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Discrete symmetries in semi-realistic orientifold compactifications

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work in progress

with P. Anastasopoulos, M. Cvetic, J. Halverson,and P. Vaudrevange.

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Plan of the talk

- Motivation
- Discrete gauge symmetries in the MSSM
- Intersecting D-Brane Models
- Bottom-up search
- Discrete symmetries in D-brane compactifications
- Discrete symmetries in local D-brane setups

Motivation

- Superpotential terms invariant under the Standard model gauge symmetries

$$W = Q_L H_u u_R + Q_L H_d d_R + L H_d E_R + Q_L L d_R + d_R d_R u_R + L L E_R$$

- the first three terms gives masses to quarks and leptons
- the last three terms, if present, lead to dangerously high proton decay rate
- there are allowed dangerous dimension 5 operators $Q_L Q_L Q_L L$ and $u_R u_R d_R E_R$
- discrete symmetries may explain the absence of these undesired terms
- However, global discrete symmetries are expected to be violated in consistent theory of quantum gravity
- an exception are discrete symmetries that have a gauge symmetry origin

$\rightsquigarrow$ discrete gauge symmetries

Motivation

- Those discrete gauge symmetries are subject to discrete anomaly cancellation conditions, just as normal gauge symmetries are.

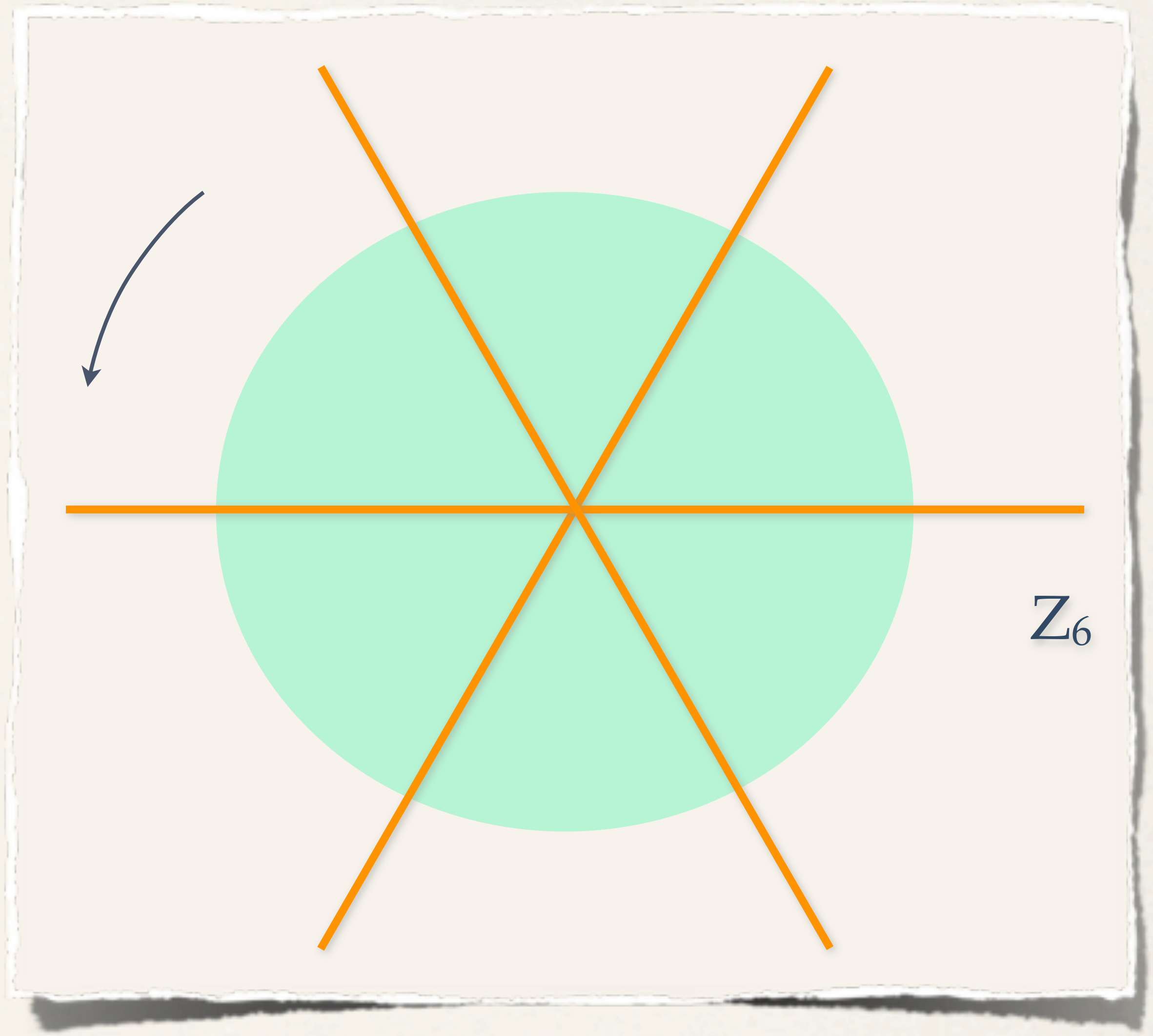
- Intriguing discrete gauge symmetries:

  - Matter parity: M_2 forbids the R-parity violating terms

  - Baryon triality: B_3 forbids the dangerous dimension 5 operators

  - Proton hexality: P_6 forbids the R-parity violating terms and the dangerous dimension 5 operators

- In this talk we will investigate the presence and the role of discrete gauge symmetries in local D-brane realizations of the MSSM.



Discrete gauge symmetries in the MSSM

Discrete gauge symmetries in the MSSM

- Discrete gauge symmetries are subject to discrete anomaly cancellations
- Classification of all family independent discrete gauge symmetries within the MSSM:

Dreiner, Luhn, Thormeier

  - Cancellation of mixed, cubic and gravitational anomalies:

$$A_{SU(3) \times SU(3) \times Z_N}\ , \quad A_{SU(2) \times SU(2) \times Z_N}\ , \quad A_{Z_N \times Z_N \times Z_N}\ , \quad A_{G \times G \times Z_N}$$

  - Allowed Yukawa couplings : $Q_L H_u U$, $Q_L H_d D$, $L H_d E$

- Any family independent discrete gauge symmetry of the MSSM can be expressed as:

$$g_N = R_N^m \times A_N^n \times L_N^p \quad \text{where} \quad m, n, p = 0, 1, \ldots N-1$$

Ibanez, Ross

Discrete gauge symmetries in the MSSM

- The MSSM particles are charged under these independent Z_N gauge symmetries:

	Q^i	U^i	D^i	L^i	E^i	N^i	H_u	H_d
R	0	−1	1	0	1	−1	1	−1
A	0	0	0	−1	1	1	0	0
L	0	0	−1	−1	0	1	0	0

