Non-geometric fluxes in higher dimensions

based on 1106.4015, 1202.3060 and 1204.1979 in collaboration with

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ARNOLD SOMMERFELD CENTER FOR THEORETICAL PHYSICS

Outline

What is non-geometry?

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1. Non-geometry
    - Introduction
    - Examples
2. Non-geometric fluxes
    - Non-geometric flux compactification
    - Algebraic structures in higher dimensions

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    - Supergravity
    - Double field theory
    - Connecting both

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Non-geometry

General notion

- String theory has more symmetries than point particles
  – in particular: T-duality

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General notion

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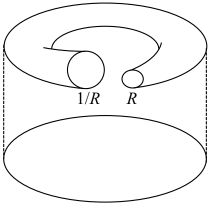
What does this mean geometrically?

More concrete

- Structure group includes $O(d, d)$ transformations [Dabholkar, Hull: 2005] [Hellerman, McGreevy, Williams: 2002]
- Target space not a manifold anymore
- Still consistent string backgrounds [Hull: 2004]



Non-geometry

Features of non-geometry

- Non-trivial monodromies
  $\approx$ "target space with ill-defined fields"
- Non-commutativity, non-associativity
- Constructions with no straightforward target space interpretation (T-folds, etc.)

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Examples

1. T-dual of a geometric background
2. Asymmetric orbifold

Example 1: Torus with H-flux

Construction

- T^2 fibration over S^1, coordinates X^1, X^2 and X^3
- Linear b-field with $H_{123} = 1$

$$g = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad b = \begin{pmatrix} 0 & X^3 & 0 \\ -X^3 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

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- $X^3 \to X^3 + 1$ induces b-field gauge transformation, model **globally well-defined**

Example 1: Torus with H-flux

Non-geometric frame

- Perform **two T-dualities** in the isometry directions of X^1, X^2

$$g = \frac{1}{1+(X^3)^2}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1+(X^3)^2 \end{pmatrix}, \quad b = \frac{1}{1+(X^3)^2}\begin{pmatrix} 0 & -X^3 & 0 \\ X^3 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

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- $X^3 \to X^3 + 1$ cannot be patched by diffeomorphism or gauge transformation
  $\rightarrow$ fields globally **ill-defined**

Example 1: Non-commutativity and non-associativity

Target space?

- Investigate monodromies of complex structure τ and Kähler parameter ρ
- Expect **non-commutativity** for this background:

$$\left[X^1,\ X^2\right] \sim N^3$$

[Lüst: 2010] [Andriot, Larfors, Lüst, Patalong: to appear]

- Similar to open string case [Seiberg, Witten: 1999]
- After a third (formal) T-duality, expect **non-associativity**

$$\left[\left[X^1,\ X^2\right],\ X^3\right] + \text{perm.} \neq 0$$

[Blumenhagen, Deser, Lüst, Plauschinn, Rennecke: 2011]

Example 2: Asymmetric orbifold

[Condeescu, Florakis, Lüst: 2012]

- Twisted three-torus, freely acting $\mathbb{Z}_4$ orbifold
- Asymmetric boundary conditions on $Z \sim X^1 + \mathrm{i}X^2$:

$$Z_L(\tau, \sigma + 2\pi) = e^{2\pi\mathrm{i}\theta} Z_L(\tau, \sigma)$$
$$Z_R(\tau, \sigma + 2\pi) = e^{-2\pi\mathrm{i}\theta} Z_R(\tau, \sigma)$$

- Induced by transformation of the Kähler parameter

$$\rho(X^3) \to -\rho(X^3)^{-1}$$

  that is a non-trivial element of the T-duality group
- **Non-commutative coordinates**

$$\left[X^1,\ X^2\right] = \mathrm{i}\Theta(\theta, N^3) \neq 0$$

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Non-geometric fluxes: 4 dimensions

Non-geometric flux compactification

- In gauged supergravity: [Shelton, Taylor, Wecht: 2006]

$$
\begin{aligned}
[Z_a, Z_b] &= H_{abc}X^c + f^c{}_{ab}Z_c \\
[Z_a, X^b] &= -f^b{}_{ac}X^c \\
[X^a, X^b] &= 0
\end{aligned}
$$

