Lifshitz Scaling in String Theory

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based on: *JHEP1012(2010)047 and JHEP1205(2012)122*

with: *L.Barclay, R.Gregory, G.Tasinato, S.Parameswaran*

Introduction

- AdS/CFT has yield useful insights into strongly coupled field theories, in particular QCD.
- Moreover it has triggered very fruitful investigation in condensed matter physics systems, which present strongly coupled phenomena.
- AdS/CFT concerns scale invariant bdy. theory. However in condensed matter systems (where Lorentz invariance is not a particularly natural symmetry), more general scaling properties such as dynamical *Lifshitz scaling, z*:

$$t \to \lambda^z t \quad , \quad x^i \to \lambda x^i \quad , \quad r \to r/\lambda.$$

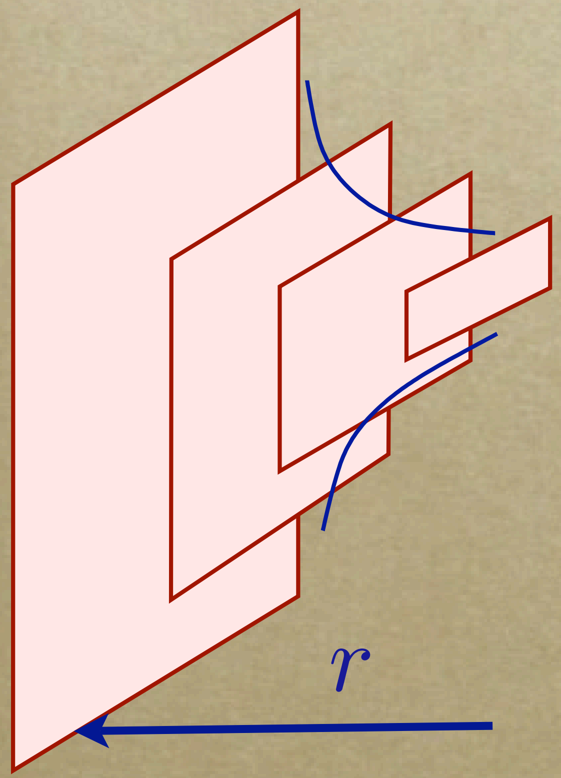
are quite common.

Lifshitz geometry

- A simple way to geometrize scale invariant but non-Lorentz invariant metrics, is the *Lifshitz metric*:

[Kachru-Liu-Mulligan]

$$ds^2 = L^2 r^2 \left[r^{2(z-1)} dt^2 - d\underline{x}^2 \right] - L^2 \frac{dr^2}{r^2}$$



anisotropic deformation of adS space. Time and space warp differently across the bulk.

- Solutions of this form do not arise in pure Einsten gravity with a negative CC. Extra matter is required to support this metric:

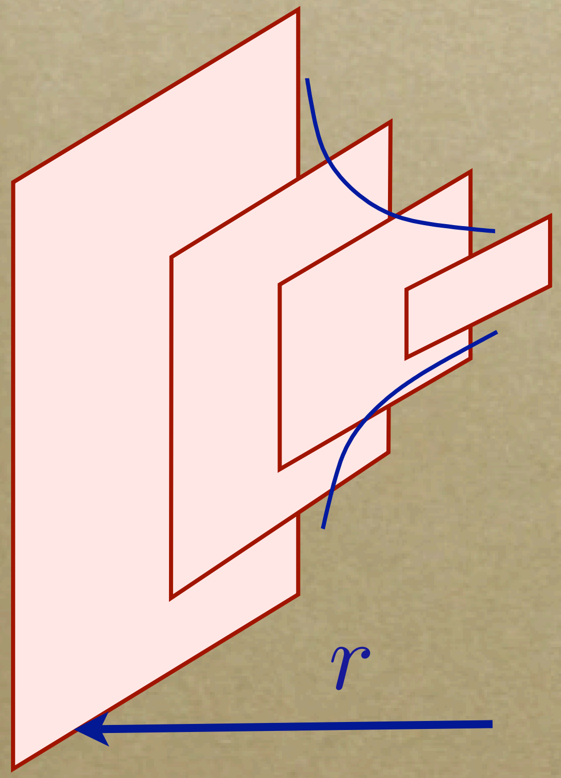
$$R^t{}_t = z(z+2)\,, \qquad R^x{}_x = z+2\,, \qquad R^r{}_r = z^2+2$$

Bottom up models

- First empirical models built in 4D with coupled 1 and 2-form gauge fields (A_1, B_2), besides Λ. [Kachru-Liu-Mulligan]

$$S = \int d^4x\sqrt{-g}\,(R-2\Lambda) - \frac{1}{2}\int\left(\frac{1}{e^2}F_{(2)}\wedge *F_{(2)} + F_{(3)}\wedge *F_{(3)}\right) - c\int B_{(2)}\wedge F_{(2)}.$$

System dual to a massive vector field theory: scaling is supported by a massive vector flux, with very specific values for q, m, Λ



$$\mathcal{L} = -R - 2\Lambda - \frac{1}{4}F^2 + \frac{m^2}{2}A^2$$

[Taylor]

$$A = qr^z dt \qquad q = L\sqrt{\frac{2(z-1)}{z}}$$

$$m^2 = (D-2)\frac{z}{L^2}$$

$$ds^2 = L^2r^2\left[r^{2(z-1)}dt^2 - d\underline{x}^2\right] - L^2\frac{dr^2}{r^2}$$

$$\Lambda = -\frac{1}{2L^2}\left(z^2 + (D-3)z + (D-2)^2\right)$$

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- Lifshitz scaling is problematic as $r \to 0$.
- Ideally one would like to embed Lifshitz scaling within string theory to have confidence on holographic dual FT and explore the range of possible geometries such as BH's (thermal states in FT side).
- In spite of relatively simple field content, no string theory model found with specific values of m and Λ, though some models with specific z later found.