- All possible family independent discrete gauge symmetry have been classified
- They belong to classes of $\mathbf{Z}_2$, $\mathbf{Z}_3$, $\mathbf{Z}_6$, $\mathbf{Z}_9$, $\mathbf{Z}_{18}$ Dreiner, Luhn, Thormeier
  - Matter parity: $M_2 = R_2$
  - Baryon triality: $B_3 = R_3 L_3$
  - Proton hexality: $P_6 = R_6^5 L_6^2$

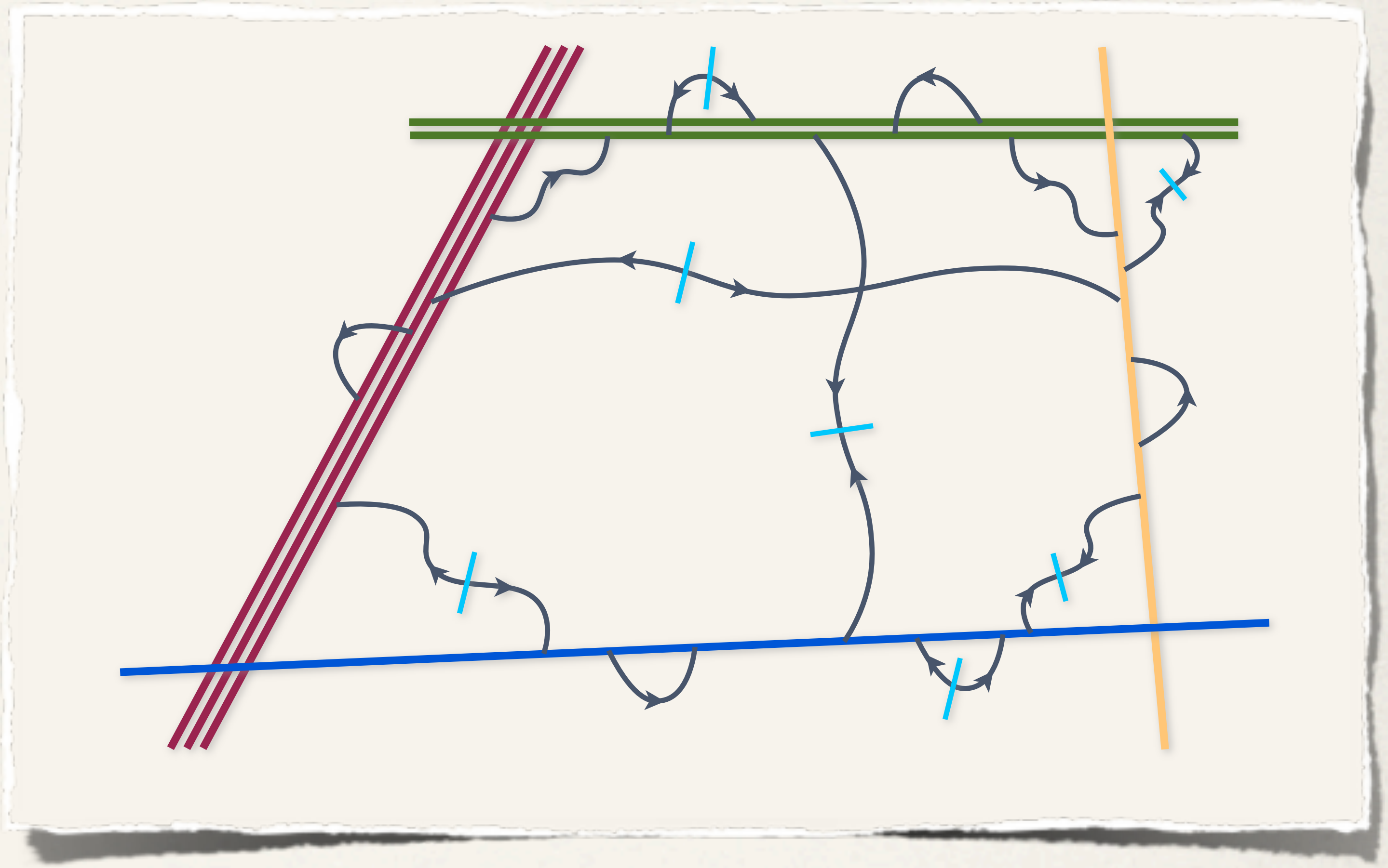
Discrete gauge symmetries vs couplings

- Physical consequences of the discrete gauge symmetries:

	R_2	R_3L_3	R_3	L_3	$R_3^2L_3$	$R_6^5L_6^2$	R_6	$R_6^3L_6^2$	$R_6L_6^2$	all Z_9 & Z_{18}
H_uH_d	✓	✓	✓	✓	✓	✓	✓	✓	✓	
H_uL		✓								
$LL\bar{E}$		✓								
$LQ\bar{D}$		✓								
$\bar{U}\bar{D}\bar{D}$				✓						
$QQQL$	✓		✓				✓			
$\bar{U}\bar{U}\bar{D}\bar{E}$	✓		✓				✓			
LH_uLH_u	✓	✓				✓				

Dreiner Luhn Thormeier

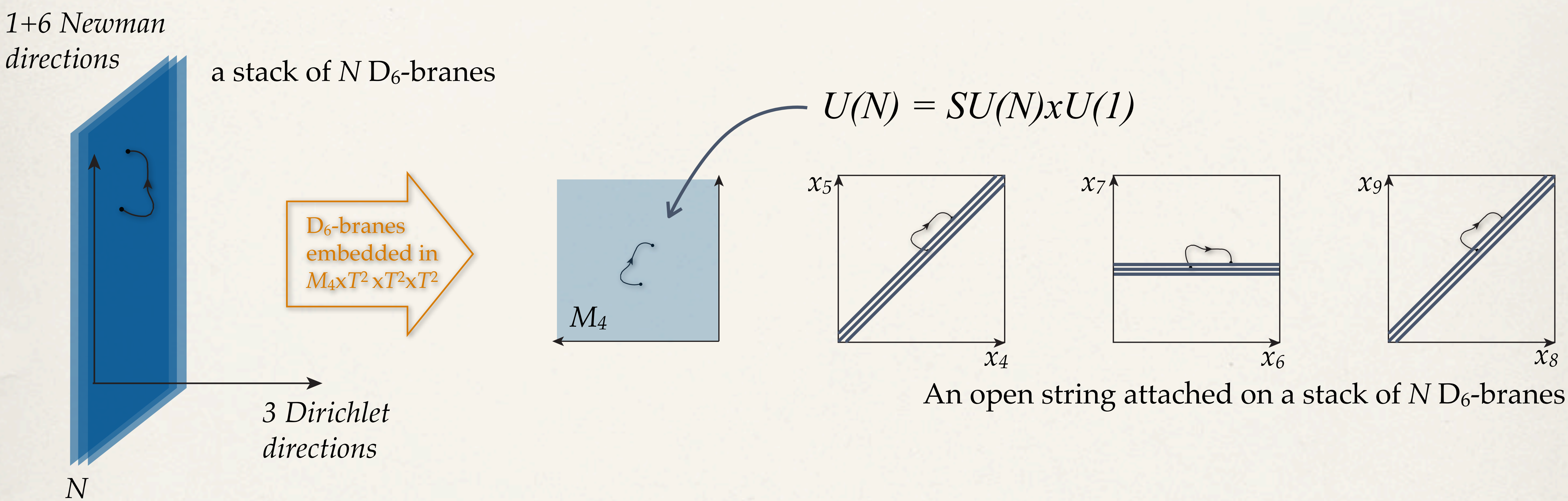
- The Yukawas couplings Q_LH_uU , Q_LH_dD, LH_dE are allowed for each of the above.



