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Singularities from large volume

Refs

Yamazaki, Kennaway reviews (0803.4474) (0706.1660)

D-branes @ large volume

Aspinwall (th/0403166)

Sharpe (th/0307245)

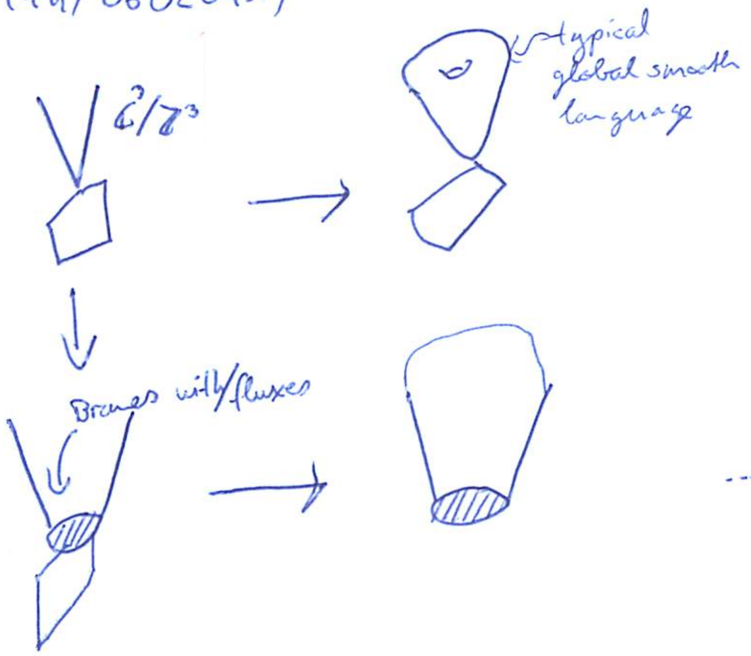
General geometry: Griffith & Harris

Z-map (th/0602041)

Flavour branes (1201.5379)

Orientifolds (Diaconescu et al. th/0606180)

The picture



large vol. persp.

... restrict to toric sing.

typically no CFT description (e.g. dP_3) but techniques don't need it

Content

+ Toric geometry
+ Dimer models
+ Dictionary sing. $\leftrightarrow$ LV
+ Flavour branes
+ Orientifolds

Toric geometry

+ Generalization of $\mathbb{P}^n$

$$\mathbb{P}^2 \quad (x_1, x_2, x_3) \simeq (\lambda x_1, \lambda x_2, \lambda x_3) \quad ; \lambda \in \mathbb{C}^*$$
$$(x_1, x_2, x_3) \neq (0,0,0)$$

+ Weighted proj. spaces

$$(x_1, x_2, x_3) \simeq (\lambda^{\omega_1} x_1, \lambda^{\omega_2} x_2, \lambda^{\omega_3} x_3)$$

+ Toric spaces:

$$dP_1 \begin{cases} \mathbb{C}_1^*: (x_1, x_2, x_3, x_4) \to (\lambda_1 x_1, \lambda_1 x_2, \lambda_1 x_3, x_4) \\ \mathbb{C}_2^*: (x_1, x_2, x_3, x_4) \to (x_1, x_2, \lambda_2 x_3, \lambda_2 x_4) \end{cases}$$

equiv. notation

	x_1	x_2	x_3	x_4
$\mathbb{C}_1^*$	1	1	1	0
$\mathbb{C}_2^*$	0	0	1	1

GLSM - viewpoint

	x_1	x_2	x_3	z
$U(1)$	1	1	1	-3

charge matrix

$$V_D = \left(\sum |x_i|^2 - 3|z|^2 - \xi\right)^2$$

Moduli space of this theory

$\xi < 0$. $\quad z \neq 0$

$$|z|^2 = \frac{1}{3}\left(\sum |x_i|^2 - \xi\right)$$

$z \in \mathbb{R}_+$ (phase fixed by choice of gauge)