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\begin{aligned}
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- T-duality chain: $H_{abc} \;\rightarrow\; f^b{}_{ac} \;\rightarrow\; Q_c{}^{ab} \;\rightarrow\; R^{abc}$

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Phenomenological impact

- Moduli fixing [Micu, Palti, Tasinato: 2007]
- De Sitter vacua [de Carlos, Guarino, Moreno: 2009]

Non-geometric fluxes: higher dimensions

Main idea

- Use **generalised complex geometry**
  - T-duality group $O(D, D)$ is structure group of $TM \oplus T^*M$
  - Metric g and b-field b embedded in one object $\mathcal{H}$
  - Convenient framework in type II flux compactifications

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- **Courant bracket** for generalised vectors and basis $\mathcal{E}_A = \{e_a, e^a\}$ to define algebra

$$[\mathcal{E}_A, \mathcal{E}_B] = F^C{}_{AB}\mathcal{E}_C$$

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- Non-geometric fluxes are **components** of structure coefficients

[Graña, Minasian, Petrini, Waldram: 2008]

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Refined investigation

- Turn $TM \oplus T^*M$ into a Courant algebroid with

$$
\begin{aligned}
[\![e_a, e_b]\!] &= \mathcal{H}_{abc}\; e^c + \mathcal{F}^c{}_{ab}\; e_c \\
[\![e_a, e^b]\!] &= -\mathcal{F}^b{}_{ac}\; e^c + \mathcal{Q}_a{}^{bc}\; e_c \\
[\![e^a, e^b]\!] &= \mathcal{Q}_c{}^{ab}\; e^c + \mathcal{R}^{abc}\; e_c
\end{aligned}
$$

- Recover algebraic structure from 4 dimensions

[Blumenhagen, Deser, Plauschinn, Rennecke: 2012]

Central questions

Connection between the two central ideas?

$$\boxed{\text{Non-geometry}} \quad \stackrel{?}{\longleftrightarrow} \quad \boxed{\text{Non-geometric fluxes}}$$

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- Supergravity, double field theory, generalised geometry, doubled worldsheet theories, T-folds, matrix models, ...

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Our approach

What?

- Introduce non-geometric fluxes manifestly
- Relate them to the algebraic structures shown before

How?

- Central object in non-geometry: bivector β
- Perform **field redefinition** to reveal β

Supergravity: Procedure

$$\boxed{\hat{\mathcal{L}}(\hat{g}, \hat{b}, \hat{\phi}) \;\rightarrow\; \tilde{\mathcal{L}}(\tilde{g}, \tilde{\beta}, \tilde{\phi})}$$

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Where is β?

- Inspiration from generalised geometry: use **generalised metric**

$$\mathcal{H} = \mathcal{E}^T \mathcal{E}$$

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$$\mathcal{H} = \mathcal{E}^T \mathcal{E} = \begin{pmatrix} \hat{g} - \hat{b}\hat{g}^{-1}\hat{b} & \hat{b}\hat{g}^{-1} \\ -\hat{g}^{-1}\hat{b} & \hat{g}^{-1} \end{pmatrix}$$

- Like any metric, parametrise by **different vielbeins**

$$\hat{\mathcal{E}} = \begin{pmatrix} \hat{e} & 0 \\ -\hat{e}^{-T}\hat{b} & \hat{e}^{-T} \end{pmatrix}$$

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$$\hat{\mathcal{E}} = \begin{pmatrix} \hat{e} & 0 \\ -\hat{e}^{-T}\hat{b} & \hat{e}^{-T} \end{pmatrix}, \quad \tilde{\mathcal{E}} = \begin{pmatrix} \tilde{e} & \tilde{e}\tilde{\beta} \\ 0 & \tilde{e}^{-T} \end{pmatrix}$$