D-brane Model Building

Intersecting D-brane models

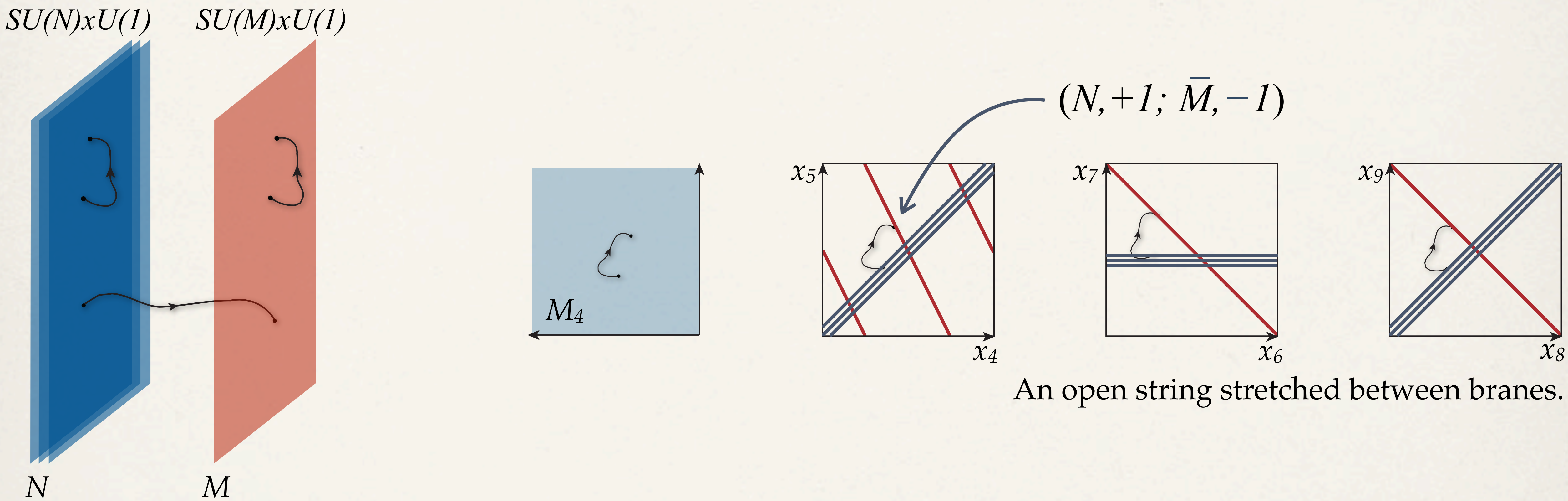
- We focus on type IIA constructions with intersecting D6 branes:
- D6 branes fill out 4D space-time and wrap three-cycles π_a in the internal manifold



- Strings with both ends on a stack of branes give rise to U(N) = SU(N)xU(1) group.

Intersecting D-brane models

- Chiral matter and family replication:



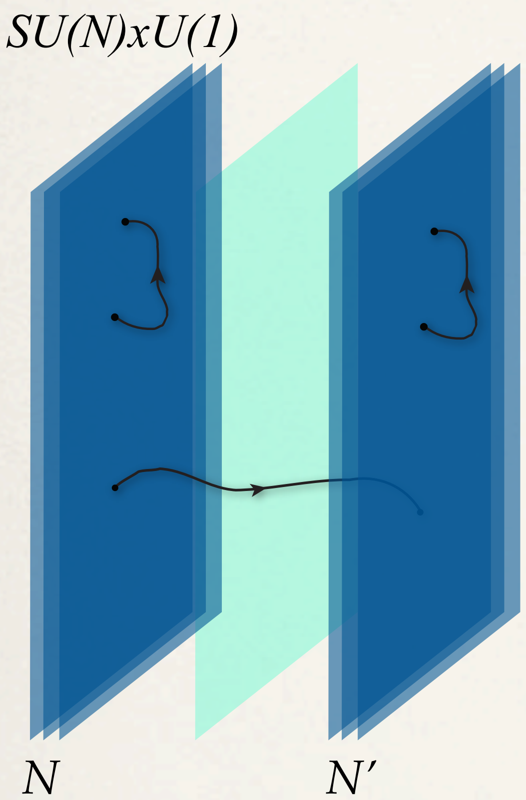
An open string stretched between branes.

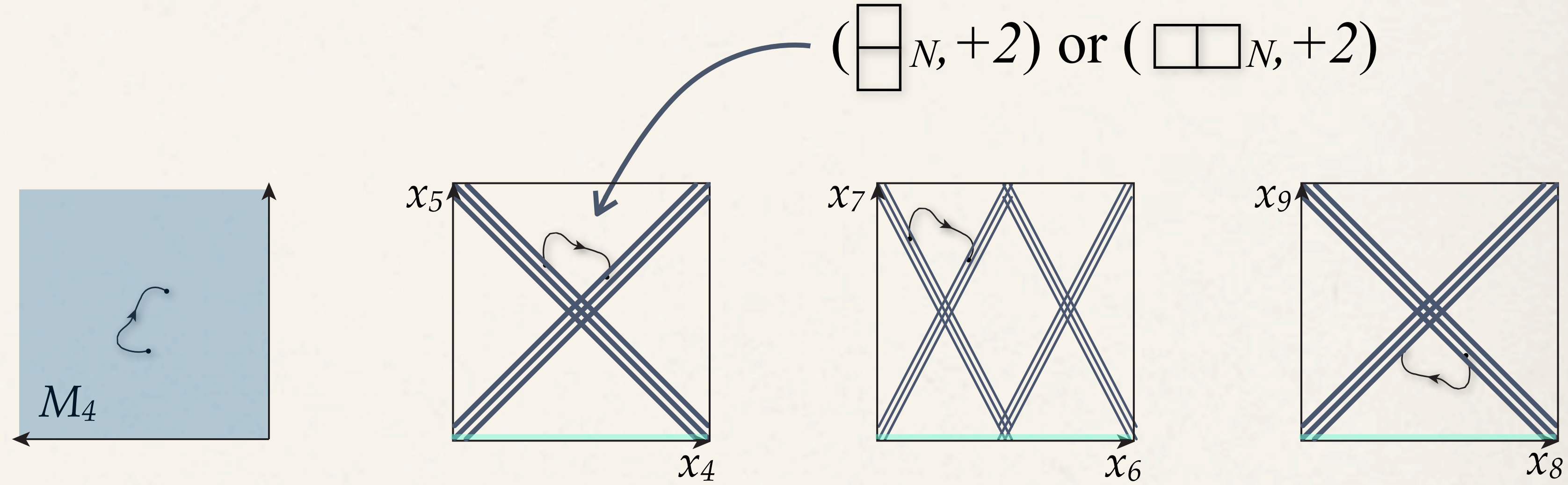
- At each intersection of two stacks of D6-branes appears a chiral fermion transforming as bifundamentals