$U(1) \to \mathbb{Z}_3$

$(x_1, x_2, x_3) \to (\omega x_1, \omega x_2, \omega x_3)$ $\quad (x_1, x_2, x_3) \in \mathbb{C}^3$ $\quad \omega^3 = 1$

we are left with $\mathbb{C}^3/\mathbb{Z}^3$

$\xi > 0$:

$(x_1, x_2, x_3) \neq (0,0,0)$

Assume $z = 0$ $\quad V_D = \left(\sum |x_i|^2 - \xi\right)^2$

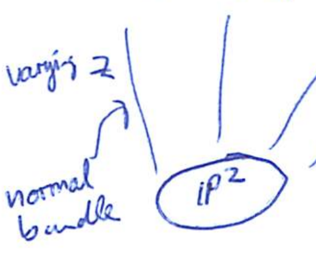
	x_1	x_2	x_3
$U(1)$	1	1	1

$\mathbb{C}^* = U(1) \times \mathbb{R}_+$

V_D fixes $\mathbb{R}_+$

$\mathbb{C}^* : (x_1, x_2, x_3] \leftrightarrow (\lambda x_1, \lambda x_2, \lambda x_3) \quad \cong \mathbb{P}^2$

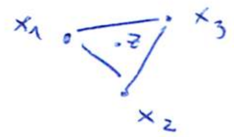
$\mathcal{O}(-3) \to \mathbb{P}^2$



Toric diagrams

charge matrix $\to Q \cdot v = 0$

$$v = \begin{array}{c} \begin{array}{cccc} x_1 & x_2 & x_3 & z \end{array} \\ \begin{pmatrix} -1 & 0 & 1 & 0 \\ 0 & -1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{pmatrix} \end{array}$$
$\leftarrow$ can always be chosen to be 1



Divisor: zero loci of homogeneous polynomials.

Linear equivalence $\simeq$ Polynomials of the same weight

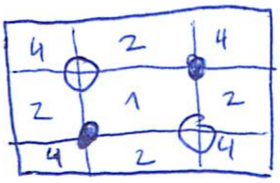
;

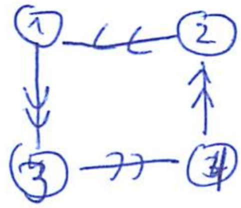
3]

Divisors of $\mathbb{P}^2$: $\mathcal{O}(n)$ (Polyn. of certain degrees (i.e. certain charges ~~Aff~~ under $U(1)$))