Top down approach

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- For string motivated spacetimes, we will have to compactify in such a was as to preserve Chern-Simons structure of the phenomenological model. However Λ can be replaced by a false vacuum.

- Can achieve this via consistent truncations of type IIA and IIB 10D supergravity.

- We embedded general (z) Lifshitz spacetimes in string theory via consistent truncation of IIA/IIB supergravity using 6D/5D Romans gauged SUGRA with a flux compactification on H_2.

Lifshitz solutions in string theory

- 6D Romans gauged SUGRA can be obtained from 10D massive IIA SUGRA on an S4. Contains dilaton, 1-form, massive 2-form and non-Abelian gauge field:

$$-\frac{R}{4}+\frac{1}{2}(\partial\phi)^2-\frac{e^{-\sqrt{2}\phi}}{4}\left(\mathcal{H}_2^2+(F_2^{(i)})^2\right)+\frac{e^{2\sqrt{2}\phi}}{12}G_3^2+V(\phi)$$

$$+\frac{1}{8}\epsilon^{ABCDEF}B_{AB}\left(\mathcal{F}_{CD}\mathcal{F}_{EF}+m\,B_{CD}\mathcal{F}_{EF}+\frac{m^2}{3}B_{CD}B_{EF}+F_{CD}^{(i)}F_{EF}^{(i)}\right)$$

where:

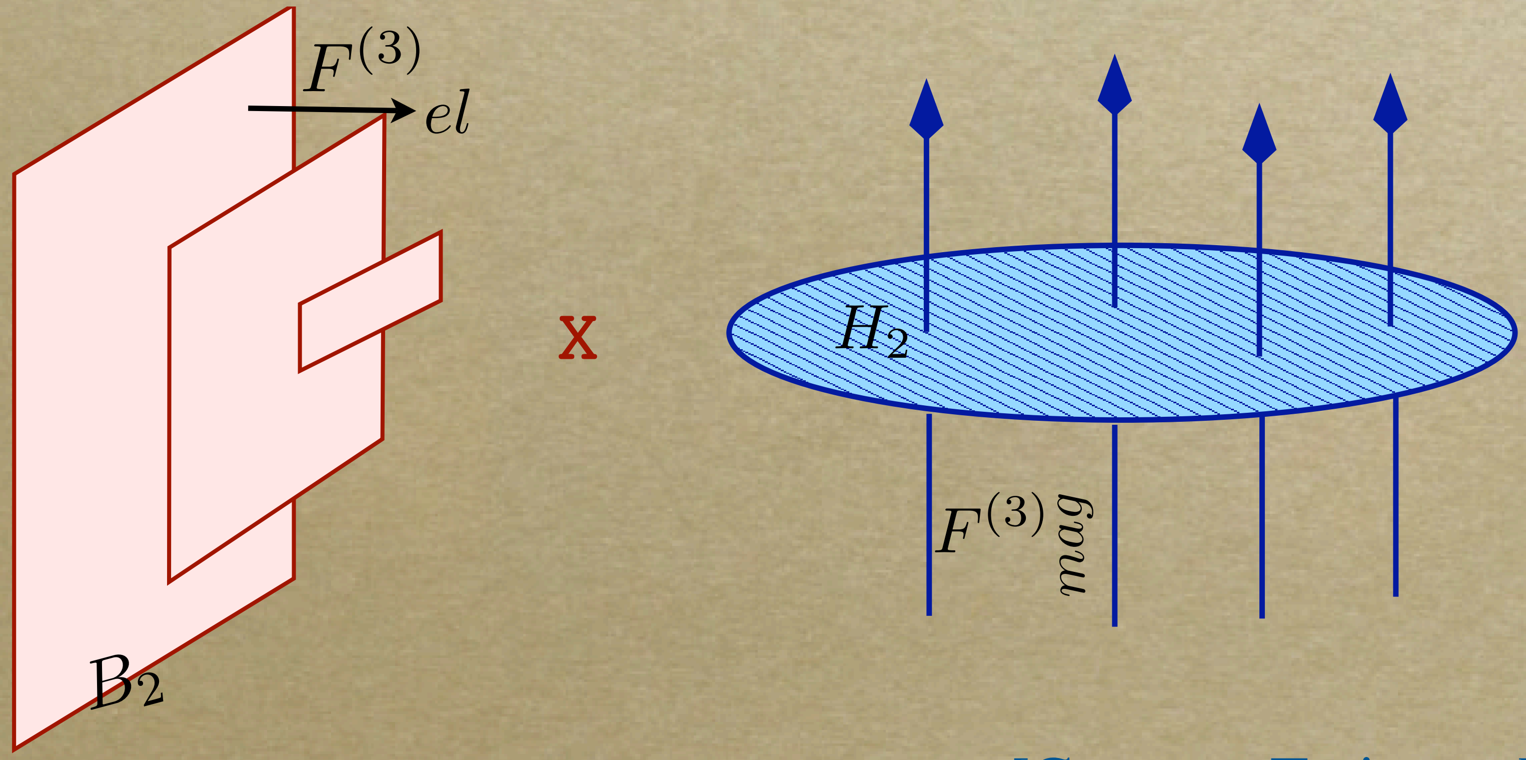
$$\begin{cases} G_3=dB_2\,, \qquad \mathcal{H}_2=\mathcal{F}_2+m\,B_2 \\ V(\phi)=\frac{1}{8}\left(g^2e^{\sqrt{2}\phi}+4g\,m\,e^{-\sqrt{2}\phi}-m^2e^{-3\sqrt{2}\phi}\right)\end{cases}$$