- Their multiplicity is given by the number of intersections: $\#(\square_a, \overline{\square}_b) = \pi_a \circ \pi_b$

Intersecting D-brane models

- Orientifold:





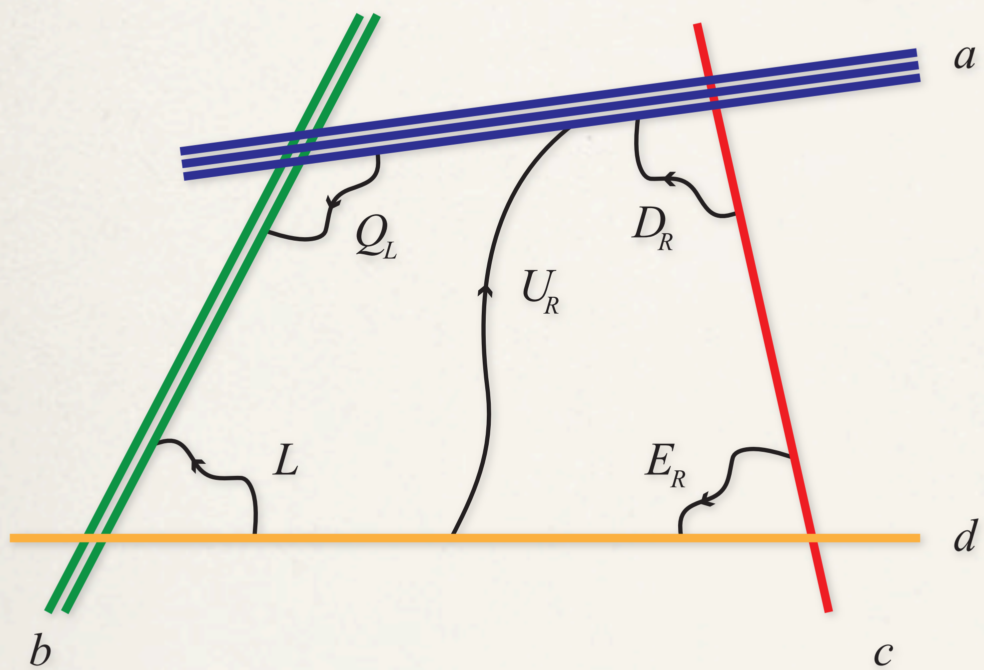
An open string stretched between branes.

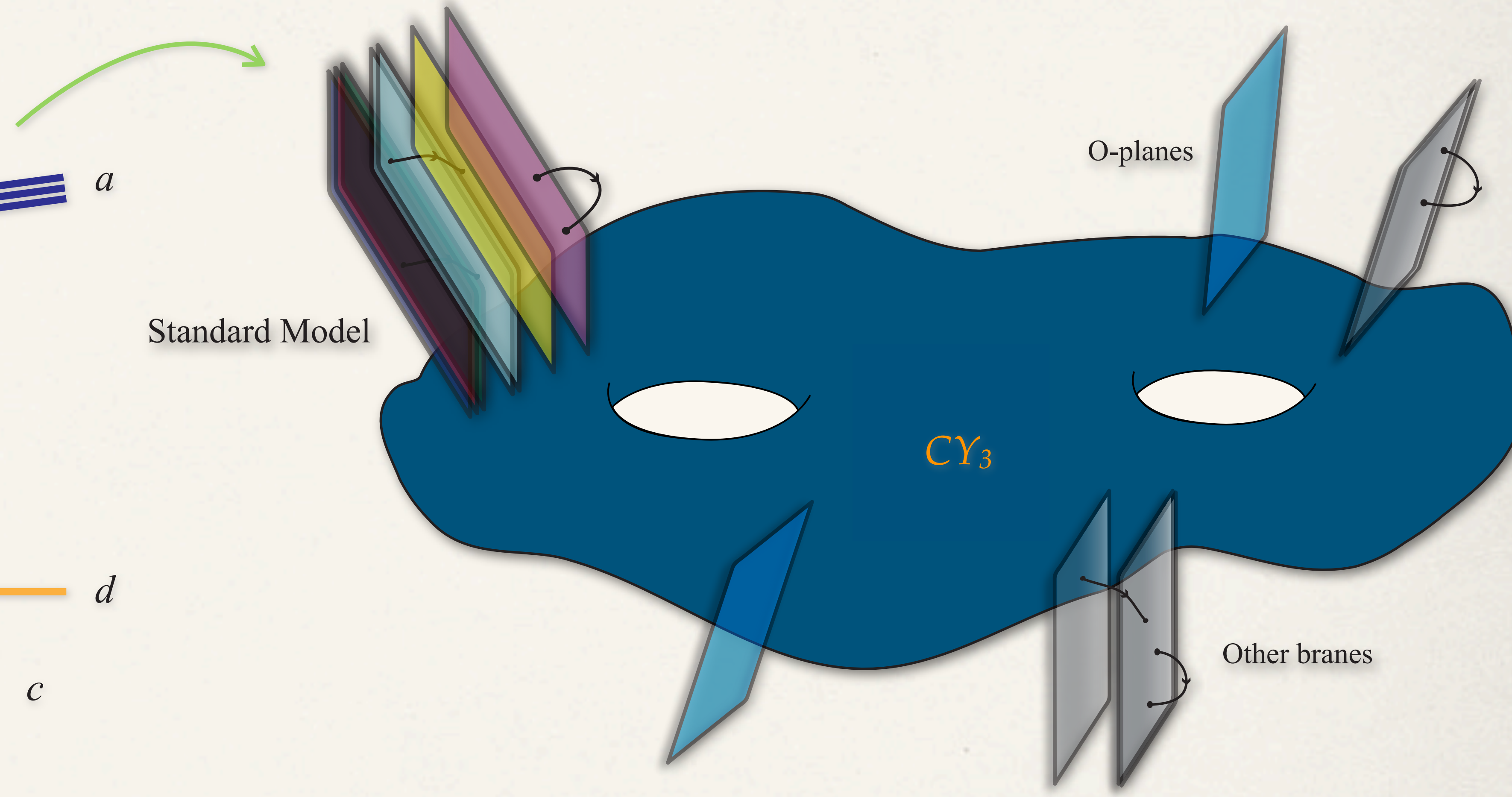
- Strings stretched between a brane and its image transform as (anti)symmetric reps.
- Their multiplicity is given by:

$$\#(\,\Box\Box_a) = \frac{1}{2}\left(\pi_a \circ \pi'_a - \pi_a \circ \pi_{O6}\right) \qquad \#(\begin{smallmatrix}\Box\\\Box\end{smallmatrix}_a) = \frac{1}{2}\left(\pi_a \circ \pi'_a + \pi_a \circ \pi_{O6}\right)$$

Bottom-Up models

- D-branes allow for a bottom-up building approach:





- Local models: set of D6-branes, which are localized at some region of the CY_3.

