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Anomaly Cancelation & Abelian Gauge Symmetries in F-theory

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arXiv:1210.nnnn [hep-th]: M. Cvetič, T.W. Grimm, D.K.

Consistency of low-energy effective field theories

INTRODUCTION

EFFECTIVE FIELD THEORIES & CONSISTENCY

Low-energy physics from UV theory

- Start with **consistent UV theory** and **choice of vacuum**.
- Effective theory **automatically consistent** by consistency in UV.
  - Understand **mechanisms for consistency in IR** from UV.
  - **Learn about UV theory and its vacua** in return.

GOALS OF THIS TALK

Consistency of effective physics in 4D F-theory compactifications

Conditions arising from anomaly cancellation in 4D F-theory

Points addressed:

- Calculation of **chirality of 4D matter** on compact fourfolds.
- **Green-Schwarz-mechanism** in 4D F-theory.
- Construction of **U(1)-symmetries** in 4D F-theory.

Results:

- Relations between **4D and 3D physics**.
- Anomaly constraints ⇒ **geometric conditions on fourfolds**
- **Anomalies canceled** in concrete cases.

GOALS OF THIS TALK

Complications to obtain 4D F-theory effective action:

- **no 12-dimensional effective action of F-theory** as starting point for dimensional reduction
  ⇒ formulate **F-theory via dual M-theory.**
- Address **F-theory anomaly cancelation in M-theory**

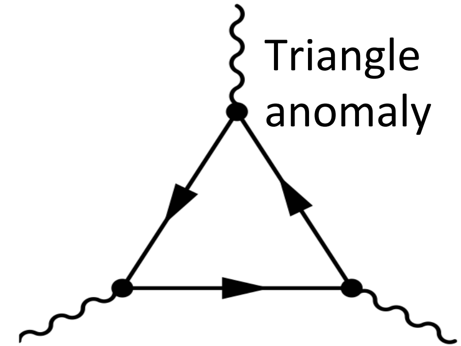
Weak coupling limit Type IIB with D7+O7:
[Many works,…, Plauschinn]

Anomalies and the Green-Schwarz mechanism

REVIEW OF ANOMALIES

Anomalies in field theory and gravity

- Anomaly is **breakdown of classical symmetry in quantum theory**
  - Violation of current conservation law at one loop
  - One-loop exact
  - Anomalies of **gauge & diffeomorphism symmetry**



- <u>Consider here:</u> four-dimensional gauge theory coupled to gravity
  - Gauge group $\widehat{G} = G \times U(1)_1 \times \cdots \times U(1)_{n_{U(1)}}$
    - Non-Abelian group G (product of simple groups)
    - **Abelian factors** $U(1)_m, \; m = 1, \ldots, n_{U(1)}$
  - Left-chiral Weyl fermions:
    - **Representation** $\mathbf{R}$ under G
    - U(1)-charges q_m under → **charge vector** $\underline{q}$
    - **Multiplicity** $n(\mathbf{R}_{\underline{q}})$

- Anomaly polynomial: only **left-chiral Weyl fermion**

$$I_6 = \sum_{\mathbf{R}_{\underline{q}}} n(\mathbf{R}_{\underline{q}}) \left(\frac{1}{6} \mathrm{tr}_{\mathbf{R}_{\underline{q}}} (iF^3_{\widehat{G}}) + \frac{1}{48} \mathrm{tr}_{\mathbf{R}_{\underline{q}}} (iF_{\widehat{G}}) \mathrm{tr} R^2 \right)$$

  - Gauge and regulator **invariant characterization of anomalies** (charact. class)
  - Regulator-dep. gauge **variation of quantum eff. action by descend-equation**

- Four qualitatively **different types of anomalies in 4D**: $F_{\widehat{G}} \longrightarrow F_G \, , F^m$

(A) Pure anomalies

- non-Abelian $\mathrm{tr}(F^3_G)$: $\quad \mathcal{A}^G = \sum_{\mathbf{R}_{\underline{q}}} n(\mathbf{R}_{\underline{q}}) V(\mathbf{R})$
- Abelian $F^m F^n F^k$: $\quad \mathcal{A}^{U(1)^3}_{mnk} = \frac{1}{6} \sum_{\underline{q}} n(\underline{q}) q_m q_n q_k$

