SU(3) Structure Compactifications and Calabi-Yau Model Building in Heterotic

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with M. Larfors And D. Lüst: arXiv:1205.6208

with L. Anderson, A. Lukas and E. Palti: arXiv: 1202.1757
1106.4804

and with A. Constantin: to appear.

SU(3) Structure Backgrounds:

- Consider compactification on a six manifold admitting an SU(3) structure.

Torsion classes:

$$\begin{aligned} dJ &= -\frac{3}{2}\mathrm{Im}(W_1\overline{\Omega}) + W_4 \wedge J + W_3 \\ d\Omega &= W_1 J \wedge J + W_2 \wedge J + \overline{W}_5 \wedge \Omega \end{aligned}$$

- SU(3) Holonomy: Calabi-Yau $W_i = 0 \; \forall \; i$
- SU(3) Structure $\mathcal{N} = 1$ vacuum: Strominger System

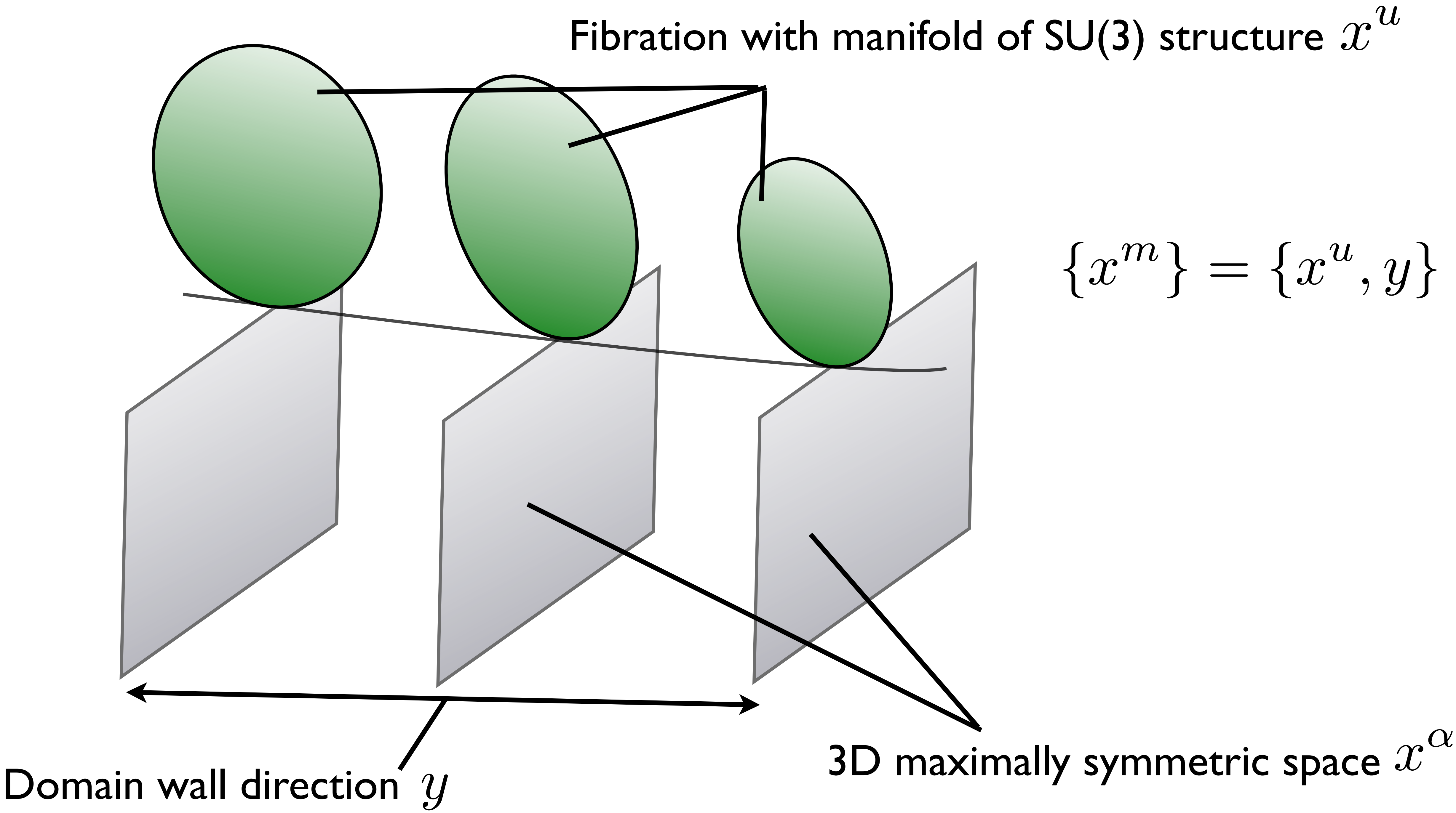
$W_1 = W_2 = 0 \qquad W_4 = \frac{1}{2} W_5 = d\hat{\phi}$ Lopes et al: hep-th/0211118