The geometry

Analytic Lifshitz solution has compact H2 carrying flux and constant dilaton. 1-parameter family of solutions determined by z for ANY z.

$$ds^2 = L^2 \left[r^{2z} dt^2 - r^2 \left(dx_1^2 + dx_2^2 \right) - \frac{dr^2}{r^2} \right] - a^2 dH_2^2$$

$$F^{(3)}_{tr} = q\, b\, L^3\, r^{z-1}\,, \qquad F^{(3)}_{y_1 y_2} = q\, \epsilon_{12}\,, \qquad B_{x_1 x_2} = \frac{bL^3}{2} r^2\,, \qquad \phi = \phi_0$$



[Gregory, Tasinato, Parameswaran, IZ, ‘10]

The equations of motion

- With this form for flux in H2, the gauge equations reduce to one function, which gives the needed structure for the energy momentum and Einstein equations become algebraic relations.

$$d * e^{-\sqrt{2}\phi} F^{(3)} = qG$$

$$d * e^{2\sqrt{2}\phi} G = m^2 e^{-\sqrt{2}\phi} * B + qF^{(3)}$$

$$\Rightarrow F^{(3)} = -q e^{\sqrt{2}\phi} * B$$

$$R^t_t = V + e^{2\sqrt{2}\phi} G_{r12} G^{r12} + \frac{q^2}{2a^4} e^{-\sqrt{2}\phi} + 6q^2 B_{12} B^{12} \frac{e^{\sqrt{2}\phi}}{a^4} + \frac{m^2}{2} B_{12} B^{12} e^{\sqrt{2}\phi}$$

$$R^r_r = V - e^{2\sqrt{2}\phi} G_{r12} G^{r12} + \frac{q^2}{2a^4} e^{-\sqrt{2}\phi} + 6q^2 B_{12} B^{12} \frac{e^{\sqrt{2}\phi}}{a^4} + \frac{m^2}{2} B_{12} B^{12} e^{\sqrt{2}\phi}$$

$$R^l_l = V - e^{2\sqrt{2}\phi} G_{r12} G^{r12} + \frac{q^2}{2a^4} e^{-\sqrt{2}\phi} - 2q^2 B_{12} B^{12} \frac{e^{\sqrt{2}\phi}}{a^4} - \frac{3m^2}{2} B_{12} B^{12} e^{\sqrt{2}\phi}$$

The solution

- Given the SUGRA parameters g and m, L and the dilaton can be tuned to give a solution for any z

- Two branches of solutions (but not for all z values)

$$\hat{b}^2 = z - 1$$

$$\hat{g}^2 = 2z(4+z)$$

$$\hat{m}^2 = \frac{2}{z}\left[6 + z \mp 2\sqrt{z(4+z)}\right]$$

$$\hat{q}^2 = \frac{1}{2z}\left[(z+2)(z-3) \pm 2\sqrt{z(4+z)}\right]$$

$$\frac{1}{\hat{a}^2} = \left[6 + 3z \mp 2\sqrt{z(4+z)}\right]$$

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The solution

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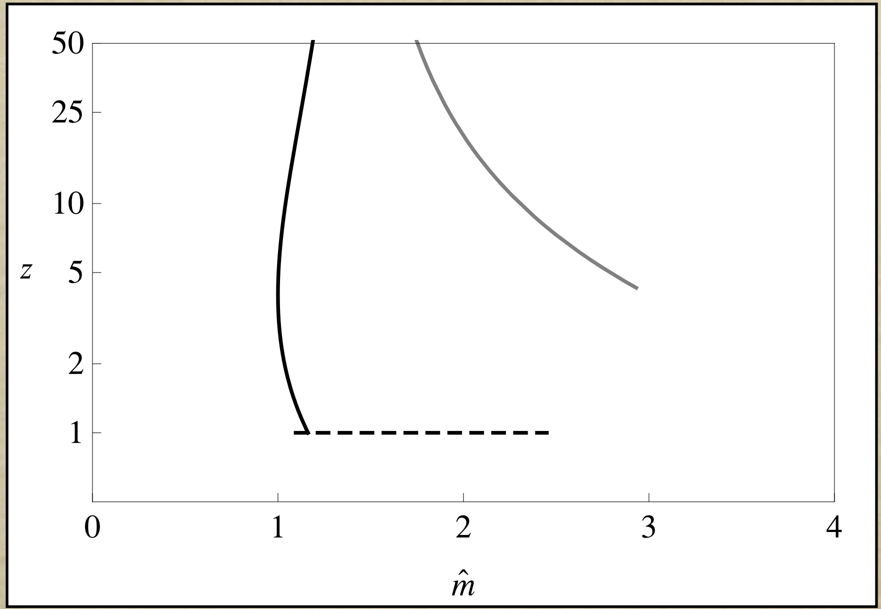
- There is also a 1-parameter family of adS solutions.