(B) Mixed anomalies

- Abelian/non-Abelian $F^m \mathrm{tr}(F^2_G)$: $\mathcal{A}^{G-U(1)}_m = \frac{1}{2} \sum_{\mathbf{R}_{\underline{q}}} n(\mathbf{R}_{\underline{q}}) U(\mathbf{R}) q_m$
- Abelian/gravitational $F^m \mathrm{tr} R^2$: $\mathcal{A}^{\text{grav-U(1)}}_m = \frac{1}{48} \sum_{\underline{q}} n(\underline{q}) q_m$

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$\widehat{G}$-field strength   Riemann tensor

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Generalized Green-Schwarz mechanism

1. <u>Green-Schwarz counter term:</u> **topological term** for axions ρ_α

$$S_{\mathrm{GS}}^{(4)} = \frac{1}{8}\int \frac{2}{\lambda_G} b^{\alpha}_{(G)} \rho_\alpha \, \mathrm{tr}_{\mathbf{f}}(F_G \wedge F_G) + 2 b^{\alpha}_{mn} \rho_\alpha \, F^m \wedge F^n + \frac{1}{2} a^{\alpha} \rho_\alpha \, \mathrm{tr}(R \wedge R)$$

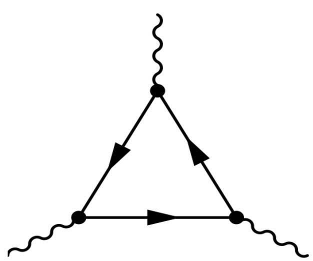
2. <u>Gaugings of axions:</u> **gauged shift symmetry**

$$\nabla \rho_\alpha = d\rho_\alpha + \Theta_{\alpha m} A^m,$$

$\Theta_{\alpha m}$ determines gauged symmetry

Green-Schwarz-mechanism

- Anomalous variation of $S_{\mathrm{GS}}^{(4)}$.
- **Cancel triangle anomaly by tree-level effect.**



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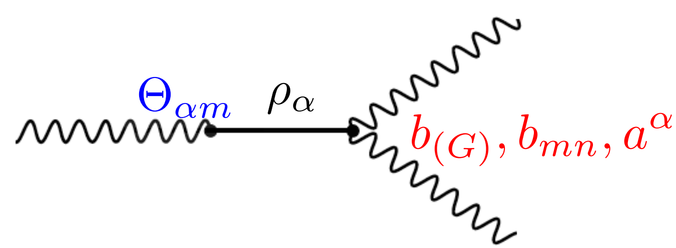


Abelian: Not factorizable

Abelian/non-Abelian: $\mathcal{A}^{U(1)^3}_{mnk} = \frac{1}{4\lambda_G} b^{\alpha}_{(mn} \Theta_{k)\alpha}$

Abelian: $\mathcal{A}^{grav-U(1)}_{mnk} = \frac{1}{16} a^{\alpha} \Theta_{m\alpha}$

Abelian/gravitational: $\mathcal{A}^{G-U(1)}_{m} = \frac{1}{4\lambda_G} b^{\alpha}_{(G)} \Theta_{\alpha m}$

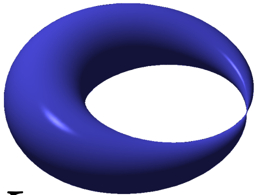
An introduction

F-THEORY AS M-THEORY

F-theory via Type IIB: Basic ingredients

- F-theory is geometric **SL(2,Z) invariant** formulation of Type IIB compactification on B_3: invariant object is **two-torus T^2** [Vafa; Review: Denef]

  ⇨ **singular fibration** of T^2 (auxiliary) over B_3.