- Does not care about global aspects of compactification.

Antoniadis, Kiritsis, Tomaras
Aldazabal, Ibanez, Quevedo, Uranga

- Global construction is more satisfying, but local setups are more efficient and also sufficient to address various phenomenological issues.

Tadpole condition

- Consistency and stability of D-brane models require cancellations of tadpoles:

$$\sum_x N_x(\pi_x + \pi'_x) - 4\pi_{O6} = 0$$

- This constraint is a condition on three-cycles the D-branes wrap

- Using the intersection formulae we can translate cycle- to representation-language:

$$\#\left(\begin{smallmatrix}\Box\\\Box\end{smallmatrix}_a\right) = \frac{1}{2}\left(\pi_a \circ \pi'_a + \pi_a \circ \pi_{O6}\right) \qquad \#(\Box_a, \overline{\Box}_b) = \pi_a \circ \pi_b$$

$$\#(\Box\!\Box_a) = \frac{1}{2}\left(\pi_a \circ \pi'_a - \pi_a \circ \pi_{O6}\right) \qquad \#(\Box_a, \Box_b) = \pi_a \circ \pi'_b$$

For each U(N) stacks one obtains:

$$\sum_{x \neq a} N_x \left(\#(\Box_a, \overline{\Box}_x) + \#(\Box_a, \Box_x)\right) + (N_a - 4)\#\left(\begin{smallmatrix}\Box\\\Box\end{smallmatrix}_a\right) + (N_a + 4)\#(\Box\!\Box_a) = 0$$

This is the usual anomaly cancellation for non-abelian gauge symmetries

Tadpole condition

- However, there are additional constraints arising from single D-brane stacks, so called U(1) stacks.
- For each U(1) stack one has

$$\sum_{x\neq a}\left(\#(\Box_a,\overline{\Box}_x)+\#(\Box_a,\Box_x)\right)+5\#(\Box\!\Box_a)=0 \mod 3$$

- These constraints have no four-dimensional field theory analogue.
- They are related to anomalies in higher dimensions.
- Both types of constraints are only necessary conditions but not sufficient.

U(1) masslessness condition

- For MSSM constructions we need a massless U(1) identified with the hypercharge.
- Each D-brane carries a U(1) which typically is anomalous and becomes massive via the Green-Schwarz mechanism
- The U(1) survives as a global symmetry that is respected by all perturbative quantities

$\rightsquigarrow$ D-instantons can break global symmetries and induce forbidden couplings

- A linear combination $U(1) = \sum_x q_x U(1)_x$ remains massless (no coupling to axions) if:

$$\frac{1}{2} \sum_x q_x N_x (\pi_x - \pi'_x) = 0$$

Aldazabal, Ibanez, Quevedo, Uranga

U(1) masslessness condition

- Again we translate the cycle-constraint into a constraint on the representation behavior

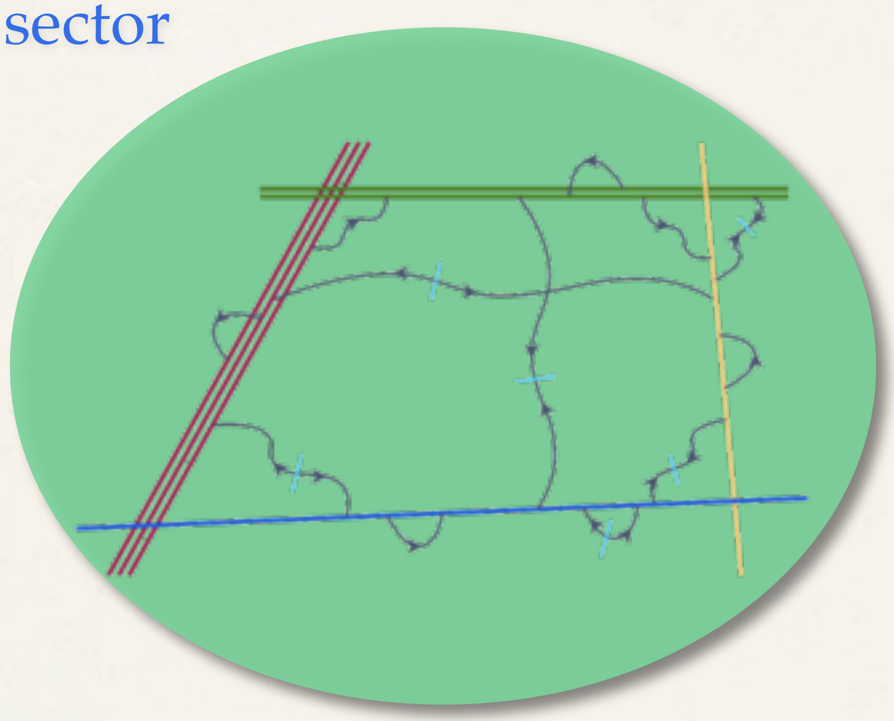
$$\frac{1}{2}\sum_{x\neq a} q_x\, N_x \#(\square_a, \overline{\square}_x) - \frac{1}{2}\sum_{x\neq a} q_x\, N_x \#(\square_a, \square_x)$$
$$- \frac{q_a N_a}{2(4-N_a)} \left(\sum_{x\neq a} N_x \left(\#(\square_a, \overline{\square}_x) + \#(\square_a, \square_x)\right) + 8\#(\,\square\!\square_a) \right) = 0$$

- Here, we have used tadpole condition to substitute for the antisymmetric reps.
- Constraint implies cancellation of 4D mixed, cubic and gravitational anomalies
- Constraint is stronger $\rightsquigarrow$ related to higher dimensional anomalies
- Constraints are only necessary conditions but not sufficient.

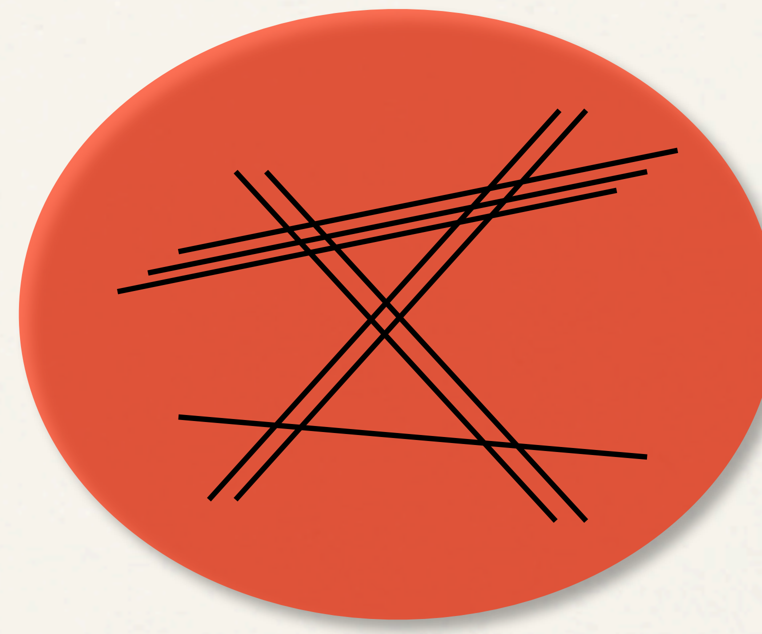
Local D-brane quiver

- Assume we have a local D-brane setup on which the MSSM is realized

Visible sector



Hidden sector that does not intersect chirally the visible sector



⇝ within the visible sector the constraints arising from tadpole cancellation and masslessness of the hypercharge have to be satisfied