Supergravity: Field redefinition

- Read off **transformation rules**

$$\hat{g} = (\tilde{g}^{-1} + \tilde{\beta})^{-1}\tilde{g}^{-1}(\tilde{g}^{-1} - \tilde{\beta})^{-1}$$
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- Recognise nasty inverse, nevertheless plug into

$$\hat{\mathcal{L}} = e^{-2\hat{\phi}}\sqrt{|\hat{g}|}\left(\hat{\mathcal{R}} + 4|\partial\hat{\phi}|^2 - \frac{1}{2}|\hat{H}|^2\right)$$

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- Lots of work...

$$
\begin{aligned}
|\mathrm{d}\hat{\phi}|^2 - |d\tilde{\phi}|^2 = {} & \tfrac{1}{4}\tilde{g}^{km}\tilde{g}^{pq}\tilde{g}^{uv}\partial_m\tilde{g}_{pq}\partial_k\tilde{g}_{uv} \\
& + \tfrac{1}{2}\tilde{g}^{km}\tilde{g}^{pq}(G^{-1})_{uv}\partial_m\tilde{g}_{pq}\partial_k G^{vu} \\
& + \tfrac{1}{4}\tilde{g}^{km}(G^{-1})_{pl}(G^{-1})_{uv}\partial_m G^{lp}\partial_k G^{vu} \\
& - \tilde{g}^{km}\tilde{g}^{pq}\partial_k\tilde{g}_{pq}\partial_m\tilde{\phi} \\
& - \tilde{g}^{km}(G^{-1})_{pq}\partial_k G^{qp}\partial_m\tilde{\phi}
\end{aligned}
$$

$$
\begin{aligned}
|\mathrm{d}\hat{\phi}|^2 - |d\tilde{\phi}|^2 = {} & \tfrac{1}{4}\tilde{g}^{km}\tilde{g}^{pq}\tilde{g}^{uv}\partial_m\tilde{g}_{pq}\partial_k\tilde{g}_{uv} \\
& + \tfrac{1}{2}\tilde{g}^{km}\tilde{g}^{pq}(G^{-1})_{uv}\partial_m\tilde{g}_{pq}\partial_k G^{vu} \\
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& - \tilde{g}^{km}(G^{-1})_{pq}\partial_k G^{qp}\partial_m\tilde{\phi}
\end{aligned}
$$

$$
\begin{aligned}
\hat{\mathcal{R}} - \tilde{\mathcal{R}} = {} & + \partial_k\tilde{g}_{su}\partial_m\tilde{g}_{pq}\Big(2\tilde{g}^{km}\tilde{g}^{uq}\tilde{g}^{ps} + 2\tilde{g}^{pq}\tilde{g}^{ks}\tilde{g}^{mu} + \frac{1}{2}\tilde{g}^{uq}\tilde{g}^{sm}\tilde{g}^{kp}\Big) \\
& - \tilde{g}_{pq}\partial_k\tilde{\beta}^{pk}\partial_m\tilde{\beta}^{qm} - \frac{1}{2}\tilde{g}_{pq}\partial_k\tilde{\beta}^{qm}\partial_m\tilde{\beta}^{pk} \\
& + 2\tilde{g}^{km}\tilde{g}^{pq}\partial_k\partial_m\tilde{g}_{pq} + 2\tilde{g}^{km}(G^{-1})_{pq}\partial_k\partial_m G^{qp} \\
& + \partial_m G^{vl}\Big(-2\tilde{g}^{mr}\tilde{g}^{ks}(G^{-1})_{lv}\partial_k\tilde{g}_{rs} - \tilde{g}^{rs}\tilde{g}^{km}(G^{-1})_{lv}\partial_k\tilde{g}_{rs} \\
& \qquad + \tilde{g}^{ms}\tilde{g}^{ru}(G^{-1})_{lu}\partial_v\tilde{g}_{rs} - \tilde{g}^{km}\tilde{g}^{rs}(G^{-1})_{ls}\partial_k\tilde{g}_{vr}\Big) \\
& + \partial_m G^{vl}\Big((G^{-1})_{lq}\partial_v G^{qm} + \frac{1}{2}\hat{g}_{lq}\partial_v G^{mq}\Big) \\
& - \partial_m G^{vl}\partial_k G^{ps}\frac{1}{2}\tilde{g}^{km}\Big(2(G^{-1})_{lv}(G^{-1})_{sp} + 5(G^{-1})_{sv}(G^{-1})_{lp} + \hat{g}
\end{aligned}
$$

