightarrow$ Anomaly cancelled!

Discrete Green-Schwarz mechanism

- α is $\mathbb{Z}_N$ transformation
- Chiral superfields transform

$$\Phi^{(f)} \mapsto e^{-2\pi i q^{(f)} \frac{1}{N}} \Phi^{(f)}$$

- Assume $\int \mathrm{d}^4 x A(\alpha) = 2\pi \frac{\text{integer}}{N}$
- Again, add Dilaton superfield S with discrete shift trafo

$$S \mapsto S + \frac{i}{2}\Delta_{\mathrm{GS}} \quad \text{with} \quad \Delta_{\mathrm{GS}} = \frac{1}{\pi N} A_{G-G-\mathbb{Z}_N}$$

$\Rightarrow$ Anomaly cancelled!