Bottom-up search

- Bottom-up search for realistic D-brane quivers: Cvetic Halverson Richter
  - Top-down constraints: tadpole cancellation

    massless hypercharge
  - Bottom-up constraints: MSSM spectrum

    Yukawa couplings are pert. or non-pert. realized

    R-parity violating terms are absent on pert. and (non-pert.) level

    no dim. 5 proton decay operators

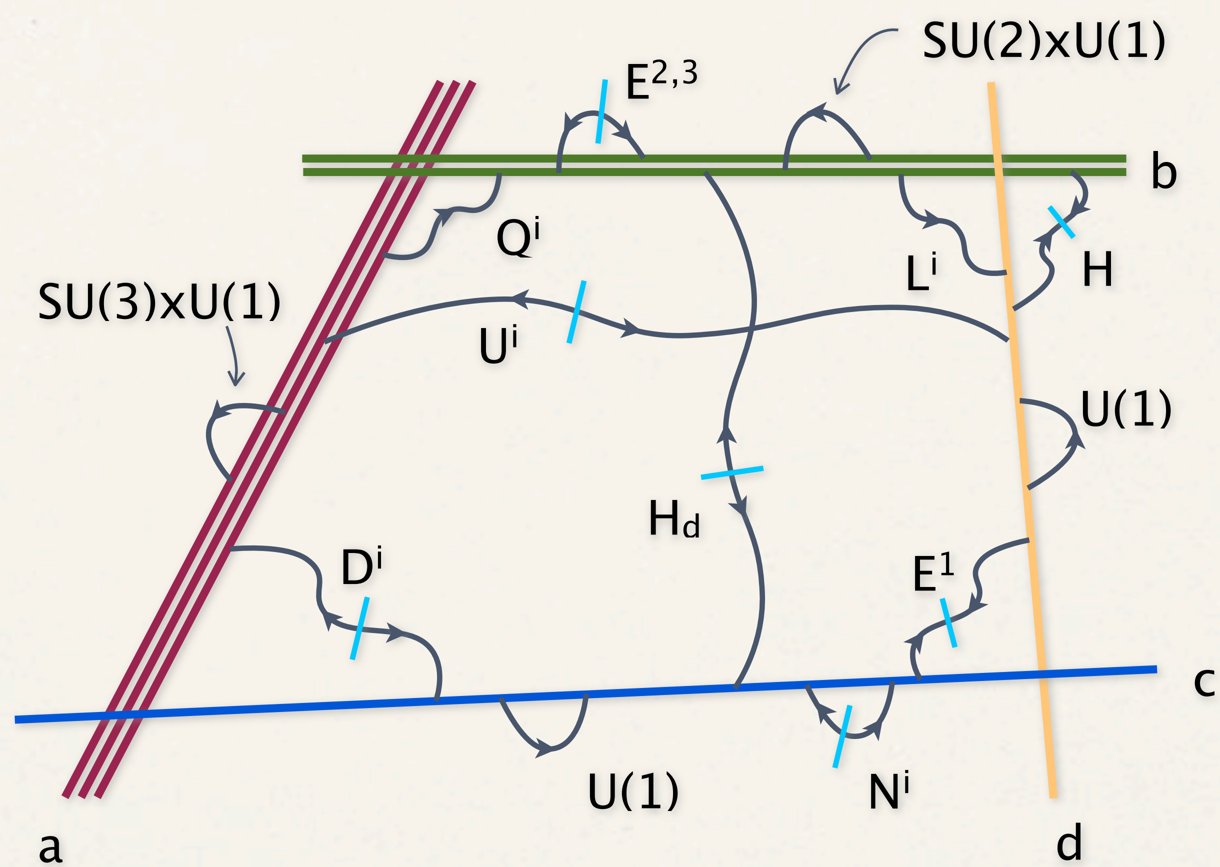
    realistic CKM matrix

    mechanism to generate small Neutrino masses

A D-brane Standard Model

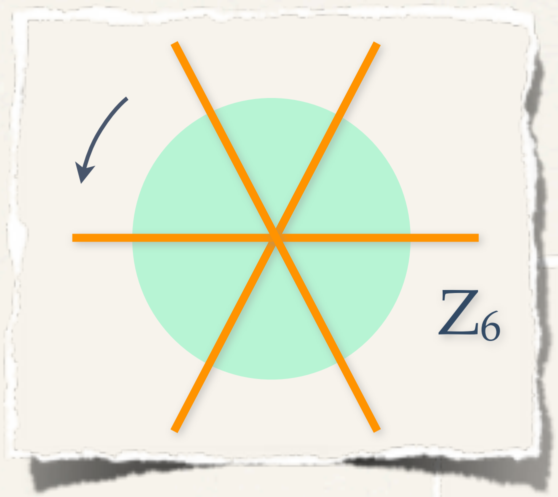
- Consider a specific D-brane Standard Model with: $Y = -\frac{1}{3}U(1)_a - \frac{1}{2}U(1)_b + U(1)_d$

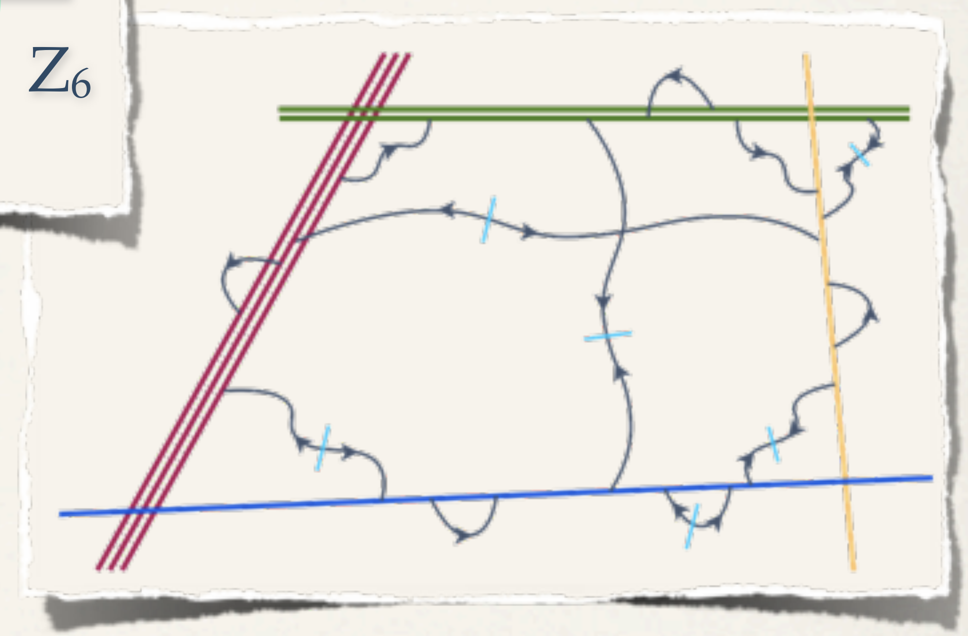
$Q^i\ (\mathbf{3},\mathbf{2})_{1/6}$	$U^i\ (\bar{\mathbf{3}},1)_{-2/3}$	$D^i\ (\bar{\mathbf{3}},1)_{1/3}$	$L^i\ (1,\mathbf{2})_{-1/2}$	$E^i\ (1,1)_1$	$N^i\ (1,1)_0$	$H_u\ (1,\mathbf{2})_{+1/2}$	$H_d\ (1,\mathbf{2})_{-1/2}$
$3(a,\bar{b})$	$3(\bar{a},\bar{d})$	$3(\bar{a},\bar{c})$	$3(b,\bar{c})$	$(c,d), 2\,\bar{\boxminus}_b$	$3\,\square\!\square_c$	(b,d)	(b,c)