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$$\hat{g} \in \left[\sqrt{6}, 3\sqrt{\frac{6}{5}}\right]$$

$$\hat{m} = \hat{g} - \sqrt{\hat{g}^2 - 6}$$

$$\hat{q}^2 = -\frac{3}{4}\left(\hat{g}^2 - 6\right) + \frac{\hat{g}}{2}\sqrt{\hat{g}^2 - 6}$$

$$\frac{1}{\hat{a}^2} = \frac{3}{2}\hat{g}^2 - 6 - \hat{g}\sqrt{\hat{g}^2 - 6}$$

[Gregory, Tasinato, Parameswaran, IZ, ‘10]

More general solutions

- Consider more general static solutions. Alterations need only have radial dependance for all fields:

$$ds^2 = L^2 \left[e^{2f(r)} dt^2 - e^{2c(r)} \left(dx_1^2 + dx_2^2 \right) - e^{2d(r)} dr^2 \right] - e^{2h(r)} dH_2^2$$

$$F^{(3)}_{tr} = L^2 Q(r) \,, \qquad F^{(3)}_{y_1 y_2} = q\, \epsilon_{12} \,, \qquad \phi = \phi(r)$$

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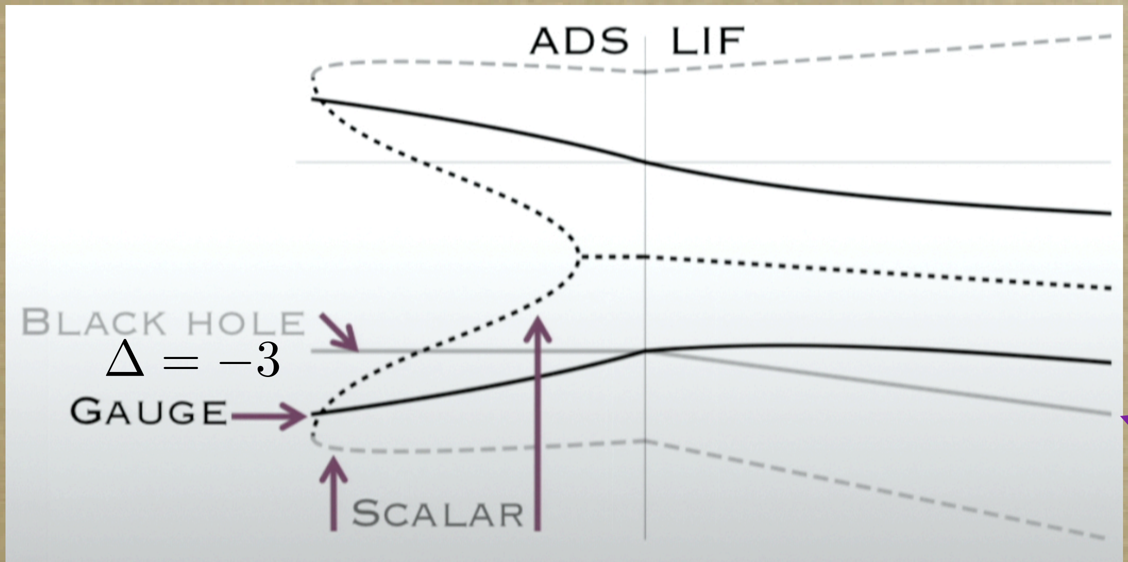
- Study the general radial eom's from dynamical system perspective. This reveals space of solutions is 7 dimensional. Critical points correspond to pure adS/Li solutions.

- Perturbing around exact solutions allows us to characterize all possible asymptotic behaviours. Eigenvalues give the fields' radial fall off's

$$\delta\Phi_i = V_{ij}\, r^{\Delta_j} \qquad \left(r\frac{d}{dr}\delta\Phi = A\,\delta\Phi \right)$$

- System is characterized by horizon size, gauge and scalar charge. Can identify adSBH in phase diagram, but Lifshitz is more complex: eigenvectors degenerate at crossing point



Lifshitz structure

$\left(\frac{r_+}{r}\right)^{z+2}$

Black Holes

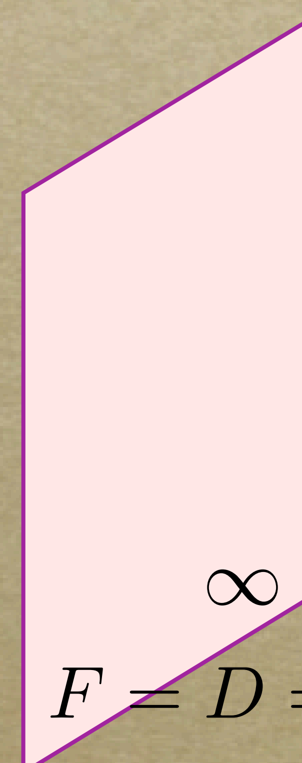
- To find black holes, we need to solve radial equations with a horizon. From eigenvalue analysis we know that *all fields* are active.

$$ds^2 = L^2 \left[r^{2z} F(r) dt^2 - r^2 \left(dx_1^2 + dx_2^2 \right) - \frac{dr^2}{r^2 D(r)} \right] - a^2 H(r) dH_2^2$$