[wikipedia]

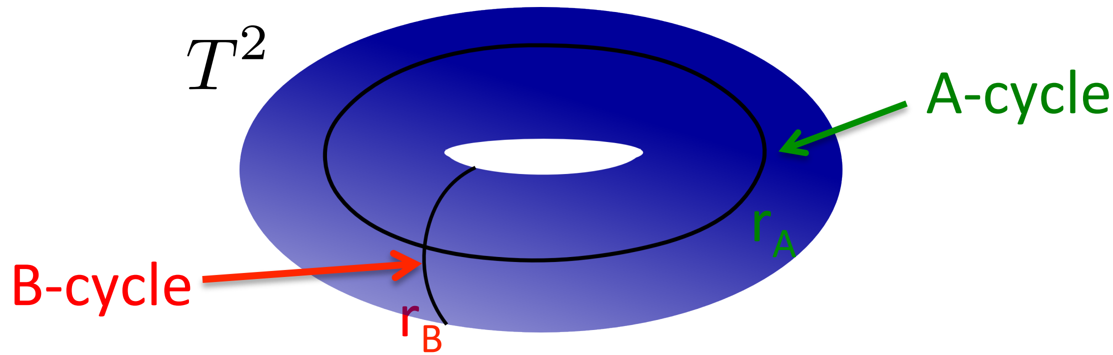
- **Singular elliptically fibered Calabi-Yau fourfold X_4:**
  - **gauge theory (ADE…)** in co-dim. one on S in B_3 (**7-branes**). [Morrison,Vafa,I&II; Many works]
  - **matter** in co-dim. two on $S \cap S'$ in B_3 (**intersecting 7-branes**). [Katz,Vafa]
  - **flux** on (G_4-flux) induces 4d matter

⇨ Four-dimensional N=1 effective theory specified by fourfold X_4.

- Gauge group $\widehat{G}$ and 4D matter in representations of $\widehat{G}$.

F-Theory via M-theory: Basic idea

- 12-dimensional **F-theory geometry is physical in M-theory**
- M-theory on $T^2 \times M_9$ is F-theory on $S^1 \times M_9$



- **M-/F-theory duality in 9D:**
  - A-cycle: M-theory circle radius r_A ⟷ Type IIA string coupling g_{IIA}
  - B-cycle: T-duality circle radius r_A/r_B ⟷ Type IIB string coupling g_{IIB}

(more generally to axio-dilaton)

- F-theory limit $\mathrm{vol}(T^2) = r_A \cdot r_B \to \infty$: grow 10[th] dimension in Type IIB

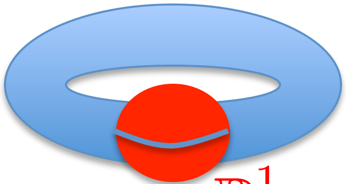
F-Theory via M-theory: Details

Generalization to **3D duality**:

M-theory on X_4 = F-theory on $X_4 \times S^1$.

➡ Study F-theory effective action in 3D M-theory

- **3D N=2 SUSY gauge theory** has **Coulomb branch**: $\widehat{G} \to U(1)^{\mathrm{rank}(\widehat{G})}$
  - VEV to Coulomb branch scalars in F-theory
  - Resolution $X_4 \to \hat{X}_4$ in M-theory



Massive states in M-theory and F-theory

M-theory on resolved $\hat{X}_4$:		F-theory on $X_4 \times S^1$:
M2-branes on resolution $\mathbb{P}^1$'s over S	⟺	massive W-bosons
M2-branes on resolving $\mathbb{P}^1$'s over $S \cap S'$	⟺	massive charged matter
M2-branes on elliptic fiber + $\mathbb{P}^1$'s	⟺	Kaluza-Klein states of matter

F-Theory via M-theory: Details

- M-theory on **smooth** $\hat{X}_4$: **11D SUGRA approximation**
  ⇒ membrane states automatically integrated out.

- F-theory on 3D Coulomb branch: **integrate out by hand**

Match of **3D Wilsonian effective actions** on Coulomb branch:

F-theory on X_4.

4D N=1 SUGRA, gauge group G,
massless matter in rep. R

M-theory/11D SUGRA on $\hat{X}_4$.