- there are many more D-instantons that may induce undesired couplings

$\rightsquigarrow$ discrete gauge symmetries may help





Discrete gauge symmetries in D-brane SM

Discrete symmetry in D-brane models

- D-brane compactifications generically give rise to multiple anomalous U(1) gauge symmetries
- Those anomalous U(1)'s become massive via the Green-Schwarz mechanism and survive as global symmetries on the perturbative level.
- D-instanton effects can break those global symmetries inducing sometimes desired, but perturbatively forbidden, couplings (Majorana mass terms, Yukawa couplings etc)
- However, we have to ensure that other instantons do not induce dangerous couplings
- Discrete gauge symmetries are an efficient way to guarantee it
- Our aim is to do an analysis over semi-realistic D-brane Standard Model configurations by the effect of all allowed discrete gauge symmetries

Discrete gauge symmetries

- Consider a discrete gauge symmetry $\mathbf{Z}_N = \sum_x k_x U(1)_x$
- This symmetry survives in the low energy effective action if:

$$\frac{1}{2}\sum_x k_x N_x (\pi_x - \pi'_x) = 0 \mod N$$

Berasaluce-Gonzalez, Ibanez, Soler, Uranga

- This condition becomes:

$$\frac{1}{2}\left(\sum_{x\neq a} k_x\, N_x \#(\Box_a, \overline{\Box}_x) - \sum_{x\neq a} k_x\, N_x \#(\Box_a, \Box_x) - \#(\,\Box\!\Box_a) - \#(\begin{array}{c}\Box\\[-4pt]\Box\end{array}_a)\right) = 0 \mod N$$

- Using tadpole conditions, we can substitute again the antisymmetrics and obtain

$$\frac{1}{2}\sum_{x\neq a} k_x\, N_x \#(\Box_a, \overline{\Box}_x) - \frac{1}{2}\sum_{x\neq a} k_x\, N_x \#(\Box_a, \Box_x)$$
$$- \frac{k_a N_a}{2(4 - N_a)}\left(\sum_{x\neq a} N_x \left(\#(\Box_a, \overline{\Box}_x) + \#(\Box_a, \Box_x)\right) + 8\#(\,\Box\!\Box_a)\right) = 0 \mod N$$

An additional discrete symmetry condition

- The fact that discrete symmetries require 0 mod N instead of 0 brings troubles…

- One can compensate that by requiring an additional constraint:

$$\sum_a k_a N_a \left(\#\left(\genfrac{}{}{0pt}{}{\square}{\square}_a\right) - \#\left(\square\square_a\right)\right) = 0 \mod N$$

arising from multiplying the homology class of the orientifold with the discrete symmetry constraint.

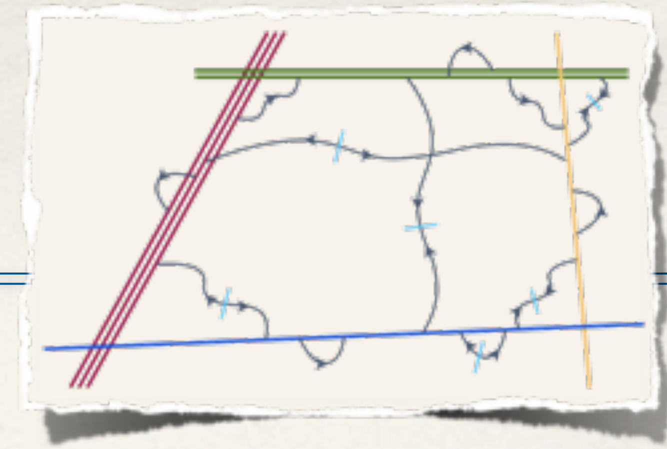
- After replacing again the antisymmetrics we get:

$$\sum_a \frac{k_a N_a}{4 - N_a} \left(\sum_{x \neq a} N_x \left(\#(\square_a, \overline{\square}_x) + \#(\square_a, \square_x)\right) + 2N_a\left(\square\square_a\right) \right) = 0 \mod N$$

- One can prove the absence of mixed, cubic and gravitaional anomalies in 4 dimensions

$SU(N) \times SU(N) \times \mathbf{Z}_N$ $\qquad$ $\mathbf{Z}_N \times \mathbf{Z}_N \times \mathbf{Z}_N$ $\qquad$ $G \times G \times \mathbf{Z}_N$

D-brane Standard Models



- Consider again the previous embedding:

$Q^i\,(\mathbf{3},\mathbf{2})_{1/6}$	$U^i\,(\bar{\mathbf{3}},1)_{-2/3}$	$D^i\,(\bar{\mathbf{3}},1)_{1/3}$	$L^i\,(1,\mathbf{2})_{-1/2}$	$E^i\,(1,1)_1$	$N^i\,(1,1)_0$	$H_u\,(1,\mathbf{2})_{+1/2}$	$H_d\,(1,\mathbf{2})_{-1/2}$
$3(a,\bar{b})$	$3(\bar{a},\bar{d})$	$3(\bar{a},\bar{c})$	$3(b,\bar{c})$	$(c,d), 2\,\overline{\begin{smallmatrix}\square\\\square\end{smallmatrix}}_b$	$3\,\square\!\square_c$	(b,d)	$(\bar{b},\bar{c})$

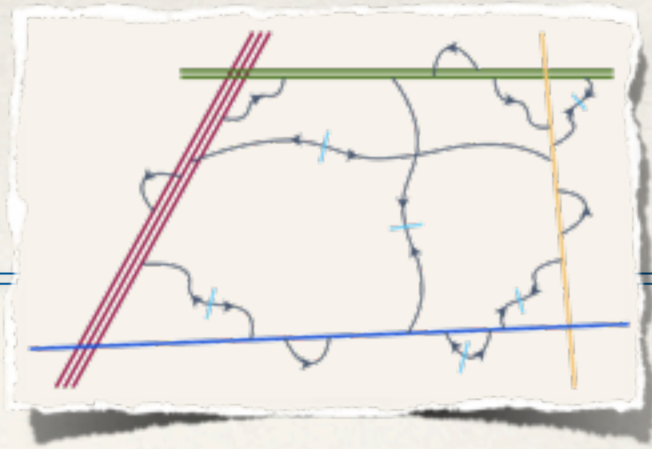
- With the discrete charges: $Q_{discrete} = k_a Q_a + k_b Q_b + k_c Q_c + k_d Q_d$

- We want to find all (k_a, k_b, k_c, k_d) that for various $\mathbf{Z}_N$ satisfy:

$$\frac{1}{2}\sum_x k_x N_x(\pi_x - \pi'_x) = 0 \mod N$$

- Each k take values from 0, 1, ... 2N (due to the 1/2 overall factor).