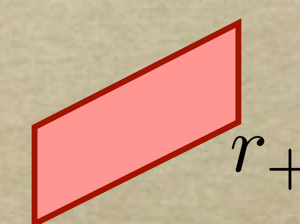
$$B_{x_1 x_2} = L^2 P(r) r^2 / 2$$

$$F^{(3)}_{tr} = L^2 e^{\sqrt{2}\phi}\, r^{z-1} \frac{\sqrt{F}}{\sqrt{D}\, H} P(r)$$

$$\phi = \phi(r)$$



$$F = D = H = P = 1$$



$$F \sim f_1 (r - r_+) + \dots$$

$$D \sim d_1 (r - r_+) + \dots$$

$$P \sim P_0 + \dots$$

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$$\varphi \sim \varphi_0 + \dots$$

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- Solutions characterised by 2 parameters: scalar and vector initial data at horizon (fix r+=1).

$$B_{x_1 x_2} = L^2 P(r) r^2/2$$

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∞

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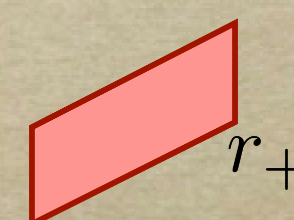

r_+

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- Solutions characterised by 2 parameters: scalar and vector initial data at horizon (fix r+=1).

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$$F^{(3)}_{tr} = L^2 e^{\sqrt{2}\phi}\, r^{z-1}\frac{\sqrt{F}}{\sqrt{D}\,H}P(r)$$

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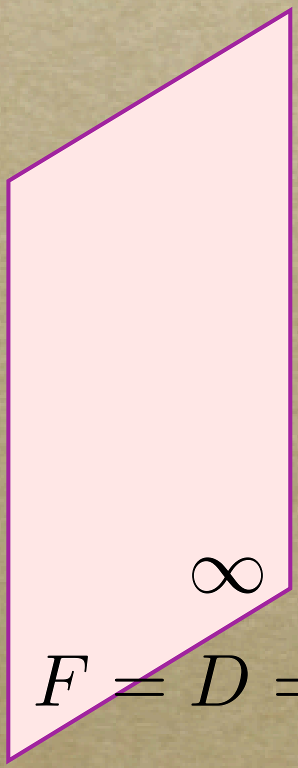
- General solutions found numerically, integrating out from horizon to look for regular solutions.



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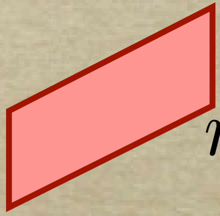