Resolved fourfold $\hat{X}_4$, G_4-flux

S^1-reduction
+integrate out

Classical
reduction

3D N=2 gauge theory on Coulomb branch,
$U(1)^{\mathrm{rank}(\widehat{G})}$ **massive states integrated out**

- Address questions about **4D anomalies on 3D Coulomb branch**:
  ⇒ Relevance of integrating out massive states!

The 3D Coulomb branch and geometric relations on elliptic fourfolds

ANOMALIES IN 4D F-THEORIES

The program

Agenda for study of anomaly cancelation in F-theory

In a given F-theory fourfold $\hat{X}_4$ **identify**

- the **gauge symmetry** realized.
- **chiral matter** and **4D chirality**: anomalies.
- **GS-terms** and **gaugings**: GS-mechanism.

Physical data encoded in key **geometrical data of resolved fourfold** $\hat{X}_4$ (new divisors) and G_4**-flux** (new forms).

Spin-Off: Learn something about **construction of U(1)'s** in F-theory from Abelian and mixed anomalies.

Gauge symmetry on resolved fourfolds

Structure of resolved fourfolds $\hat{X}_4$ for M-/F-theory compactifications

- Divisors and forms: $D_A = (B, D_\alpha, D_i, D_m)$, $\omega_A = (\omega_0, \omega_\alpha, \omega_i, \omega_m)$

Base B, vertical divisors (inherited) New resolution divisors

1. Non-Abelian gauge group: from new divisors D_i $i = 1, \ldots, \mathrm{rank}(G)$

$$D_i \cdot D_j \cdot D_\alpha \cdot D_\beta = -C_{ij} S_{(G)} \cdot B \cdot D_\alpha \cdot D_\beta$$

2. Abelian U(1)-factors: from new divisors D_m $m = 1, \ldots, n_{U(1)}$

$$D_m \cdot D_n \cdot D_\alpha \cdot D_\beta = -\delta_{mn} S_m^{U(1)} \cdot B \cdot D_\alpha \cdot D_\beta$$

- Mathematical origin of U(1)'s: D_m obtained from **Mordell-Weil group of rational sections** of $\hat{X}_4$ via Shioda map.

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Cartan matrix of G Divisor wrapped by rank(G) 7-branes

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No kinetic mixing Divisor wrapped by Abelian 7-brane

- Mathematical origin of U(1)'s: D_m obtained from **Mordell-Weil group of rational sections** of $\hat{X}_4$ via Shioda map.

Identification of Green-Schwarz-terms

Green-Schwarz terms are **determined topologically**

1. <u>7-brane gauge coupling functions:</u> **cpx. volumes** of $S_{(G)}$, $S^{U(1)}_{(m)}$

Expansion of 7-brane divisors

Non-Ab. 7-branes: $S_{(G)} = b^{\alpha}_{(G)} D_\alpha$

Ab. 7-branes: $S^{U(1)}_{(m)} \delta_{mn} = b^{\alpha}_{mn} D_\alpha$

GS-term with gauge fields

$$\int b^{\alpha}_{(G)} \rho_\alpha \mathrm{tr}_{\mathbf{f}}(F_G \wedge F_G) + b^{\alpha}_{mn} \rho_\alpha F^m \wedge F^n$$

2. <u>Curvature couplings:</u> (from comparison to **D7-brane CS-action**)

Canonical class of base

$$K_B = [-c_1(B)] = a^\alpha D_\alpha$$

GS-term with Riemann²

$$\int a^\alpha \rho_\alpha \mathrm{tr} R^2$$

- Gaugings of ρ_α involves **extra ingredient**: G_4-flux.

G_4-flux on resolved fourfolds for F-theory

- **Topologically non-trivial** vacuum expectation value of field strength G_4
  - Locally: $G_4 = \langle dC_3 \rangle$ - quantized [Witten]
- Consider **special fluxes** of the form (vertical flux)

[Greene,Morrison,Plesser]

$$G_4 = m^{AB} \omega_A \wedge \omega_B$$

G_4-fluxes in M-theory induce **effects in 3D action**

1. 3D Cherns-Simons terms:

$$S^{3D}_{\text{CS}} = \int \frac{1}{2} \Theta_{AB} A^A \wedge F^B \qquad \Theta_{AB} = \int_{\hat{X}_4} G_4 \wedge \omega_A \wedge \omega_B$$

[Haack,Louis]

   – Have to require some $\Theta_{AB} = 0$ **for F-theory lift** with unbroken group G.