$$|\mathrm{d}\hat\phi|^2 - |d\tilde\phi|^2 = \frac14 \tilde g^{km}\tilde g^{pq}\tilde g^{uv}\partial_m\tilde g_{pq}\partial_k\tilde g_{uv} + \frac12 \tilde g^{km}\tilde g^{pq}(G^{-1})_{uv}\partial_m\tilde g_{pq}\partial_k G^{vu} + \frac14 \tilde g^{km}(G^{-1})_{pl}(G^{-1})_{uv}\partial_m G^{lp}\partial_k G^{vu} - \tilde g^{km}\tilde g^{pq}\partial_k\tilde g_{pq}\partial_m\tilde\phi - \tilde g^{km}(G^{-1})_{pq}\partial_k G^{qp}\partial_m\tilde\phi$$

$$\begin{aligned}\hat{\mathcal R} - \tilde{\mathcal R} = &+ \partial_k\tilde g_{su}\partial_m\tilde g_{pq}\left(2\tilde g^{km}\tilde g^{uq}\tilde g^{ps} + 2\tilde g^{pq}\tilde g^{ks}\tilde g^{mu} + \frac12\tilde g^{uq}\tilde g^{sm}\tilde g^{kp}\right)\\ &- \tilde g_{pq}\partial_k\tilde\beta^{pk}\partial_m\tilde\beta^{qm} - \frac12\tilde g_{pq}\partial_k\tilde\beta^{qm}\partial_m\tilde\beta^{pk}\\ &+ 2\tilde g^{km}\tilde g^{pq}\partial_k\partial_m\tilde g_{pq} + 2\tilde g^{km}(G^{-1})_{pq}\partial_k\partial_m G^{qp}\\ &+ \partial_m G^{vl}\Big(-2\tilde g^{mp}\tilde g^{ks}(G^{-1})_{lv}\partial_k\tilde g_{ps} - \tilde g^{rs}\tilde g^{km}(G^{-1})_{lv}\partial_k\tilde g_{rs}\\ &\quad + \tilde g^{ms}\tilde g^{ru}(G^{-1})_{lu}\partial_v\tilde g_{rs} - \tilde g^{km}\tilde g^{rs}(G^{-1})_{ls}\partial_k\tilde g_{vr}\Big)\\ &+ \partial_m G^{vl}\left((G^{-1})_{lq}\partial_v G^{qm} + \frac12 (G^{-1})_{lq}\partial_v G^{mq}\right)\\ &- \partial_m G^{vl}\partial_k G^{ps}\frac12\tilde g^{km}\Big(2(G^{-1})_{lv}(G^{-1})_{sp} + 5(G^{-1})_{sv}(G^{-1})_{lp} + \hat g_{\dots}\end{aligned}$$

$$\begin{aligned}|\hat H|^2 = &\frac12\tilde g_{p_1p_2}\tilde g_{q_1q_2}\tilde g^{s_1s_2}\partial_{s_1}\tilde\beta^{p_1q_1}\partial_{s_2}\tilde\beta^{p_2q_2}\\ &+ (\hat g_{q_1q_2} - \tilde g_{q_1q_2})\partial_{p_1}\tilde g^{p_1q_1}\partial_{p_2}\tilde g^{p_2q_2}\\ &+ \tilde g^{s_1s_2}\partial_{s_1}G_\epsilon^{p_1q_1}\left(2\tilde g_{p_1p_2}(G_\epsilon^{-1})_{q_1q_2} + \frac12\hat g_{q_1q_2}\partial_{s_2}\tilde g^{p_2q_2}\right) \left(\tilde g_{p_1p_2}\hat g_{q_1q_2} + (G_\epsilon^{-1})_{q_2\dots}\right.\end{aligned}$$