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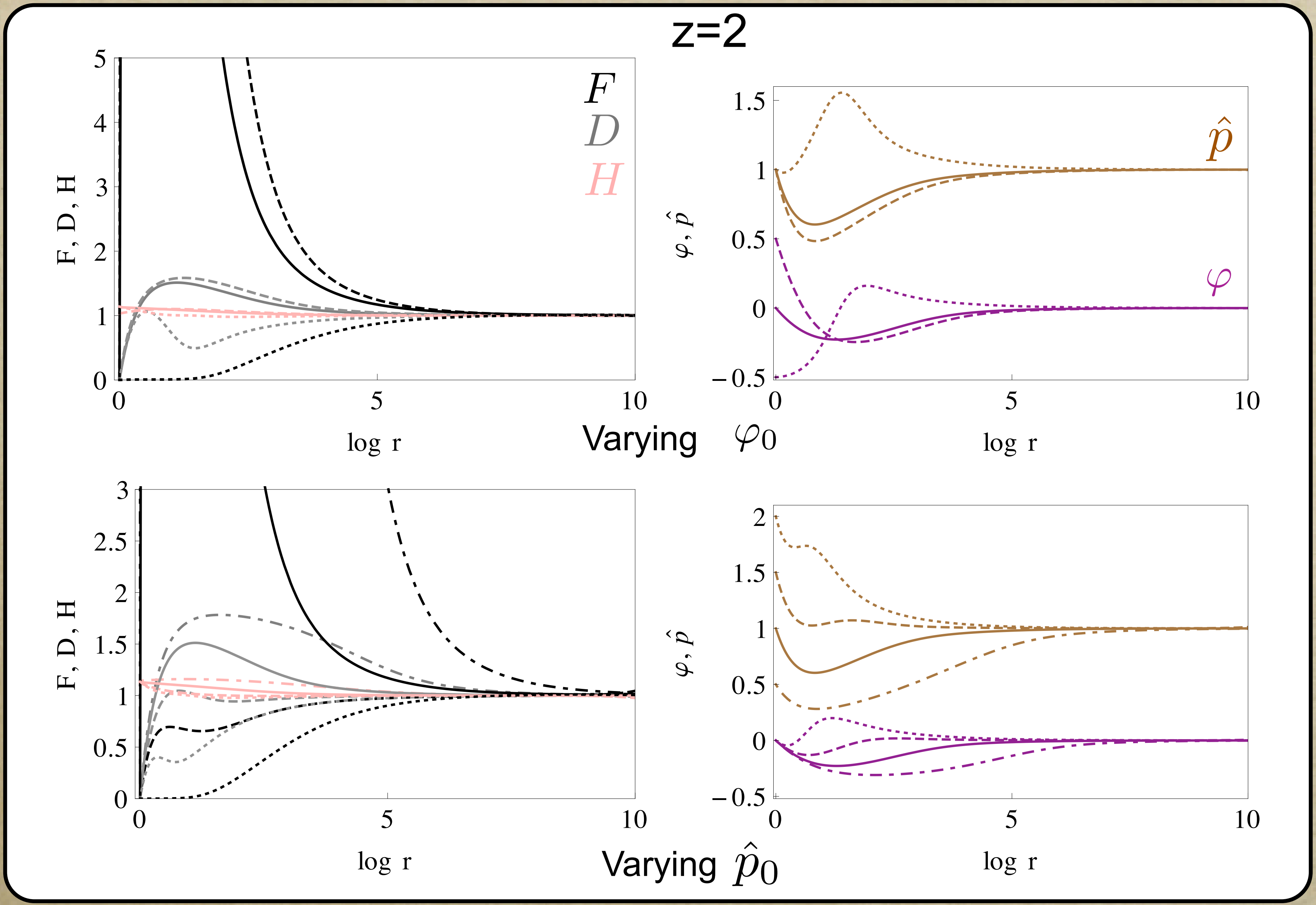
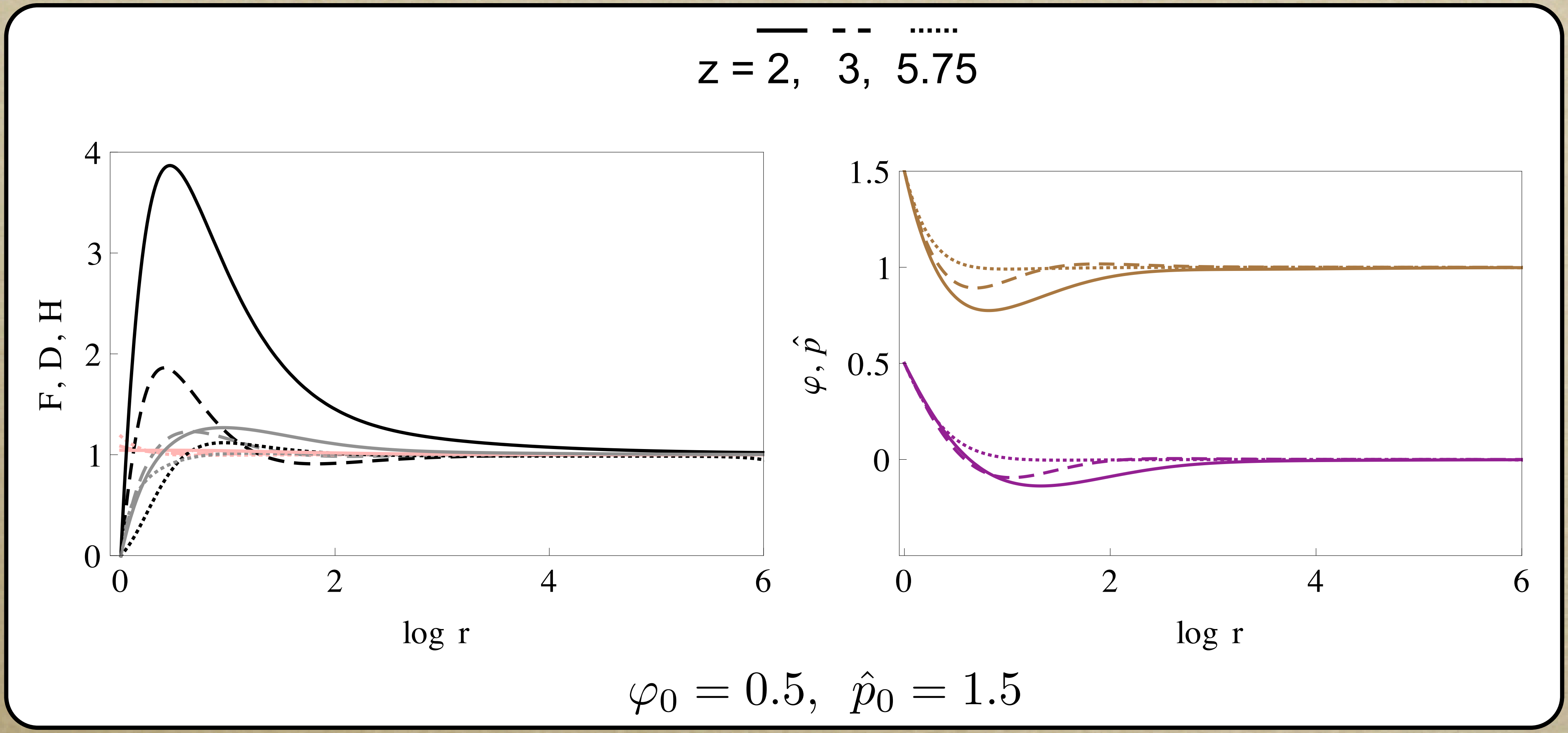
Numerical Solutions I



- Generic LiBH tends to have sharp peak in Newtonian potential g_{tt}. By contrast, radial metric function, relatively well behaved.
- As scalar charge drops, better behaviour. As gauge charge drops things get worst near horizon. In contrast to adSBH, more features near horizon.