- SU(3) Structure $\mathcal{N} = 1/2$ vacuum: Generalized half-flat

$W_{1-} = W_{2-} = 0 \quad W_4 = \frac{1}{2} W_5 = d\hat{\phi}$ Lukas et al: hep-th/1005.5302

We will add extra fluxes to the analysis, and provide solutions for the supergravity fields.

The setup:



Metric and associated field ansatzes

$$ds_{10}^2 = e^{2A(x^m)}\left(ds_3^2 + e^{2\Delta(x^u)}dydy + g_{uv}(x^m)dx^u dx^v\right)$$

$$H_{\alpha\beta\gamma} = f\epsilon_{\alpha\beta\gamma} \qquad H_{\alpha mn} = H_{\alpha\beta n} = 0 \qquad \partial_\alpha\hat{\phi} = 0$$

- Three dimensional space is maximally symmetric.
- New fluxes: f and H_{yuv}
- Gravitino variation in x^α directions

$$\Longrightarrow A(x^m) = \text{constant}$$

- Define $\Theta = d\Delta$

The Killing spinor equations and Bianchi Identities become...

Consistency at fixed y

$$J\wedge dJ = J\wedge J\wedge d\hat{\phi} \ , \quad d\Omega_- = 2d\hat{\phi}\wedge\Omega_- - e^{-\Delta} * H_y - \frac{1}{2} f J\wedge J \ ,$$

$$0 = \frac{1}{2} * f - \Omega_+ \wedge H - \frac{1}{2} e^{-\Delta} H_y \wedge J\wedge J \ , \quad e^{\Delta} * d\hat{\phi} = \frac{1}{2} H_y \wedge \Omega_- - \frac{1}{2} e^{\Delta} H\wedge J \ ,$$

$$dH = 0 \ , \quad d(*e^{-2\hat{\phi}-\Delta} H_y) = 0 \ , \quad df = 0$$

Flow eqns

$$J\wedge J' = e^{\Delta} d\Omega_+ - \frac{1}{2} e^{\Delta} * (H\wedge\Omega_-) J\wedge J - 2e^{\Delta} d\hat{\phi}\wedge\Omega_+ - e^{\Delta}\Omega_+\wedge\Theta$$

$$\Omega_-' = e^{\Delta} dJ - e^{\Delta} * (H\wedge\Omega_-)\Omega_- - 2e^{\Delta} d\hat{\phi}\wedge J + e^{\Delta} J\wedge\Theta - *He^{\Delta} - fe^{\Delta}\Omega_+$$

$$\hat{\phi}' = -\frac{1}{2} e^{\Delta} * (H\wedge\Omega_-)$$

$$H' = dH_y \ , \quad (*e^{-2\hat{\phi}-\Delta} H_y)' = -d * (e^{-2\hat{\phi}+\Delta} H) \ , \quad f' = 0$$

reduces correctly to previous cases.

Rewrite fluxes and y derivatives

Helps with solving equations in a construction independent manner

$$
\begin{aligned}
H &= A_{1+}\Omega_+ + A_{1-}\Omega_- + A_{2+}\wedge J + A_{3+} \\
H_y &= B_1 J + B_2 + B_{3+} \ .
\end{aligned}
$$

such that

$$
\begin{aligned}
A_{3+}\wedge\Omega_\pm &= 0 \\
A_{3+}\wedge J &= 0 \\
B_2\wedge J\wedge J &= 0 \ .
\end{aligned}
$$

and write:

$$
\begin{aligned}
J' &= \gamma_1 J + \gamma_{2+} + \gamma_3 \\
0 &= \gamma_{2+}\wedge J\wedge J = \gamma_3\wedge J\wedge J \ .
\end{aligned}
$$

$$
\begin{aligned}
\Omega'_- &= \alpha_{1+}\Omega_+ + \alpha_{1-}\Omega_- + \alpha_{2+}\wedge J + \alpha_3 \ , \\
\Omega'_+ &= \beta_{1+}\Omega_+ + \beta_{1-}\Omega_- + \beta_{2+}\wedge J + \beta_3 \ , \\
0 &= \Omega_\pm\wedge\alpha_3 = J\wedge\alpha_3 \ , \\
0 &= \Omega_\pm\wedge\beta_3 = J\wedge\beta_3 \ .
\end{aligned}
$$

- The quantities α, β and γ can easily be found in any given example (see paper for many worked cases).

Solving consistency conditions:

$$d\hat{\phi} = W_4$$

$$H_y = e^{\Delta}(-f - 2W_{1-})J - e^{\Delta}W_{2-} + \frac{1}{2}e^{\Delta}((2W_4 - W_5)\llcorner\overline{\Omega} + \mathrm{c.c})$$

Also specifies some of the components of H

- Setting new fluxes to zero we recover the generalized half-flat conditions

$$W_{1-} = W_{2-} = 0 \quad W_4 = \frac{1}{2}W_5 = d\hat{\phi}$$

In general all but one of these conditions is relaxed.

Solving flow equations:

$$H = -\frac{1}{2}e^{-\Delta}\hat{\phi}'\Omega_+ + (\frac{7}{8} + \frac{3}{2}W_{1-})\Omega_- + *((3W_4 - 2W_{5+}) \wedge J - W_3 + e^{-\Delta}\alpha_3)$$

- We also get equations for the flow itself.
  For example:

$$\gamma_3 = e^{\Delta} W_{2+} \quad \text{and} \quad \alpha_{1+} = -3e^{\Delta} W_{1-} - \frac{15}{8} e^{\Delta} f$$

- The explicit expressions for H allow us to check the Bianchi Identities and form field equations of motion trivially in any case.

- The equations for the flow yield the y dependence of the parameters in the SU(3) structure when used with any explicit construction.

Please see paper for egs:

- CY with flux
- Cosets
- Toric varieties (SCTV's)

Calabi-Yau Model building:

- Traditionally in heterotic model building we choose a Calabi-Yau threefold and an irreducible rank 3,4,5 gauge bundle over it as our background.

e.g. rank 5: $E8 \supset SU(5) \times SU(5)$ $E8 \to SU(5)$

- Break GUT group to the standard model with Wilson lines (requires non-simply connected Calabi-Yau) $SU(5) \to SU(3) \times SU(2) \times U(1)$

- Must also ensure we have a solution to the theory and the standard model particle spectrum.

Hard: 4 known examples.

Bouchard and Donagi hep-th/0512149
Braun, He, Ovrut and Pantev hep-th/0501070
Anderson, Gray, He and Lukas arXiv/0911.1569
Braun, Candelas, Davies, Donagi arXiv/1112.1097

- We take the visible sector gauge bundle to be a sum of line bundles ⟶ simpler!

$$V = \bigoplus_a \mathcal{L}_a$$

- Any additional $U(1)$ symmetries then broken by Green-Schwarz or by deforming the bundle
- This sum must obey a series of conditions to provide a good heterotic vacuum:
  - We must be able to solve the Bianchi Identity

$$\mathrm{Ch}_2(TX) - \mathrm{Ch}_2(V) = [C] + \mathrm{Ch}_2(V')$$

  - The sum must be holomorphic (automatic)
  - The sum must be polystable and slope zero

    -each piece of sum must be stable (automatic)

    -each piece of sum must be slope zero

Manifolds: Favourable CICY's

Symmetries: arXiv:1003.3235 (Braun)

CICY 6784: $\left(\begin{array}{c|ccc} \mathbb{P}^1 & 1 & 1 & 0 \\ \mathbb{P}^1 & 0 & 0 & 2 \\ \mathbb{P}^1 & 2 & 0 & 0 \\ \mathbb{P}^3 & 1 & 1 & 2 \end{array}\right)^{4,36}$