Search for discrete symmetries

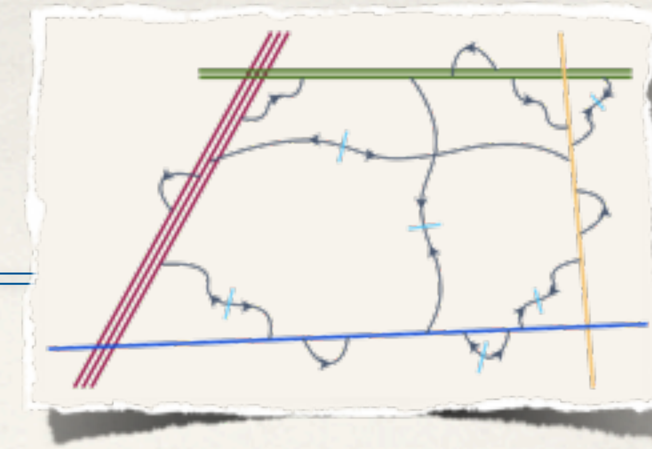


- Not all sets of (k_a, k_b, k_c, k_d) are independent.
- To avoid overcounting , we have to remember that one solution gives others:
  - by a Hypercharge shift: $(k_a, k_b, k_c, k_d) + m(q_a, q_b, q_c, q_d) \mod N$
  - there is an overall freedom so we fix the discrete charge of Q_L to zero by: $k_a = k_b$.
- Within independent vectors (k_a, k_b, k_c, k_d) we check which of them satisfy:
  - the discrete symmetry condition
  - the Symmetric-Antisymmetric condition

  } Anomalies

  - allow Yukawa terms.
- For all $\mathbf{Z}_N$ with $N \in 2, 3, 4, \ldots 20$.

Results



- for concrete example we find the discrete gauge symmetries
  - The $\mathbf{Z}_2$: $R_2 = U(1)_a + U(1)_b + U(1)_c + U(1)_d$ is the usual matter parity.
  - The $\mathbf{Z}_3$: $L_3 R_3 = U(1)_a + U(1)_b + U(1)_d$ is the baryon triality.
  - The $\mathbf{Z}_6$: $L_6^2 R_6^5 = U(1)_a + U(1)_b + 9U(1)_d + 13U(1)_d$ is the proton hexality.
- Therefore, the above discrete gauge symmetries ensure for:
  - all desired Yukawa couplings,
  - allowed μ-term, Weinberg operator,
  - No bad terms (like R-violating, no proton decay operators).
- We have extended this analysis over all semi-realistic 4 stack D-brane models.

More results

- From the systematic search over all realistic 4 stack quivers (40) we find:
  - No family dependent discrete symmetries.
  - No $\mathbf{Z}_9$ and $\mathbf{Z}_{18}$ realizations.
  - All $\mathbf{Z}_2$, $\mathbf{Z}_3$ and $\mathbf{Z}_6$ can be realized.
  - Madrid embedding $\left(\frac{1}{6}, 0, \frac{1}{2}, -\frac{1}{2}\right)$ favors matter parity, but disfavors Baryon triality.
  - Embeddings $\left(-\frac{1}{3}, \frac{1}{2}, 0, 0\right)$ and $\left(-\frac{1}{3}, \frac{1}{2}, 0, 1\right)$ favors Baryon triality, disfavors matter parity.
  - Only in few realizations proton hexality realized (4/40).

Spectrum	Hypercharge	Solution	R_2	R_3L_3	R_3	L_3	$R_3^2L_3$	$R_6^5L_6^2$	R_6	$R_6^3L_6^2$	$R_6L_6^2$
MSSM	$(-\frac{1}{3},-\frac{1}{2},0,1)$	1		✓							
		2		✓							
		3									
	$(-\frac{1}{3},-\frac{1}{2},0,1)$	1									
MSSM+3N	$(\frac{1}{6},0,\frac{1}{2},-\frac{3}{2})$	1			✓						
	$(-\frac{1}{3},-\frac{1}{2},0,0)$	1	✓								
		2	✓								
		3	✓								
		4									
		5									
		6	✓								
		7	✓								
		8	✓								
		9	✓								
		10	✓								
		11	✓								
		12	✓								
	$(-\frac{1}{3},-\frac{1}{2},0,1)$	1	✓	✓				✓			
		2	✓			✓				✓	
		3	✓								
		4	✓		✓				✓		
	$(\frac{1}{6},0,\frac{1}{2},-\frac{1}{2})$	1	✓	✓	✓	✓	✓	✓	✓	✓	✓
		2		✓							
		3		✓							
		4		✓							
		5		✓							
		6		✓							
		7		✓							
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		9		✓							
		10		✓							
		11		✓							
		12	✓	✓	✓	✓	✓	✓	✓	✓	✓
	$(\frac{1}{6},0,\frac{1}{2},-\frac{1}{2})$	1	✓	✓	✓	✓	✓	✓	✓	✓	✓
NMMSM+3N	$(\frac{1}{6},0,\frac{1}{2},-\frac{1}{2})$	1									
		2									
		3		✓							

Conclusions

- We have analyzed all four D-brane stack Standard Model configurations with interesting phenomenology (around 40 local configurations) with respect to discrete symmetries

- No family dependence, not in the quark sector which is somewhat expected and also desired, but also not in the lepton sector.

- Would be interesting to see whether the family independence holds true also for 5 stack realizations.

- No $\mathbf{Z}_9$ and $\mathbf{Z}_{18}$ realizations

- All $\mathbf{Z}_2$, $\mathbf{Z}_3$ and $\mathbf{Z}_6$ can be realized

- Matter parity is tied to Madrid embedding $\left(\frac{1}{6}, 0, \frac{1}{2}, -\frac{1}{2}\right)$

- Baryon triality is tied to embeddings $\left(-\frac{1}{3}, \frac{1}{2}, 0, 0\right)$ and $\left(-\frac{1}{3}, \frac{1}{2}, 0, 1\right)$

- Proton hexality appears very rarely.