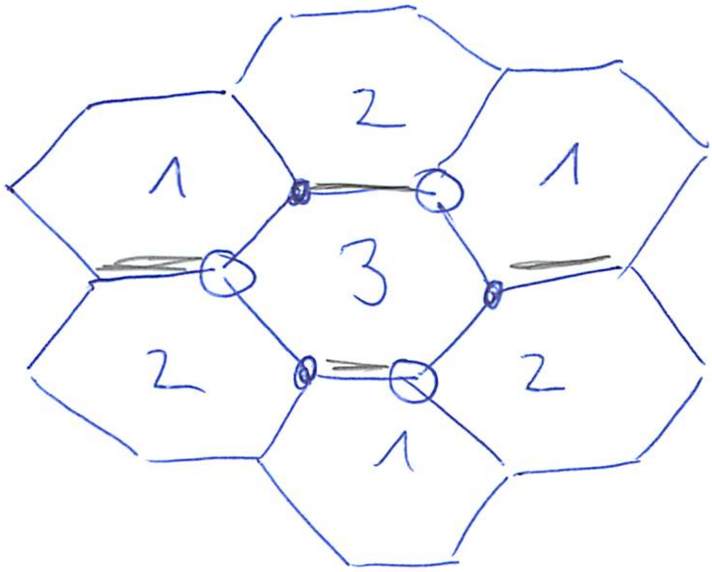
Dimer models

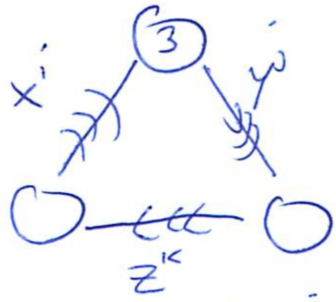
Def.: Bipartite tiling of T^2





$$W = \pm \sum \mathrm{Tr}\left(X_{21} X_{13} X_{34} X_{42}\right)$$





$$W = \sum_{ijk} \mathrm{Tr}\left(x^i y^j z^k\right)$$

Dimer → Geometry

Perfect matching

Subset of edges

s.t. every node is reached by exactly one edge in the set

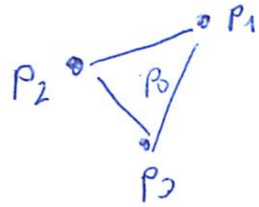
$\boxed{\text{th/0601063}}$

$$\text{winding}\,(p_1 - p_0) = (1,1)$$

$$\text{winding}\,(p_2 - p_1) = (-1,0)$$

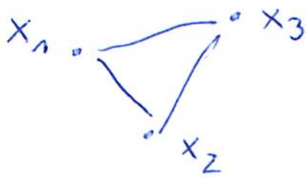
$$\text{winding}\,(p_3 - p_0) = (0,-1)$$

$$\text{winding}\,(p_0 - p_0) = (0,0)$$

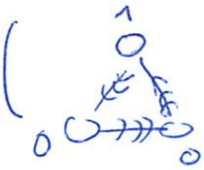


Recall

$$\mathrm{Ker}(Q) = \begin{pmatrix} -1 & 0 & 1 & 0 \\ 0 & -1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{pmatrix}$$



Construct map:

$D\Big($ 

$\Big) = \mathcal{E}_1$ = Bundle on $\mathbb{P}^2$

F_1: 

$0 \to P_3 \to P_2 \to P_1 \to F_1 \to 0$

$\boxed{\text{Hanany, Herzog, Vegh}}$

$\mathcal{T}$ map: Dimer $\to P_i$

$$\mathrm{Ext}^p(P_i, P_j) = \begin{cases} 0 & i > j \\ \mathbb{C} & p = 0 \quad i = j \\ 0 & p \neq 0 \quad i < j \\ \cdot & p = 0 \quad i < j \end{cases}$$
$$\left(\approx H^p(P_i^\vee \otimes P_j)\right)$$

Think of P_i's in terms of line bundles

P_i's will allow to calculate $\mathcal{E}_i$'s:

$$0 \to \sum c_{i_1} P_i \to \sum c_{i_2} P_i \to \ldots \to P_i \to \mathcal{E}_i \to 0$$

$$ch(\mathcal{E}_i) = (S^{-1})_{ji}\, ch(P_j)$$

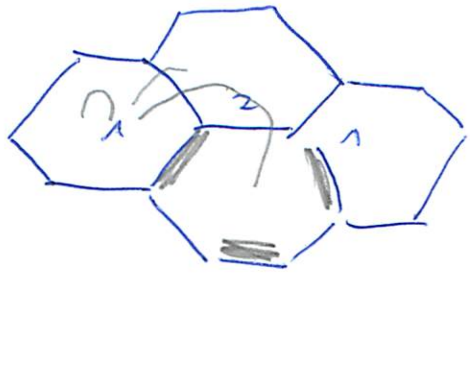
P_i are line bundles of the form $\mathcal{O}(D)$ where D is given by the $\mathcal{T}$ map

· There is a P_i for every node in the quiver (resp. for any face in the dimer)

The $\mathcal{T}$-map:
1) Choose a reference perfect matching (internal point), here p_0
2) ~~The reach face~~ Choose a reference face (here "1")
3) Construct paths in the dimer that go from $\mathcal{F}(ace)_j$ to $\mathcal{F}(ace)_i$. (allowed ways are obtained from Beilinson quiver). "Beilinson" quivers
4) See which perfect matchings have the edges that you crossed.