Numerical Solutions II

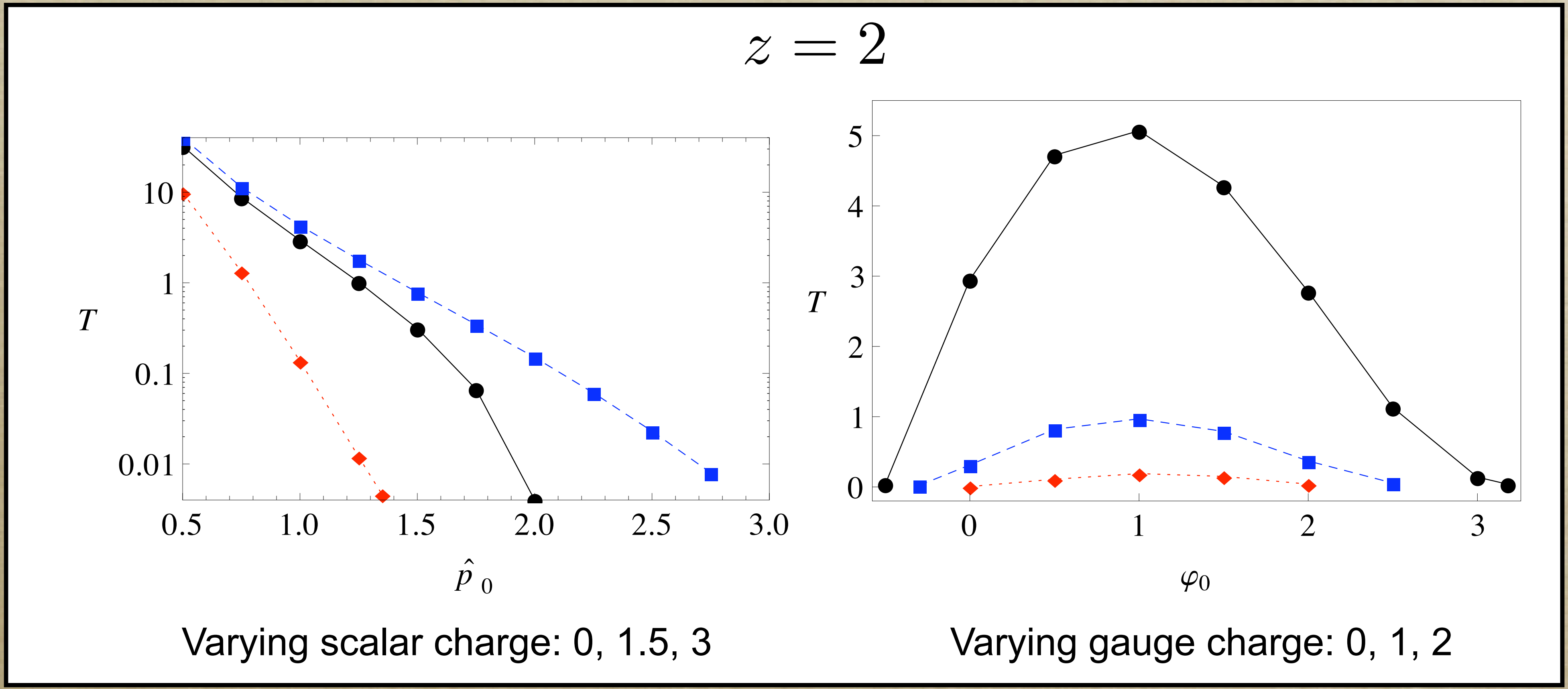
z dependence



- Increasing z calms down potentials (due to z powers).
- Fields drop to asymptotic values faster (again z power).
- All the fields have strong modulation near horizon. Area gauge might not be best and numerics miss exotic features. Suggest possible instability.

Thermodynamics

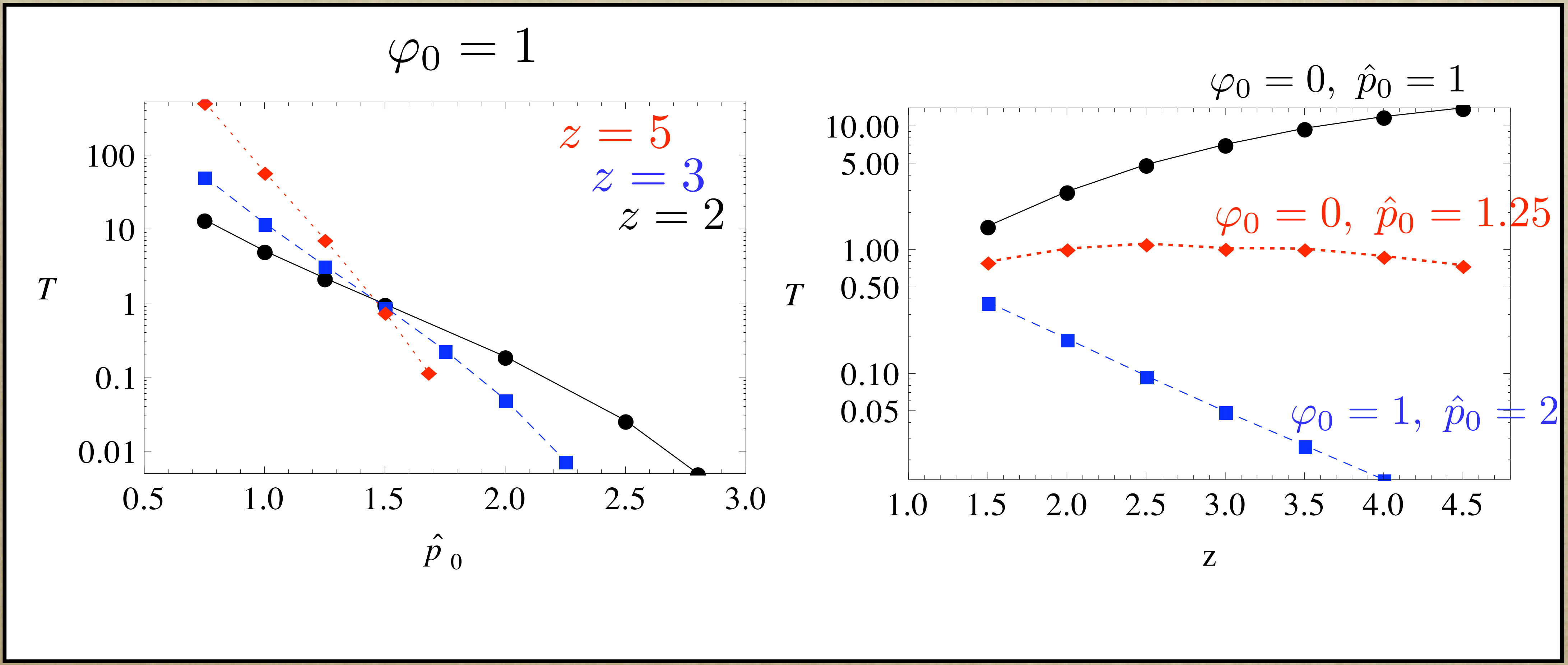
Temperature scales as r_+^z: $T = \frac{r_+^{z+1}}{4\pi}\sqrt{D'(r_+)F'(r_+)}$