2. Gaugings of axions: most importantly of ρ_α for GS-mechanism

$$D\rho_\alpha = d\rho_\alpha + \Theta_{\alpha m} A^m_{U(1)}$$

[Grimm;Grimm,Kerstan,Palti,Weigand]

4D Chirality from 3D CS-terms

- Determination of F-theory chiral index from M-theory:
  relevance of quantum effects on 3D Coulomb branch

4D F-theory from X_4	M-theory on resolved $\hat{X}_4$
4D massless chiral matter in R	11D SUGRA, G_4-flux
↓ S^1-reduction	↓ reduction

3D N=2 theory on Coulomb branch

- 3D massive charged matter
  $$m = \zeta^\Lambda \cdot q_\Lambda$$
- No 3D CS-terms classically
  $$\Theta^{\text{cl.}}_{\Sigma\Lambda} = 0$$

- No 3D matter
  integrated out automatically
- 3D CS-terms classically
  $$\Theta^{\text{cl.}}_{\Sigma\Lambda} = \Theta^{\text{flux}}_{\Sigma\Lambda}$$

- **Mismatch at classical level**: massive chiral matter, no CS-terms.

4D Chirality from 3D CS-terms

Match at quantum level: one-loop effective action

3D N=2 theory on Coulomb branch:

- 3D massive charged matter
  integrate out by hand
- 3D CS-terms at one loop

- No 3D matter
  integrated out automatically
- 3D CS-terms classically

$$\Theta^{\text{loop}}_{\Sigma\Lambda} = \frac{1}{2}\sum_{\mathbf{R}} n(\mathbf{R}) \sum_{q\in\mathbf{R}} q_\Sigma\, q_\Lambda\, \mathrm{sign}(q\cdot\zeta) \Longleftrightarrow \Theta^{cl.}_{\Sigma\Lambda} = \Theta^{\text{flux}}_{\Sigma\Lambda}$$

[Grimm,Hayashi]

Form of loop-correction:
[Aharony,Hanany,Intriligator,Seiberg,Strassler]

- **4D chirality encoded in 3D CS-terms.**
- Determine **4D chirality algorithmically** for general gauge groups [Grimm,DK]

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Charges under $U(1)^{\text{rk}(G)} \times U(1)^{n_{U(1)}}$

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SU(5), SU(5)xU(1)
Agreement with:

$$\int_{S_{\mathbf{R}}} G_4 = \chi(\mathbf{R}),$$

$S_{\mathbf{R}}$ =Matter surface

[Braun,Collinucci,Valandro;Marsano,Schaefer-Nameki;Krause,Mayrhofer,Weigand;Grimm,Hayashi]

Related: [Esole,Yau]

Checks of F-theory anomaly cancelation

[Grimm,DK,Cvetic]

Explicitly: **Toric examples**

1. Elliptic fourfold with SU(5)-singularity over $B_3 = \mathrm{Bl}_x(\mathbb{P}^3)$
   – Non-factorizable non-Abelian anomaly: No GS-mechanism

$$\chi(\mathbf{10}) + \chi(\mathbf{5}) = 0$$

   – **Automatically obeyed** for allowed fluxes.

2. Elliptic fourfold with SU(5)xU(1)-singularity: $B_3 = \mathrm{Bl}_x(\mathbb{P}^3),\ \mathbb{P}^1 \times \mathbb{P}^2$
   – Non-factorizable non-Abelian anomaly canceled automatically.
   – All mixed **anomalies canceled** by GS-mechanism.

Generally:

- Anomaly cancelation implies **relations of 3D Chern-Simons terms**
  – Implicit and hard to check because of loop-formula.
- Proof of anomaly cancelation for Abelian-gravitational anomaly.