Supergravity: Result

- Magic cancellations! Reveal non-geometric fluxes

$$\tilde{\mathcal{L}} = e^{-2\tilde{\phi}}\sqrt{|\tilde{g}|}\left(\tilde{\mathcal{R}} + 4|\partial\tilde{\phi}|^2 - \frac{1}{2}|Q|^2 - \frac{1}{2}|R|^2 + \ldots\right)$$

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with fluxes

$$Q_p{}^{mn} = \partial_p \tilde{\beta}^{mn}\ , \quad R^{mnp} = 3\tilde{\beta}^{k[m}\nabla_k\tilde{\beta}^{np]}$$

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- Contact with 4d: **dimensional reduction**

$$V_\omega \sim \sigma^{-2}\rho^{-1} \;, \quad V_H \sim \sigma^{-2}\rho^{-3} \;, \quad V_Q \sim \sigma^{-2}\rho$$

[Hertzberg, Kachru, Taylor, Tegmark: 2007]

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Tame non-geometry?

- Total derivative is important

$$\hat{\mathcal{L}}(\hat{g}, \hat{b}, \hat{\phi}) \;=\; \tilde{\mathcal{L}}(\tilde{g}, \tilde{\beta}, \tilde{\phi})$$

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- For problematic monodromies

ill-defined $\hat{g}, \hat{b}$

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ill-defined $\hat{g}, \hat{b} \quad \rightarrow \quad$ well-defined $\tilde{g}, \tilde{\beta}$

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- For problematic monodromies

$$\text{ill-defined } \hat{g}, \hat{b} \quad \longrightarrow \quad \text{well-defined } \tilde{g}, \tilde{\beta}$$

- Costs: total derivative becomes ill-defined

Supergravity: Result

Tame non-geometry?

- Total derivative is important

$$\hat{\mathcal{L}}(\hat{g},\hat{b},\hat{\phi}) \;=\; \tilde{\mathcal{L}}(\tilde{g},\tilde{\beta},\tilde{\phi}) + (\textbf{total derivative})$$

- For problematic monodromies

$$\text{ill-defined } \hat{g},\hat{b} \quad \rightarrow \quad \text{well-defined } \tilde{g},\tilde{\beta}$$

- Costs: total derivative becomes ill-defined
- **Prescription**: drop the total derivative, take $\tilde{\mathcal{L}}$ as the correct theory

Supergravity: Problem

$$\tilde{\mathcal{L}} = e^{-2\tilde{\phi}}\sqrt{|\tilde{g}|}\left(\tilde{\mathcal{R}} + 4|\partial\tilde{\phi}|^2 - \frac{1}{2}|Q|^2 - \frac{1}{2}|R|^2 + \dots\right)$$

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- Not all terms identified
- Q not a tensor
  $\rightarrow$ Must be part of $\widetilde{\mathcal{R}}$, entering like torsion

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Try to find a geometric interpretation for non-geometric Q-flux!

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Strategy

- Perform field redefinition on the DFT action
- Find Q as connection, R as tensor

DFT: Field redefinition 1

- **Step 1**: Relation from the supergravity context

$$\tilde{\mathcal{E}}(X) = \left(\tilde{g}^{-1} + \beta\right)(x, \tilde{x}) = (g + b)^{-1}(x, \tilde{x}) = \mathcal{E}^{-1}(X)$$

Problematic inverse!

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- **Step 2**: T-duality in Double Field Theory

$$\mathcal{E}'(X') = (a\mathcal{E}(X) + b)(c\mathcal{E}(X) + d)^{-1}, \quad X' = hX$$

with

$$h = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in O(D, D) \quad \text{and} \quad X = (x, \tilde{x})$$

DFT: Field redefinition 2

- Special case: T-duality in **all directions**

$$h = \begin{pmatrix} 0 & \mathbb{1} \\ \mathbb{1} & 0 \end{pmatrix} \Rightarrow \mathcal{E}'(\tilde{x}, x) = \mathcal{E}^{-1}(x, \tilde{x})$$

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Rules for the field redefinition

$$g' \to \tilde{g}^{-1} \;, \quad b' \to \beta \;, \quad \partial_i \leftrightarrow \tilde{\partial}^i$$