$P_1 = \mathcal{O}(0) = \mathcal{O}$
$P_2 = \mathcal{O}(p_3) = \mathcal{O}(1)$
$P_3 = \mathcal{O}(p_3 + p_1) = \mathcal{O}(2)$

identify p_i's with divisors, obtained previously.



$\rightarrow$ $S_{ij} = \dim \mathrm{Hom}(P_i, P_j)$

$= \dim H^0(P_i^\vee \otimes P_j)$

$S_{12} = \dim H^0(P_1^\vee \otimes P_2) = \dim(\mathcal{O} \otimes \mathcal{O}(1))$

$= \dim H^0(\mathcal{O}(1))$

$= \dim \{x_1, x_2, x_3\} = 3$

(Polynomials of degree 1 i.e. x_1, x_2, x_3)

$S_{21} = \dim H^0(\mathcal{O}(1)^\vee \otimes \mathcal{O})$

$= \dim H^0(\mathcal{O}(-1))$

$= 0$

$$S_{ij} = \begin{pmatrix} 1 & 3 & 6 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{pmatrix}$$

$$S^{-1} = \begin{pmatrix} 1 & -3 & 3 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{pmatrix}$$

$\rightsquigarrow$ Recall general formulae for Chern-characters

$$\Gamma_{\mathcal{E}_i} = [\underset{\wedge}{\overset{\mathbb{P}^2}{S}}] \wedge ch(\mathcal{E}_i) \wedge \sqrt{\frac{Td(T_S)}{Td(N_S)}}$$

$ch(\mathcal{E}_1) = 1$

$Td(E) = 1 + \frac{1}{2} c_1(E) + \frac{1}{12}(c_1^2(E) + c_2(E)) + \ldots$

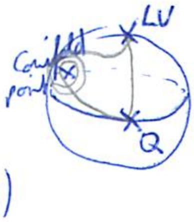
$\Gamma_{\mathcal{E}_1} = [S] \wedge (1 + \frac{3}{2} \ell + \frac{5}{4} \ell^2)$

$\Gamma_{\mathcal{E}_2} = [S] \wedge (-2 - 2\ell - \frac{1}{2} \ell^2)$

$\Gamma_{\mathcal{E}_3} = [S] \wedge (1 + \frac{1}{2} \ell + \frac{1}{4} \ell^2)$

Note: $\sum \Gamma_{\mathcal{E}_i} = [S] \wedge \ell^2$

Aside: $\Gamma_{\mathcal{E}_1}$ seems stable in quiver locus & @ large volume. However this does not have to be the case:



DSZ: $\langle \Gamma_{\mathcal{E}_1}, \Gamma_{\mathcal{E}_2} \rangle = \sum_{i=0}^{3} (-1)^i \int_X \Gamma_{\mathcal{E}_1}^{(2i)} \wedge \Gamma_{\mathcal{E}_2}^{(6-2i)}$

$= -\int_X \Gamma_{\mathcal{E}_1}^{(2)} \wedge \Gamma_{\mathcal{E}_2}^{(4)} + \int_X \Gamma_{\mathcal{E}_1}^{(4)} \wedge \Gamma_{\mathcal{E}_2}^{(2)}$

$= -\int_X ([S]) \wedge ([S] \wedge (-2\ell)) + \int_X [S] \wedge \frac{3}{2} \ell \wedge (-2[S])$

$= -\int_S [S] \wedge (-2\ell) + \int_S \frac{3}{2} \ell \wedge (-2[S])$ $\quad \hat{=} \quad [S]|_S = \mathcal{O}(-3)$

$= -\int_S (-3\ell) \wedge (-2\ell) + \int_S \frac{3}{2} \ell \wedge (+6\ell) = (-6+9) \int_S \ell^2 = 3$