Abelian-gravitational anomaly cancelation from M-/F-theory duality

[Grimm,DK,Cvetic]

- 3D Chern-Simons terms sensitive to **all charged massive states**
- Distinguished geo**metric 3D Chern-Simons term**

$$S_{\rm CS}^{(3D)} = \frac{1}{2}\int \Theta_{0m} A^0 \wedge F^m$$

1. Interpretation in F-theory
   - **Graviphoton A^0** and **4D U(1)-vectors A^m**:

$$ds^2_{\rm 4D} = r^2(dy - A^0)^2 + ds^2_{\rm 3d}$$

   - **Not generated classically** in S^1-reduction: one-loop effect

2. Non-vanishing in M-theory: $C_3 \supset A^0 \wedge \omega_0 + A^m \wedge \omega_m$,

$$\Theta_{0m} = \int_{\hat{X}_4} \omega_0 \wedge \omega_m \wedge G_4 = \frac{1}{2}\int_{\hat{X}_4} c_1(B_3) \wedge \omega_m \wedge G_4 = -\frac{1}{2} a^\alpha \Theta_{\alpha m}$$

- Generated by loops of graviphoton A^0-charged states

⇨ **Kaluza-Klein-states** of 4D charged matter.

Relevance of KK-states in 6D/5D: [Bonetti,Grimm,Hohenegger]

Abelian-gravitational anomaly cancelation from M-/F-theory duality

[Grimm,DK,Cvetic]

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1. Interpretation in F-theory
   - **Graviphoton A^0** and **4D U(1)-vectors A^m**:

$$ds^2_{4\text{D}} = r^2(dy - A^0)^2 + ds^2_{3\text{d}}$$

   - **Not generated classically** in S^1-reduction: one-loop effect [Park; Grimm,Savelli]

2. Non-vanishing in M-theory: $C_3 \supset A^0 \wedge \omega_0 + A^m \wedge \omega_m \,, \omega_0 = [B] + \frac{1}{2}c_1(B_3)$

Subtlety

$$\Theta_{0m} = \int_{\hat{X}_4} \omega_0 \wedge \omega_m \wedge G_4 = \frac{1}{2}\int_{\hat{X}_4} c_1(B_3) \wedge \omega_m \wedge G_4 = -\frac{1}{2}a^\alpha \Theta_{\alpha m}$$

- Generated by loops of graviphoton A^0-charged states

→ **Kaluza-Klein-states** of 4D charged matter.

Relevance of KK-states in 6D/5D: [Bonetti,Grimm,Hohenegger]

Abelian-gravitational anomaly from Kaluza-Klein states

[Grimm,DK,Cvetic]

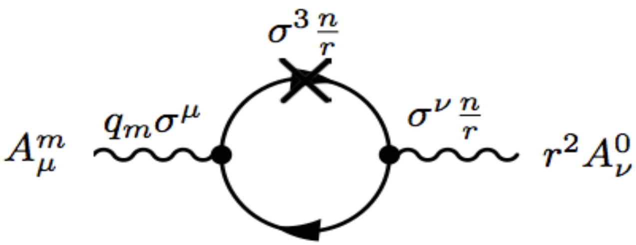
- **One-loop result** for Θ_{0m} from F-theory:

$$\Theta^{\text{loop}}_{0m} = \frac{1}{2}\sum_f \sum_{n=-\infty}^{\infty} n\, q^f_m \,\text{sign}(\zeta \cdot q^f + \frac{n}{r}) = -\frac{1}{12}\sum_{\underline{q}} n(\underline{q}) q_m$$

1) 3D loop of Kaluza-Klein states = 4D Abelian-gravitational anomaly
2) Identification with M-theory result → **4D anomaly cancelation**.

$$\frac{1}{3}\sum_{\underline{q}} n(\underline{q}) q_m = a^\alpha \Theta_{\alpha m}$$