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- Each term of $S_{\text{DFT}}(\mathcal{E}, d)$ is **invariant separately**
- Dilaton density e^{-2d} remains unchanged

DFT: Field redefinition 3

Effectively: **Simply invert all indices!** No calculation at all!

$$e^{-2d}\Big[-\frac{1}{4}\, g^{ik} g^{jl}\, g^{pq} \Big(\mathcal{D}_p\mathcal{E}_{kl}\,\mathcal{D}_q\mathcal{E}_{ij} - \mathcal{D}_i\mathcal{E}_{lp}\mathcal{D}_j\mathcal{E}_{kq} - \bar{\mathcal{D}}_i\mathcal{E}_{pl}\,\bar{\mathcal{D}}_j\mathcal{E}_{qk}\Big)$$
$$+\, g^{ik} g^{jl}\big(\mathcal{D}_i d\;\bar{\mathcal{D}}_j\mathcal{E}_{kl} + \bar{\mathcal{D}}_i d\;\mathcal{D}_j\mathcal{E}_{lk}\big) + 4g^{ij}\mathcal{D}_i d\,\mathcal{D}_j d\;\Big]$$

$$\downarrow$$

$$e^{-2d}\Big[-\frac{1}{4}\, \tilde{g}_{ik}\tilde{g}_{jl}\tilde{g}_{pq} \Big(\tilde{\mathcal{D}}^p\tilde{\mathcal{E}}^{kl}\,\tilde{\mathcal{D}}^q\tilde{\mathcal{E}}^{ij} - \tilde{\mathcal{D}}^i\tilde{\mathcal{E}}^{lp}\tilde{\mathcal{D}}^j\tilde{\mathcal{E}}^{kq} - \bar{\tilde{\mathcal{D}}}^i\tilde{\mathcal{E}}^{pl}\,\bar{\tilde{\mathcal{D}}}^j\tilde{\mathcal{E}}^{qk}\Big)$$
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- **Strategy**: rewrite the action in terms of covariant objects

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- Define new **Riemann tensor** and scalar curvature

DFT: Rewritten action

$$S_{\mathrm{DFT}} \;=\; \int dx d\tilde{x}\, \sqrt{|\tilde{g}|}\, e^{-2\tilde{\phi}} \Big[\mathcal{R} + \check{\mathcal{R}} - \frac{1}{12} R^{ijk} R_{ijk} \\ + 4(\partial\tilde{\phi})^2 + 4\big(\tilde{D}^i \tilde{\phi} + \mathcal{T}^i\big)^2 \Big]$$

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Geometric interpretation for non-geometric fluxes

Connection to the literature

Formalism matches nicely with recent results:

- R-flux expression in **Scherk-Schwarz reductions** of DFT

  [Aldazabal, Baron, Marques, Nunez: 2011][Geissbühler: 2011]

$$R^{ijk} = 3\left(\tilde{\partial}^{[i}\beta^{jk]} + \beta^{p[i}\partial_p\beta^{jk]}\right)$$

- **Bianchi** identity [Blumenhagen, Deser, Plauschinn, Rennecke: 2012]

$$\tilde{\nabla}^{[i}R^{jkl]} = 0$$

$\rightarrow$ Recover algebraic structure known before

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$\boxed{\text{DFT} \quad \mathcal{E}_{ij}}$

Connection to supergravity

$$\begin{tikzcd}
\boxed{\text{DFT} \quad \mathcal{E}_{ij}} \arrow[d, "\tilde{\partial}^i = 0"] \\
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- **Non-geometric fluxes** can be made visible in supergravity and DFT.
- The appearance of **non-geometry** is connected to the appearance of non-geometric fluxes.
- DFT allows for a **geometric** interpretation of non-geometric fluxes.

Outlook

- Connection to **doubled worldsheets**? (SGN's talk on Friday)

  [Groot Nibbelink, Patalong: 2012]

- Connection to supergravity as **generalised geometry**?

  [Coimbra, Strickland-Constable, Waldram: 2011]

- String theory on a **Courant algebroid**?