Symmetry: $\mathbb{Z}_2 \times \mathbb{Z}_2$

- Simple ambient space
- CY defined as intersection of vanishing loci of polynomials
- All Kahler forms descend from ambient space
- Manifold can be quotiented by freely acting symmetries to obtain non-trivial π_1 $\longrightarrow$ wilson lines possible

Bundles: Sums of Line Bundles

- Line bundles on a CY are defined by their first Chern class

$$c_1 = \frac{1}{2\pi}[\mathrm{tr}F]$$

- For favourable CICYs we may write

$$c_1(\mathcal{L}) = \frac{1}{2\pi}[\mathrm{tr}F] = \sum_{i=1}^{h^{1,1}} c_1^i(\mathcal{L}) J_i$$

  - here $c_1^i(\mathcal{L})$ are integers and the J_i are the Kahler forms descending from the ambient space factors

- We denote:

$$\mathcal{L} = \mathcal{O}(c_1^i(\mathcal{L}))$$

Line bundle standard models:

- Time to scan! In addition to those already discussed what conditions must our bundles satisfy?
  - Must quotient CICY by freely acting symmetry to allow Wilson lines

    $$\rightarrow V = \bigoplus_a \mathcal{L}_a$$ must be EQUIVARIANT

  - We must get the right spectrum!
    - $h^1(X,V) = 3|\Gamma|$
    - $h^1(X,V^*) = 0$

    → 3 SU(5) $\mathbf{10}$ families, no $\overline{\mathbf{10}}$ anti-families after quotienting

- $h^1(X, \wedge^2 V) - h^1(X, \wedge^2 V^*) = 3|\Gamma|$

  $\longrightarrow$ Chiral asymmetry of 3 $\overline{5}$'s after quotienting

- $h^1(X, \wedge^2 V^*) > 0$

  $\longrightarrow$ At least one Higgs $5\ \overline{5}$ pair before quotient

- One additional condition (a little more complicated) which ensures that all Higgs triplets are removed by the Wilson line and at least 1 pair of Higgs doublets survives

So what do we get?...

- Scanned ~10^{12} models (desktop only for now. Algorithm improvements underway with Andrei Constantin.)
  - There are 23 CICYs which are favourable, have $h^{1,1} = 5$ and have freely acting symmetries. We scan over integers between -2 and 2 in the line bundles for these.
  - There are 19 CICYs which are favourable, have $h^{1,1} = 4$ and have freely acting symmetries. We scan over integers between -3 and 3 in the line bundles for these.

$\longrightarrow$ 202 models on 13 Cicys

  - The 6 such CICYs with $h^{1,1} = 2$ and 12 with $h^{1,1} = 3$ gave nothing, even scanning for integers as large as 10 in the first case.

Note that when I give the number of models I am not including different possible choices of Wilson line and equivariant structure for each one - so there are in fact many more than I am saying (between 100 and 1000 choices for each model - not all phenomenologically viable).

- Keeping just one example of each spectrum generated each time: 2122 standard models.
- Keeping just one example of models which look identical at this level of detail on each Calabi-Yau: 407 standard models.

Full database available here:

http://www-thphys.physics.ox.ac.uk/projects/CalabiYau/linebundlemodels/index.html

Some example statistics:

standard models	no mass-less $U(1)$	1 Higgs pair	2 Higgs pairs	3 Higgs pairs	$\mathrm{rk}(Y^{(u)}) > 0$	no proton decay, $\lambda = \lambda' = 0$	1 Higgs, $\mathrm{rk}(Y^{(u)}) > 0$, $\lambda = \lambda' = 0$, $U(1)$s massive
407	237	262	77	63	45	198	13

Table 1: *Statistics of basic properties in the standard model database [42].*

In conclusion:

- One can create very large numbers of heterotic standard models in this manner.
- One can push the phenomenological analysis of these models beyond merely getting the correct spectrum.

Summary

- SU(3) structure backgrounds:
  - Showed how to generalise the torsion classes giving rise to a good heterotic background.
  - Gave explicit solutions for supergravity fields: especially important for solving Bianchi Identities.
- Calabi-Yau model building:
  - Have constructed a few thousand standard models on smooth Calabi-Yau compactifications of heterotic.
  - Technical trick was to use line bundles rather than higher rank vector bundles in the construction.
  - Large number of models allows us to aim for more detailed phenomenology.