- Diagrammatic: 4D triangle diagram → **infinite sum of 3D diagrams**

$\sum_n$ 

for each KK-state $\Psi(x,y) = \sum_n \Psi_n(x) e^{iyn}$

- Identify **3D Feynman rules**: $\mathcal{D}_\mu \Psi_n = (\partial_\mu + i q_\Lambda A^\Lambda_\mu + i n A^0_\mu)\Psi_n$

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Abelian-gravitational anomaly from Kaluza-Klein states

[Grimm,DK,Cvetic]

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ζ-function regularization

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A^0-charge $q_0=n$ Kaluza-Klein mass

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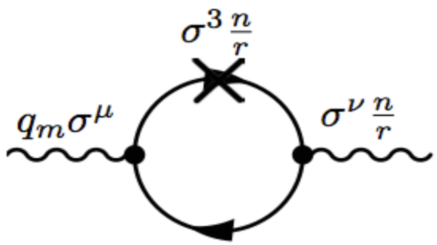
$\sum_n A_\mu^m$ 

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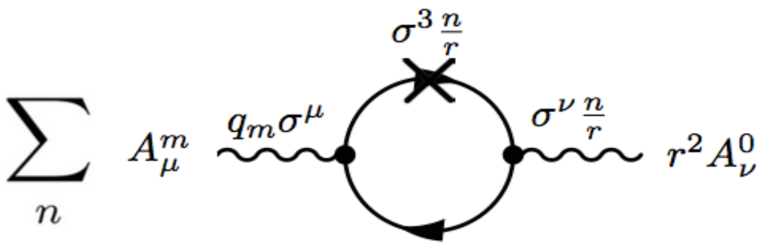

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3D vertex   3D KK-mass

Geometric relations on elliptic fourfolds from anomalies

[Grimm,DK,Cvetic]

- Explicit proof of anomaly cancelation hard
  - **Anomaly cancelation implicit** in structure of **3D Chern-Simons terms**.

- Reverse logic: **impose geometric relations** by anomaly cancelation

1. Pure non-Abelian, Abelian and mixed Abelian-non-Abelian:

$$\frac{1}{6}\sum_{S_{\mathbf{R}}}\sum_{c\subset S_{\mathbf{R}}}(S_{\mathbf{R}}\cdot[G_4])(c\cdot D_\Lambda)(c\cdot D_\Sigma)(c\cdot D_\Gamma)=\frac{1}{8}[G_4]\cdot D_{(\Gamma}\cdot\pi_*(D_\Lambda\cdot D_{\Sigma)})$$

2. Mixed Abelian-gravitational:

$$\frac{1}{48}\sum_{S_{\mathbf{R}}}\sum_{c\subset S_{\mathbf{R}}}(S_{\mathbf{R}}\cdot[G_4])(c\cdot D_\Lambda)=-\frac{1}{32}[G_4]\cdot[c_1(B_3)]\cdot D_\Lambda$$

→ Have to **hold on any resolved elliptic fourfold** for any F-theory G_4

- Purely geometric relations in 6D F-theory compactifications

[Park; Morrison,Park]

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  - c shrinkable wrapped by M2-branes ⟶ chiral matter in F-theory
  - G_4 allowed F-theory fluxes, D_Λ resolution divisors

⇒ Have to **hold on any resolved elliptic fourfold** for any F-theory G_4

- Purely geometric relations in 6D F-theory compactifications

[Park; Morrison,Park]

Summary

- **4D chiralities** effectively determined **from 3D CS-terms**.
- **Green-Schwarz mechanism** from fourfold intersection on $\hat{X}_4$ and by chirality-inducing G_4-flux.
- Improved understanding of **U(1)'s on fourfolds**.
- **Anomaly cancelation checked** explicitly in compact F-theory GUT-models with U(1)'s **Local models: additional conditions [Palti]**
  ⇒ **straightforward extension** to SO(10), E6, more U(1)'s…
- Anomaly cancelation **implies relations among 3D CS-terms**.
- Abelian-graviational anomaly derived from 3D **Kaluza-Klein loop-calculation** of CS-terms & match to **classical M-theory**.
- Anomaly cancelation implies **geometric relations** on all elliptically fibered Calabi-Yau fourfolds.
  ⇒ Mathematical proof?