- Scalar field first increases and then decreases the temperature. Finite amount of scalar charge you can add.
- Temperature decreases as gauge field increases near the horizon: "charging up" the BH. Looks like extremal limit.

Thermodynamics

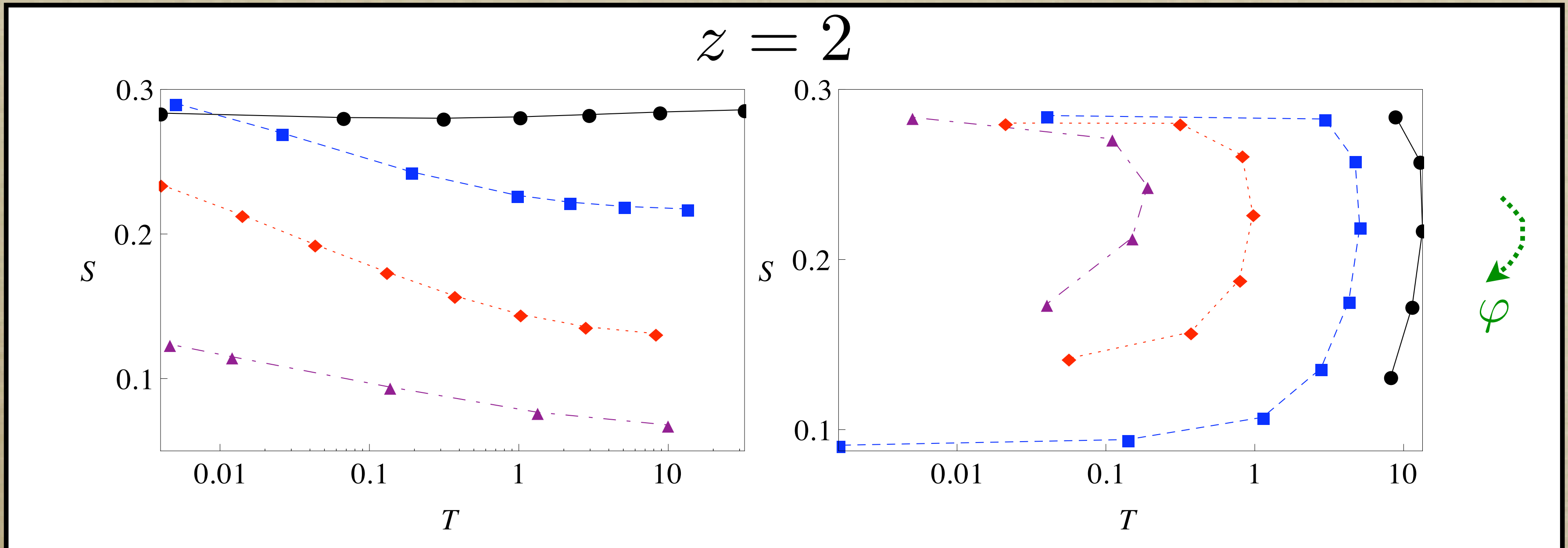
z dependence



- Changing z scales T away from T=1: though geometries seem smoother for higher z (previous plots), the range of charge becomes smaller.

Entropy

$$S = \frac{1}{4} r_+^2 H(r_+)$$



Varying gauge for scalar= 0, 1, 2, 3 Varying scalar for gauge= 0.75, 1, 1.5, 2

- Increasing scalar or gauge charge initial data lowers the entropy, though response to changing scalar is more dramatic.

- The presence of two equal temperature solutions with different entropy suggest that black hole will shed scalar charge to increase its entropy ⇒ indication of black hole instability.

Summary

- We have developed a prescription for embedding Lifshitz solutions into string theory for any z. (though could be issues with quantization from compactification of H_2)
- Found rich structure of black holes, though mostly has to be done numerically
- LiBH's are rather involved, with all fields switched on, and strongly distorted geometries near the horizon
- Analytic solutions? We found analytic solutions for similar systems with constant dilaton and Louville potential. However conditions on parameters don't match SUGRA we studied

$$\mathcal{L} = R - \frac{1}{2}(\partial\phi)^2 - V(\phi) - \frac{e^{-\lambda\phi}}{4}F_2^2 + \frac{e^{-\lambda\phi}\, m^2}{2}A_1^2 - \frac{e^{-\sigma\phi}}{4}\mathcal{F}_2^2$$

- Stability?